Physics Content Structure

Topic Heading	Sub-topic (with approximate instructional time)
1 Basic scientific skills (5% weighting)	1.1 Physical quantities, units and measurement (1 hour) 1.2 Problem solving techniques (1 hour) 1.3 Graph drawing, analysis and interpretation (2 hours)
2 Mechanics (55% weighting)	2.1 Vectors (4 hours) 2.2 Motion in 1-dimension (12 hours) 2.3 Force, Newton's Laws (10 hours) 2.4 Momentum and impulse (3 hours) 2.5 Work, power and energy (3 hours)
3 Waves (10% weighting)	3.1 Transverse and longitudinal waves (3 hours) 3.2 Geometrical optics (2 hours)
4 Electricity and Magnetism (30% weighting)	4.1 Electrostatics (5 hours) 4.2 Electric circuits (7 hours) 4.3 Magnetism (1 hour) 4.4 Electromagnetism (4 hours)

Topic 1. Basic Scientific Skills

Introduction

The basic scientific skills that you learn in this section will be applied in all of the other sections of the Physics component of this course. You should try to spend enough time on this section at the beginning of your studies, to make sure that you are confident in the basic skills needed to be able to tackle the rest of the course.

In the curriculum, this topic is assigned 4 hours of contact time (and about 1 ½ hours self-study time). However, if you are very unfamiliar with these basic scientific skills, you might need to spend more time on this topic than is shown in the curriculum.

Sub-topic 1. Physical quantities, units and measurement

Content:

Unit 1: Scientific notation

Unit 2: Physical quantities, SI units and conversions

Unit 1. Scientific notation

Learning outcomes:

When you have completed this unit, you should be able to:

- display numerical values in scientific notation.

1.1. Exponents

The number 1 000 can be written in three ways:

$1\ 000 = 10 \times 10 \times 10 = 10^3$.

The small 3 written after the 10 is called an *exponent*, and tells us how many times the number 10 has been multiplied by itself.

Exponent: the small raised integer that is written after a number to tell us how many times the number is multiplied by itself. e.g. $y^2 = y \times y$.

1.2. Scientific notation

Scientific notation is a way of writing very large or small numbers so that they are more readable. In scientific notation, the number is written as a number between 1 and 10 multiplied by a power of 10. The following example shows how to write a large number in scientific notation:

Example:

Write the number 750 000 in scientific notation.

Solution

750 000 = 7,5 × 100 000 = $7{,}5 \times 10^5$

HINT: We can work out the exponent by counting how many places the decimal comma needs to move to the **left** to go from 750 000 to 7,5. In this example the decimal comma needs to move 5 spaces to the left, so this is our exponent:

$$7\,5\,0\,0\,0\,0 = 7{,}5 \times 10^5$$

We can also write a very small number, such as 0,001, as a power of ten:

$0{,}001 = \frac{1}{1000} = \frac{1}{10^3} = 10^{-3}$.

Example:

Write the number 0,0005 in scientific notation.

Solution

$0{,}0005 = \frac{5}{10\,000} = 5 \times 10^{-4}$

HINT: Another way of working out the exponent is to work out how many places the decimal comma needs to move to the right to go from 0,0005 to 5. In this example the decimal comma needs to move 4 spaces to the right, so this is our exponent:

$$0{,}0\,0\,0\,5 = 5 \times 10^{-4}$$

In Science, we also use exponents when we describe units. For example, the units for velocity are m/s or $m{\cdot}s^{-1}$. In this way, we treat units like any other algebraic variables.

From these examples, you can see that a large number always has a positive exponent when it is written in scientific notation, and a small number always has a negative exponent.

Activity 1: Practice scientific notation

1. Convert the following numbers into scientific notation:
   a. 22
   b. 8 100
   c. 26 550 000
   d. 5,5
   e. 0,1
   f. 0,00068
   g. 20 010 000

2. Write the following numbers out fully:
   a. $6{,}5 \times 10^{-2}$
   b. 7×10^{1}
   c. $1{,}693 \times 10^{6}$
   d. $2{,}78 \times 10^{-5}$

1.3. Scientific notation using a calculator

If you use your calculator to enter a number that is written in scientific notation, you use the EXP button. For example, to enter the number $6{,}5 \times 10^{4}$ on your calculator you will push the following buttons:

[6] [.] [5] [EXP] [4]

To enter the number 3×10^{-2} you will push the following buttons on your calculator:

[3] [EXP] [2] [+/-]

You can use your calculator to convert numbers into scientific notation. Different calculators have different ways of doing this. Some calculators do this conversion if you push the button that looks like this: [F-E] or [ENG]. This button will help you to convert backwards and forwards between ordinary numbers and scientific notation.

Take some time to explore your calculator, and make sure that you can enter numbers in scientific notation, and convert between scientific notation and ordinary numbers.

MAIN IDEA: Numbers can be written in scientific notation, e.g. $1{,}5 \times 10^{3}$

Unit 2. Physical quantities, SI units and conversions

Learning outcomes:

When you have completed this unit, you should be able to:

- write physical quantities as a numerical magnitude and a unit;
- recall the following base quantities and their SI units: mass (kg), length (m), time (s), current (A), temperature (K);
- convert between various scales of measurement: temperature (Celsius and Kelvin), length (km, m, cm, mm), mass (kg, g), pressure (kPa, atm);
- use the following prefixes and their symbols: nano (n), micro (μ), milli (m), centi (c), deci (d), kilo (k), mega (M), giga (G);
- relate the different orders of magnitude to the sizes and masses of common objects, ranging from an atom to the Earth.

2.1. Physical quantities and their units

In Science we measure different physical quantities, for example mass, length or temperature. When we write down our measurements, it is important that we use the correct units to show exactly what the measurement means. In this course we will use ***SI units***. The table below shows some of the SI units for different scientific quantities.

SI units: the international system of unit measurements, from "Le **S**ysteme **I**nternational d'Unites"

Quantity	SI unit	
	Name	Symbol
Length	metre	m
Mass	kilograms	kg
Time	seconds	s
Force	newton	N
Pressure	pascal	Pa
Current strength	ampere	A
Temperature	Kelvin	K

Activity 1: Practice writing the correct units for scientific quantities

For each of the measurements below, fill in the correct **symbol** for the SI units:

1. Thembi's mass is 90 ____
2. The temperature of the air outside is 300 _____
3. The length of a plank is 6 ______
4. The force that pulls Mandla toward the earth is 980 ________
5. The time for an egg to boil is 600 ________
6. The air pressure is 101 000 ________
7. The current is 0,5 _____

MAIN IDEA: Physical quantities are measured using **SI units**, e.g. meters (m), seconds (s) and newtons (N).

2.2. Conversion of units

Sometimes the SI units as they stand are not the most suitable units for measurements that are very large, or very small. It is therefore sometimes necessary to convert them into more appropriate units.

For example, if you measure the diameter of a nail to be 0,002 meters, this can be written more neatly as 2 millimeters (mm). Here "milli-" is called a *prefix*. It is a short way of saying "1000 times smaller", or "$\times 10^{-3}$". We call this way of writing numbers **prefix notation**.

prefix: part of a word that is added at the beginning of a word.

Another example is the measurement of the distance between Gauteng and Cape Town. Instead of writing this as 1 400 000 meters, you could rather express it as 1 400 kilometres (km), where the prefix is "kilo-", which means "1000 times bigger", or "$\times 10^{3}$".

The following table shows some of the prefix conversions that are often used by scientists.

Name (prefix)	Prefix Symbol	Meaning	Exponent
giga	G	1 000 000 000 times bigger than the SI unit	10^9
mega	M	1 000 000 times bigger than the SI unit	10^6
kilo	k	1 000 times bigger than the SI unit	10^3
deci	d	10 times smaller than the SI unit	10^{-1}
centi	c	100 times smaller than the SI unit	10^{-2}
milli	m	1 000 times smaller than the SI unit	10^{-3}
micro	μ	1 000 000 times smaller than the SI unit.	10^{-6}
nano	n	1 000 000 000 times smaller than the SI unit	10^{-9}

Here is an example of how to convert between different units:

Example: *(Try to solve this problem* ***on your own or with a fellow student*** *while covering the solution, and then check your work using the solution below).*

1. 600 mm = _____ m
2. 5 m = _____ cm

Solution

1. The conversion factor from mm to m is $\times 10^{-3}$.

   So 600 mm = 600×10^{-3} m = 0,6 m.

2. The conversion factor from cm to m is $\times 10^{-2}$. Therefore to convert from m to cm the conversion factor will be $\times 10^{2}$.

So 5 m = 5×10^{2} cm = 500 cm.

Activity 2: Practice converting units

Develop your skill of converting between units by doing the following exercises:

(a) 80 cm = _______ m
(b) 0,25 m = ________ mm
(c) 1 kg = ________ g
(d) 2 500 mg = ________ g
(e) 1500 Pa = _________ kPa
(f) 150 000 000 nm = _________ m
(g) 8×10^{-7} m = ________ nm
(h) 5×10^{8} μm = _________ m
(i) 15 nm = ________ m
(j) 3 000 nm = _________ μm

MAIN IDEA: We use prefix notation to write small or large numbers, e.g.

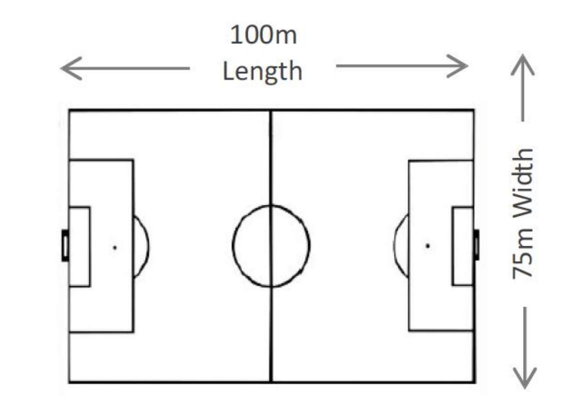
- 1 *milli*meter = 1×10^{-3} m, meaning "1000 times smaller"
- 1 *kilo*pascal = 1×10^{3} Pa, meaning "1000 times bigger".

2.3. Units of area and volume

Area describes the size of a surface, and is calculated by multiplying two lengths together. For a rectangle, area = length × breadth. Since the SI units of length are m, the SI units of area are m^2.

Example: *(Try to solve this problem* ***on your own or with a fellow student*** *while covering the solution, and then check your work using the solution below).*

Calculate the area of the soccer field shown in the picture below.



Solution

Area = length × breadth

= 100 m × 75 m

= 7500 m^2

Volume describes the amount of space an object takes up. The volume of a box is measured as length × breadth × height. Each of these is a length, measured in m, so volume is measured in m × m × m = m^3.

The units of area (m^2) and volume (m^3) are **derived** units, since they are formed from base SI units (m).

2.4. Other scientific units

When working in certain fields, scientists have sometimes found it useful to derive a new unit that is very specific for the quantity that is being described. One example of this is **temperature**. In our everyday language, we express temperature in units of degrees Celsius (^{o}C). In Science, the SI unit for temperature is Kelvin (K). The temperature of 0 K is defined as **absolute zero**, and is equal to -273 ^{o}C.

To convert from degrees Celsius into Kelvin we write:

Temperature (K) = Temperature (^oC) + 273.

<u>Example</u>: *(Try to solve this problem* ***on your own or with a fellow student*** *while covering the solution, and then check your work using the solution below).*

1. Convert the temperature 25 ^oC into units of Kelvin.
2. Convert the temperature 200 Kelvin into units of ^oC.

<u>Solution</u>

1. Temperature = 25 ^oC + 273 = 298 K.
2. Temperature = 200 K – 273 = -73 ^oC.

Another example of this is **pressure**. In Science, the SI unit for pressure is Pascal (Pa). In some applications, for example atmospheric science, pressure is measured in units of atmospheres (atm). 1 atm = 101 325 Pa = 101,325 kPa.

<u>Example:</u> *(Try to solve this problem* ***on your own or with a fellow student*** *while covering the solution, and then check your work using the solution below).*

a. What is a pressure of 15 atm in Pa?

b. What is a pressure of 100 Pa in atm?

<u>Solution</u>:

a. If 1 atm = 101 325 Pa, then 15 atm = 15 × 101 325 Pa = 1 519 875 Pa

b. 100 kPa = 100 × 10^3 Pa

If 1 atm = 101 325 Pa, then 1 Pa = $\frac{1}{101\ 325}$ atm

So 100 Pa = $\frac{100}{101\ 325}$ atm = 9,87 × 10^{-4} atm.

When recording **time** measurements in science, the SI units are seconds (s). Sometimes we measure time in minutes (min) or hours (hr). There are 60 seconds in 1 minute, and 60 minutes in 1 hour. Using units, we write this as 60 min/hour, or as 60 $min \cdot hr^{-1}$.

When converting between different units, if we keep a record of the units through all steps of a problem, then if we haven't made a mistake, our final answer should have the correct units for the kind of quantity that it is. In this way we can use our units as a guide when trying to work out how to solve a problem. This is called **dimensional analysis**. The following example shows how to use dimensoinal analysis when converting time units.

Example: *(Try to solve this problem* ***on your own or with a fellow student*** *while covering the solution, and then check your work using the solution below).*

Work out how many minutes there are in 1 day.

Solution:

There are 60 minutes in 1 hour. Using units, we can write this as 60 min·hr^{-1}.

There are 24 hours in one day. We can write this as 24 hours·day^{-1}.

To work out how many minutes there are in 1 day, we do the following calculation:

60 min·hr^{-1} × 24 hours·day^{-1}

At this point we can treat the units as variables, and cancel them using the rules of algebra:

60 min·~~hr^{-1}~~ × 24 ~~hrs~~·day^{-1} = 1440 min·day^{-1}

We can also use dimensional analysis to convert from units of km·hr^{-1} to m·s^{-1}:

Example: *(Try to solve this problem* ***on your own or with a fellow student*** *while covering the solution, and then check your work using the solution below).*

A taxi travels with a speed of 72 km·hr^{-1}. Write this speed in units of m·s^{-1}.

Solution:

The conversion factor from km to m is $\times 10^3$

Therefore speed = 72 km·hr^{-1} × 10^3 = 7,2 × 10^4 m·hr^{-1}

There are 60 minutes in 1 hour, and 60 seconds in 1 minute

Therefore 1 hour = 3600 seconds, which we can write as 3600 sec·hr^{-1}

Therefore speed = $\frac{7{,}2\times10^4 \text{ m}\cdot\cancel{\text{hr}^{-1}}}{3600 \text{ sec}\cdot\cancel{\text{hr}^{-1}}}$ = 20 m·s^{-1}

Activity 3: Practice converting between units

Test your understanding of physical quantities, units and measurement by doing the following practice exercises:

1. The length of a pencil is 12 cm. Write this length in mm and m.
2. The mass of a cup of tea is 220 g. What is this mass in mg and in kg?
3. The diameter of a hydrogen atom is 0,1 nm. What is this diameter in m? (Write your answer in scientific notation).
4. The distance between the Earth and the Sun is 1,5 × 10^{11} m. What is this distance in km?

5. If the mass of a grain of sand is 2×10^{-6} g, what is this mass in μg?
6. The mass of the Earth is $5{,}97 \times 10^{27}$ g. What is this mass in kg?
7. Convert the temperature 25 ^{o}C into units of Kelvin.
8. Convert the temperature 200 Kelvin into units of ^{o}C.
9. How old is someone (in years) if their age is 7 884 000 minutes?
10. The pressure inside a balloon is 22 atm. What is this pressure in Pa, and in kPa?
11. The earth travels around the sun at a speed of $1{,}08 \times 10^{6}$ $km \cdot hr^{-1}$. How fast does the earth travel around the sun in $m \cdot s^{-1}$? Show how you get your answer using dimensional analysis.

Sub-topic 2. Problem solving techniques

Content:

Unit 1: Problem solving strategies

Unit 1. Problem-solving strategies

Learning outcomes:

When you have completed this unit, you should be able to:

- use diagrams as problem solving tools;
- apply steps in problem solving procedures to solve single- and multi-step problems;
- reflect on and interpret answers to calculations..

1.1. Useful steps in problem solving

When you are trying to solve a problem in science, the solution will not always be obvious straight away. It is therefore useful to have a strategy that you follow which will help you in tackling a difficult problem. The list of steps below describes some possible strategies that you could try.

1. Draw a diagram of the scenario
2. Write a list of the given information
3. Read the question carefully to decide what is being asked
4. Select an appropriate equation or scientific concept for solving this problem
5. Do the calculation carefully
6. Reflect on your answer, making sure that the value you obtained is sensible, and that you have answered the question identified in Step 3. Check that the units of your answer are correct.

You will not always need to use all of these steps, but you will find some of these strategies useful at different times. In the following section you will see some examples of how you could use these strategies.

1.2. Using scientific formulae

We use scientific formulae to show the mathematical relationship between scientific concepts.

For example, for an object moving with a constant speed, the mathematical relationship between speed (s), distance (D) and time (t) is that the speed of an object is the distance traveled divided by the time. This is given by the scientific formula:

$$s = \frac{D}{t}$$

If we know the speed and the time, we can calculate the distance covered by changing the subject of the formula to D:

$$D = s \times t$$

If we know the distance traveled and the speed, we can calculate the time by changing the subject of the formula to t:

$$t = \frac{D}{s}$$

<u>Example</u> *(Try to solve this problem* ***on your own or with a fellow student*** *while covering the solution, and then check your work using the solution below).*

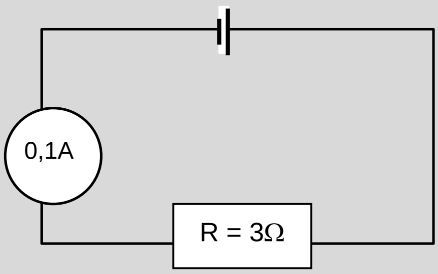
The mathematical relationship between current (I), potential difference (V) and resistance (R) for a resistor in an electric circuit is:

$$R = \frac{V}{I}$$

Calculate the potential difference across the resistor if is has a resistance of 3 Ω and a current of 0,1 A through it.

<u>Solution:</u>

Step 1 – Draw a diagram



Step 2 – Given: R = 3 Ω and I = 0,1 A

Step 3 – Work out what is being asked:

We are asked to calculate the potential difference (V) across the resistor.

Step 4 – Select equation or concept:

The equation that we need to use in this case is:

$$R = \frac{V}{I}$$

Step 5 – Do the calculation:

We need to change the subject of the formula to V, we multiply both sides of the equation by I.

$$R \times I = \frac{V}{I} \times I$$

The I's on the RHS cancel, and we can rewrite this as:

$$V = R \times I$$

$$= 3\ \Omega \times 0{,}1\ A$$

$$= 0{,}3\ V$$

Step 6 – Reflect on your answer:

This is a very important step in problem solving, since it is easy to make a mistake when entering numbers into your calculator. Study the answer that you have found, and check that your value for the voltage looks reasonable. If it is, write down your final answer:

The potential difference across the resistor is 0,3 V.

Activity 1 – *Working with scientific formulae*

The relationship between the density (d), mass (m) and volume (V) for an object is given by the formula:

$$d = \frac{m}{V}$$

Use this formula to answer the following questions:

1. Ayanda finds the mass of a stone to be 15 g, and its volume is 5 cm^3. What is the stone's density?
2. Fizile measures a cup of flour (250 cm^3) and wants to know its mass. She looks up the density of flour, and finds that it is 0,4 $g{\cdot}cm^{-3}$. What is the mass of the flour?
3. Fizile now wants to measure the volume of 500 g of flour. How many cups of flour does she need?

Sub-topic 3. Graph drawing, analysis and interpretation

Content:

Unit 1: Constructing and analysing straight line graphs

Unit 2: Interpreting the shapes of non-linear graphs

Unit 1. Constructing and analysing straight line graphs

Learning outcomes:

When you have completed this unit, you should be able to:

- select appropriate variables and scales for graph plotting;
- accurately construct a straight-line graph from given data;
- analyse a graph to extract meaningful information, namely:
  - value and physical meaning of the gradient of a straight-line graph;
  - value and physical meaning of the intercepts of a straight-line graph;
  - physical interpretation of the shape of a non-linear graph.

Introduction

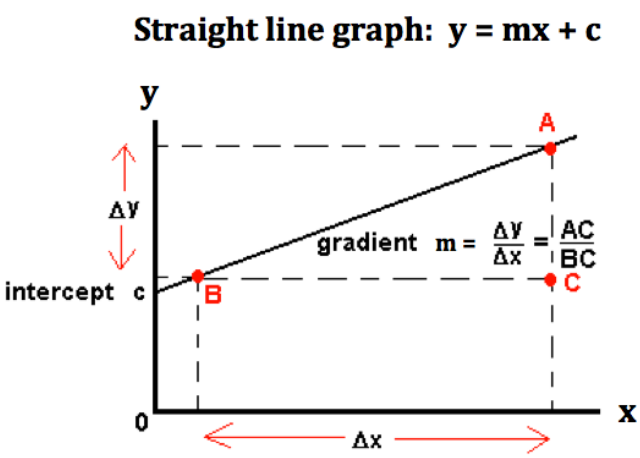
Many relationships in science can be represented using graphs. It is therefore important to be able to draw clear and accurate graphs, as well as being able to read and interpret graphs. There are many different types of graph which can be used, but we will only look at straight-line graphs in this introduction.

1.1. Properties of a straight line graph

A straight line graph in science shows that there is a clear mathematical relationship between the two quantities that we are showing on the graph. We can use the mathematical equation to represent the straight line:

$$y = mx + c$$

Here m is the gradient of the graph, and c is the y-intercept. We plot the dependent variable on the y-axis, and the independent variable on the x-axis.



(If you are unsure about this equation, revise the section of your mathematics course on straight line graphs).

When finding the gradient of a straight line, choose two points that lie on the line, but which are quite far apart. Then the gradient is found in the following way:

$$\text{gradient} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

In science, once you have calculated the gradient, the units give you an idea of the quantity that this is describing.

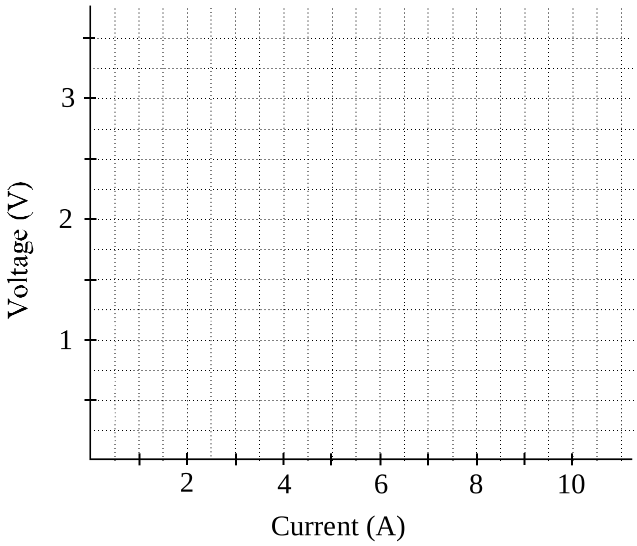
1.2. Drawing a straight line graph using experimental readings

When you plot a graph in science, it is important to choose a set of axes that allows you to plot the data points accurately. Here are some hints for choosing your axes:

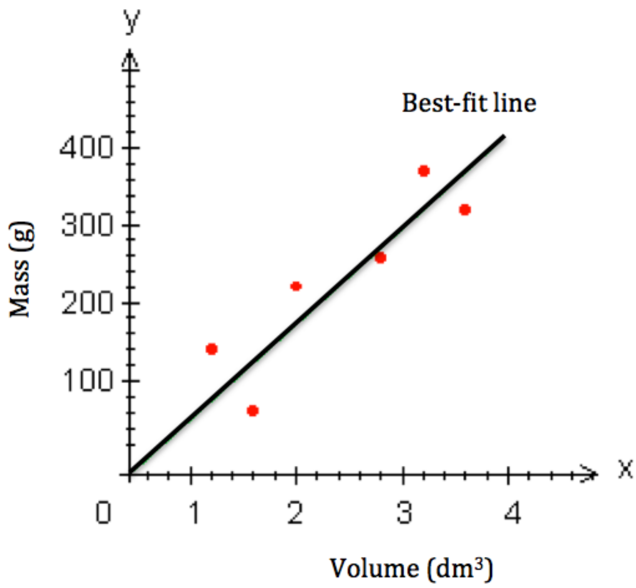
- Determine a **scale** (the value for each square on your x- and y-axis) that best fits the range of each variable.
- Use a scale that lets you plot your data **as large as possible** in the space you have.
- The range of each axis might be different, but each one must be **consistent**. If one box represents one metre at the beginning of the graph, then one box must always represent one metre for that axis.
- Number and label each axis,including the units of measurement.

consistent: unchanging

The picture below shows a set of numbered axes with labels.



When we draw a graph in science, we are plotting experimental (measured) values on our graphs. In other words, the readings are not always exactly accurate, but contain some uncertainty. As a result, when we plot a graph they will not always lie in a perfect straight line. For this reason, we do not connect the dots exactly as they lie, but we draw a **best-fit line**, which is our best estimate of the line that is represented by these points. In drawing our best fit line, we should try to have an equal number of points above the line as below the line.



The example below shows how to draw a straight line graph and then calculate and interpret the gradient of the graph.

Example *(Try to solve this problem **on your own or with a fellow student** while covering the solution, and then check your work using the solution below).*

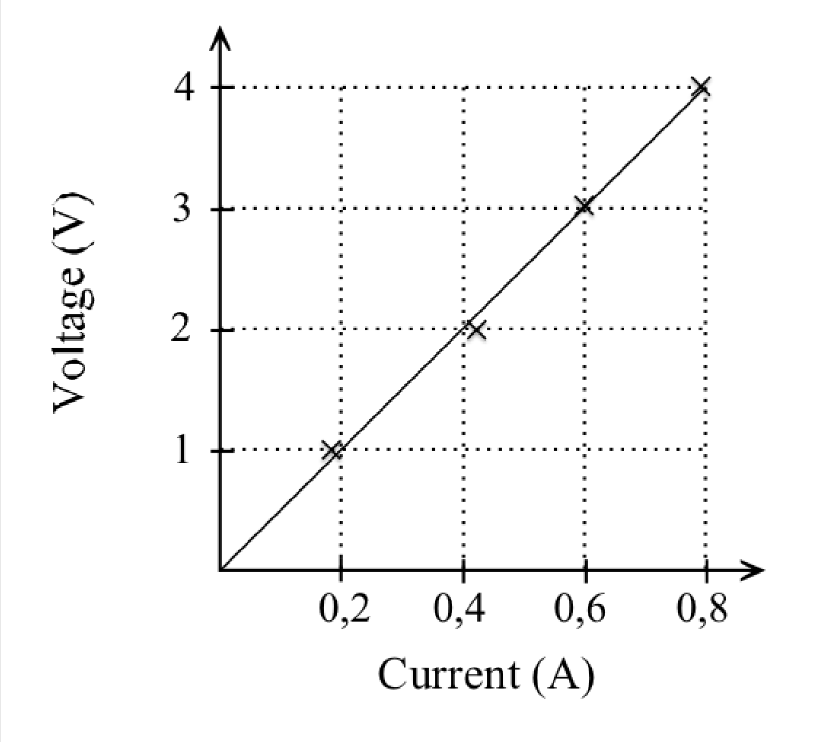
Nathi measured the voltage across a resistor and the current through that resistor. The table below contains the data from his measurements.

Voltage (V)	Current (A)
1,0	0,18
2,0	0,42
3,0	0,60
4,0	0,79

1. Plot a graph of the voltage and current values given in this table. Draw a best-fit line through the points.
2. Calculate the gradient of this line. What physical quantity does this gradient tell you about?

<u>Solution</u>:

1. The graph of the voltage vs current for the resistor is shown below, together with the best fit line:



2. To find the gradient of this line, we choose two points that lie exactly on the line. In this case we could choose the points:
(x_1, y_1) = (0 A, 0 V) and (x_2, y_2) = (0,8 A, 4 V).

$$\text{gradient} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4\text{ V} - 0\text{ V}}{0{,}8\text{ A} - 0\text{ A}} = 5\ \Omega.$$

This tells us that the resistance of the resistor is 5 Ω.

A straight line graph that goes through the origin tells us that the values that we have plotted on the x- and y-axis are **directly proportional** to each other. In the example above, we can write this as "voltage is proportional to current", or $V \propto I$.
Another way of writing this is:

$$\frac{V}{I} = k,$$

where *k* is the constant of proportionality. In this example the constant of proportionality is the resistance of the resistor.

Some straight line graphs don't go through the origin, but have an intercept on the x- or y-axis. This intercept can give us useful information.

Unit 2. Interpreting the shapes of non-linear graphs

Learning outcomes:

When you have completed this unit, you should be able to:

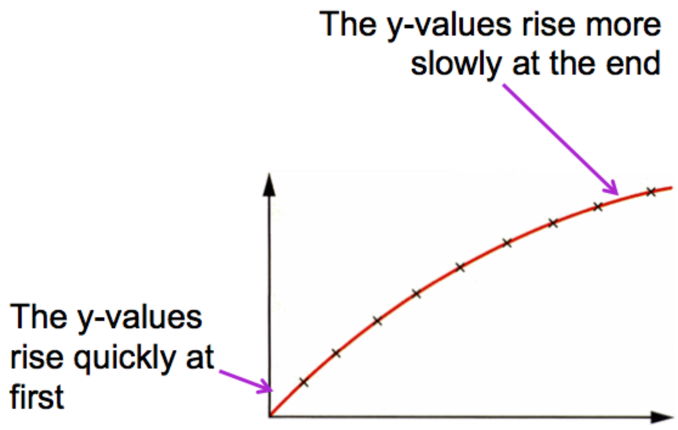
- analyse a graph to extract meaningful information, namely:
  - physical interpretation of the shape of a non-linear graph.

Introduction

Not all scientific graphs are straight line graphs. In this unit you will be introduced to the shape and meaning of some of the different kinds of graphs.

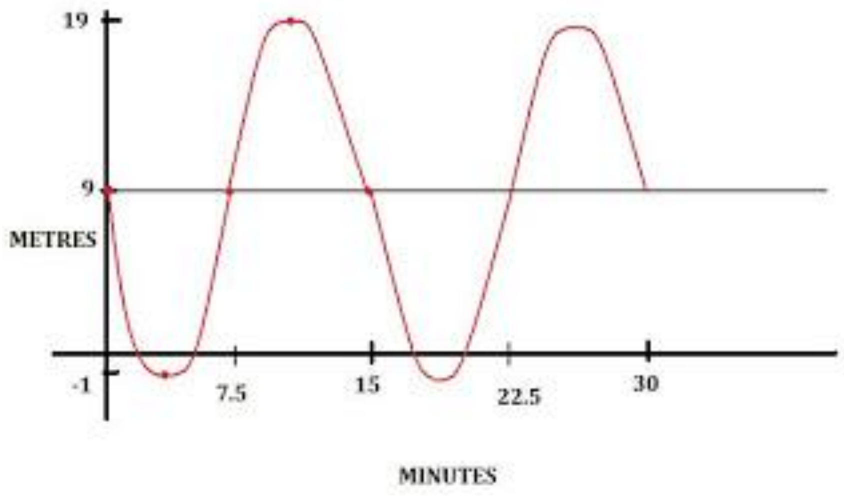
2.1. Curved graphs

Sometimes the y-values don't rise linearly with the x-values, but form a curve. We need to look carefully at the shape of the curve to see how the relationships between the y and x values change. For example, in the graph below, the y-values rise more quickly at the beginning, and more slowly at the end.

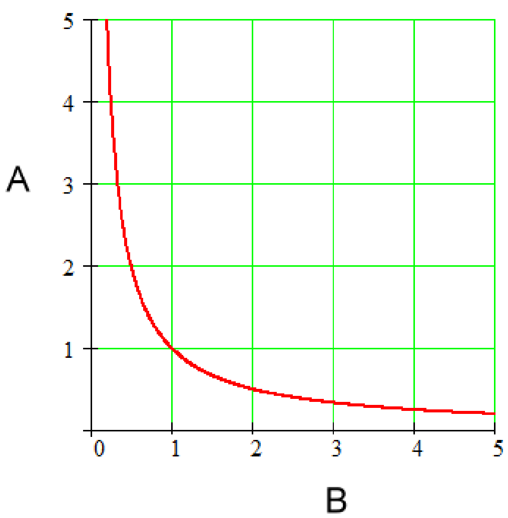


2.2. Periodic graphs

There are many situations in nature where a quantity will change periodically with time, in other words there is a regular repeated pattern. An example is when a ball attached to a rope swings backwards and forwards repeatedly. The graph below is an example of a periodic graph.



2.3. Inverse proportion graphs



The graph on the right has a hyperbola shape, and this shows that the quantities on the graph are inversely proportional to each other. What this means is that, as the x-value increases, the y-value decreases in value.

For the graph shown on the right, we say that A is **inversely proportional** to B. We can write this mathematically as:

$$A \propto \frac{1}{B} \quad \text{or} \quad B \propto \frac{1}{A}$$

Assessment Activity: Basic Scientific Skills **Total marks = 30**

Assess your understanding of this topic by answering the following questions.

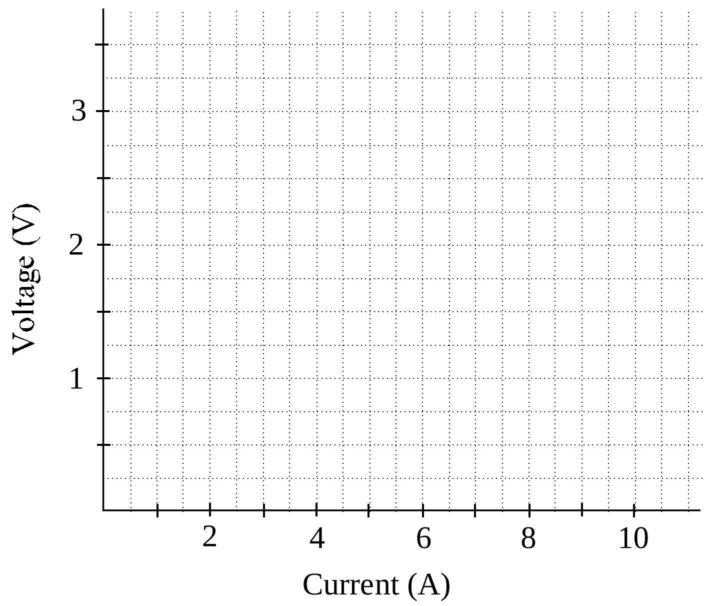
1. The diameter of a needle is 0,1 mm. What is this diameter in the following units (write your answers in scientific notation): (6)
   a. nm
   b. µm
   c. km

2. Use the steps in the problem solving strategy to answer the following question: A glass container has a mass of 0,12 kg. This container is filled with 250 cm^3 of water. A teaspoon of salt (5 cm^3) is added to the water. The density of water is 1 $g \cdot cm^{-3}$, and the density of salt is 2 $g \cdot cm^{-3}$. What is the total mass of the water, the salt and the container together? Give your answer in g and in kg. (Use the equation $d = \frac{m}{V}$). (6)

3. Sandile did an experiment with an electric circuit consisting of a variable power supply, a resistor and an ammeter. He attached the voltmeter in parallel to the resistor. He took the current reading on the ammeter for different settings on the power supply. His results are shown in the table below:

Voltage across R (V)	Current through R (A)
1,5	4,2
2,0	5,3
2,5	6,7
3,0	8,3
3,5	9,3

a. Plot these points on a set of axes like the one below. (5)

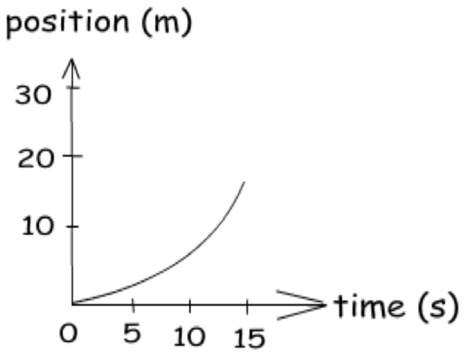


b. Draw a best-fit line through your points. (1)
c. Find the gradient of the best-fit line. (5)

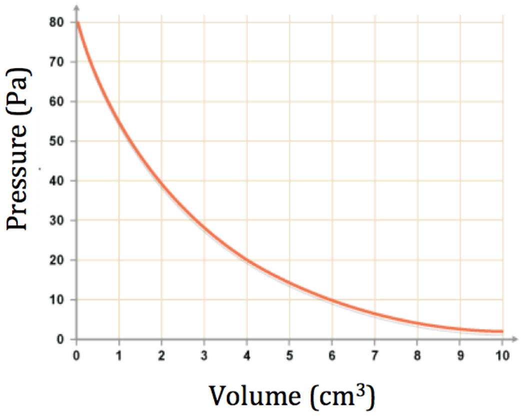
d. What physical quantity does this gradient tell us about? (1)

4. In your own words, describe the relationship between the two variables shown in the following graphs.

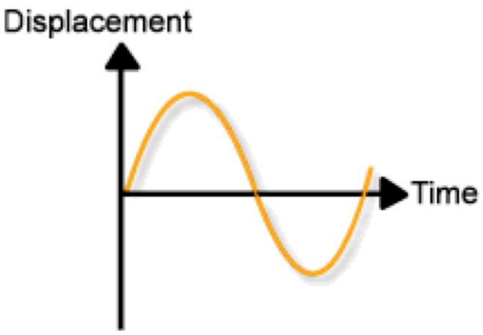
a. (2)



b. (2)



c. (2)



My Notes

Use this space to write your own questions, comments or key points.

Summary of key learning:

- Numerical values can be expressed in **scientific notation**, e.g.
  - 280 can be written as $2{,}8 \times 10^2$
  - 0,005 can be written as 5×10^{-3}
- Physical quantities are measured using **SI units**, e.g.
  - meters (m), seconds (s) and newtons (N).
- We use prefix notation to write small or large numbers, e.g.
  - 1 *milli*meter = 1×10^{-3} m, meaning "1000 times smaller".
  - 1 *kilo*pascal = 1×10^{3} Pa, meaning "1000 times bigger".
- When solving problems in science it is important to use steps to help us to answer the problems clearly and carefully. Important steps are:
  - Draw a diagram
  - Write a list of the given information
  - Read the question carefully to decide what is being asked
  - Select an appropriate equation or scientific concept for solving this problem
  - Do the calculation carefully
  - Reflect on your answer.
- The shape of scientific graphs tells us important information about the relationships between the physical quantities. Some graphs are:
  - straight line graphs
  - curved graphs
  - periodic graphs
  - inverse proportion graphs

Topic 2. Mechanics

Introduction

Mechanics is the study of motion and its causes. The greatest contribution to the development of mechanics was by Isaac Newton, who developed the three laws of motion and his law of universal gravitation to predict and explain phenomena. He showed that the physical world can be explained by a few special laws that can be expressed using mathematical formulae.

Sub-topic 1. Vectors

Content:

Unit 1: Introduction to vectors and scalars

Unit 2: Vectors in 2-dimensions

Unit 1. Introduction to vectors and scalars

Learning outcomes:

When you have completed this unit, you should be able to:

- state what is meant by scalar and vector quantities, and give examples of each;
- add vectors that are co-linear (in 1-dimension) using a graphical method (head-to-tail) as well as by calculation;

Introduction

In Physics there are different types of physical quantities that we can measure. These can be divided into two categories, scalars and vectors. In this unit you will learn about the difference between scalars and vectors, and you will learn how to represent these and solve problems using them. In this unit you will also learn about adding vectors in 1 dimension, that is, vectors that are in a straight line. In the following unit you will extend your knowledge to adding vectors in 2 dimensions.

1.1. The difference between scalars and vectors

scalar – a physical quantity that has a magnitude (size) but not a direction

vector – a physical quantity that has a magnitude (size) and a direction

In Physics there are two kinds of quantities, scalars and vectors.

A ***scalar*** is a physical quantity that has a magnitude (size), but not a direction.

- Example: temperature, because it does not have a direction.

A ***vector*** is a physical quantity that has a magnitude (size) and a direction.

- Example: force, because it has a clear direction.

When we write a vector quantity we need to write it with the magnitude and the direction.

Activity 1: Identify vectors and scalars

Identify whether the following physical quantities are scalars or vectors:

a. Temperature
b. Force
c. Mass
d. Density
e. Volume
f. Weight
g. Heat

MAIN IDEA: A scalar is a physical quantity that has a magnitude but not a direction. A **vector** is a physical quantity that has a magnitude and a direction.

1.2. Representing vectors

We represent vectors using arrows to show that they have a direction. The length of the arrow shows the magnitude of the vector. We use a scale to explain what the length means (see the example below). We show the direction of the vector by the direction that the arrow is pointing in.

A negative vector has the same magnitude as the original vector, but it has the opposite direction.

Example:

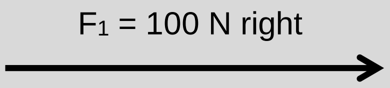
Show the following forces using vector diagrams.

a) A force F_1 of 100 N is pulling towards the right.
b) A force F_2 of 40 N is pulling towards the left.
c) The force $-F_1$

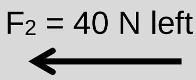
Solution:

Scale: 1 cm = 20 N

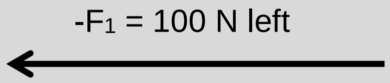
a)



b)



c)



Notes from the example:

- Choose a scale for your vector diagram, and clearly show the scale.
- It is important to clearly label your vector diagrams.

Activity 2 – *Representing vectors*

Draw the following vectors:

a) A force F_A of 600 N to the left.

b) A force F_B of 300 N to the right.

c) Force $-F_A$.

d) Force $-2F_B$.

MAIN IDEA: A vector can be represented by an **arrow**, where:

- the **length** of the arrow shows the magnitude
- the **direction** of the arrow shows the direction

A **negative** vector has the same magnitude as the original vector, with opposite direction.

1.3. Adding vectors in 1 dimension

When adding vectors in 1 dimension (in a straight line), we can then find the resultant vector by placing the tail (end) of one of the vectors next to the head (start) of the other vector. This is called the ***head-to-tail method***. The resultant vector is the final vector that we draw from the tail of the first vector to the head of the last vector.

Example:

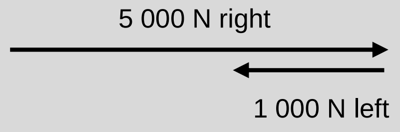
A truck pulls a trailor with a force of 5 000 N to the right. There is a force of friction of 1 000 N to the left on the trailor. What is the resultant force on the trailor?

Solution:

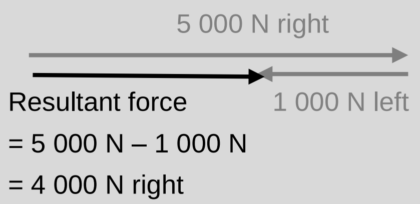
We can draw a vector diagram of our forces, showing their magnitude and direction.

Scale: let 1 cm = 1 000 N

We draw the vectors by placing the tail of one of the vectors next to the head of the other vector (the head-to-tail method):



We can now find the resultant force:



The resultant force is therefore 4 000 N to the right.

Activity 3 – *Adding vectors in 1 dimension*

You are given the following force vectors:

F_A = 90 N to the left F_B = 30 N to the right F_C = 60 N to the left

Use vector diagrams to find the resultant vectors:

a. $F_A + F_B$

b. $F_A - 2F_C$

MAIN IDEA: When adding vectors in 1 dimension we use the ***head-to-tail method:***

- place the tail (end) of one of the vectors next to the head (start) of the other vector;
- the resultant vector is the final vector that we draw from the tail of the first vector to the head of the last vector.

Unit 2. Vectors in 2 dimensions

Learning outcomes:

When you have completed this unit, you should be able to:

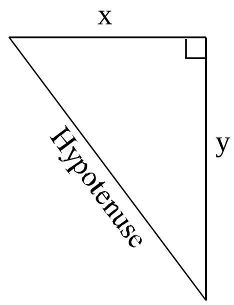
- add two vectors that are at right angles to determine the resultant using a graphical method (head-to-tail or tail-to-tail) as well as by calculation;
- determine the x- and y-components of a vector on the Cartesian plane.

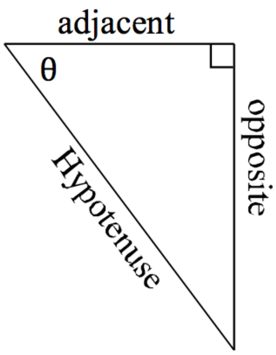
Introduction

In the previous unit you learnt about adding vectors in 1 dimension (in a straight line). In this unit you will extend your knowledge to adding vectors in 2 dimensions (at right angles to each other).

2.1. Mathematical tools needed for 2 dimensional vectors

In a right-angled triangle, if we know the lengths of any two sides of the triangle, we can find the length of the remaining side using Pythagoras' theorem:





Given a right-angled triangle with one known side and a known angle θ, we can calculate the length of any of the other side using the following trigonometric identities:

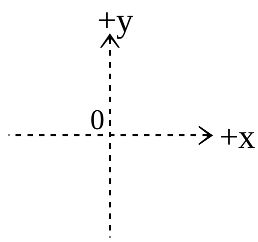
$$\sin\theta = \frac{opposite}{hypotenuse} \qquad \cos\theta = \frac{adjacent}{hypotenuse}$$

$$\tan\theta = \frac{opposite}{adjacent}$$

We can also use these identities to calculate an unknown angle if we know the length of any two sides of the triangle.

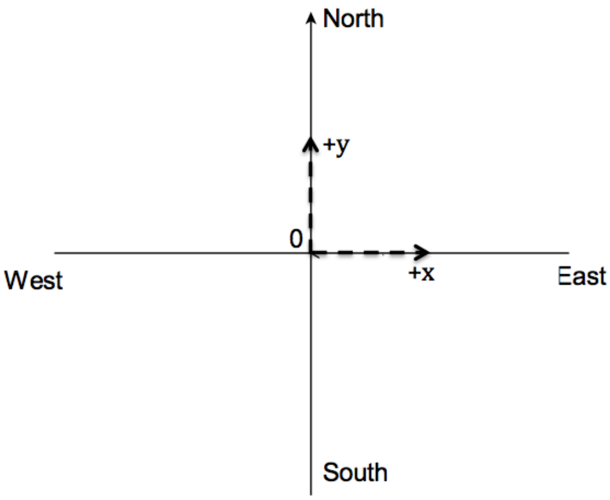
2.2. Frame of reference for vectors in 2 dimensions

When we work with vectors in 2 dimensions, we need to set a frame of reference, which includes a zero point and a positive direction for each of the dimensions that we are looking at. In other words, our



Frame of reference in 2 dimensions

frame of reference will look like a small coordinate system, showing the zero point for each axis at the origin, and showing the positive x and y directions. This is called the Cartesian plane. The diagram on the right shows what our frame of reference would look like in 2 dimensions.



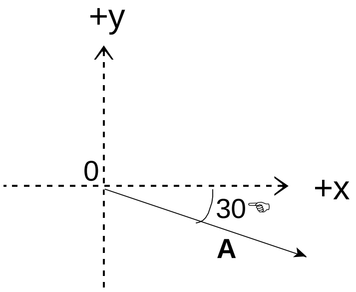
The compass directions of North, East, South and West can be expressed in terms of the Cartesian plane. We can choose to let the +y direction correspond to North, and then the East direction will be the +x direction, as the diagram shows.

Example 1:

Describe the direction of vector **A** shown on the right on the Cartesian plane.

Solution:

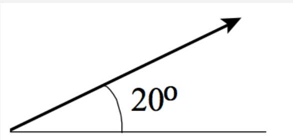
The direction of **A** is 30º below the +x axis.



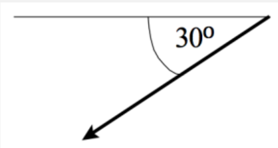
Activity 1 – *Directions of vectors in 2 dimensions*

Give the direction of the following vectors:

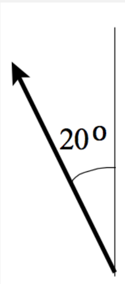
a.



b.



c.



2.3. The resultant of two perpendicular vectors using the tail-to-head method

When we add two vectors that are perpendicular to each other, we can sketch them on the Cartesian plane, and then use Pythagoras' theorem to find the resultant vector. One method that we can use the **tail-to-head** method, where we place the tail (end) of one of the vectors next to the head (start) of the other vector. The resultant vector is the arrow from the tail of the first vector to the head of the second vector.

If you construct the diagram carefully, you can measure the resultant vector from your diagram using a ruler and a protractor. You can also use trigonometry to calculate the magnitude and direction of the resultant vector.

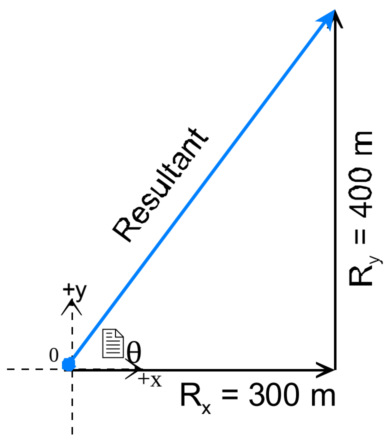
Example:

Boipelo walked 300 m toward the East, then 400 m toward the North. What was her resultant displacement?

Solution:

Frame of reference: Let the +y direction be North.

We can then draw a head-to-tail vector diagram of Boipelo's movements R_x and R_y. This is shown on the right. (The frame of reference is shown on the diagram). Her resultant displacement is therefore:



$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{300^2 + 400^2} = 500 \text{ m}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{R_y}{R_x} = \frac{400 \text{ m}}{300 \text{ m}}$$

Therefore $\theta = 53{,}1^{\circ}$

Boipelo's resultant displacement is therefore 500 m in a direction of $53{,}1^{\circ}$ above the +x-axis.

MAIN IDEA: To find the resultant of two vectors using the tail-to-head method:

- The vectors are joined tail-to-head.
- The resultant vector is the arrow from the tail of the first vector to the head of the second vector.

2.4. The resultant of two perpendicular vectors using the tail-to-tail method

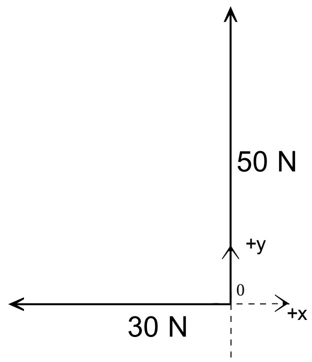
We can also use a **tail-to-tail** method to find the resultant vector. Here we place both of the tails of the vectors together at the origin of the Cartesian plane, and we then complete a parallelogram to find the resultant.

Example:

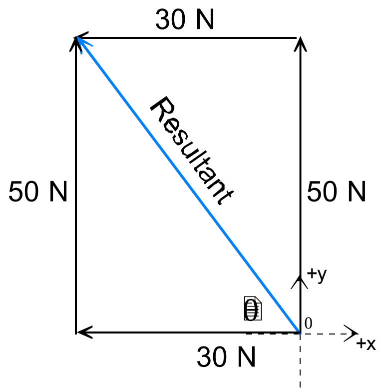
Two ropes pulled on an object. One rope pulled with a force of 30 N in the -x direction, and the second rope pulled with a force of 50 N in the +y direction. What was the resultant force on the object?

Solution:

The diagram on the right shows the forces on the object, connected tail-to-tail. (The frame of reference is shown on the diagram)



In the second diagram the vectors have been redrawn to form a parallelogram. We can use this parallelogram to find the resultant vector.



Magnitude of resultant = $\sqrt{(30\text{ N})^2 + (50\text{ N})^2} = 58{,}3\text{ N}$

Using trigonometry, we know that $\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{50\text{ N}}{30\text{ N}}$

Therefore $\theta = 59^{o}$

The resultant force on the object is therefore 58,3 N at an angle of 59^{o} above the –x axis.

MAIN IDEA: To find the resultant of two vectors using the tail-to-tail method:

- The vectors are joined tail-to-tail. A parallelogram is then formed using these vectors.
- The resultant vector is the arrow from the position where the tails join towards the opposite corner of the parallelogram.

Activity 2: *Practice adding vectors in 2 dimensions*

Solve the following problem using a vector diagram and calculations:

A jogger travels 6 km toward the South. He then jogs 5 km toward the West. What is his resultant displacement? Use the tail-to-head method.

2.5. Finding vector components

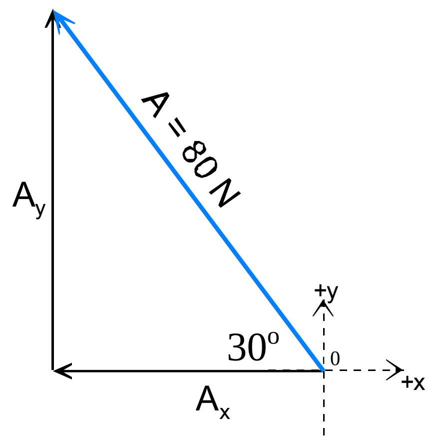
If we are given the magnitude and direction of a vector relative to the Cartesian plane, we can use trigonometry to work out the x- and y-components of the vector. The following example shows you how this is done.

Example:

A vector A has a magnitude of 80 N, and a direction of 30º above the -x axis. Find the x- and y-components of this vector.

Solution:

The diagram on the right shows the vector and its components on the Cartesian plane.



Using trigonometry, we know that

$$\sin\ \theta = \frac{\text{opposite}}{\text{hypotenuse}} \quad \text{and} \quad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

Therefore $\sin 30^o = \frac{A_y}{A}$

So $\mathbf{A}_y$ = + A sin 30º = + 80 N sin 30º = + 40 N

Similarly, $\mathbf{A}_x$ = - A cos 30º = - 80 N cos 30º = - 69,3 N

(This component has a negative sign since it is in the –x direction.)

Once we have found the components of a set of vectors, we can add the vectors by adding their components in the following way:

- add together all of the components that are parallel to the x-direction to find the resultant component in the x-direction ($\mathbf{R}_x$)
- add together all of the components that are parallel to the y-direction to find the resultant component in the y-direction ($\mathbf{R}_y$)
- find the resultant of $\mathbf{R}_x$ and $\mathbf{R}_y$ using Pythagoras and trigonometry.

MAIN IDEA: For a vector with magnitude A and direction θ from the x axis:

- The magnitude of the x-component of the vector can be found using the equation $A_x = A \cos(\theta)$
- The magnitude of the y-component of the vector can be found using the equation $A_y = A \sin(\theta)$
- We can find the resultant of two or more vectors by adding together the x-components ($\mathbf{R}_x$) and the y-components ($\mathbf{R}_y$), and finding the resultant of $\mathbf{R}_x$ and $\mathbf{R}_y$.

Activity 3 - *Finding components of vectors*

Find the x- and y-components of the following vectors, and hence find the resultant vector A + B:

a. Vector A that has a magnitude of 40 N and a direction of 45^{o} above the +x axis.

b. Vector B that has a magnitude of 65 N and a direction of 30^{o} above the –x axis.

Assessment Activity: Vectors **Total marks = 30**

Answer the following questions to assess your understanding of vectors:

1. What is the difference between a scalar and a vector? (3)

2. List two examples of a scalar, and two examples of a vector quantity. (4)

3. Nonhle walked 300 m in the +x direction, and then 600 m in the +y direction. What was her resultant displacement? Use the tail-to-head method. (6)

4. Force A has a magnitude of 48 N and points towards the West. Force B has the same magnitude, but points towards the South. Use the tail-to-tail method to determine the magnitude and direction of A + B. (6)

5.

   a. Find the x- and y-components of the following vectors: (4)

      i. Vector A = 30 N at a direction of 15^{o} above the +x axis.

      ii. Vector B = 40 N at a direction of 25^{o} below the –x axis.

   b. Find the resultant of vectors A and B. (7)

My Notes

Use this space to write your own questions, comments or key points.

Summary of key learning:

- **A scalar** is a physical quantity that has a magnitude but not a direction.
- A **vector** is a physical quantity that has a magnitude and a direction.
- A vector can be represented by an arrow, where:
    - the length of the arrow shows the magnitude
    - the direction of the arrow shows the direction
- A negative vector has the same magnitude as the original vector, with opposite direction.
- When adding vectors in 1 dimension we use the ***head-to-tail method:***
    - place the tail (end) of one of the vectors next to the head (start) of the other vector;
    - the resultant vector is the final vector that we draw from the tail of the first vector to the head of the last vector.
- To find the resultant of two vectors using the tail-to-head method:
    - The vectors are joined tail-to-head.
    - The resultant vector is the arrow from the tail of the first vector to the head of the second vector.
- To find the resultant of two vectors using the tail-to-tail method:
    - The vectors are joined tail-to-tail. A parallelogram is then formed using these vectors.
    - The resultant vector is the arrow from the position where the tails join towards the opposite corner of the parallelogram.
- The x- and y-components of a vector with magnitude A and direction θ from the x axis are:
    - $A_x = A \cos(\theta)$
    - $A_y = A \sin(\theta)$

Sub-topic 2. Motion in 1-dimension

Content:

Unit 1: Position, displacement, distance

Unit 2: Speed, velocity, acceleration

Unit 3: Graphs of motion

Unit 4: Equations of motion

Unit 5: Projectile motion

Unit 1. Position, displacement, distance

Learning outcomes:

When you have completed this unit, you should be able to:

- define position, displacement and distance;
- define position relative to a frame of reference;
- determine displacement and distance for 1-dimensional motion.

1.1. Frame of Reference

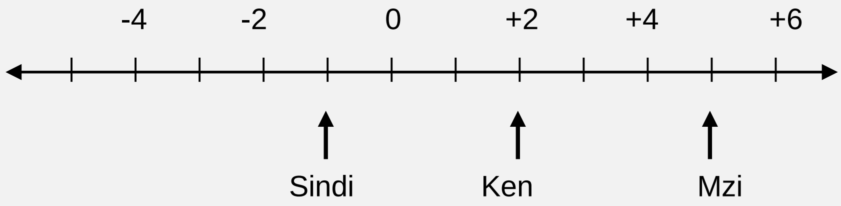
When we describe the position or movement of an object, we need to have a ***frame of reference*** to tell us where our zero point is, and what direction we are moving in. When we are describing our direction, we could use terms like East or West, up or down, backwards or forwards. In 1 dimensional motion it is useful to choose one direction as positive, and then the opposite direction is negative.

1.2. Position

An object's position describes where it is relative to a frame of reference. Position is a length reading, and so we measure it in units of meters (m). In one dimension, positions are either positive or negative, depending on which side of the zero point they lie.

Activity 1 – Finding positions

Study the diagram below and answer the questions that follow:



1. <u>Frame of reference</u>: Positions to the left of the zero point are negative, and positions to the right of the zero point are positive.
   a. What are the positions of Sindi, Ken and Mzi?

2. <u>Frame of reference</u>: Let Mzi be the reference point. Positions to the left of Mzi are negative, and positions to the right of Mzi are positive.
   a. What is Sindi's position relative to Mzi?
   b. What is Ken's position relative to Mzi?

3. <u>Frame of reference</u>: Let Ken be the reference point. Positions to the left of Ken are positive, and positions to the right of Ken are negative.
   a. What is Sindi's position relative to Ken?
   b. What is Mzi's position relative to Ken?

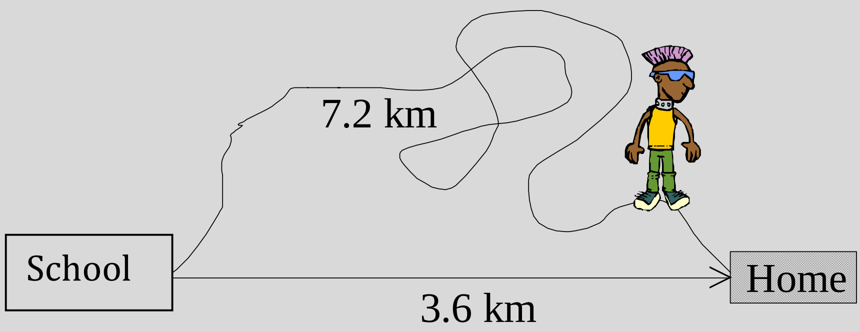
MAIN IDEA: *Position* describes where an object is relative to a frame of reference.

1.3. Distance and Displacement

As an introduction to some of the terms we use to describe movement, we will use an example.

<u>Example:</u> *(Try to solve this problem* ***on your own or with a fellow student*** *while covering the solution, and then check your work using the solution below).*

Thabo walked home after school. His home was 3,6 km east of his school. The path that he took was not a very straight one, and he ended up walking quite a long way! The diagram shows the path that he took.



1. What is the *total length of the path* that Thabo walked?
2. What is the *length between Thabo's starting point and his ending point*?

Solution

1. The total length of the path that Thabo walked is **7,2 km**.
2. The length between Thabo's starting point and his ending point is **3,6 km**.

From this example you can see that we need two different words to describe these two lengths, ***distance*** and ***displacement***.

We use the word ***distance*** for the **total length of the path**. The symbol that we use for distance is D, and it is measured in SI units of meters (m).
From the example of Thabo walking home, you can see that his path was in lots of different directions at different times. The total distance that he covered was 7,2 km.
Distance is a **scalar** quantity because it does not have a specific direction.

We use the word ***displacement*** to measure the **change in position** from the starting point to the ending point of the motion. This displacement will always be in a straight line. It is a **vector** quantity because it has a very clear direction.
From the example of Thabo walking home, you can see that his displacement from the pub to his home was 3,6 km East.
You can see that the distance covered can often be much greater than the displacement.

Since displacement is defined as the change in position, the symbol that we use for displacement is Δx. It is measured in SI units of meters (m).

We calculate displacement in the following way: $\Delta x = x_f - x_i$

where x_f is the final position, and x_i is the initial position.

<u>Example</u>: *(Try to solve this problem* ***on your own or with a fellow student*** *while covering the solution, and then check your work using the solution below).*

a) Keletso walked from a position of 0,6 m to a position of -1,2 m. What was her displacement?

b) Keletso then walked from her position of -1,2 m to a position of 1 m. What was her displacement?

c) What was Keletso's total distance covered in (a) and (b)?

<u>Solutions:</u>

a) **<u>Given:</u>** x_i = 0,6 m and x_f = -1,2 m

Displacement: $\Delta x = x_f - x_i$ = -1,2 m – 0,6 m = -1,8 m

b) **<u>Given:</u>** x_i = -1,2 m and x_f = 1 m

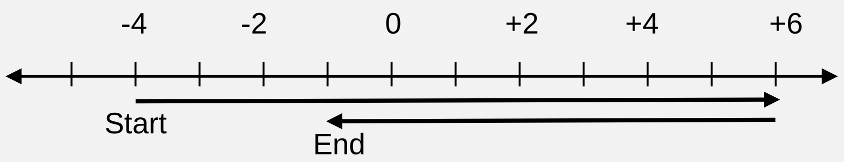
Displacement: $\Delta x = x_f - x_i$ = 1 m – (-1,2 m) = +2,2 m

c) Total distance D = 1,8 m + 2,2 m = 4 m

Activity 2 – Finding displacements

Answer the following questions. Show your working clearly.

1. Sindisiwe starts at a position of +2,5 m. She walks to a position of -4 m. What is her displacement? (Remember to show the direction with a + or a – sign)
2. Themba walks from a position of -5 m to a position of -3,5 m. What is his displacement?
3. If Celani starts at a position of -4 m and walks in the positive direction for 2 m, what is her new position?
4. The number line below shows Felix's movement.



a. What was Felix's starting position?

b. What was the position where Felix changed his direction of movement?

c. What was his ending position?

d. What was the total **distance** covered by Felix?

e. What was Felix's **displacement**?

MAIN IDEAS:

- **Distance** (D) is the total length of the path of movement, measured in units of meters (m).
- **Displacement** (Δx) is the change in position, measured in units of meters (m).
  $\Delta x = x_f - x_i$

Unit 2. Speed, velocity, acceleration

Learning outcomes:

When you have completed this unit, you should be able to:

- define speed, instantaneous velocity and average velocity;
- determine speed, instantaneous velocity and average velocity for 1-dimensional motion;
- define acceleration;
- determine acceleration for 1-dimensional motion with uniform acceleration.

Introduction

When we work with motion, we need to be able to describe how fast something is moving. In the same way that there are two terms to describe path lengths, there are also two terms that we use when we are measuring the fastness of an object: ***speed*** and ***velocity***. We also need to work out how the velocity of the object is changing. This is called ***acceleration***. In this unit you will learn about speed, velocity and acceleration.

2.1. Speed

If we want to calculate the ***speed***, we divide the total distance (D) covered by the time (Δt) that it took him to travel that distance.

$$\text{speed} = \frac{\text{distance covered}}{\text{total time taken}} = \frac{\text{D}}{\Delta \text{t}}$$

Since distance is a scalar quantity, it means that the speed is also a scalar. Scientifically we say that speed is the *rate at which distance is covered*. Because the SI unit for distance is meters (m), and for time is seconds (s), speed is measured in units of meters per second (m/s or $m{\cdot}s^{-1}$).

When we have units like m/s, what we mean is the number of meters traveled in one second (or the number of meters per second). So the word "*per*" just means "*in one*".

2.2. Average Velocity

The ***average velocity*** is found by dividing the displacement (or change in position) by the time taken:

$$\text{average velocity} = \frac{\text{displacement}}{\text{total time taken}}$$

You will notice that velocity is also measured in m·s^{-1}. Displacement is a vector quantity, in other words, it has a specific direction. This means that the velocity is also a vector quantity, as it has the **same direction** as the displacement.

Mathematically we write the equation for the average velocity as: $v = \frac{\Delta x}{\Delta t}$

2.3. Instantaneous Velocity

> **instantaneous** – done in an instant (right now)

> **infinitesimal** - very small

The ***instantaneous velocity*** is found by dividing the displacement (or change in position) by an infinitesimal time interval:

$$\text{instantaneous velocity} = \frac{\text{displacement}}{\text{small time interval}}$$

Instantaneous velocity is also measured in m·s^{-1}, and it is a vector quantity that has the **same direction** as the displacement.

Mathematically we write the equation for the instantaneous velocity as: $v = \frac{dx}{dt}$

where dx is the small displacement, measured in m, and dt is the small time interval, measured in s.

The instantaneous speed of the object is the magnitude of the instantaneous velocity. So the instantaneous speed has a magnitude but does not have a direction, and it is therefore a scalar quantity.

Examples: *(Try to solve this problem* ***on your own or with a fellow student*** *while covering the solution, and then check your work using the solution below).*

Zama walked with a constant pace from a position of +2 m to a position of +6 m in 2s.

1. What was her instantaneous velocity at the time 1 second?

Zama then walked with a constant pace to a position of +3m in a time of 3 s.

2. What was Zama's instantaneous velocity after 1 second of this motion?
3. What was her instantaneous speed?
4. What was Zama's average velocity for the total movement in parts 1 and 2?

Solution:

1. **Given:** x_i = +2 m and x_f = +6 m;

The velocity is constant from 0 s to 2 s, so we use this time interval to find the instantaneous velocity at 1 s.

Instantaneous velocity at 1 s: $v = \frac{dx}{dt} = \frac{6\ m - 2\ m}{2\ s - 0\ s} = \frac{+4\ m}{2\ s}$ = +2 m·s^{-1}

2. **Given:** x_i = +6 m and x_f = +3 m

The velocity is constant from for this 3 s, so we use this time interval to find the instantaneous velocity after 1 s.

Instantaneous velocity after 1 s: $v = \frac{dx}{dt} = \frac{3\text{ m} - 6\text{ m}}{3\text{ s} - 0\text{ s}} = \frac{-3\text{ m}}{3\text{ s}}$ = -1 m·s^{-1}

3. Instantaneous speed after 1 s = $\frac{D}{dt} = \frac{3\text{ m}}{3\text{ s} - 0\text{ s}}$ = 1 m·s^{-1}

   OR Instantaneous speed =magnitude of Instantaneous velocity =1 m·s^{-1}

4. **Given:** x_i = +2 m and x_f = +3 m; Δt = 2 s + 3 s = 5 s

Average velocity: $v = \frac{\Delta x}{\Delta t} = \frac{3\text{ m} - 2\text{ m}}{5\text{ s}} = \frac{1\text{ m}}{5\text{ s}}$ = +0,2 m·s^{-1}

Activity 1 – Test your understanding of speed and velocity

Answer the following questions.

1. Look back at the example of Thabo walking home from school.
   (a) If Thabo took 30 minutes to get to his home, calculate the average speed of his total journey in m·s^{-1}.
   (b) Calculate Thabo's average velocity in m·s^{-1}.
   (c) Calculate Thabo's average velocity in km·hr^{-1}.

2. Isihle walked with a constant velocity from a position of +3 m to a position of -3 m in 6 seconds.
   (a) Calculate her instantaneous velocity after 3 seconds of this motion.

Isihle then ran with a constant velocity to a position of +6 m in 2 seconds.

   (b) What was her instantaneous velocity after 1 second of this motion?
   (c) What was Isihle's average velocity for her total movement described in (a) and (b) above?
   (d) What was Isihle's average speed for her total movement?

Isihle then moved from her position of +6 m with a velocity of -3 m·s^{-1} for 6 seconds.

   (e) What was Isihle's final position after this movement?

MAIN IDEAS:

- **Speed** is the total distance covered divided by the time taken:

$$\text{speed} = \frac{D}{\Delta t}$$

- Speed is a scalar and is measured in units of meters per second ($m{\cdot}s^{-1}$).
- **Average velocity** is found by dividing the displacement by the time:

$$v = \frac{\Delta x}{\Delta t}$$

- **Instantaneous velocity** is found by dividing the displacement by an infinitesimal time interval:

$$v = \frac{dx}{dt}$$

- Velocity is a vector and is measured in meters per second ($m{\cdot}s^{-1}$).

2.4. Acceleration

Acceleration is the *rate of change of velocity*. It is found by dividing the change in velocity by the time it takes for that velocity change:

$$\text{acceleration} = \frac{\text{change in velocity}}{\text{total time taken}}$$

Acceleration is a vector, and its symbol is a. Mathematically we can write the equation as:

$$a = \frac{\Delta v}{\Delta t}$$

Acceleration is measured in meters per second per second (m/s/s), or meters per second squared (m/s^2, or $m{\cdot}s^{-2}$).

<u>Example:</u> *(Try to solve this problem* ***on your own or with a fellow student*** *while covering the solution, and then check your work using the solution below).*

1. Carol starts from rest, and pedals her bike for 4 seconds until it has a velocity of 8 $m{\cdot}s^{-1}$. What is her average acceleration?
2. She now rides at a constant speed for 6 seconds. What is her average acceleration during this time? (Justify your answer using a calculation).
3. She applies her breaks for 3 seconds and slows down so that her speed becomes 2 $m{\cdot}s^{-1}$. What is her average acceleration during this time?

Solution:

1. $a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t} = \frac{8\ m{\cdot}s^{-1} - 0\ m{\cdot}s^{-1}}{4\ s} = +2\ m{\cdot}s^{-2}$

2. $a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t} = \frac{8\ m{\cdot}s^{-1} - 8\ m{\cdot}s^{-1}}{6\ s} = 0\ m{\cdot}s^{-2}$

3. $a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t} = \frac{2\ m{\cdot}s^{-1} - 8\ m{\cdot}s^{-1}}{3\ s} = \frac{-6\ m{\cdot}s^{-1}}{3\ s} = -2\ m{\cdot}s^{-2}$

Activity 2 – Investigating acceleration

Calculate the acceleration for each of the following:

1. A car travels in the positive direction with a constant velocity of 4 $m{\cdot}s^{-1}$ for 2 seconds.
2. A car starts from rest ($v = 0\ m{\cdot}s^{-1}$), and after 2 seconds it is travelling with a velocity of 4 $m{\cdot}s^{-1}$.
3. A car is moving with a velocity of 8 $m{\cdot}s^{-1}$. After 2 seconds it has slowed down to a velocity of 4 $m{\cdot}s^{-1}$.
4. A car is travelling with a velocity of -4 $m{\cdot}s^{-1}$. It slows down to a stop in 2 s. Calculate its acceleration.
5. A car is travelling with a velocity of -4 $m{\cdot}s^{-1}$. It increases its speed, and reaches a velocity of -8 $m{\cdot}s^{-1}$ in 2 s.

From your calculations, write a summary of the kind of movement which will give you an acceleration that is:

- Zero
- Positive
- Negative

Acceleration is a vector, and so we have to write it with a magnitude and a direction. For 1 dimensional motion the direction is given by the sign + or -.

From the previous activity you should notice the following:

- Acceleration is ***zero*** when there is no motion, or when motion is uniform (constant velocity).
- Acceleration is ***positive*** when the movement is getting faster in a positive direction, or when the movement is getting slower in a negative direction
- Acceleration is ***negative*** when the movement is becoming slower in a positive direction, or when the movement is getting faster in a negative direction..

It is important to note that we cannot use the word deceleration to describe negative acceleration. The word deceleration describes the motion of an object that is slowing down, but negative acceleration can mean that an object is speeding up in the negative direction. To avoid making mistakes, it is a good idea to never use the word deceleration in Physics.

You will also notice that the acceleration does not tell us any information about the direction of the motion. It only describes how the velocity is changing.

MAIN IDEAS: ***Acceleration*** is the change in velocity divided by the total time, and is a vector quantity:

$$\text{acceleration a} = \frac{\text{change in velocity}}{\text{total time taken}} = \frac{\Delta v}{\Delta t}$$

Acceleration is measured in units of $m{\cdot}s^{-2}$

Unit 3. Graphs of motion

Learning outcomes:

When you have completed this unit, you should be able to:

- plot graphs of position vs time, velocity vs time and acceleration vs time;
- interpret and determine information from graphs of position vs time, velocity vs time and acceleration vs time for 1-dimensional motion with uniform acceleration.

Introduction

There are three different kinds of graphs that describe the motion of an object:

- Graphs of position vs time
- Graphs of velocity vs time
- Graphs of acceleration vs time

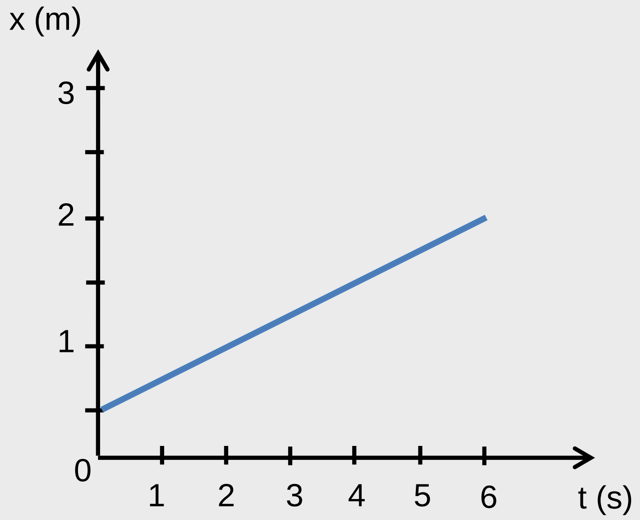
We will explore these graphs in this unit.

3.1. Graphs of position vs time

A position vs time graph is constructed with position values given on the y-axis, and time values given on the x-axis. A typical position-time graph is shown in the example below.

Example: *(Try to solve this problem* ***on your own or with a fellow student*** *while covering the solution, and then check your work using the solution below).*

The position-time graph below describes Andile's movement while he is walking.



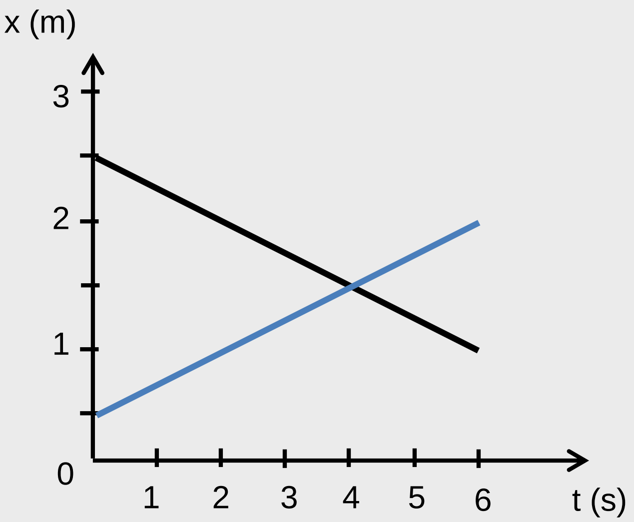
1. What was Andile's position at a time of 6 seconds?
2. Describe Andile's motion in words.
3. What was Andile's total displacement?

Sindi started from a position of 2,5 m and walked for 6 seconds with the same speed as Andile, but in the opposite direction to Andile's motion.

4. On the same graph, draw a sketch to show what the position-time graph would look like for Sindi.
5. Will Sindi and Andile pass each other? At what time?

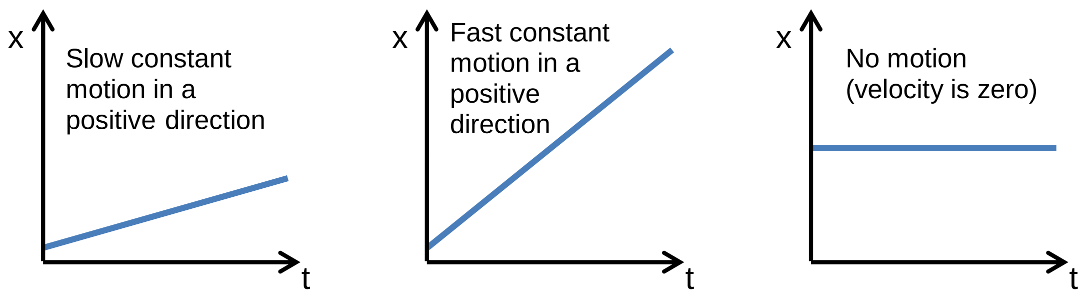
<u>Solution:</u>

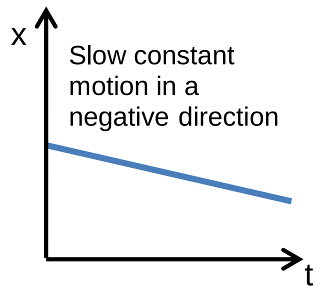
1. Andile's position at a time of 6 seconds was 2 m.
2. Andile moved from a starting point of 0,5 m with a uniform (constant) velocity in the positive direction.
3. $\Delta x = x_f - x_i = 2 \text{ m} - 0{,}5 \text{ m} = 1{,}5 \text{ m}$.
4. The position-time graph for Sindi's motion is shown with the dark line below:

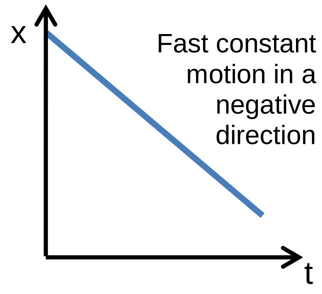


5. Yes, Sindi and Andile pass each other at a time of 4 seconds (we can see this from where the graphs intersect).

The position-time graphs for different types of constant motion are shown below:







The position-time graph is a straight line for constant motion. On a position-time graph, the **value** gives the object's position at a specific time. The **gradient** (slope) of the graph tells us the **velocity** (how fast the movement is, and in what direction).

The example below shows how to calculate velocity from a position-time graph.

Example: *(Try to solve this problem **on your own or with a fellow student** while covering the solution, and then check your work using the solution below).*

The position-time graph below describes the motion of a man on a bicycle.

1. Calculate the man's velocity between 0 s and 4 s.
2. Calculate the man's velocity between 4 s and 6 s.

Solution:

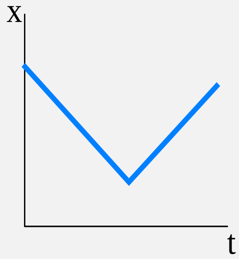
1. Velocity $= \frac{\Delta x}{\Delta t} = \frac{2\text{ m} - 2\text{ m}}{4\text{ s} - 0\text{ s}} = 0\text{ m}\cdot\text{s}^{-1}$

2. Velocity $= \frac{\Delta x}{\Delta t} = \frac{0\text{ m} - 2\text{ m}}{6\text{ s} - 4\text{ s}} = -1\text{ m}\cdot\text{s}^{-1}$

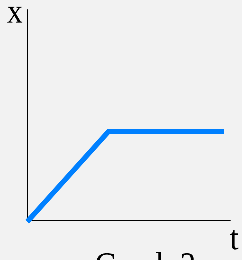
Activity 1 – Position-time graphs for uniform motion

Answer the following questions:

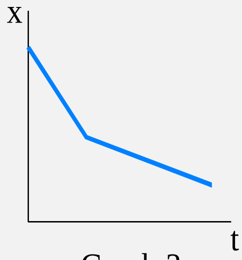
1. How would you need to walk to create the following position-time graphs?



Graph 1



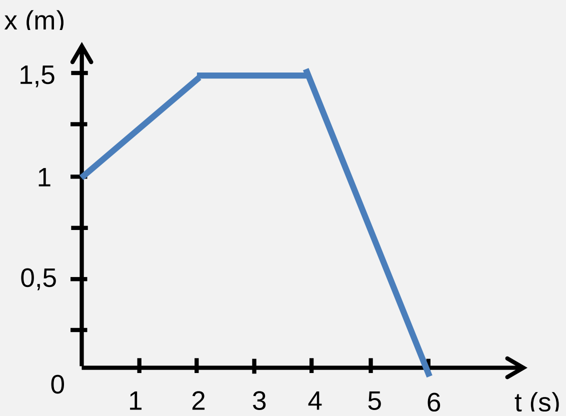
Graph 2



Graph 3

2. Sketch the shape of the position-time graphs for the following motion:
    a. Start at the zero position, and walk in a positive direction with a constant velocity.
    b. Start at some positive position, and walk in a negative direction with a uniform velocity.
    c. Stand still for some time at a positive position, then walk in a negative direction with uniform velocity.

3. The position-time graph below describes the motion of a woman who is running.



a. Describe the woman's movement during the 6 seconds.

b. Calculate the woman's velocity between

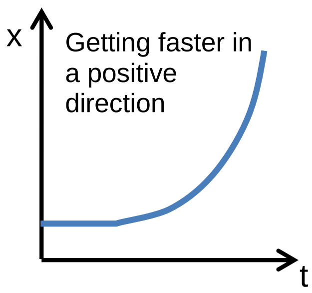
i. 0 s and 2 s

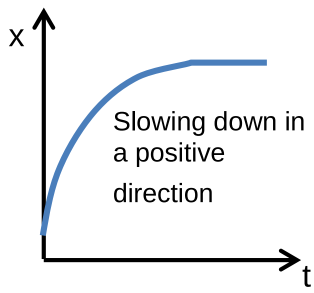
ii. 2 s and 4 s

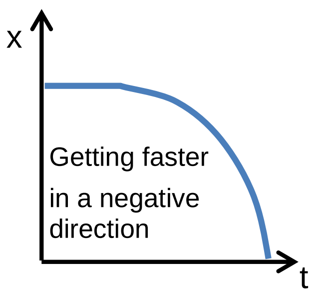
iii. 4 s and 6 s

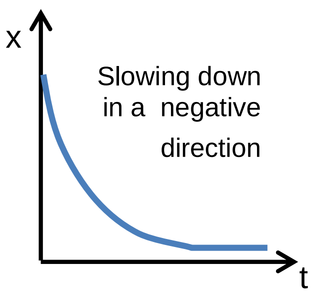
Position-time graphs for changing motion

Motion does not always have a constant velocity. When an object is accelerated with constant acceleration, we call this **uniformly accelerated motion**. Here the velocity is changing at a constant rate. The position-time graphs for uniformly accelerated motion have a curved shape. Some examples of position-time graphs for uniformly accelerated motion are shown below:









MAIN IDEA:

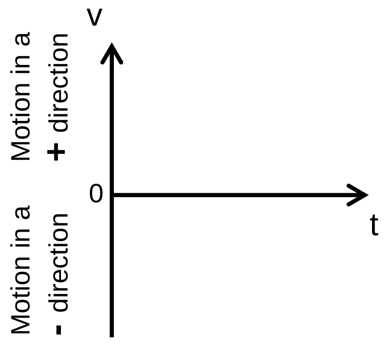
- On a position-time graph, the **value** gives the object's position at a certain time
- The **gradient** (slope) of the graph tells us the **velocity**:

$$\text{Velocity} = \text{gradient of x-t graph} = \frac{\Delta x}{\Delta t}$$

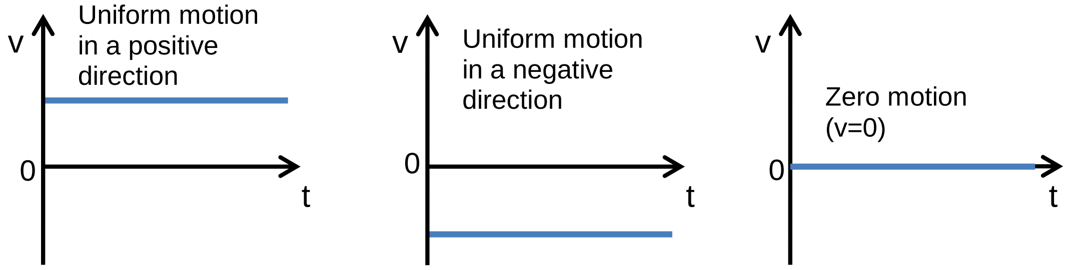
Go to https://www.youtube.com/watch?v=x2ve5yucNPQ to see a video on position-time graphs.

3.2. Graphs of velocity vs time

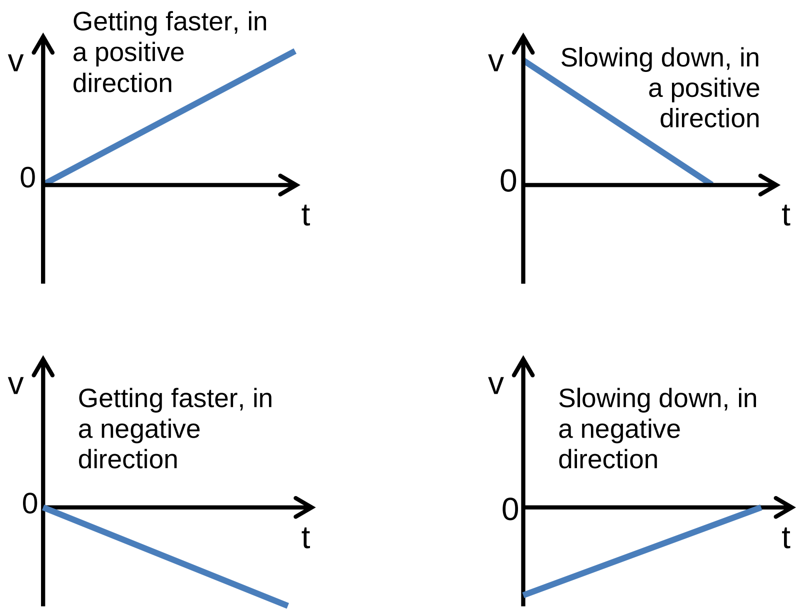
We can also use graphs of velocity vs time to represent the motion of an object. The velocity vs time graph is constructed with velocity values given by the y-axis, and time values given on the x-axis. Velocity values that are above the time-axis are positive, which means that the direction of the motion is positive. Velocity values that are below the time-axis are negative, which means that the direction of the motion is negative.



For ***uniform*** motion (motion with a constant velocity) the velocity-time graph is a straight horizontal line. The velocity-time graphs for different types of ***uniform*** motion are shown below:



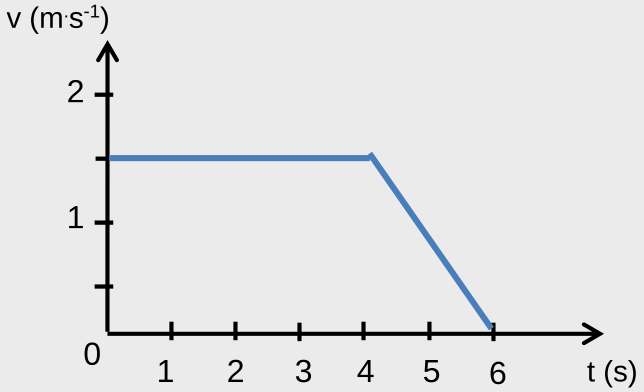
For ***uniformly accelerated*** motion the velocity-time graph is a straight line that is not horizontal. The ***gradient*** of the velocity-time graph tells us the value of the acceleration of the object. The velocity-time graphs for different types of ***uniformly accelerated*** motion are shown below:



For **constant motion**, the velocity-time graph is a straight, horizontal line. The graph is **above** the time-axis for movement in the positive direction, and **below** the time-axis for movement in the negative direction. When there is **no movement (zero velocity)**, the velocity-time graph is on the time-axis. The **gradient** of the velocity-time graph tells us the **acceleration** of the object.

<u>**Example**</u>: *(Try to solve this problem* ***on your own or with a fellow student*** *while covering the solution, and then check your work using the solution below).*

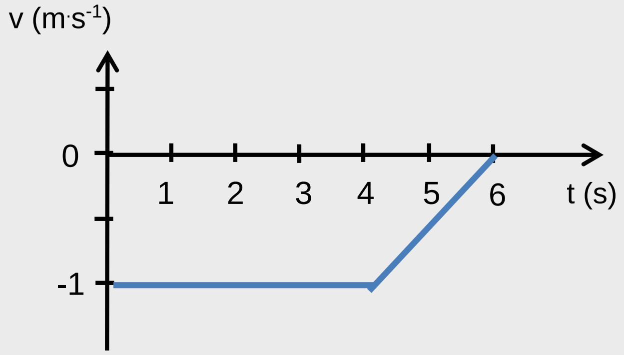
The velocity-time graph below describes a the motion of a taxi.



1. Describe the taxi's motion in words.
2. What was the taxi's acceleration between 0 s and 4 s?
3. What was the taxi's acceleration between 4 s and 6 s?
4. A second taxi is moving with a constant velocity of -1 m·s^{-1} for 4 seconds, and then slows to a stop in a time of 2 seconds. Draw its velocity-time graph.

<u>Solution:</u>

1. The taxi moved with a constant positive velocity of 1,5 m·s^{-1} for the first 4 seconds. It then slowed to a stop in a time of 2 seconds.
2. a = gradient of v-t graph = $\frac{v_f - v_i}{\Delta t} = \frac{1{,}5 \text{ m·s}^{-1} - 1{,}5 \text{ m·s}^{-1}}{4 \text{ s} - 0 \text{ s}}$ = 0 m·s^{-2}
3. a = $\frac{v_f - v_i}{\Delta t} = \frac{0 \text{ m·s}^{-1} - 1{,}5 \text{ m·s}^{-1}}{6 \text{ s} - 4 \text{ s}}$ = -0,75 m·s^{-2}
4. The velocity-time graph for the second taxi is:

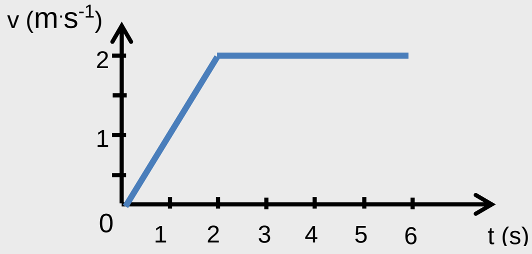


Finding displacement from the velocity-time graph

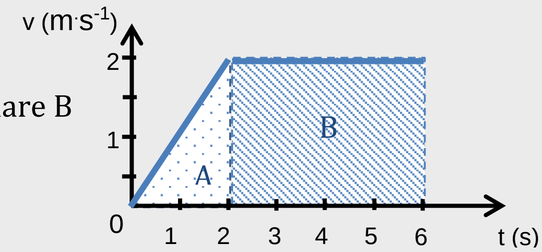
The ***area underneath*** the velocity-time graph tells us the value of the displacement of the object during that time interval. This does not give us any information about the starting position of the object, it only tells us the change in position of the object.

Example: *(Try to solve this problem* ***on your own or with a fellow student*** *while covering the solution, and then check your work using the solution below).*

For the velocity-time graph given below, calculate the total displacement of the object.



Solution:



Δx = area underneath v-t graph

= area under triangle A + area under square B

$= (\frac{1}{2}\text{ base} \times \text{height}) + (\text{base} \times \text{height})$

$= (\frac{1}{2} \times 2\text{ s} \times 2\text{ m·s}^{-1}) + (4\text{ s} \times 2\text{ m·s}^{-1})$

= 2 m + 8 m

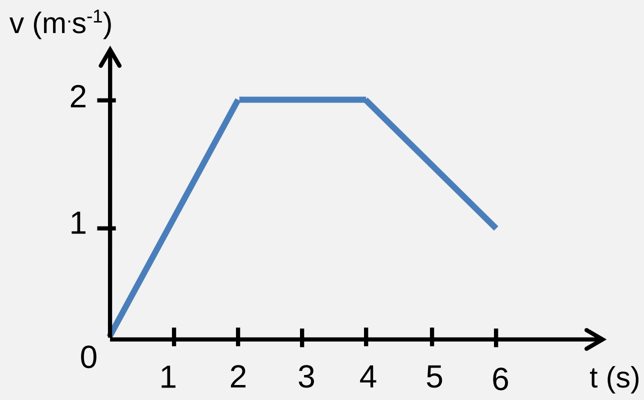
= 10 m

Activity 3 – Velocity-time graphs

Answer the following questions:

1. Draw the velocity-time graphs for the following movements:
    a. Walk in a positive direction with a constant but slow velocity
    b. Walk in a negative direction with a constant but fast velocity
    c. Walk in a negative direction, starting from rest and speeding up
    d. Walk in a positive direction, with a high velocity at first, and gradually slowing to a stop.

2. Cameron rode his bicycle. The velocity-time graph for his motion is shown below:



a. Describe Cameron's motion in words.
b. Calculate Cameron's acceleration in the following time intervals:
   i. between 0 s and 2 s
   ii. between 2 s and 4 s
   iii. between 4 s and 6 s
c. What was Cameron's total displacement?
d. Draw the position-time graph for the first 2 seconds of Cameron's motion.

MAIN IDEAS:

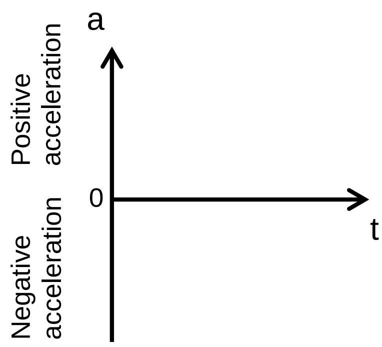
- For **uniform motion**, the velocity-time graph is a straight, horizontal line. The line is above the time-axis for movement in the positive direction, and below the time-axis for movement in the negative direction.
- When there is **no motion (zero velocity)**, the velocity-time graph is on the time-axis.
- On a velocity-time graph, the **value** gives the object's velocity at a certain time
- The **area underneath** the velocity-time graph gives a value for the **displacement**.
- The **gradient** of the velocity-time graph gives a value for the **acceleration**.

$$\text{acceleration} = \text{gradient of v-t graph} = \frac{\Delta v}{\Delta t}$$

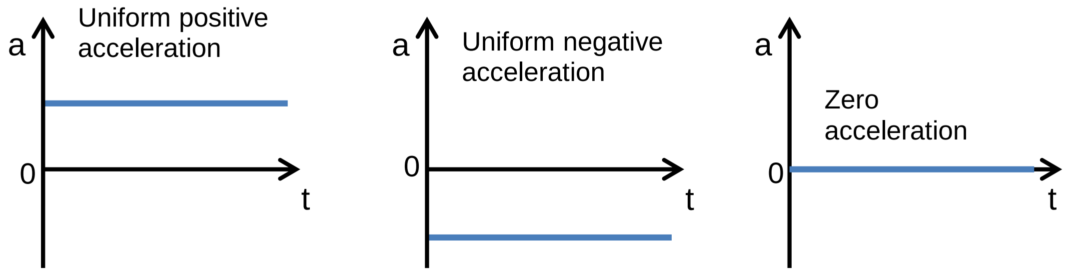
Go to https://www.youtube.com/watch?v=rD0tmgMdbQg to see a video on velocity-time graphs.

3.3. Graphs of acceleration vs time

In addition to using graphs of position vs time and velocity vs time to represent motion, we can also use graphs of acceleration vs time. An acceleration vs time graph has acceleration values on the y-axis, and time values on the x-axis.



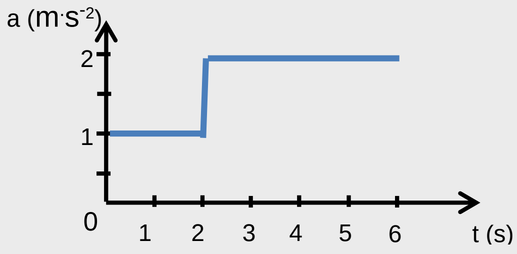
Since we will only look at motion that has uniform acceleration, the acceleration-time graph is always a horizontal line. The sketches of the three acceleration-time graphs for uniformly accelerated motion are shown below:



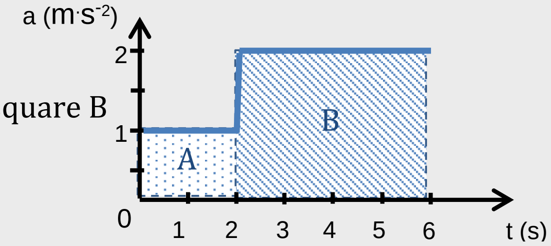
The area underneath the acceleration-time graph tells us the change in the object's velocity during that time interval (Δv).

Example: *(Try to solve this problem* ***on your own or with a fellow student*** *while covering the solution, and then check your work using the solution below).*

The acceleration-time graph given below describes the motion of an object. Calculate the change in the object's velocity.



Solution:



Δv = area underneath a-t graph

= area under square A + area under square B

= (base × height) + (base × height)

= $(2\ s \times 1\ m{\cdot}s^{-2}) + (4\ s \times 2\ m{\cdot}s^{-2})$

= $2\ m{\cdot}s^{-1} + 8\ m{\cdot}s^{-1}$ = $10\ m{\cdot}s^{-1}$

Activity 4 – Acceleration-time graphs

Answer the following questions:

An object started from rest and accelerated uniformly for 5 seconds, as shown in the graph below:

a. What was the object's velocity after 5 seconds?
b. Sketch the shapes of the velocity-time and position-time graphs for this object's motion.

The object now comes to rest after slowing down for 2 seconds,

c. Calculate the acceleration of the object in the 2 seconds.
d. Draw the acceleration-time graph and velocity-time for this part of the object's motion.

MAIN IDEAS:

- The gradient of the velocity-time graph gives the acceleration of the object.
- For an object with a **constant positive acceleration**, the acceleration-time graph will be a straight horizontal line above the x-axis.
- For an object with a **constant negative acceleration**, the acceleration-time graph will be a straight horizontal line beneath the x-axis.
- The area underneath the acceleration-time graph gives a value for the amount that the velocity has changed as a result of that acceleration.

Go to the website https://www.youtube.com/watch?v=JrFfi3t9FG4 to see a video on linking position graphs with velocity and acceleration graphs.

Unit 4. Equations of motion

Learning outcomes:

When you have completed this unit, you should be able to:

- use the kinematics equations of motion to solve problems for 1-dimensional motion.

Introduction

4.1. The equations of motion

We can represent motion that has constant acceleration using equations. These equations give us a way of linking displacement, velocity and acceleration. They are very useful, as they give us a way of solving problems for different situations involving motion. (These are sometimes called the "kinematics equations of motion").

The four equations of motion for objects that have constant acceleration are:

$v_f = v_i + a\Delta t$

$\Delta x = v_i\Delta t + ½a\Delta t^2$

$v_f^2 = v_i^2 + 2a\Delta x$

$\Delta x = ½(v_f + v_i)\Delta t$

In these equations, v_i is the initial velocity, in units of $m{\cdot}s^{-1}$

v_f is the final velocity, in $m{\cdot}s^{-1}$

a is the acceleration, in $m{\cdot}s^{-2}$

Δt is the time interval of the motion, in seconds (s)

Δx is the displacement of the object, in meters (m)

Below is an example of how to solve a problem using these equations of motion. (Here we have used the steps for problem solving given in Topic 1 of this workbook).

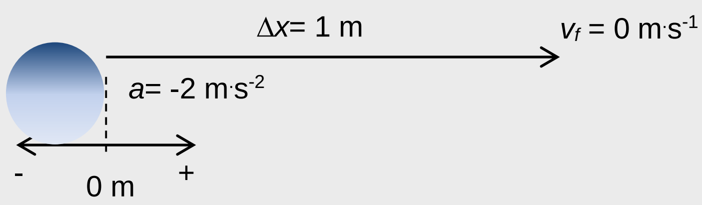
<u>Example</u>: *(Try to solve this problem* ***on your own or with a fellow student*** *while covering the solution, and then check your work using the solution below).*

A marble was rolled to the right along a rough surface, and came to rest after it rolled for 1 metre. The acceleration of the marble was -2 m·s^{-2}.

a. Calculate the marble's initial velocity.
b. Find the time that the marble took to come to a stop.

<u>Solution</u>

Step 1: Diagram of the scenario:



Step 2: <u>Given</u>: $a = -2$ m·s^{-2} and $\Delta x = 1$ m and $v_f = 0$ m·s^{-1}

Step 3: a. We are asked to find the marble's initial velocity, v_i.

b. We are asked to find the time for the marble to stop, Δt

Step 4: a. The equation that we will use is $v_f^2 = v_i^2 + 2a\Delta x$

b. The equation that we will use is $v_f = v_i + a\Delta t$

Step 5: Calculation:

a. From the equation $v_f^2 = v_i^2 + 2a\Delta x$

We can solve for v_i:

$v_i^2 = v_f^2 - 2a\Delta x = 0 - 2 \times (-2 \text{ m·s}^{-2}) \times 1 \text{ m} = 4 \text{ m·s}^{-1}$

b. From the equation $v_f = v_i + a\Delta t$

We can solve for Δt:

$\Delta t = (v_f - v_i) / a = (0 - 4 \text{ m·s}^{-1}) / (-2 \text{ m·s}^{-2}) = 2 \text{ s}$

Step 6: a. The marble had an initial velocity of +4 m·s^{-1} to the right.

b. The time for the marble to stop is 2 seconds.

Activity 1 – Solving Problems with Equations of Motion

Use the equations of motion to solve the following motion problems.

1. A car is travelling with a velocity of 40 $m{\cdot}s^{-1}$ and applies its brakes. It comes to a stop after breaking for 10 seconds. Calculate the acceleration of the car.
2. A skier starts from rest, and accelerates down a slope at 1,6 $m{\cdot}s^{-2}$ for 5 s.
    a. How far does she travel during the first 5 seconds?
    b. She then continues with a constant velocity for another 5 s. How far does she travel during this time?
    c. After this she skis up a slight slope and comes to a stop after traveling for 8 m. What is her acceleration as she travels up the slope?

MAIN IDEAS: The four equations of motion for objects that have constant acceleration are:

$$v_f = v_i + a\Delta t$$

$$\Delta x = v_i\Delta t + \frac{1}{2}a\Delta t^2$$

$$v_f^2 = v_i^2 + 2a\Delta x$$

$$\Delta x = \frac{1}{2}(v_f + v_i)\Delta t$$

Unit 5. Projectile motion

Learning outcomes:

When you have completed this unit, you should be able to:

- verify that the gravitational acceleration of objects in free-fall is constant using experimental data of ball-bearings of different masses in free-fall;
- explain that objects in free-fall (projectiles) accelerate towards the earth with a constant acceleration of 9,8 m·s^{-2};
- apply graphs and kinematics equations of motion to objects in free-fall, in familiar and novel contexts..

5.1. Objects in free-fall

Projectile motion describes the motion of objects that are falling freely under the influence of gravity. In the following exercise you will calculate the gravitational acceleration of a ball that is dropped in the air.

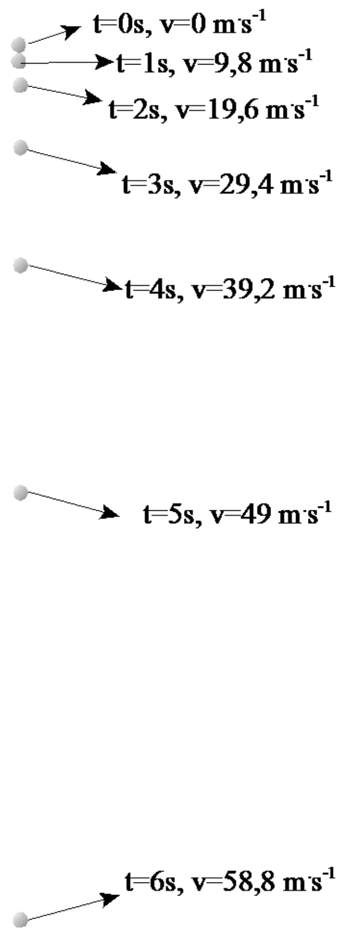
Activity 1 – Gravitational acceleration of a falling object

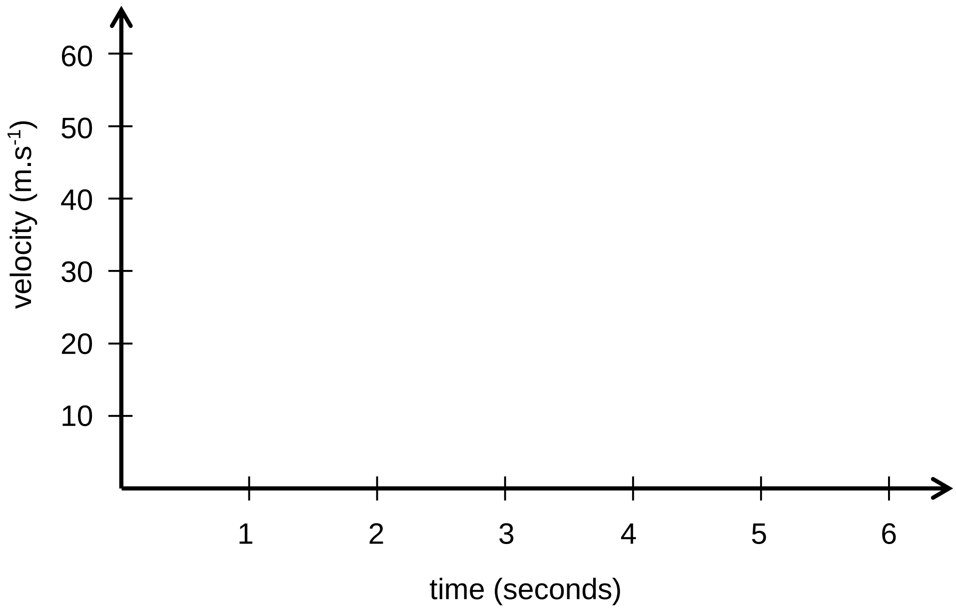
On the right you will see a picture of a ball that is falling through the air. The ball's velocity and time values are shown at various times.

<u>Frame of reference</u>: The downward direction is positive, and the starting position of the ball is zero.

Use this information to answer the following questions:

1. Below is a set of axes that will allow you to draw a graph of velocity on the y-axis against time on the x-axis for these values. On this set of axes, plot each of the points of velocity and time from the diagram on the right.



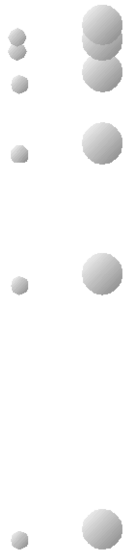


2. Draw a best-fit line through your points.
3. Describe the shape of your velocity vs time graph.
4. From the information given in the diagram on the right, calculate the acceleration of the ball between the following times:
   a. between 0s and 1s
   b. between 2s and 3s
   c. between 3s and 6s
5. Use your calculations to complete the following table:

Acceleration ($m{\cdot}s^{-2}$)	Time (s)
	1
	2
	3
	4
	5
	6

6. Construct a set of axes that will allow you to draw a graph of acceleration on the y-axis against time on the x-axis. Label your axes carefully. On your set of axes, plot each of the points of acceleration and time from your table. Draw a best-fit line through your points.
7. What do you notice about the shape of your acceleration vs time graph for an object that is falling through the air? (1)
8. The diagram below shows the positions of two balls at various times. Ball B

has three times the mass of Ball A. What can you conclude about the relationship between the acceleration of the balls and their masses? (2)



What you should have found in this activity is that the acceleration of the ball is 9,8 $m{\cdot}s^{-2}$, and that the acceleration does not depend on the mass of the ball. We call this the **acceleration due to gravity**, which has the symbol **g.** Its direction is always downwards (towards the earth).

MAIN IDEAS: For projectile motion (objects in free-fall) the acceleration due to gravity is g = 9,8 $m{\cdot}s^{-2}$ downward.

5.2. Strategies for solving different scenarios for projectile motion

There are three scenarios for projectile motion that we will explore by working through examples:

Case 1: An object is dropped from rest

<u>Example</u>:

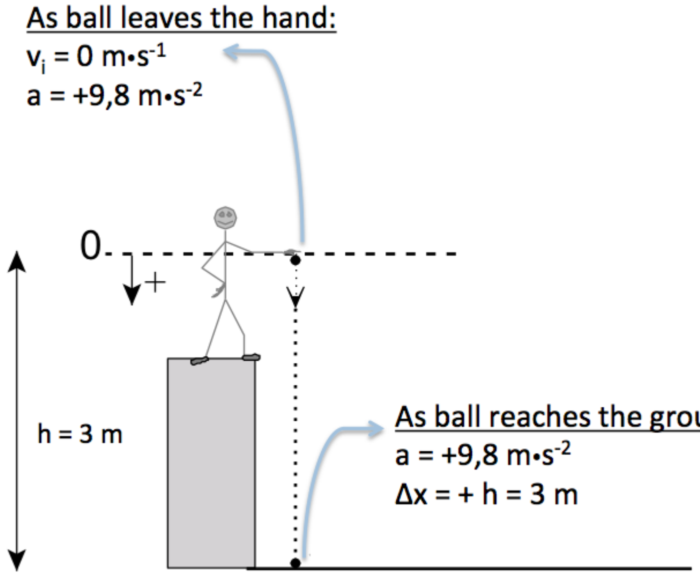
Ayanda was standing on a bridge, and dropped a stick into the river, which was 3 m below the height that he dropped the stick from.

a. Calculate the final velocity of the stick just before it hit the water.
b. Calculate the amount of time that it would take for the stick to hit the water.

Solution:

Step 1 and 2: Diagram of the scenario with given information:

Frame of reference: Choose down as +



Step 3: a. We are asked to find the stick's final velocity, v_f.

b. We are asked to find the time taken for the stick to hit the water, Δt.

Step 4: a. The equation that we will use is $v_f^2 = v_i^2 + 2a\Delta x$

b. The equation that we will use is $v_f = v_i + a\Delta t$

Step 5: Calculation:

a. From the equation $v_f^2 = v_i^2 + 2a\Delta x = 0 + 2 \times (9,8\ m \cdot s^{-2}) \times 3\ m = 58,8\ m^2 \cdot s^{-2}$

Therefore $v_f = \sqrt{58,8\ m^2 \cdot s^{-2}} = 7,67\ m \cdot s^{-1}$

b. From the equation $v_f = v_i + a\Delta t$

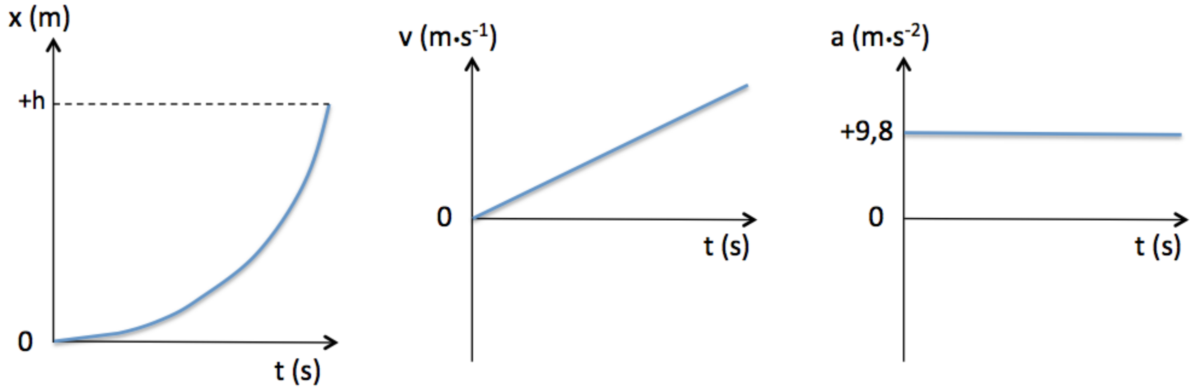
We can solve for Δt:

$\Delta t = (v_f - v_i) / a = (7,67\ m \cdot s^{-1} - 0) / (9.8\ m \cdot s^{-2}) = 0,78\ s$

Step 6: a. The stick had a final velocity of $+7,67\ m \cdot s^{-1}$ downwards.

b. The time for the stick to hit the water is 0,78 seconds.

The three graphs of motion for an object that is dropped from rest (with down as the + direction) are shown below:



Case 2: An object is thrown upward, and then falls back to the same height that it was thrown from

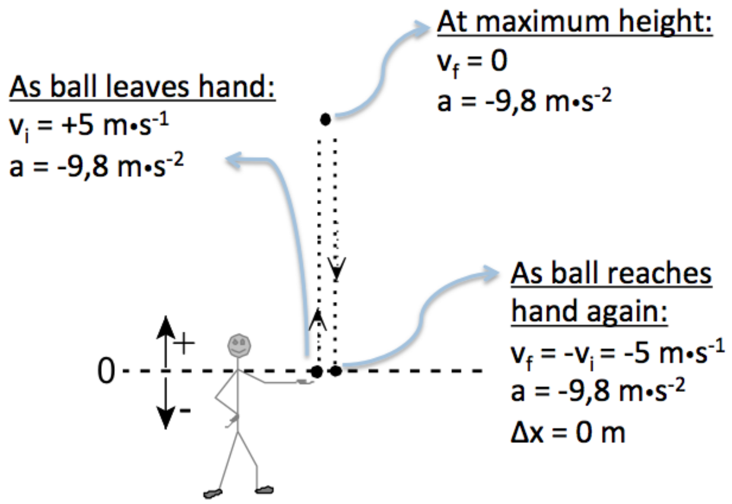
Kagiso threw a ball upward with an initial velocity of 5 $m \cdot s^{-1}$.

a. What was the maximum height reached by the ball?
b. After how much time did the ball fall back to the position that Kagiso threw it from?

Solution:

Step 1 and 2: Diagram of the scenario with given information:

Frame of reference: Choose up as +



Step 3: a. We are asked to find the maximum height, Δx, where $v_f = 0$.

b. We are asked to find the time for the full motion, Δt, where $v_f = - v_i$.

Step 4: a. The equation that we will use is $v_f^2 = v_i^2 + 2a\Delta x$

b. The equation that we will use is $v_f = v_i + a\Delta t$

Step 5: Calculation:

a. From the equation $v_f^2 = v_i^2 + 2a\Delta x$ we solve for Δx:

$\Delta x = (v_f^2 - v_i^2) / 2a = (0 - (5 \text{ m·s}^{-1})^2) / (2 \times (-9{,}8 \text{ m·s}^{-2})) = +1{,}28 \text{ m}$

b. From the equation $v_f = v_i + a\Delta t$ we can solve for Δt:

$\Delta t = (v_f - v_i) / a = (-5 \text{ m·s}^{-1} - (+5 \text{ m·s}^{-1})) / (9{,}8 \text{ m·s}^{-2}) = 1{,}02 \text{ s}$

OR we can find the time to the maximum height:

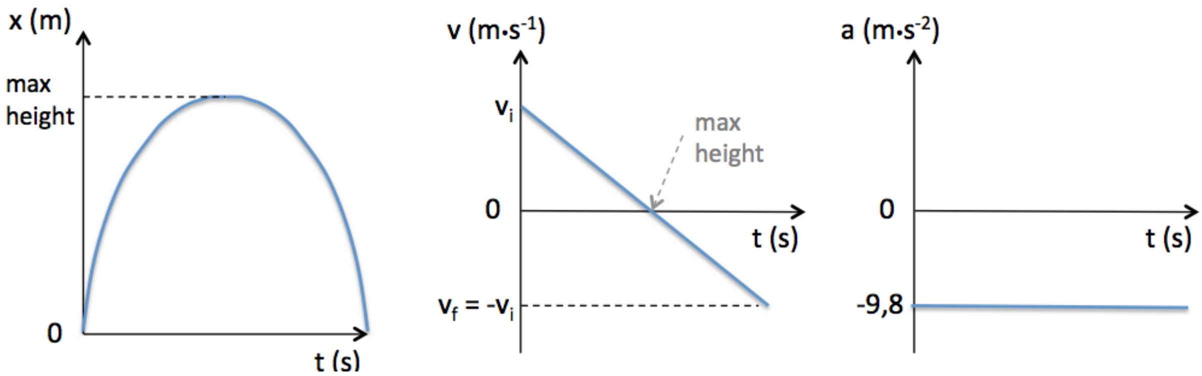
$\Delta t = (v_f - v_i) / a = (0 \text{ m·s}^{-1} - (+5 \text{ m·s}^{-1})) / (9{,}8 \text{ m·s}^{-2}) = 0{,}51 \text{ s}$

Therefore total time = 0,51s × 2 = 1,02 s

Step 6: a. The maximum height reached is 1,28 m above the starting position.

b. The total time of the motion is 1,02 seconds.

The three graphs of motion for an object that is thrown upward, and then falls back to the same height that it was thrown from (with up as the + direction) are shown below:



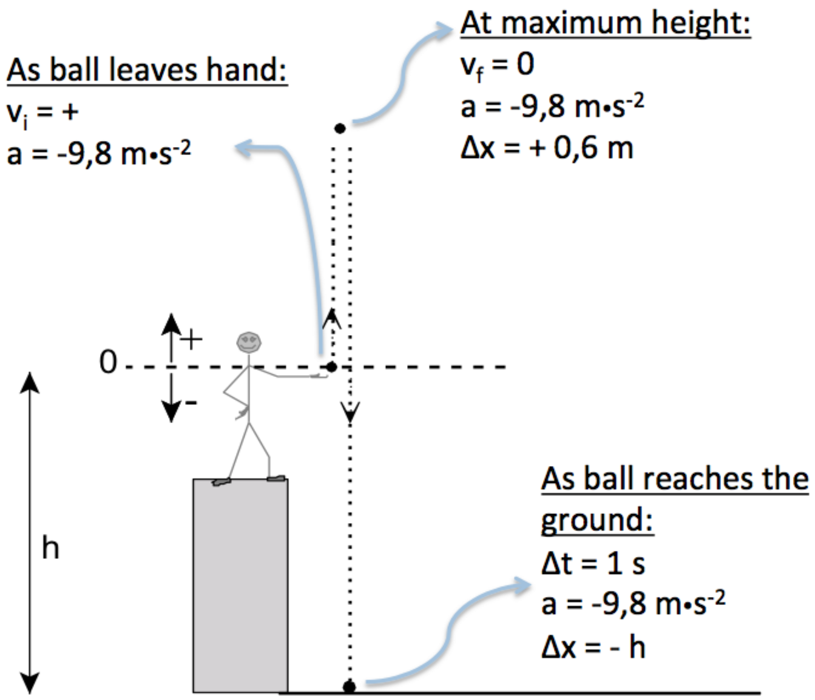
Example 3: An object is thrown up and then falls down to a position below its starting position

A flea jumps from a table to a height of 0,6 m above the table, and falls to the ground below the table 1 second after it jumped. What is the height of the table above the ground?

Solution:

Step 1 and 2: Diagram of the scenario with given information:

Frame of reference: Choose up as +



Step 3: We are asked to find the height of the table, which is the magnitude of the displacement for the flea's full motion, Δx. To be able to do this, we need to find the initial velocity, v_i.

Step 4: We will first use the equation $v_f^2 = v_i^2 + 2a\Delta x$ to find v_i

We will then use the equation $\Delta x = v_i\Delta t + ½a\Delta t^2$ to find Δx.

Step 5: Calculation:

From the equation $v_f^2 = v_i^2 + 2a\Delta x$ we solve for v_i:

$v_i^2 = v_f^2 - 2a\Delta x = 0^2 - (2 \times (-9,8\ m{\cdot}s^{-2}) \times 0,6\ m) = 11,76\ m^2{\cdot}s^{-2}$

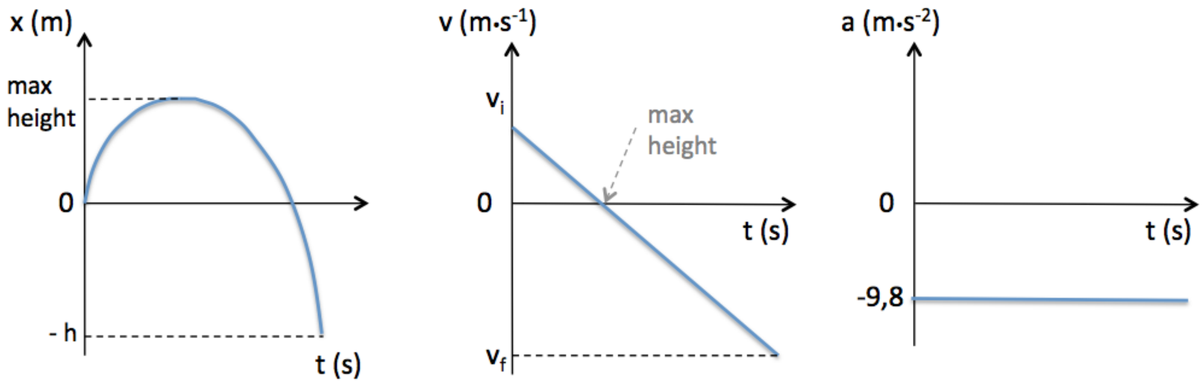
Therefore $v_i = \sqrt{11,76\ m^2 \cdot s^{-2}} = +3,43\ m{\cdot}s^{-1}$

Therefore $\Delta x = v_i\Delta t + ½a\Delta t^2 = 3,43\ m{\cdot}s^{-1} \times 1\ s + (½ \times (-9,8\ m{\cdot}s^{-2}) \times 1\ s) = -1,47\ m$

(This displacement is negative, since we chose up as the positive direction, and the ground is **below** the table.)

Step 6: The height of the table is 1,47 m above the ground.

The three graphs of motion for an object that is thrown up and then falls down to a position below its starting position (with up as the + direction) are shown below:



Assessment Activity: Motion in 1 dimension **Total marks = 100**

Assess your understanding of this topic by answering the following questions.

1. Complete the table below by giving one word for the quantity that is described, showing the units that each of the quantities is measured in, and showing whether each is a scalar or a vector quantity: (15)

Description	Name of quantity	Units	Scalar / Vector
The total length of the path of motion			
The change in position from starting point to ending point			
The rate at which distance is covered			
The rate of change of position			
The rate of change of velocity			

2. Njabulo walked with a constant velocity from a position of -4 m to a position of +5 m in 3,6 seconds.
   a. Calculate his average velocity. Show your working clearly. (2)

   Njabulo then ran with a constant velocity to a position of -6 m in 2 seconds.

   b. What was his velocity for this 2 seconds? (2)
   c. What was his average velocity for the total movement described in a and b? (3)

3. A cyclist, starting from rest, accelerates down a slope at 2 $m{\cdot}s^{-2}$.
   a. Explain in your own words what 2 $m{\cdot}s^{-2}$ means. (3)
   b. What is the cyclist's velocity at the end of
      (i) 1 second? (ii) 2 seconds? (iii) 5 seconds? (6)
   c. How far has he travelled at the end of 5 s? (4)

d. Sketch the shapes of the graphs of the cyclist's position, velocity and acceleration against time. (You don't need to include values on your graphs). (9)

4. The velocity of a train is 28 $m{\cdot}s^{-1}$. It applies its brakes, and has an average acceleration of -1,50 $m{\cdot}s^{-2}$.

a. How much time is needed for the train to reach a velocity to 10 $m{\cdot}s^{-1}$? (4)

b. Sketch the shapes of the graphs of the train's position, velocity and acceleration against time. (You don't need to include values on your graphs). (9)

5. A stone is dropped from a bridge that is 8 m above a river. What is the stone's velocity just before it hits the river? (4)

6. A ball is thrown straight upward and reaches a maximum height of 6 m.

a. Calculate the time that the ball takes to return to the height that it was thrown from. (5)

b. Sketch the shapes of the graph of the ball's velocity against time. (3)

c. Sketch the shapes of the graph of the ball's acceleration against time. (3)

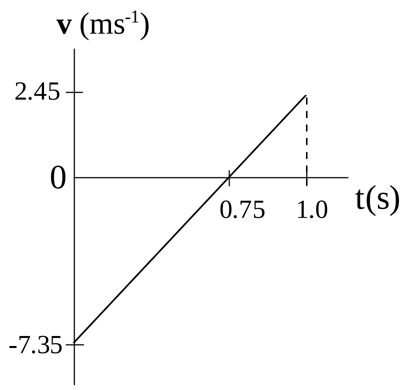
7. If a ball is thrown vertically upward, and it takes 6 s for it to return to its point of release, calculate its initial velocity. (5)

8. A ball is thrown straight upward with an initial speed of 8 $m{\cdot}s^{-1}$ from the roof of a building that is 9 m above the ground. The ball falls to the ground below the roof. Let the upward direction be positive.

a. Calculate the final velocity of the ball as it hits the ground. (5)

b. Sketch the shape of the graph of the ball's velocity against time. (3)

9. The graph below represents the motion of an object in the air.



a. Use this graph to find the object's total displacement. (4)

b. What is the object's velocity at 0,5 s? (3)

c. Sketch a graph of the object's acceleration against time. (3)

d. Describe the motion of this object in words. Include a description of the frame of reference. (5)

My Notes:

Use this space to write your own questions, comments or key points.

Summary of key learning:

- **Position** describes where an object is relative to a frame of reference. Position is measured in units of meters (m).
- **Distance** (D) is the total length of the path of movement, measured in units of meters (m). It is a **scalar** quantity.
- **Displacement** (Δx) is the change in position, measured in units of meters (m). $\Delta x = x_f - x_i$. Displacement is a **vector** quantity.
- **Speed** is the total distance divided by the time: speed $= \frac{D}{\Delta t}$
- Speed is a scalar and is measured in meters per second ($m{\cdot}s^{-1}$).
- **Average velocity** is the displacement divided by the time: $v = \frac{\Delta x}{\Delta t}$
- **Instantaneous velocity** is the displacement divided by an infinitesimal time interval: $v = \frac{dx}{dt}$
- Velocity is a vector and is measured in meters per second ($m{\cdot}s^{-1}$).
- **Acceleration** is the change in velocity divided by the total time, and is a vector quantity: $a = \frac{\Delta v}{\Delta t}$
- Acceleration is measured in units of $m{\cdot}s^{-2}$.
- On a position-time graph, the **value** gives the object's position at a certain time .
- The **gradient** (slope) of the graph tells us the **velocity**:

  v = gradient of x-t graph = $\frac{\Delta x}{\Delta t}$
- For **uniform motion**, the velocity-time graph is a straight, horizontal line.
- When there is **no motion (zero velocity)**, the velocity-time graph is on the time-axis.
- On a velocity-time graph, the **value** gives the object's velocity at a certain time.
- The **area underneath** the velocity-time graph gives a value for the **displacement**.
- The **gradient** of the velocity-time graph gives a value for the **acceleration**: a = gradient of v-t graph = $\frac{\Delta v}{\Delta t}$

- For an object with a **constant positive acceleration**, the acceleration-time graph will be a straight horizontal line above the x-axis.
- For an object with a **constant negative acceleration**, the acceleration-time graph will be a straight horizontal line beneath the x-axis.
- The area underneath the acceleration-time graph gives a value for the amount that the velocity has changed as a result of that acceleration.
- The four **equations of motion** for objects that have constant acceleration are:

  $v_f = v_i + a\Delta t$

  $\Delta x = v_i\Delta t + \frac{1}{2}a\Delta t^2$

  $v_f^2 = v_i^2 + 2a\Delta x$

  $\Delta x = \frac{1}{2}(v_f + v_i)\Delta t$

- For **projectile motion** (objects in free-fall) the acceleration due to gravity is $g = 9{,}8\ m{\cdot}s^{-2}$ downward.

Sub-topic 3. Force and Newton's laws

Content:

Unit 1: Forces

Unit 2: Newton's Laws of Motion

Unit 1. Forces

Learning outcomes:

When you have completed this unit, you should be able to:

- differentiate between mass and weight;
- calculate the weight of an object on earth;
- identify all of the forces acting on an object, including weight, normal force, applied force, frictional force and tension force;
- draw free body diagram(s) to represent the forces acting on an object.

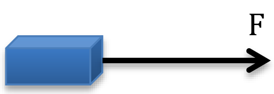
Introduction

In this unit you will be introduced to the concept of forces, and you will learn about some of the different types of forces.

1.1. What is a force?

A force is a push or a pull exerted upon an object. A force can also be a twist exerted on an object. Force is measured in units called newtons (N). Since all forces have a clear magnitude and direction, forces are vector quantities. We can therefore represent a force that acts on an object using an arrow.

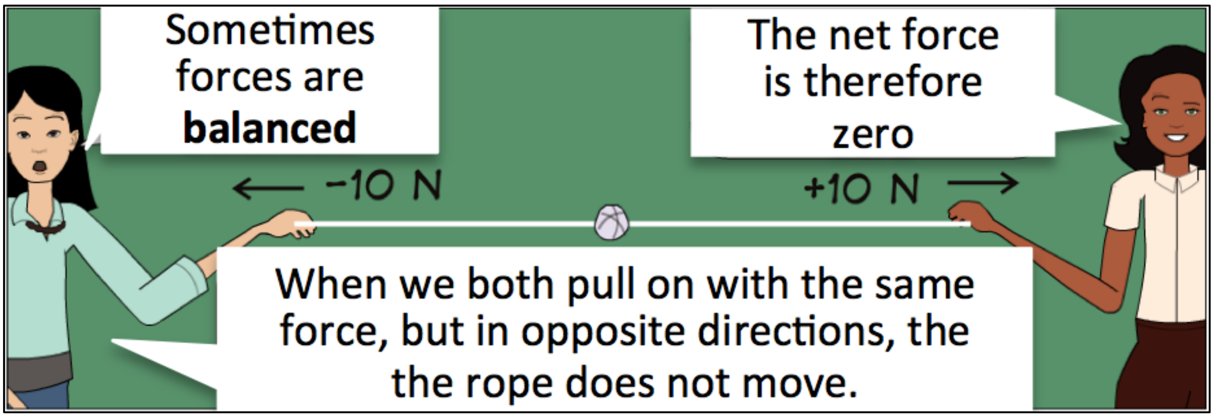
> **exerted** – applied to, or acting on an object



Some of the forces that we will look at in this unit are the net force, the gravitational force, the normal force, resistance forces (such as friction and air resistance), and tension forces. These are explained below.

1.2. The net force

The resultant or net force on an object (F_{net}) is the combined effect of all of the forces acting on that object. If forces are **balanced** there is no net force, and so the forces will not have an effect on the motion of the object. To have an effect, the forces must be **unbalanced**, so that there is a **net force** acting on an object.

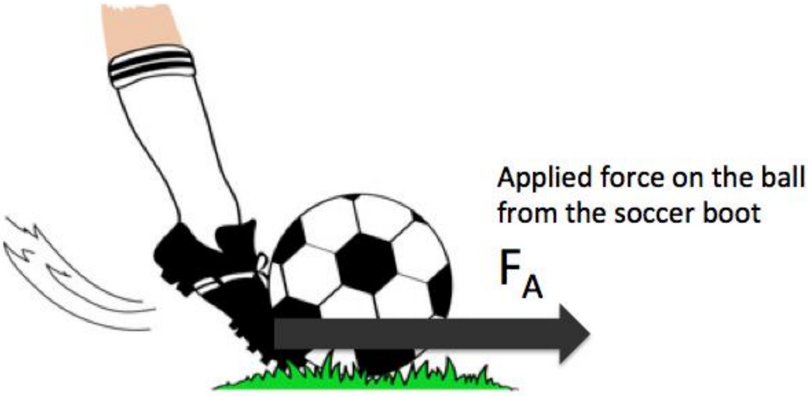


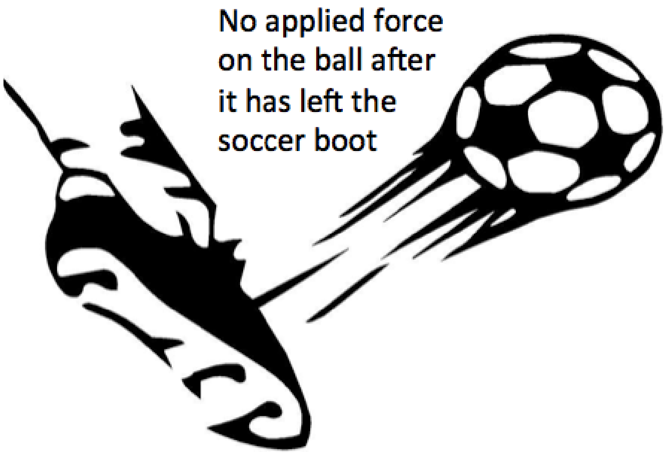
> **MAIN IDEAS:**
> - The ***net force,*** F_{net}, is the sum of all of the forces acting on an object. It is also called the resultant force.
> - If forces are **balanced** there is no net force: $F_{net} = 0$

1.3. Applied force

When an external force is exerted on an object, for example a push by a person's hand or the push from a car's engine, this is called the **applied force** (**F_A**). To correctly identify an applied force, you need to be able to identify the person or object that is exerting the force. For example, when a netball is being thrown upward, the applied force is exerted by the hand on the ball.

We only include the applied force in our calculations **while it is acting** on the object. For example, if a person kicks a soccer ball along the floor, the applied force on the ball only has an effect on the ball while the ball is in contact with the foot. After the ball has left the foot, there is no applied force on the ball. The only force on the ball is the gravitational force that attracts it toward the earth (if we ignore the effects of air resistance).





MAIN IDEAS:

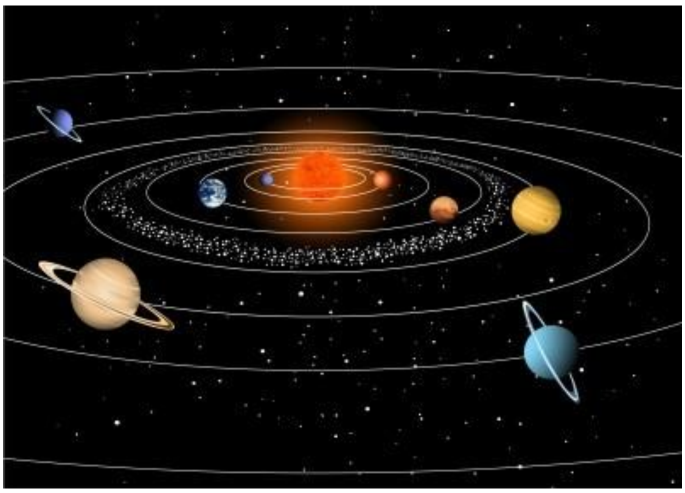
- The ***applied force,*** F_A, is an external force that is exerted on an object (push or pull).
- We only include the applied force in our calculations **while it is acting** on the object.

1.4. Gravitational force

The gravitational force is the force of attraction (pull) that objects/bodies have on one another due to their masses.

For example:

- the attraction of the Sun and planets,
- the attraction between the Earth and the Moon,
- the attraction between the Earth and objects on the surface of the Earth.



Objects with greater mass have more gravitational pull on each other. The gravitational force decreases as the distance between the objects increases.

We use the symbol F_g for the gravitational force, and it is measured in newtons (N).

Activity 1 – Observing the gravitational force

<u>You will need</u>:

A piece of scrap paper, rolled up into a ball

Throw your ball of paper up in the air and observe what happens. Answer the following questions:

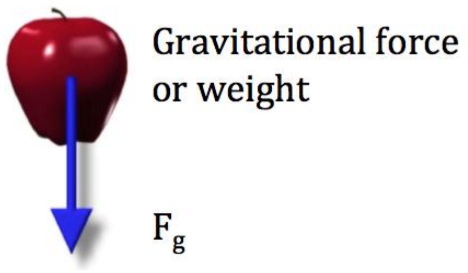
1. Describe the ball's motion.
2. Can you explain what is happening to the ball in terms of the forces acting on it? (Ignore he air resistance force).
3. Why does the ball not continue moving upward forever?

Now hold the ball and let it drop towards the Earth.

1. Describe the ball's motion.
2. Can you explain what is happening to the ball in terms of the forces acting on it?

Weight and mass

The weight of an object is the gravitational force exerted on it by the Earth (or the Moon, or another planet). Since weight is a force, it is also measured in newtons (N).



The **mass** of the object stays the same no matter where it is measured. However, the **weight** of an object will change when it is weighed in different places with different gravitational force. For example, an object on the moon has a lower weight than on Earth.

To calculate an object's weight on Earth we use the equation:

weight = mass in kg $\times$ 9,8 $m{\cdot}s^{-2}$

We can write this as a mathematical formula:

$F_g = m\,g$

<u>Note about units</u>: If you work out the units in the formula above, you will see that 1 N is equivalent to 1 $kg{\cdot}m{\cdot}s^{-2}$

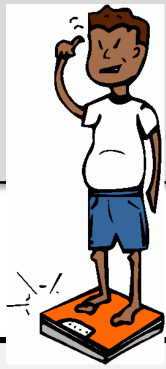
Example: *(Try to solve this problem **on your own or with a fellow student** first while covering the solution, and then check your work using the solution below).*

Nkosi has a mass of 60 kg. What is his weight on Earth?

Solution:

$F_g = m\,g = 60\text{ kg} \times 9{,}8\text{ m·s}^{-2} = 588\text{ N}$

Nkosi's weight is 588 N downward.



Think about this:

Does a bathroom scale measure mass or weight?

Activity 2 – Calculate mass and weight

Complete the following table by filling in the missing information:

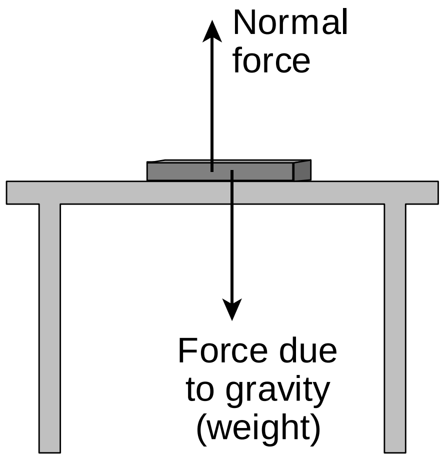
Mass (g)	**Mass (kg)**	**Weight (N)**
500 g	500 g ÷ 1000 = 0,5 kg	0,5 × 9,8 = 4,9 N
100 g		
	5 kg	
		98 N
	20 kg	
0,5 g		
	0,03 kg	
	3 500 kg	
2 500 g		
		882 N

MAIN IDEAS:

- The ***gravitational force,*** F_g, is the downward force exerted by the Earth on an object. It is also called the **weight** of the object.
- We calculate weight using the equation: $F_g = m\,g$

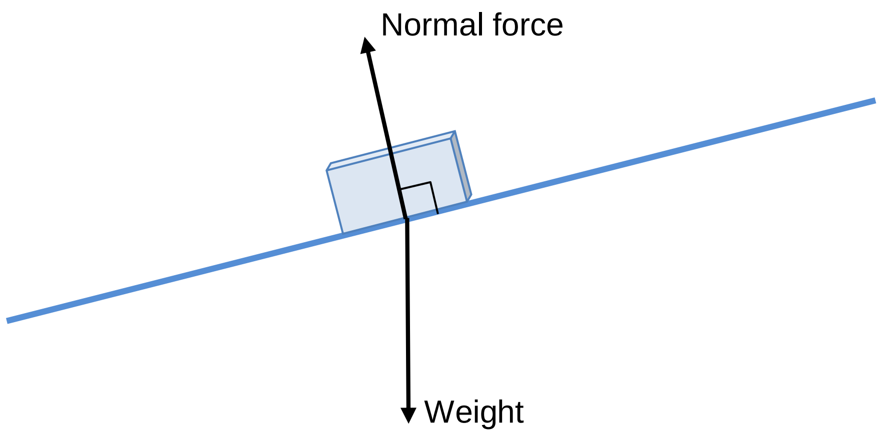
1.5. The normal force

When a book is on a table, the gravitational force acts downward on the book. But the book is not moving. This means that the downward force on the book must be balanced by an upward force, to keep the two forces balanced. Therefore the upward force must be equal in magnitude, but opposite in direction, to the weight. This upward force is exerted on the book by the table surface, and is called the **normal force, F_N**. (This force is called the normal force since it acts perpendicularly, or normally, to the surface that the object is in contact with). As a result of the downward weight being balanced by the upward normal force, there is no net force on the book.



When an object is on a surface that is not horizontal, the direction of the normal force on this object is again perpendicular to the surface. In this case you can see that the normal force is not equal and opposite to the weight.

perpendicular – at right angles

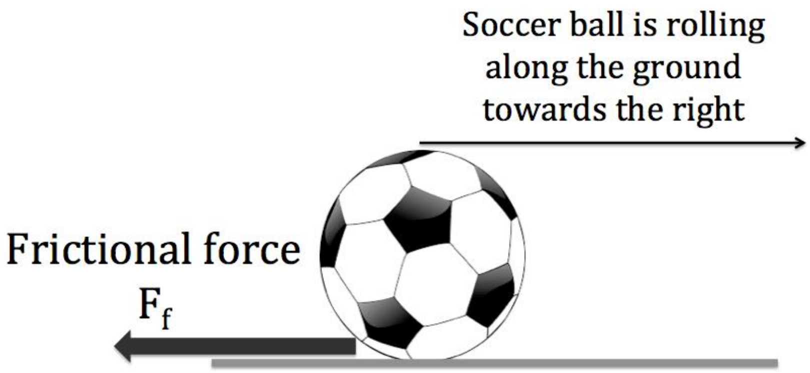


MAIN IDEAS:

- The ***normal force,*** F_N, is the force exerted by a surface on an object that is in contact with it.
- The normal force is always ***perpendicular*** to the surface.

1.6. Resistance forces

When an object moves in a certain direction, there are forces that oppose the motion of the object, in other words, they act in the opposite direction to the motion. For example, when you kick a ball it does not move long the ground forever, because there is a force of **friction** that acts in the opposite direction to its movement. This force is caused by the interaction of the two surfaces. This frictional force is parallel to the surface, and opposes the motion of the object.



Another example of a resistance force is **air resistance.** When an object is moving through the air, its motion is opposed by the air resistance force (F_a).

In the following activity you will investigate this force.

Activity 3 – The effect of air resistance on a falling object

<u>You will need</u>:
A feather
A stone

1. Hold the feather and the stone at the same height above the Earth.
2. Drop the feather and the stone at the same time. Which one hits the ground first?
3. Can you explain your observations? (Hint: identify all of the forces acting on each object)
4. If the feather and the stone were dropped at the same time on the moon, where there is no air, what would you observe?

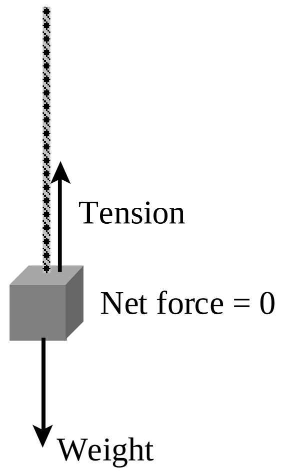
Go to https://www.youtube.com/watch?v= XJcZ-KoL9o to see … a video that shows a ball bearing and a feather falling in a vacuum.

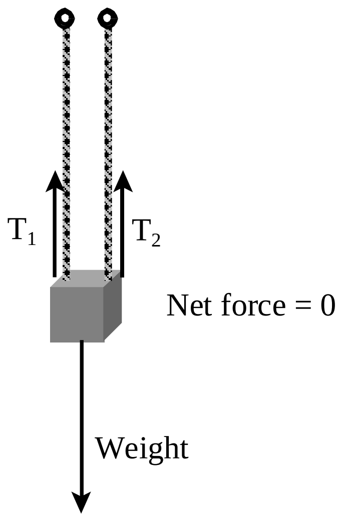
MAIN IDEAS:

- The ***frictional force***, F_f, is the force that opposes the motion of an object that is moving on a surface. The frictional force acts parallel to the surface that the object is in contact with.
- The ***air resistance force***, F_a, is the force that opposes the motion of an object that is moving through the air.

1.7. Tension forces

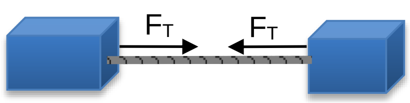
When an object that is hanging from a rope, and is not moving, the object's weight causes a downward force on the object, pulling it towards the earth. But since the object is not moving, there must be an upward force on it from the rope. This force is equal and opposite to the weight. We call this force the ***tension*** (due to the rope).





We can have more than two forces that are in equilibrium. The diagram on the left shows an object that is hung from two ropes. If the ropes have tension T_1 and T_2, then, since the net force must equal zero, the weight must be equal to $T_1 + T_2$.

When a rope is attached to two objects at each end of the rope, the tension pulls equally on the objects on the opposite ends of the rope.

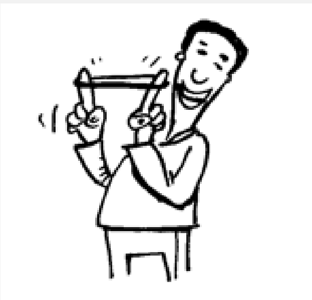


MAIN IDEAS:

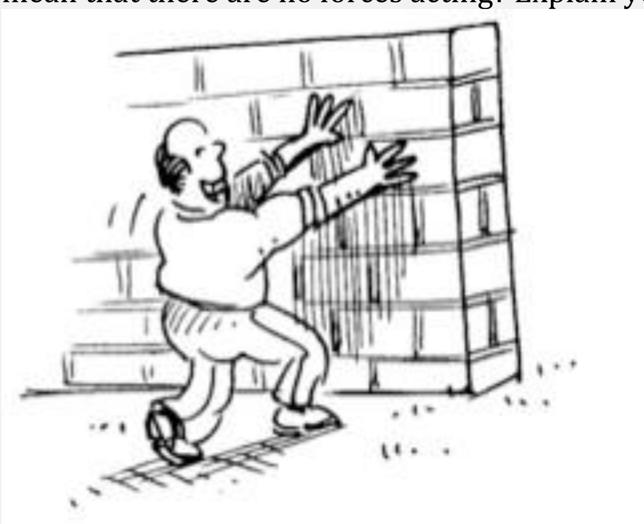
- The ***tension force,*** F_T, is the force exerted on an object by a rope or string that is attached to the object.
- The tension force pulls equally on the objects on the opposite ends of the rope.

Activity 4 – Identifying forces

1) Try to identify and name all of the forces in the pictures.

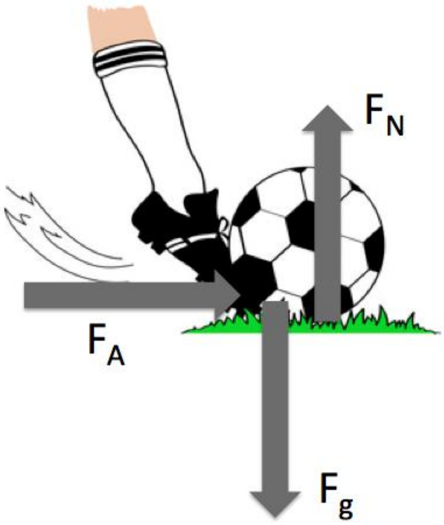


2) Can you see the effect of the force that the man is exerting on the wall? Does this mean that there are no forces acting? Explain your answer.

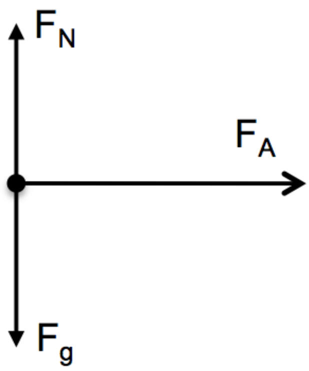


1.8. Force diagrams and free body diagrams

We can represent the forces acting on an object using a ***force diagram***, which is a picture of the object with all of the forces acting on it drawn in as arrows. We can also represent these forces in a ***free body diagram***. Here we represent the object with a dot, and all the forces acting on the object are shown as arrows pointing outward from this dot. The diagrams below show the difference between a force diagram and a free body diagram.



Force diagram



Free body diagram

Activity 5 – Free body diagrams

Answer the following questions:

Bernard threw an apple up in the air, and watched it move up to its maximum height and fall back down again. Draw a free body diagram of the forces on the apple for the following times. (Include air resistance forces in your diagrams):

a. While the apple is still in contact with the hand, which is pushing up on the apple
b. When the apple is rising in the air
c. When the apple is at its maximum height
d. When the apple is falling back down to the ground.

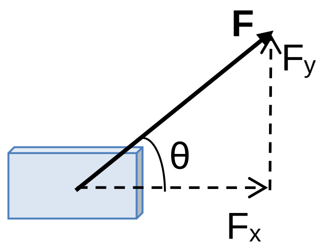
MAIN IDEAS:

- A ***force diagram*** shows a picture of an object with all of the forces acting on it drawn in as arrows.
- A ***free body diagram*** shows the object as a dot, and all the forces acting on the object are shown as arrows pointing outward from this dot.

1.9. Forces in 2 dimensions

Finding the components of a force

Not all forces act parallel or perpendicular to a surface. Some forces are at an angle to a surface. When this is the case, it is useful to find the **components** of the force. The diagram below shows the x- and y-components of a force F that makes an angle θ with the x-axis.



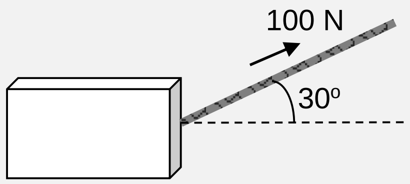
$F_x = F \cos \theta$

$F_y = F \sin \theta$

Here F_x is the component of the force in the x direction (parallel to the surface), and F_y is the component of the force in the y direction (perpendicular to the surface). Once we have found the components of a force, we can replace the force itself with these components to help us in solving problems.

Activity 6: Find the components of a force

Dave is pulling with a rope on a box with a force of 100 N at an angle of 30^o with the x-axis. Calculate the x- and y-components of the force that Dave exerts on the box.



MAIN IDEAS: The x- and y-components of a force F that makes an angle θ with the x axis are:

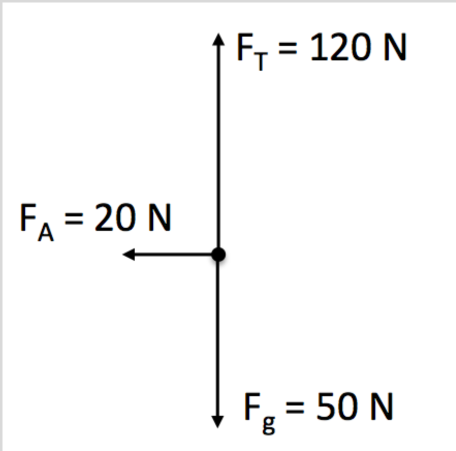
- $F_x = F \cos \theta$
- $F_y = F \sin \theta$

Finding the net force in 2 dimensions

When we are dealing with forces that are in two dimensions, then we need to find the net force in the x direction, and the net force in the y direction. The net force in the x direction $F_{net\,x}$ is a vector sum of all the components in the x direction. The net force in the y direction $F_{net\,y}$ is a vector sum of all the components in the y direction.

Example: *(Try to solve this problem **on your own or with a fellow student** first while covering the solution, and then check your work using the solution below).*

A 50 N weight that is attached to a rope is being pulled upward with a force of 120 N. An applied force is pushing on the weight with a force of 20 N to the left. What is the net force on the weight?



Solution:

The free body diagram of the forces is shown on the right.

We use the Cartesian plane as our frame of reference.

Forces in the x direction:

There is only one force in the x direction, so

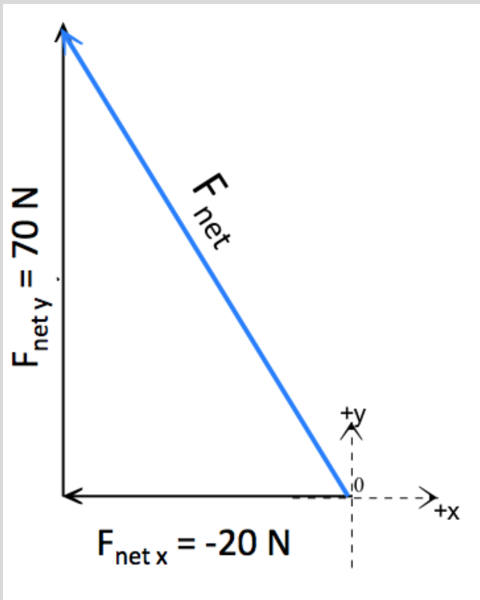
$F_{net\,x} = F_A = -20$ N (This is negative since the –x direction is left).

Forces in the y direction are:

Tension force $F_T = +120$ N

and weight $F_g = -50$ N

Therefore $F_{net\,y} = +120\text{ N} - 50\text{ N} = +70$ N



We use Pythgoras' theorem to find the magnitude of the net force.

$$F_{net} = \sqrt{(20\text{ N})^2 + (70\text{ N})^2} = 72{,}8\text{ N}$$

Using trigonometry, $\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{70\text{ N}}{20\text{ N}}$

Therefore $\theta = 74{,}1^\circ$

The net force on the weight is therefore 72,8 N at an angle of 74,1° above the -x axis.

Assessment Activity: *Types of force*

Answer the following questions to assess your understanding of forces:

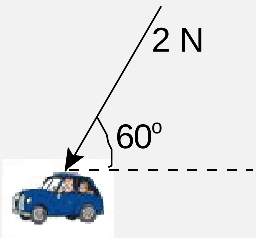
1. Describe what each of the following terms means in your own words:
    a. Gravitational force
    b. Tension force
    c. Applied force
    d. Frictional force
    e. Normal force

2. Thandi has a mass of $5{,}5 \times 10^4$ g.
    a. Calculate her mass in kg.
    b. Calculate her weight on Earth.
    c. If a 2 kg mass has a weight of 3,26 N on the moon, what is Thandi's weight on the moon?

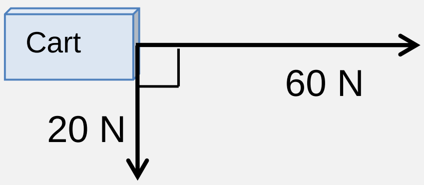
3. Two children are pulling on a box that has a weight of 20 N. The box is sitting on a smooth floor. Thabo is pulling on the box with a force of 12 N to the left. Roy is pulling on the box with a force of 8 N to the right. (Ignore friction)
    a. Draw a free body diagram that shows all of the forces on the box.
    b. What is the normal force on the box?
    c. What is the net force on the box?

4. A child is hanging from two ropes. Her weight is 250 N, and all of the forces are in equilibrium.
    a. If the tension is equal in the two ropes, what is the tension in each one?
    b. The child shifts her weight so that the tension in the rope on the left increases by 50 N. What is the tension in the rope on the right?

5. Jonas pushes his toy car with a force of 2 N at an angle of 60^o to the x-axis, as the diagram shows.
    a. Draw a free body diagram that shows all of the forces on the car.
    b. Calculate the x- and y-components of the force that Jonas exerted on the car.



6. Two donkeys are hitched to a cart and have been pulling in the same direction. They get a fright and one donkey suddenly pulls to the right, as the diagram shows. What is the net force on the cart?



My Notes:

Use this space to write your own questions, comments or key points.

Summary of key learning:

- A force is a push or a pull exerted upon an object. Force is measured in units called newtons (N).
- The ***net force,*** F_{net}, is the sum of all of the forces acting on an object. It is also called the resultant force. If forces are **balanced** there is no net force: $F_{net} = 0$
- The ***applied force,*** F_A, is an external force that is exerted on an object. We only include the applied force in our calculations **while it is acting** on the object.
- The ***gravitational force,*** F_g, is the downward force exerted by the Earth on an object. It is also called the **weight** of the object. We calculate weight using the equation: $F_g = m\,g$
- The ***normal force,*** F_N, is the force exerted by a surface on an object that is in contact with it. The normal force is always ***perpendicular*** to the surface.
- The ***frictional force***, F_f, is the force that opposes the motion of an object that is moving on a surface. The frictional force acts parallel to the surface that the object is in contact with.
- The ***air resistance force***, F_a, is the force that opposes the motion of an object that is moving through the air.
- The ***tension force,*** F_T, is the force exerted on an object by a rope or string that is attached to the object. The tension force pulls equally on the objects on the opposite ends of the rope.
- A ***force diagram*** shows a picture of an object with all of the forces acting on it drawn in as arrows.
- A ***free body diagram*** shows the object as a dot, and all the forces acting on the object are shown as arrows pointing outward from this dot.
- The x- and y-components of a force F that makes an angle θ with the x axis are: $F_x = F\cos\theta$ and $F_y = F\sin\theta$

Unit 2. Newton's Laws of Motion

Learning outcomes:

When you have completed this unit, you should be able to:

- state Newton's first, second and third laws of motion;
- apply Newton's laws of motion to various scenarios involving forces in equilibrium and non-equilibrium (include multiple coupled objects, but **exclude** object on an inclined plane), in familiar and novel contexts.

Introduction

Isaac Newton was a scientist who lived more than 350 years ago. When he was only 23 years old he developed three laws of motion which were so simple but important that they formed the basis of scientific thinking for the next 300 years, and are still used in many scientific areas today.

2.1. Newton's First Law

Newton's first law of motion states:

An object continues in a state of rest or uniform (moving with constant) velocity unless it is acted upon by an unbalanced (net or resultant) force.

In other words:

- If ***no net force*** acts on an object, the object will either remain **at rest**, or if it is moving, it will continue moving with a **constant velocity** in a straight line.
- If there ***is a net force*** acting on an object, it will cause a change in that object's velocity.
- If an object is ***at rest***, or travelling in a straight line with ***constant velocity***, then there is no net force acting on the object.

Discussion point: Discuss these questions with a fellow student, or reflect on them yourself

- When you throw a stone in the air, why does it not continue moving in a straight line forever?
- If you roll a ball along a horizontal surface, why does it not continue moving in a straight line forever?

What Newton's first law implies is that if there is a net force acting on an object, this will cause a change in that object's velocity, in other words it will cause the object to accelerate. This relationship between the force and the acceleration is described in Newton's second law.

> **MAIN IDEAS:** Newton's first law states: "*An object continues in a state of rest or uniform velocity unless it is acted upon by an external unbalanced force.*"

2.2. Newton's second law

Newton's second law of motion states:

When a net force F_{net}, is applied to an object of mass m, the object accelerates in the direction of the net force. The acceleration a is directly proportional to the net force and inversely proportional to the mass.

We can write Newton's second law as a mathematical equation:

$$F_{net} = m\,a$$

where F_{net} is the net force on an object, measured in units of newton (N)
m is the mass, measured in kg
a is the acceleration of the object, measured in m/s^2

Force is always measured in units of newton (N). 1 N of force is the amount of force that will cause an object with a mass of 1 kg to accelerate at 1 m/s^2. So 1 N = 1 $kg{\cdot}m/s^2$.

Because acceleration is a vector quantity, force is also a vector quantity. The force always has the ***same direction*** as the acceleration.

The following points should be kept in mind when we apply Newton's second law:

- When we apply this equation, we must apply it to ***one object*** at a time. Then we need to identify all of the forces acting on that object.
- It is helpful to draw ***free body diagrams*** showing all of the forces acting on the object.
- If all of the forces on an object are in ***equilibrium***, they balance each other out and the net force is zero. This means that the object will remain at rest, or will travel with a constant velocity.
- If the forces on the object do not balance each other out (***non-equilibrium***), there is a net force on the object. This means that the object will have a non-zero acceleration.
- The components of the forces in the ***vertical (y) direction*** should be looked at

separately to find $F_{net\ y}$. This can then be used to find the acceleration in the y direction: $F_{net\ y} = m \cdot a_y$

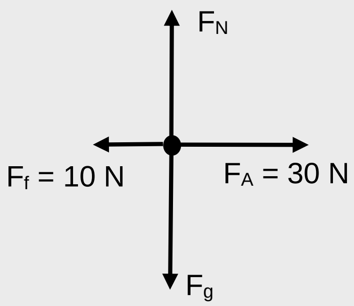
- Similarly, the forces in the ***horizontal (x) direction*** should be looked at separately to find $F_{net\ x}$. This can then be used to find the acceleration in the x direction: $F_{net\ x} = m \cdot a_x$

<u>Example 1</u>: *(Try to solve this problem **on your own or with a fellow student** first while covering the solution, and then check your work using the solution below).*

A box with a mass of 5 kg is being pushed along the ground in the +x direction with a force of 30 N. There is a frictional force of 10 N between the box and the ground. Calculate the acceleration of the box.

<u>Solution:</u>

We first draw a free body diagram of all of the forces on the box.



The forces perpendicular to the surface are in equilibrium, so $F_{net\ y} = 0$.

The net force parallel to the surface is $F_{net\ x}$ = 30 N – 10 N = +20 N.

So $a_x = \frac{F_{net\ x}}{m} = \frac{20\ N}{5\ kg} = 4\ m \cdot s^{-2}$

The acceleration is $4\ m \cdot s^{-2}$ in the +x direction.

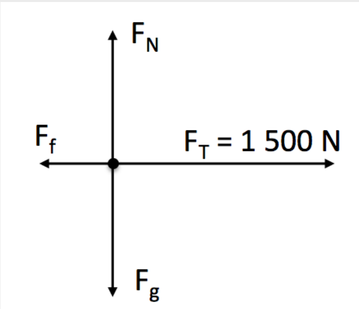
Example 2: *(Try to solve this problem **on your own or with a fellow student** while covering the solution, and then check your work using the solution below).*

A car with a mass of 800 kg was being towed using a tow rope. The tension in the tow rope was 1 500 N, and the acceleration of the car was 1,25 m/s^2. There is a fricitonal force between the car and the road surface. (Ignore the mass of the tow-rope).

a. Draw a free body diagram of all of the forces on the car.
b. Calculate the frictional force beteen the car and the road.

Solution:

a. The free body diagram shows the the forces on the car:



b. We first need to find the net force:
F_{net} = m a = 800 kg × 1,25 m/s^2 = 1 000 N
The forces in the y direction are in equilibrium, so we don't need to use these to find the frictional force.
In the x direction: $F_{net\,x} = F_T + F_f$
Therefore $F_f = F_{net\,x} - F_T$ = 1 000 N – 1 500 N = - 500 N
Therefore frictional force is 500 N in the opposite direction to the tension force.

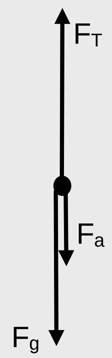
Example 3: *(Try to solve this problem* ***on your own or with a fellow student*** *while covering the solution, and then check your work using the solution below).*

A car with a mass of 1 000 kg is being lifted vertically upward at a constant speed of 5 m·s⁻¹ by a strong cable. The tension in the cable that is lifting the car is 10 000 N. There is an air resistance force acting on the car, and there are no horizontal forces acting on the car.

a. Draw a free body diagram of the forces acting on the car.
b. Calculate the magnitude of the air resistance force.

Solution:

a. The picture on the right shows the free body diagram of the forces acting on the car. (The direction of the air resistance force is down, since it opposes the upward motion of the car.)

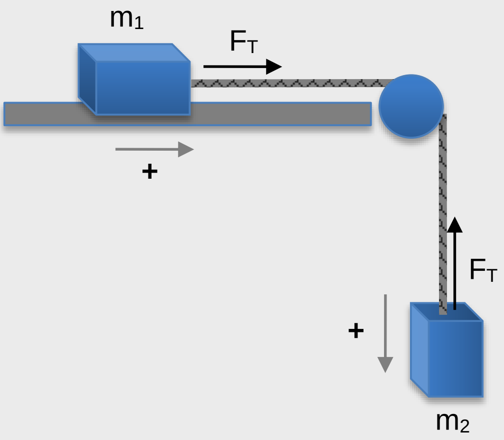


b. $F_{net\ x}$ = 0 N since there are no horizontal forces.
Since the motion is upward, we choose up as the positive direction.
The vertical speed is constant, so $a_y = 0\ m \cdot s^{-2}$
Therefore $F_{net\ y} = m \cdot a_y = 0$ N
But $F_{net\ y} = F_T + F_g + F_a$
Therefore $0 = F_T + F_g + F_a$
Therefore $F_a = -F_T - F_g = -10\ 000\ N - (1\ 000\ kg \times (-9{,}8\ m \cdot s^{-2}))$
$= -200$ N

The air resistance force is 200 N in a downward direction.

<u>Example 4</u>: *(Try to solve this problem* ***on your own or with a fellow student*** *while covering the solution, and then check your work using the solution below).*

A box with a mass of 500 g is sitting on a horizontal surface, and is connected by a rope that is threaded over a frictionless pulley to an object that has a mass of 300 g, which is hanging from the end of the rope. There is a frictional force of 1,2 N between the surface and the box. Find the tension in the rope.

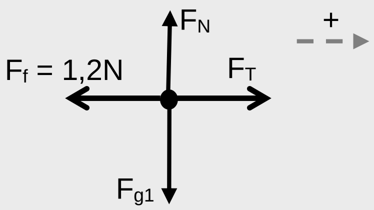


<u>Solution</u>:

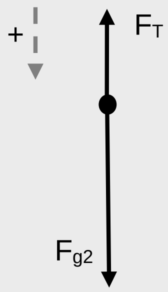
Let the box be m_1 and the mass hanging from the rope be m_2.

Frame of reference: Since m_1 will move to the right, we choose **right** as + for this object, and since m_2 will move down, we choose **down** as + for this object.

Free body diagram for m_1:



Free body diagram for m_2:



Calculation:

The net force on object m_1 can be expressed as $F_{net\ 1} = m_1a = 0,5a$

But we can also express the net force as: $F_{net\ 1} = F_T + F_f$

$\therefore 0,5a = F_T + (-1,2)$ (1)

The weight of object m_2 is: $F_{g2} = m_2g = 0,3 \times 9,8 = 2,94N$

The net force on object m_2 can be expressed as $F_{net\ 2} = m_2a = 0,3a$

But we can also express the net force as: $F_{net\ 2} = F_{g2} + F_T$

$\therefore 0,3a = 2,94 + (-F_T)$ (2)

From equation (2): $F_T = 2,94 - 0,3a$

Substitute this into equation (1): $0,5a = (2,94 - 0,3a) - 1,2$

$\therefore 0,8a = 1,74$

$\therefore a = 2,175\ m{\cdot}s^{-2}$

Substitute this back into equation (2): $F_T = 2,94 - 0,3a = 2,29$ N

The tension in the rope is 2,29 N.

MAIN IDEAS:

- Newton's second law states: "*When a net force F_{net}, is applied to an object of mass m, the object accelerates in the direction of the net force. The acceleration a is directly proportional to the net force and inversely proportional to the mass.*"
- We can write this as an equation: $F_{net} = m\,a$

Activity 1: Newton's first and second law

Answer the following questions:

1. An object with a mass of 150 kg has an acceleration of 2 $m{\cdot}s^{-2}$. Calculate the magnitude of the net force on the object.

2. The net force on a 25 kg object is 500 N. Find the acceleration of the object.

3. Thomas gave a 50 g ball a push so that it rolled along a horizontal surface, and the ball eventually slowed down to a stop.

   a. Why did the ball not continue to move with a constant velocity after it had been pushed by Thomas' hand? Use Newton's first law in your explanation.

   b. If the friction force between the ball and the surface is 0,2 N, calculate the acceleration of the ball.

4. A car with a mass of 1000 kg is pulled along a horizontal road with a force of 5000 N to the right. There is a frictional force opposing the car's movement. As a result the net force on the car is 4000 N to the right.

   a. Draw a free body diagram of all of the forces acting on the car.

   b. Calculate the acceleration of the car.

   c. What is the magnitude and direction of the frictional force?

5. A car with a mass of $1,4{\times}10^3$ kg is being towed by a tow-truck. The car is accelerating at a rate of 0,5 $m{\cdot}s^{-2}$. There is a frictional force of 200 N between the car's wheels and the road. What is the magnitude of the tension in the tow-rope?

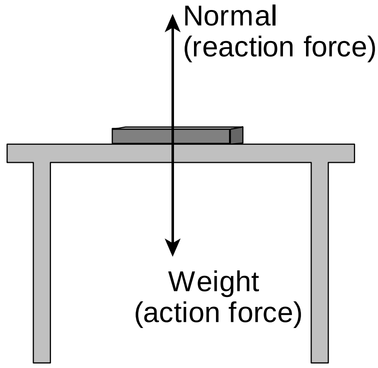
6. An object with a mass of 1,2 kg is falling in air with an acceleration of 9 $m{\cdot}s^{-2}$. What is the magnitude and direction of the force due to air resistance? (The weight of the object is 11,76 N)

2.3. Newton's Third Law

Newton's third law of motion states:

When object A exerts a force on object B, object B simultaneously exerts an oppositely directed force of equal magnitude on object A.

Another way of stating this law is: "For every action there is an equal and opposite reaction."

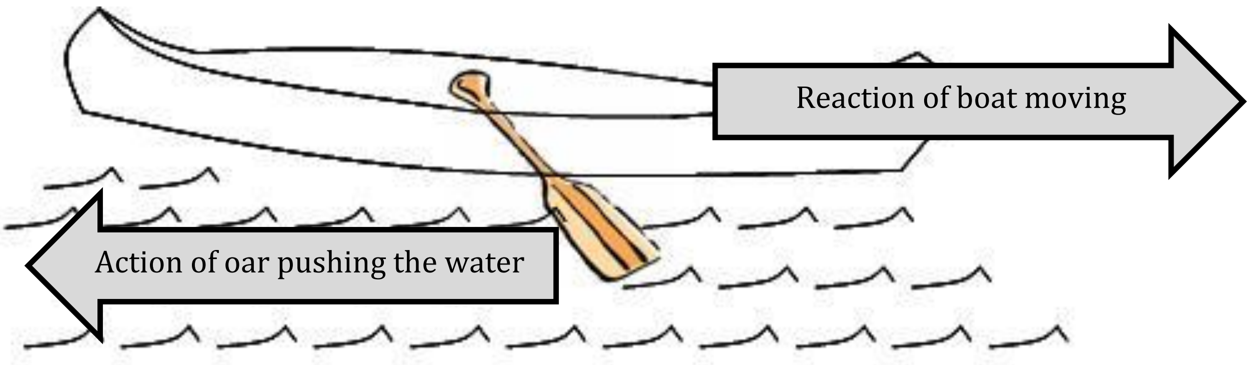


Forces always take place in pairs, which are known as "action-reaction force pairs." If we look at the example of a book that is placed on a table, the action force is the weight of the book acting downward on the table. The reaction force is the normal force that the surface of the table is exerting on the book.

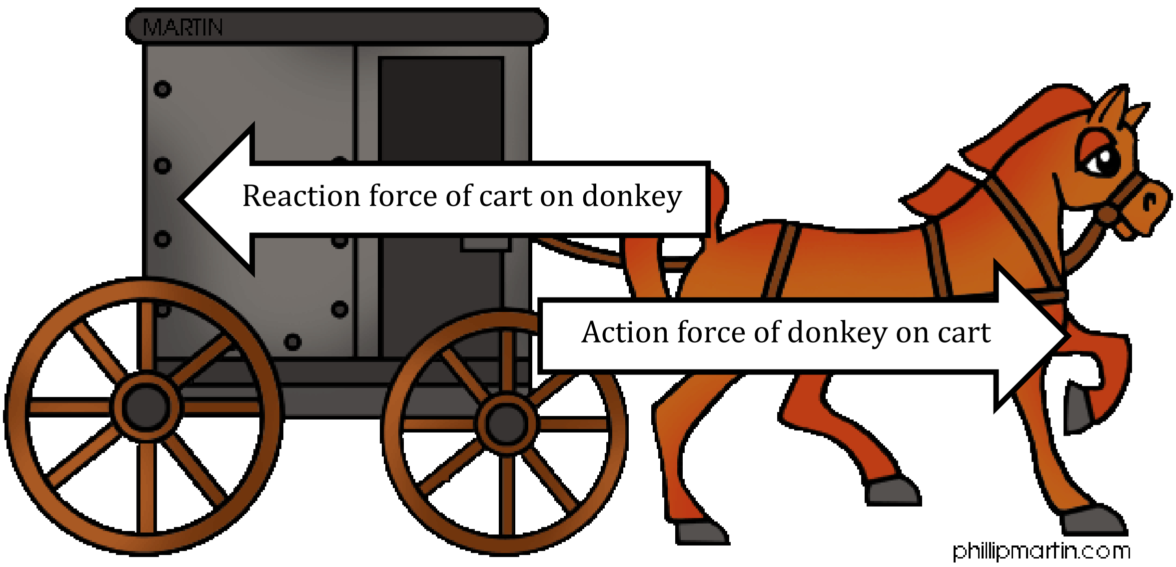
So the properties of action-reaction force pairs are:

- the reaction force is **equal in magnitude** to the action force
- the reaction force is **opposite in direction** to the action force
- the two forces take place simultaneously (at the same time)

Newton's third law is applied in many examples of motion. In a rowing boat, the action force is the force of the oar pushing backward on the water. The reaction force is the forward movement of the boat.



When a donkey pulls a cart, the force that the donkey exerts on the cart is the action force. The reaction force is the force that the cart exerts on the donkey.



Activity 2: Newton's third law

Identify the action and reaction forces in each of the pictures shown below:

1.

2.

3.

4.

MAIN IDEAS:

- Newton's third law states: "*When object A exerts a force on object B, object B simultaneously exerts an oppositely directed force of equal magnitude on object A.*"
- Action-reaction force pairs are equal in magnitude but opposite in direction to each other.

Assessment Activity: *Newton's Laws* **Total marks = 70**

Answer the following questions to assess your understanding of Newton's Laws:

1. State Newton's first law of motion. (2)

2. Thabo has attached a rope to a box, and is pulling on the rope, but the box does not move at all.
   a. What is the net force on the box? (1)
   b. Explain what is keeping the box from moving. (2)
   c. Draw a free body diagram of the box, and label all the forces acting on it. (4)

3. Mandy is riding her bicycle, and is pushing on the peddles with a force of 50 N, and as a result she is moving with a constant velocity. What is the net force on the bicycle? Explain your answer. (3)

4. A 50 g ball is sitting on the ground.
   a. What is the size of the force that the earth exerts on the ball? (3)
   b. What is the name of this force? (1)
   c. What is the size of the force that the ball exerts on the earth? Give a reason for your answer. (2)
   d. A force of 10 N is applied to this object parallel to the ground, and there is a frictional force of 2 N acting on the object. Find the object's acceleration. (5)

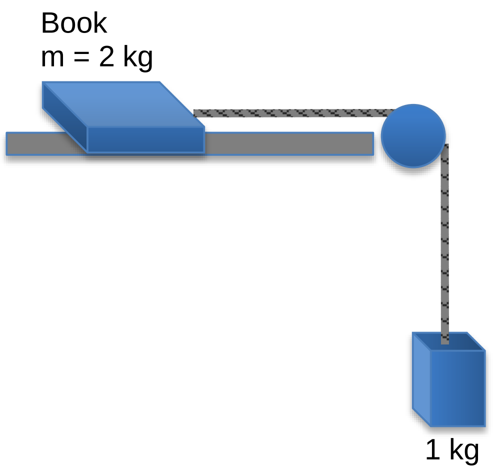
5. An object with a mass of 10 kg is being pushed, and as a result it has an acceleration of 2 $m{\cdot}s^{-2}$.
   a. Calculate the magnitude of the net force on the object. (3)
   b. If there is a frictional force of 5 N that is opposing the movement of the object, what is the magnitude of the applied force on the object? (5)

6. Two people are pulling a box that has a mass of 15 kg. Grace is pulling on the box with a force of 20 N in the +x direction. Sam is pulling on the box with a force of 12 N in the –x direction. There is a frictional force of 3 N opposing the box's movement.
   a. Draw a free body diagram that shows all the forces on the box. (5)
   b. What is the acceleration of the box? (6)

7. A child pushes his 50g toy car with a force of 0,06N at the angle of 60^{o} to the ground. As a result, the car moves forward with a constant acceleration of 0,4 $m{\cdot}s^{-2}$. Calculate the frictional force on the car. (6)

8. A 1 200 kg car is towing a trailer with a mass of 400 kg. The engine of the car applies a total pushing force of 2 000 N.
   a. What is the acceleration of the car and trailer if we ignore friction? (5)
   b. Calculate the force that is exerted on trailer. (3)
   c. The car and trailer move onto a sandy road to that there is a frictional force of 100 N on the trailer, and 150 N on the car. The car's engine applies the same pushing force of 2 000 N. Find the net force on the car. (6)

9. A book with a mass of 2 kg is sitting on a horizontal surface, and is connected by a rope that is threaded over a frictionless pulley to an object that has a mass of 1 kg, which is hanging from the end of the rope, as shown in the diagram below. There is a frictional force of 3 N between the surface and the book. Find the acceleration of the book. (8)



My Notes

Use this space to write your own questions, comments or key points.

Summary of key learning:

- Newton's first law states: "*An object continues in a state of rest or uniform velocity unless it is acted upon by an external unbalanced force.*"
- Newton's second law states: "*When a net force F_{net}, is applied to an object of mass m, the object accelerates in the direction of the net force. The acceleration a is directly proportional to the net force and inversely proportional to the mass.*"
- We can write Newton's second law as an equation: $F_{net} = m\ a$
- Newton's third law states: "*When object A exerts a force on object B, object B simultaneously exerts an oppositely directed force of equal magnitude on object A.*"
- Action-reaction force pairs are equal in magnitude but opposite in direction to each other.

Sub-topic 4. Momentum and impulse

Content:

Unit 1: Linear momentum and impulse

Unit 2: Momentum in collisions

Unit 1. Linear momentum and impulse

Learning outcomes:

When you have completed this unit, you should be able to:

- define linear momentum and impulse;
- calculate the momentum of a moving object;
- calculate the change in momentum of an accelerating object;
- define force as the rate of change of momentum.

1.1. Momentum

Reflection Question:

Imagine rolling a ball along a frictionless surface.

- What will you observe about the ball's velocity?
- What is the net force on the ball after it has left your hand?
- What is it that keeps the ball moving forward after it has left your hand?

Discuss your ideas with another student, or write down your ideas.

Newton's first law tells us that a ball that is moving with a constant velocity has no net force acting on it. But there is some property that is keeping it moving forward. This property is called *momentum*, which is **not a force**.

Momentum is defined as the quantity of motion of a moving body (or the property that keeps an object moving in a certain direction). The momentum of a moving object is **proportional** to its **mass,** and to its **velocity**.
We can write this mathematically as:

$$\text{Momentum} = \text{mass} \times \text{velocity}$$

The equation for this is:

$p = m v$

where p is the momentum of the object, measured in $kg \cdot m \cdot s^{-1}$,
m is the mass of the object, measured in kg, and
v is the velocity of the object, measured in $m \cdot s^{-1}$.

Momentum is a **vector** quantity. The momentum will always have the **same direction as the velocity**.

Activity 1 –Momentum

Answer the following questions:

1. A car that has a mass of $1{,}2 \times 10^3$ kg is travelling at a speed of 72 $km \cdot hr^{-1}$. Calculate the momentum of the car.
2. A 20 kg child is riding on his go-cart, which has a mass of 7,5 kg, with a speed of 8 $m \cdot s^{-1}$. What is the momentum of the child and the go-cart together?
3. The child applies the breaks of the go-cart so that it is now moving with a speed of 6 $m \cdot s^{-1}$. By how much has its momentum changed?
4. Which has more momentum: a 900 kg car moving at 100 $km \cdot hr^{-1}$ or a 1800 kg vehicle moving at 20 $m \cdot s^{-1}$?

MAIN IDEA: Momentum (symbol p) is defined as the quantity of motion of a moving body. It is the property that keeps a body in motion:

$p = mv$

Momentum is measured in units of $kg \cdot m \cdot s^{-1}$

1.2. Impulse

Newton's first law can also be written in this way: *In order to change the momentum of an object, a net force must be applied to that object*.

We can write this mathematically as:

Change in momentum = force × time interval

The equation for this is:

$\Delta p = F \Delta t$

where Δp is the change in the momentum of the object, measured in $kg \cdot m \cdot s^{-1}$,
F is the force exerted on the object, measured in newtons (N), and
Δt is the time for which the force acts, measured in seconds (s).

The name that is given to the force multiplied by the time interval is the **impulse**.
We can therefore write: Impulse = $F \Delta t$

But since Δp means the change in momentum, we can write it as $\Delta p = m\,\Delta v$
Therefore: Impulse = $F\,\Delta t = m\,\Delta v$

Example: *(Try to solve this problem **on your own or with a fellow student** while covering the solution, and then check your work using the solution below).*

A 5 kg ball falls into a sand pit with an initial velocity of 4 $m{\cdot}s^{-1}$, and comes to rest in a tenth of a second. What is the force that the ball exerted on the sand?

Solution:

Given: $m = 5$ kg; $v_i = 4\ m{\cdot}s^{-1}$; $v_f = 0\ m{\cdot}s^{-1}$; $\Delta t = 1/10\ s = 0{,}1\ s$

From the equation $F\,\Delta t = m\,\Delta v$ we can rewrite this to make F the subject of the formula:

$$F = \frac{m\,\Delta v}{\Delta t} = \frac{5kg \times (4\ m{\cdot}s^{-1} - 0\ m{\cdot}s^{-1})}{0{,}1\ s} = 200\ N$$

The force that the ball exerted on the sand was 200 N downward.

Example: *(Try to solve this problem **on your own or with a fellow student** while covering the solution, and then check your work using the solution below).*

A 0,02 kg ball is travelling towards a wall with a velocity of 20 $m{\cdot}s^{-1}$. It bounces off the wall, and returns in the opposite direction with a velocity of 18 $m{\cdot}s^{-1}$. The ball is in contact with the wall for 0,2 seconds.

a. Calculate the net force exerted on the ball.
b. Calculate the acceleration of the ball.

Solution

Frame of reference: Let the direction of the initial velocity of the ball be positive

Given: $m = 0{,}02$ kg; $v_i = 20\ m{\cdot}s^{-1}$; $v_f = -18\ m{\cdot}s^{-1}$; $\Delta t = 0{,}2\ s$

$p_i = 0{,}02 \times 20 = 0{,}4\ kg{\cdot}m{\cdot}s^{-1}$; $p_f = 0{,}02 \times (-18) = -0{,}36\ kg{\cdot}m{\cdot}s^{-1}$

Calculation:

a. $F_{net} = \frac{\Delta p}{\Delta t} = \frac{-0{,}76}{0{,}2} = -\,3{,}8\ N$

The net force exerted on the ball is 3,8 N in the opposite direction to its initial velocity.

b. $a = \frac{\Delta v}{\Delta t} = \frac{-18\ m{\cdot}s^{-1} - 20\ m{\cdot}s^{-1}}{0{,}2\ s} = -190\ m{\cdot}s^{-2}$

OR from Newton's second law $a = \frac{F}{m} = \frac{-3{,}8\ N}{0{,}02\ kg} = -190\ m{\cdot}s^{-2}$

The acceleration of the ball is 190 $m{\cdot}s^{-2}$ in the opposite direction to its initial velocity.

Activity 2 –Momentum and Impulse

1. A 50 g bullet hits a block of wood with an initial velocity of 500 $m{\cdot}s^{-1}$, and comes to rest in 0,025 seconds.
    a. What is the force that the block exerted on the bullet?
    b. What is the force that the bullet exerted on the block?
2. During a bungee jump, a person jumps off a high bridge with a very strong elastic rope tied to their ankles. Explain, in terms of impulse and momentum, why a bungee rope can not be made out of normal non-elastic rope.
3. A 0,1 kg ball is falling towards the floor with a velocity of 10 $m{\cdot}s^{-1}$. It bounces off the floor with a velocity of 8 $m{\cdot}s^{-1}$. If the ball was in contact with the floor for 0,2 seconds, what was the force exerted on the ball by the floor?

MAIN IDEA:

- **Impulse** is the change in the momentum of an object, which is equal to the force on an object multiplied by the time interval:

$$\text{Impulse} = F\,\Delta t = m\,\Delta v$$

Unit 2. Momentum in collisions

Learning outcomes:

When you have completed this unit, you should be able to:

- state the principle of conservation of momentum;
- apply the principle of conservation of momentum to solve problems involving collisions between two bodies in 1-dimension, in familiar and novel contexts.

2.1. The momentum of colliding objects

When two objects come into contact with one another, we say that they are involved in a collision. In the following activity you will explore the combined momentum of objects before and after they collide with one another.

Activity 1: Exploring the momentum of colliding objects

<u>You will need:</u>

2 marbles or balls that have the same size and mass
A piece of wood or desktop that has a groove cut into it
2 stop-watches
A mass measuring scale
A ruler or measuring tape

1. Measure the mass of each marble (or ball), and record the mass in the table below. (If you do not have a mass measuring scale, assume the mass of each marble or ball to be 10 g).

m_1 (kg)	Δt_1 (s)	v_1 ($m \cdot s^{-1}$)	m_1v_1 ($kg \cdot m \cdot s^{-1}$)	m_2 (kg)	Δt_2 (s)	v_2 ($m \cdot s^{-1}$)	m_2v_2 ($kg \cdot m \cdot s^{-1}$)

2. Place one marble (m_1) at one end of the groove, and the other marble (m_2) half way along the groove.

3. Measure the distance from marble m_1 to marble m_2, and from marble m_2 to the end of the groove.
4. Start one of your stop-watches as you flick marble m_1 towards marble m_2, and stop the stop-watch as it collides with marble m_2.
5. Start your second stop-watch as marble m_1 strikes marble m_2, and stop the stop-watch as marble m_2 reaches the end of the groove.
6. Use your values for the distances and times to calculate the velocity and momentum of marble m_1 before the collision, and the velocity and momentum of marble m_2 after the collision. Fill these values into the table above. (Let the direction of marble m_1 be positive).
7. Repeat this experiment 3 times to verify your findings.
8. Answer the following questions:
   a) How did the combined momentum of the marbles before the collision compare with the combined momentum of the marbles after the collision for each experiment?
   b) Do your answers verify the law of conservation of momentum? If not, can you explain any experimental errors in your results?

In this activity you should have found that the combined momentum of the objects before the collision was equal to the combined momentum of the objects after the collision. The total amount of momentum in this system is therefore conserved during the collision. This is called the **Principle of Conservation of Momentum**, which states that in an isolated system the total linear momentum remains constant, in magnitude and direction. (An isolated system is one where there are no external forces acting on the objects.)

In other words, the total momentum before the collision is equal to the total momentum after the collision. We can write this mathematically as:

$$p_{before} = p_{after}$$

Therefore $m_1v_{i1} + m_2v_{i2} = m_1v_{f1} + m_2v_{f2}$

where v_i refers to the initial velocity and v_f the final velocity.

NOTE: When solving problems with momentum it is very important to choose a **<u>frame of reference</u>** to make sure that the directions are correct!

<u>Example</u>: *(Try to solve this problem **on your own or with a fellow student** while covering the solution, and then check your work using the solution below).*

An 40 g green block is moving with a velocity of 5 m·s^{-1} towards a yellow 50 g block that is standing still. After hitting the yellow block, the green block bounces backwards along the path it has travelled with a velocity of 1 m·s^{-1}. Calculate the velocity of the yellow block after the collision.

<u>Solution:</u>

Frame of Reference: Let the direction of the green block's initial velocity be positive.

Given: Let the mass of the green block be m_1 and of the yellow block be m_2.

Then m_1 = 40 g = 0,04 kg ; m_2 = 50 g = 0,05 kg ;

Before the collision $\mathbf{v}_{i1}$ = 5 m·s^{-1}; $\mathbf{v}_{i2}$ = 0 m·s^{-1};

After the collision $\mathbf{v}_{f1}$=-1 m·s^{-1}; $\mathbf{v}_{f2}$=?

Calculation:

From the equation $m_1v_{i1} + m_2v_{i2} = m_1v_{f1} + m_2v_{f2}$

We get: $v_{f2} = ((m_1v_{i1} + m_2v_{i2}) - m_1v_{f1}) / m_2$

$$v_{f2} = \frac{(0{,}04 \times 5) + (0{,}05 \times 0) - (0{,}04 \times (-1))}{0{,}05} = 4{,}8 \text{ m·s}^{-1}$$

The final velocity of the yellow block is 4,8 m·s^{-1} in the direction that the green block was initially travelling in.

Activity 2 – Conservation of momentum

1. Thandi, who has a mass of 50 kg, is standing still on her ice skates, and throws a 1,5 kg ball to her friend. If the ball leaves her hands with a speed of 8 m·s^{-1}, what was Thandi's velocity as a result of the throw?

2. Karl is running along the passage with a speed of 3 m·s^{-1}, and runs straight into his father, who is walking in the opposite direction to Karl with a speed of 2 m·s^{-1}. As a result of the collision, his father comes to a stop. If Karl has a mass of 65 kg, and his father has a mass of 130 kg, what was Karl's final velocity?

MAIN IDEA: Momentum (symbol p) is defined as the quantity of motion of a moving body. It is the property that keeps a body in motion:

$$p = mv$$

Momentum is measured in units of $kg{\cdot}m{\cdot}s^{-1}$

Assessment Activity: Momentum and impulse **Total marks = 40**

Answer the following questions to assess your understanding of momentum and impulse:

1. A ball is thrown upward in the air, and after it has reached its maximum height it falls back down to its starting point. Complete the following table for the different parts of the ball's motion by writing only **UP / ZERO / DOWN** in each block: (9)

Movement of the ball	**Net force on the ball (UP / ZERO / DOWN)**	**Momentum of the ball (UP / ZERO / DOWN)**	**Acceleration of the ball (UP / ZERO / DOWN)**
Ball is moving upward			
Ball is at its maximum height			
Ball is moving downward			

2. Which has more momentum, a 150 g cricket ball with a velocity of 100 $km{\cdot}hr^{-1}$, or a 10 kg child walking at 0,5 $m{\cdot}s^{-1}$? Show your calculations clearly. (5)

3. What do we mean by the term *impulse*? (2)

4. A 4 g dart is thrown at a dartboard that is hanging on a wall, and it strikes the board with an initial velocity of 50 $m{\cdot}s^{-1}$. If comes to rest in 0,05 seconds, what is the force that the dartboard exerted on the dart? (4)

5. State the law of conservation of momentum. (2)

6. Kopano throws a 20 g ball at a wall with a velocity of 15 $m{\cdot}s^{-1}$. It bounces off the wall, and returns back to him with a velocity of 14 $m{\cdot}s^{-1}$. The ball is in contact with the wall for 0,1 seconds.
    a. Calculate the force exerted on the ball by the wall. (4)
    b. What is the net force exerted on the wall by the ball? (1)

c. Calculate the acceleration of the ball. (3)

Solution

Frame of reference: Let the direction of the initial velocity of the ball be positive

Given: m = 0,02 kg; $v_i = 20\ m{\cdot}s^{-1}$; $v_f = -18\ m{\cdot}s^{-1}$; $\Delta t = 0,2\ s$

$p_i = 0,02 \times 20 = 0,4\ kg{\cdot}m{\cdot}s^{-1}$; $p_f = 0,02 \times (-18) = -0,36\ kg{\cdot}m{\cdot}s^{-1}$

Calculation:

a. $F_{net} = \frac{\Delta p}{\Delta t} = \frac{-0,76}{0,2} = -3,8\ N$

The net force exerted on the ball is 3,8 N in the opposite direction to its initial velocity.

b. $a = \frac{\Delta v}{\Delta t} = \frac{-18\ m{\cdot}s^{-1} - 20\ m{\cdot}s^{-1}}{0,2\ s} = -190\ m{\cdot}s^{-2}$

OR from Newton's second law $a = \frac{F}{m} = \frac{-3,8\ N}{0,02\ kg} = -190\ m{\cdot}s^{-2}$

The acceleration of the ball is $190\ m{\cdot}s^{-2}$ in the opposite direction to its initial velocity.

7. A 5 g bullet is fired from a gun that has a mass of 400 g. The bullet has a velocity of $360\ m{\cdot}s^{-1}$ as it leaves the gun barrel. What is the recoil velocity of the gun? (5)

ANS: Let the bullet be m_1, and let the gun be m_2. Choose the bullet's direction as positive.

Given: $m_1 = 0,005kg$; $v_{i1} = 0m{\cdot}s^{-1}$; $m_2 = 0,400kg$; $v_{i2} = 0m{\cdot}s^{-1}$; $v_{f1} = 360m{\cdot}s^{-1}$

What is being asked: v_{f2} = ?

Calculation:

$m_1 v_{i1} + m_2 v_{i2} = m_1 v_{f1} + m_2 v_{f2}$

But since both v_{i1} and v_{i2} are 0, then the initial momentum is zero.

The equation becomes: $0 = m_1 v_{f1} + m_2 v_{f2}$

So $v_{f2} = m_1 v_{f1} / m_2 = -0,005kg \times 360m{\cdot}s^{-1} / 0,400kg = = -4,5\ m{\cdot}s^{-1}$

Therefore the gun's recoil velocity is $4,5\ m{\cdot}s^{-1}$ in the opposite direction to that of the bullet.

8. A 35 kg child running at $6\ m{\cdot}s^{-1}$ jumps onto a 10 kg go-cart which was initially at rest. What will be the velocity of the child and the go-cart together immediately after the child jumps on the go-cart? (5)

My Notes:

Use this space to write your own questions, comments or key points.

Summary of key learning:

- **Momentum** (symbol p) is defined as the quantity of motion of a moving body. It is the property that keeps a body in motion: $p = mv$
- **Impulse** is the change in the momentum of an object, which is equal to the force on an object multiplied by the time interval:
- Impulse = $F \Delta t = m \Delta v$
- Momentum and impulse are measured in units of $kg{\cdot}m{\cdot}s^{-1}$.
- The **Principle of Conservation of Momentum** states that in an isolated system the total linear momentum remains constant, in magnitude and direction: $m_1v_{i1} + m_2v_{i2} = m_1v_{f1} + m_2v_{f2}$

Sub-topic 5. Work, power and energy

Content:

Unit 1: Energy conversion and conservation

Unit 2: Work and power

Unit 1. Energy conversion and conservation

Learning outcomes:

When you have completed this unit, you should be able to:

- list examples of different forms of energy, including kinetic energy, potential energy (gravitational, chemical, elastic), electrical energy, light energy, thermal energy and nuclear energy;
- define kinetic energy and gravitational potential energy in words and using mathematical expressions: $E_k = ½ mv^2$ and $E_p = mgh$;
- define mechanical energy;
- state the principle of the conservation of mechanical energy;
- apply the principle of conservation of mechanical energy to various contexts, including objects that are dropped or thrown vertically upwards, and the motion of a swing or pendulum.

Introduction

Energy is a very important concept. Everything we do in our everyday lives requires some kind of energy. In this unit you will learn the scientific meaning of energy.

All forms of energy can be categorised into two main types of energy – ***potential energy*** and ***kinetic energy***.

- Potential energy is energy that is stored in some way
- Kinetic energy is the energy of movement.

We will begin by looking at gravitational potential energy, as this helps us to understand the concept of potential energy.

1.1. Gravitational Potential Energy

When we lift an object, the effort, or work, that we put into lifting it is stored in this object as energy. This stored energy is called ***gravitational potential energy***. Since gravitational potential energy is a form of energy, it is measured in the units of energy, namely joules (J). In the following activity you will explore the factors that affect the gravitational potential energy of an object.

Activity 1: Exploring gravitational potential energy

<u>You will need:</u>

2 objects, one that has a larger mass than the other
A scale for measuring mass
A tape measure or ruler
A chair and a desk

1. Measure the length from the floor to the seat of the chair. Record this length as h_1. Now measure the length from the floor to the desktop. Record this length as h_2.

   Length from floor to chair: h_1 = ____________

   Length from floor to desk: h_2 = ____________

2. Measure the mass of each of your objects, and record these as m_1 for the lighter object and m_2 for the heavier object.

   Lighter object: m_1 = ____________

   Heavier object: m_2 = ____________

3. Lift object m_1 to the height of the chair. Now lift object m_2 to the height of the chair. Which do you think needed more effort (work) to lift it to this height?
4. The work done in lifting an object to a height above the ground is stored in the object as gravitational potential energy. Which object has the greater gravitational potential energy, m_1 or m_2?
5. Return your objects to the floor. Now lift object m_2 to the height of the desk (h_2). Did this require more work to be done than lifting object m_2 to the height of the chair (h_1)?
6. Which object has the greater gravitational potential energy, the one lifted to h_1 or to h_2?
7. From your investigations, complete the following sentences:
   - The greater the mass of an object that is lifted, the ______________ the gravitational potential energy of the object.
   - The greater the height that an object is lifted to, the ______________ the gravitational potential energy of the object.

When an object is lifted upward from a reference point, for example the floor, the greater the mass of the object being lifted, the more effort needs to be done in lifting it, and therefore the greater the amount of stored energy. In other words, the gravitational potential energy is proportional to the mass of an object.

When an object is lifted to a greater height above the reference point, the effort needed to lift the object is greater than if the object is lifted a small height from the reference point. So the gravitational potential energy is proportional to the height of an object.

We can calculate the gravitational potential energy of an object using the following equation:

$$E_p = m\,g\,h$$

where E_p is the gravitational potential energy, measured in joules (J)
m is the mass of the object, in kg
g is the gravitational acceleration (on earth g = 9,8 $m{\cdot}s^{-2}$)
h is the height of the object above the zero point, in metres (m)

If you study the units in this equation, you can see that 1 J = 1 $kg{\cdot}m^2{\cdot}s^{-2}$.

<u>Example:</u> *(Try to solve this problem **on your own or with a fellow student** while covering the solution, and then check your work using the solution below).*

A professor lifts a pile of books that has a mass of 6 kg from the floor to the top of his bookshelf, which is at a height of 1,5 m above the floor. By how much has the gravitational potential energy of the pile of books increased?

<u>Solution</u>

Given: m = 6 kg , h = 1,5 m , g = 9,8 $m.s^{-2}$

$$E_p = m \cdot g \cdot h$$

$$= 6\ kg \times 9{,}8\ m.s^{-2} \times 1{,}6\ m$$

$$= 88{,}2\ kg{\cdot}m^2{\cdot}s^{-2} = 88{,}2\ J$$

<u>Example:</u> *(Try to solve this problem **on your own or with a fellow student** while covering the solution, and then check your work using the solution below).*

When a mass *m* is lifted to a height *h* above its starting point, its gravitational potential energy increases by 1 000 J. If a mass of 2*m* is lifted to a height of 3*h* above this same starting point, by how much will its gravitational potential energy increase?

<u>Solution</u>

The equation $E_p = m\,g\,h$ shows that E_p is proportional to m and h.

So if we double the mass, and the height is increased by a factor of 3, the potential energy will increase by a factor of 3× 2 = 6.

Therefore E_p = 6 × 1 000 J = 6 kJ.

Activity 2 – *Gravitational potential energy*

Answer the following questions.

1. Nonhlanhla lifts her child straight up onto a platform. Cindi pushes her child in a pram up a ramp onto the same platform so that her child is at the same height as Nonhlanhla's child. The two children have the same mass. How does the potential energy of these two children compare with each other?
2. A taxi has a mass of 1 200 kg. It drives up a hill so that its vertical height has increased by 20 m. Calculate the increase in the taxi's gravitational potential energy.
3. When a mass *m* is lifted to a height *h* above its starting point, its gravitational potential energy has increased by 1 500 J. If an object with half the mass is lifted by four times the height of the first object, by how much will its gravitational potential energy increase?

MAIN IDEAS:

- **Gravitational potential energy** is the stored energy of an object because of its position above some reference point.
- The gravitational potential energy of an object can be calculated by the equation: $E_p = m\,g\,h$

1.2. Kinetic Energy

Kinetic energy is the energy of movement. The greater the mass of a moving object, the higher the value of the kinetic energy when it moves. Similarly, the faster the object is moving, the greater its kinetic energy.

The kinetic energy can be calculated using the following equation:

$$E_k = \frac{1}{2} m\,v^2$$

where E_k is the kinetic energy of the object, measured in joules (J), m is the mass of the object, in kg, and v is the velocity of the object, in $m{\cdot}s^{-1}$.

Example: *(Try to solve this problem **on your own or with a fellow student** while covering the solution, and then check your work using the solution below).*

A woman is running with a netball that has a mass of 500 g. The woman's mass is 82 kg and she is running with a velocity of 6 m·s⁻¹. What is the total kinetic energy of the woman and the ball together?

Solution

Given: m = 82 kg + 0,5 kg = 82,5 kg, v = 6 $m.s^{-1}$

$E_k = \frac{1}{2} m v^2$

$= \frac{1}{2} \times 82{,}5\ kg \times (6\ m{\cdot}s^{-1})^2$

$= 1485\ J$

Activity 3 – *Kinetic energy*

Answer the following questions:

1. You are standing by the roadside waiting for a taxi, while holding a heavy suitcase. Does this involve kinetic energy?
2. The taxi that you get into has a mass of 900 kg (including your mass). When it is traveling at a speed of 100 $km{\cdot}hr^{-1}$, what is its kinetic energy in kJ?
3. If this taxi reduces its speed to 50 $km{\cdot}hr^{-1}$, by what factor has its kinetic energy decreased?

MAIN IDEAS:

- The **kinetic energy** of an object is the energy of the object's movement.
- The kinetic energy is calculated by the equation: $E_k = \frac{1}{2} m v^2$

1.3. Mechanical Energy

The mechanical energy is the sum of the gravitational potential energy and the kinetic energy of the object. We can write this as an equation:

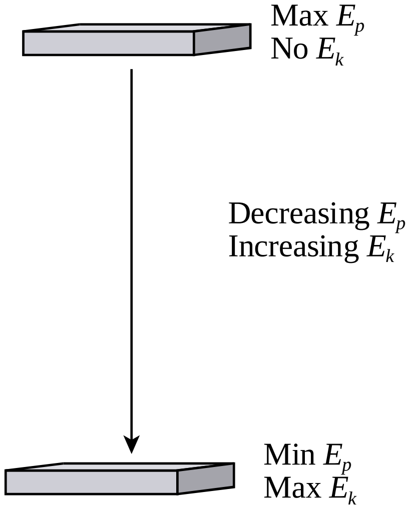
$$E_M = E_p + E_k$$

where E_M is the total mechanical energy, E_p is the potential energy, and E_k is the kinetic energy of the object, all measured in units of joules (J).

To understand this we will look at the example of a book that falls to the ground. When you hold the book above your head, it has gravitational potential energy as a result of its position. While the book is being held by the hand, the velocity is zero, and therefore the kinetic energy is zero.

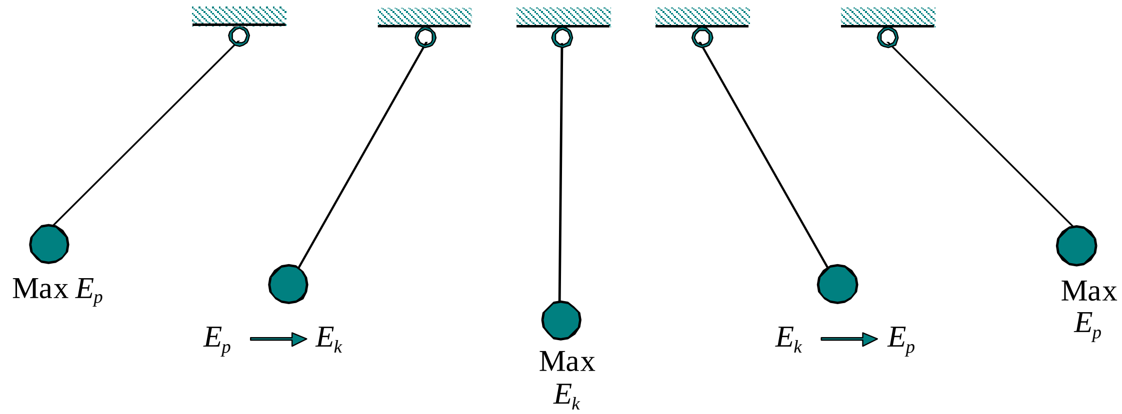
As the book is dropped, its velocity increases in a downward direction. At the same time its height above the ground is decreasing. Therefore its kinetic energy is increasing, while its gravitational potential energy is decreasing.

Just above the floor the book will have zero gravitational potential energy, as its height above the floor is zero. All of its gravitational potential energy has been converted into kinetic energy.



What this means is that the **mechanical energy** of the object is **conserved**. The gravitational potential energy and kinetic energy are converted from one form to another, but the total remains constant.

This conversion of energy from potential energy to kinetic energy and back again takes place in many forms of motion. Another example of this is a ball that is swinging from a rope. The diagram below shows how energy is converted between potential energy and kinetic energy as the ball swings.



We can use an equation to show the conservation of mechanical energy.

Since the mechanical energy before some change is equal to the mechanical energy afterwards, we can write this as:

$$E_{M_1} = E_{M_2}$$

where E_{M_1} is the total mechanical energy before the change, and E_{M_2} is the total mechanical energy after the change.

We can write this equation using symbols for the kinetic and potential energy:

$$E_{p_1} + E_{k_1} = E_{p_2} + E_{k_2}$$

where E_{p_1} is the initial potential energy, E_{k_1} is the initial kinetic energy, E_{p_2} is the final potential energy after the change, and E_{k_2} is the final kinetic energy.

In reality, we know that a ball like this one can not keep swinging forever. Air resistance and frictional forces cause the ball to lose some of its total mechanical energy. In this way, energy is ***dissipated*** from the system, and as a result, the ball swings to a slightly lower height each time, until it eventually stops.

dissipated – lost, or given out

<u>Example:</u> *(Try to solve this problem **on your own or with a fellow student** while covering the solution, and then check your work using the solution below).*

A trailer with a mass of 2 000 kg is at rest on a slope at a height of 50 m above the ground. The trailer is released and rolls downhill until it has a height of 25 m above the ground. What is the speed of the trailer at this new height? (Assume that there is no energy dissipation.)

<u>Solution</u>

Given: $m = 2\times10^3$ kg; $h_1 = 50$ m; $h_2 = 25$ m; $g = 9{,}8$ m·s^{-2}

$v_1 = 0$ m·s^{-1}; $v_2 = ?$

From the equation: $E_{p_1} + E_{k_1} = E_{p_2} + E_{k_2}$

We first find the total initial mechanical energy:

$$E_{p_1} + E_{k_1} = m\,g\,h_1 + \frac{1}{2} m\,v_1^2$$

$$= (2\times10^3 \text{ kg} \times 9{,}8 \text{ m·s}^{-2} \times 50 \text{ m}) + 0$$

$$= 9{,}8\times10^5 \text{ J}$$

Therefore total initial mechanical energy = $9{,}8\times10^5$ J = $E_{p_2} + E_{k_2}$

Therefore $9{,}8\times10^5 \text{ J} = m\,g\,h_2 + \frac{1}{2} m\,v_2^2$

$$= (2\times10^3 \text{ kg} \times 9{,}8 \text{ m·s}^{-2} \times 25 \text{ m}) + (0{,}5 \times 2\times10^3 \text{ kg} \times v_2^2)$$

$$= 4{,}9\times10^5 \text{ J} + (0{,}5 \times 2\times10^3 \text{ kg} \times v_2^2)$$

So $v_2^2 = \frac{9{,}8\times10^5 \text{ J} - 4{,}9\times10^5 \text{ J}}{0{,}5\times2\times10^3 \text{ kg}} = 4{,}9 \times 10^2 \text{ m}^2\text{·s}^{-2}$

So $v_2 = 22{,}14$ m·s^{-1}

MAIN IDEAS:

- Mechanical energy = gravitational potential energy + kinetic energy:
  $E_M = E_p + E_k$
- The total mechanical energy in a system is conserved when there is no loss of energy to the environment.

Activity 4 – *Mechanical energy*

A 500 g ball is lifted to a height of 1,5 m above the ground. Answer the following questions:

a. By how much has the ball's gravitational potential energy increased?

b. If the ball is dropped from this height, what is its kinetic energy just before it hits the ground?

c. Find **two ways** of calculating the ball's velocity just before it reaches the ground.

Unit 2. Work and power

Learning outcomes:

When you have completed this unit, you should be able to:

- define work done as the force multiplied by the distance moved in the direction of the force $W = F_x \Delta x$;
- apply the relationship for work done to various related problems, in familiar and novel contexts;
- define power as the work done divided by the time taken to do the work;
- apply the relationship for power to various related problems, in familiar and novel contexts.

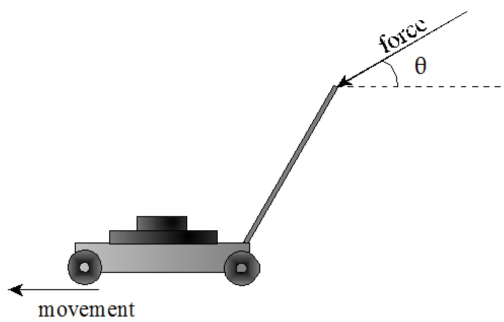
2.1. Work

Work is done on an object if a force is applied to the object, and if this force causes displacement of the object in the direction of the force.

- When a force acts at some angle θ to the direction of motion of an object, we only consider the component of the force that acts in the direction of the motion of the object. We can therefore calculate the work done by the force on the object using the equation:

  $$W = F_x \Delta x$$

  where Δx is the magnitude of the displacement.



- To calculate the x-component of the force F, we use the equation: $F_x = F \cos \theta$

2.2. Power

Power is defined as the amount of work done per unit time, or the rate at which work is done. In other words, we can calculate power by dividing the amount of work done by the time taken to do that work. In equation form, we write:

$$P = \frac{W}{\Delta t}$$

where P is the power, measured in watts (W), W is the work done, measured in joules (J), and Δt is the amount of time taken to do the work, in seconds (s).

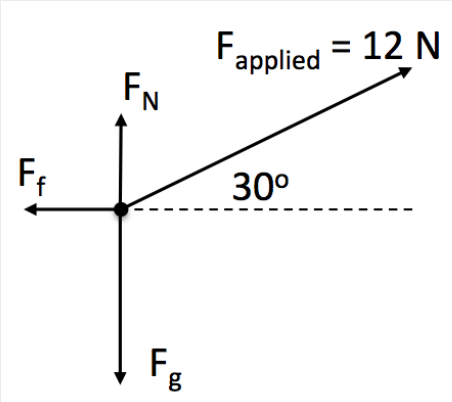
<u>Example:</u> *(Try to solve this problem* ***on your own or with a fellow student*** *while covering the solution, and then check your work using the solution below).*

Blessing pulls a box with mass 25 kg on a horizontal surface for a distance of 10 m by applying a force of 12 N to a rope that is attached to the box, at an angle of 30^o to the horizontal. A frictional force of 2 N opposes the motion of the box.

a. Calculate the work done on the box by Blessing.
b. If Blessing pulled the box for 30 seconds, what was the power that he exerted in this time?
c. Calculate the work done by the frictional force.
d. What is the work done by the gravitational force?

<u>Solution:</u>

The free body diagram of all of the forces on the box is shown on the right.



a. We first find the x-component of the applied force:

   $F_{applied\ x} = F_{applied} \cos \theta = 12 \text{ N} \times \cos 30^o = 10{,}4 \text{ N}$

   Therefore work done on the box by Blessing is:

   $W = F_x \Delta x = 10{,}4 \text{ N} \times 10 \text{ m} = 104 \text{ J}$

b. We use the equation:

   $P = \frac{W}{\Delta t} = \frac{104 \text{ J}}{30 \text{ s}} = 3{,}47 \text{ W}$

c. We first find the x-component of the frictional force:

   $F_{f\,x} = F_f \cos \theta = 2 \text{ N} \times \cos 180^o = -2 \text{ N}$

   Therefore work done on the box by the frictional force is:

   $W = F_x \Delta x = -2 \text{ N} \times 10 \text{ m} = -20 \text{ J}$

d. The work done by the gravitational force is 0 J, since this force is perpendicular to the displacement of the object, so it has no component in the direction. We can show this using a calculation:

   $F_{g\,x} = F_g \cos \theta = (25 \text{ kg} \times 9{,}8 \text{ m·s}^{-2}) \times \cos 90^o = 0 \text{ N}$

Activity 1 – Work and Power

1. Read each of the scenarios below, and answer the question for each one.

   a) Naledi spent all afternoon sitting at his desk writing an essay for homework. Did Naledi do work according to the scientific definition?

   b) Tebogo pushed her younger brother up a hill in his go-cart. Did Tebogo do work? Did her brother do work?

   c) Moses and Lydia had a tug-of-war by holding onto the opposite ends of a long piece of rope and pulling. They were evenly matched, so they stayed in one place, even though they pulled as hard as they could. Did either Moses or Lydia do work?

   d) A weight-lifter lifted a heavy weight and then held it steadily above his head.

      i) Did the weight lifter do work by lifting the weight upward?

      ii) Did the weight lifter do work by holding the weight above his head?

MAIN IDEAS:

- **Work** is done on an object if a force is applied, and causes displacement of the object in the direction of the force: $W = F_x \Delta x$
- The x-component of the force F is: $F_x = F \cos \theta$
- **Power** is defined as the amount of work done per unit time, or the rate at which work is done:

$$P = \frac{W}{\Delta t}$$

- Power is measured in watts (W).

Assessment Activity: Work, energy and power **Total marks = 60**

Answer the following questions to assess your understanding of work, energy and power:

Multiple choice questions:

1. Mandla drops a stone into a pile of soft sand from two different heights:

   Case 1: He drops the stone from a height h

   Case 2: He drops the same stone from a height 2 h.

   Which of the following is true for the mechanical energy and work in both the cases? (3)

	Mechanical energy	Work done
A.	Case 2 < Case 1	Case 2 > Case 1
B.	Case 2 < Case 1	Case 2 < Case 1
C.	Case 2 > Case 1	Case 2 > Case 1
D.	Case 2 > Case 1	Case 2 < Case 1

2. For the two cases described in Question 1, which of the following is true for the acceleration of the stone and final velocity (just before hitting the sand) in both the cases? (3)

	Acceleration	Final velocity
A.	Case 2 = Case 1	Case 2 > Case 1
B.	Case 2 = Case 1	Case 2 = Case 1
C.	Case 2 > Case 1	Case 2 > Case 1
D.	Case 2 > Case 1	Case 2 = Case 1

Written response questions:

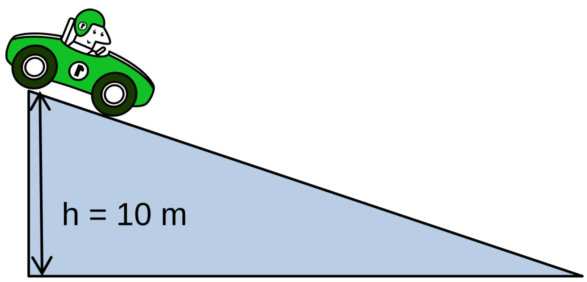
1. Complete the table below by giving one word for the quantity that is described, showing the units that each of the quantities is measured in, and writing the equation that we use to calculate each one. (12)

Description	Name of quantity	Units	Equation
Energy of movement			

Energy that an object has because of its height above the earth			
The sum of the gravitational potential energy and the kinetic energy of the object			
The rate at which work is done			

2. A 1,5 kg book is lifted from the ground to a height of 300 cm above the ground.
    a. By how much has the book's gravitational potential energy increased? (3)
    b. If the book is dropped from this height, what is its kinetic energy just before it hits the ground? (1)
    c. Calculate the book's velocity just before it strikes the ground. (3)
    d. Calculate the time taken for the book to fall to the ground. (3)

3. A 2 000 kg car is parked at the top of a hill, as the diagram shows. The car is allowed to roll down the hill. If no energy is lost through friction, what is the car's velocity at the bottom of the hill? (6)



4. A ball is attached to a rope that hangs from a hook. Kealeboga pulls back the ball to a height that gives it an increase in gravitational potential energy of $1{,}176 \times 10^{-2}$ J, and lets the ball go. The mass of the ball is 30 g. Ignore the effects of friction.
    a. Find the height that Kealeboga raised the ball to (above its rest position) (3)
    b. Calculate the kinetic energy of the ball at its lowest position (1)
    c. Calculate the speed of the ball at the lowest position (3)
    d. What is the total mechanical energy of the system? (1)
    e. Explain why the ball does not keep swinging to the same height forever in a real life situation. (3)

5.Rashid moved a 20 kg box along the floor by pulling on a rope with a force of 30 N. The rope made an angle of 40^{o} with the horizontal. There was a frictional force opposing the box's motion.

a) Draw a free body diagram showing all of the forces acting on the box. (4)

b) If Rashid moved the box a distance of 5m along the ground, how much work did he do on the box? (4)

c) If Rashid pulled the box for 2 minutes, what was the power that he exerted in this time? (3)

d) If the box moved with a constant velocity as a result of the force that he applied, what was the frictional force between the box and the floor? (5)

e) What was the work done on the box by the frictional force? (3)

f) What was the magnitude of the normal force on the box? (6)

My Notes:

Use this space to write your own questions, comments or key points.

Summary of key learning:

- **Potential energy** is energy that is stored in some way.
- **Gravitational potential energy** is the stored energy of an object because of its position above some reference point.
- The gravitational potential energy of an object can be calculated by the equation: $E_p = m\,g\,h$
- **Kinetic energy** is the energy of movement.
- The kinetic energy is calculated by the equation: $E_k = \frac{1}{2} m\,v^2$
- **Mechanical energy** = gravitational potential energy + kinetic energy:

  $E_M = E_p + E_k$
- The total mechanical energy in a system is conserved when there is no loss of energy to the environment.
- All forms of energy are measured in joules (J).
- **Work** is done on an object if a force is applied, and causes displacement of the object in the direction of the force: $W = F_x\,\Delta x$.
- Work is measured in joules (J).
- The x-component of the force F is: $F_x = F\cos\theta$
- **Power** is defined as the amount of work done per unit time, or the rate at which work is done:

  $P = \frac{W}{\Delta t}$
- Power is measured in watts (W).

Topic 3. Waves

Introduction

We experience waves all the time in the world around us, whenever we see or hear anything. We can represent waves using simple diagrams that show their structure and characteristics. By understanding waves we can predict their behaviour in different conditions, and we can put them to use in many helpful ways.

In this topic you will learn about the two different types of waves, namely transverse and longitudinal waves, and how we experience these in our everyday lives. You will also learn about geometrical optics, which looks at the behaviour of light as it reflects and bends, and how this can be applied in our everyday lives.

Sub-topic 1. Transverse and longitudinal waves

Content:

Unit 1: Transverse waves

Unit 2: Longitudinal waves

Unit 1. Transverse waves

Learning outcomes:

When you have completed this unit, you should be able to:

- describe wave motion as a vibration in a medium, resulting in the transfer of energy without matter being transferred;
- define frequency, wavelength, period and amplitude;
- draw a diagram of a transverse wave and indicate the wavelength, amplitude, particle movement and direction of propagation of the wave;
- give examples of transverse waves;
- define the wave speed as the product of the frequency and wavelength of a wave $v = f\lambda$ (the wave equation);

- apply the wave equation to solve problems involving transverse waves, in familiar and novel contexts.

In the previous unit you learnt about transverse waves. In this unit you will learn about a different kind of wave, called a longitudinal wave.

1.1. What is a wave?

> **Medium** – the substance that a wave travels through

A wave is a regular disturbance that takes place in a medium, resulting in a transfer of energy through the medium. In other words, a wave is a repeated movement, where the particles of the medium move backwards and forwards in a regular vibration.

Activity 1: Observing the movement of a wave

> You will need:
>
> A long spring (slinky), or a rope
> A piece of coloured cloth

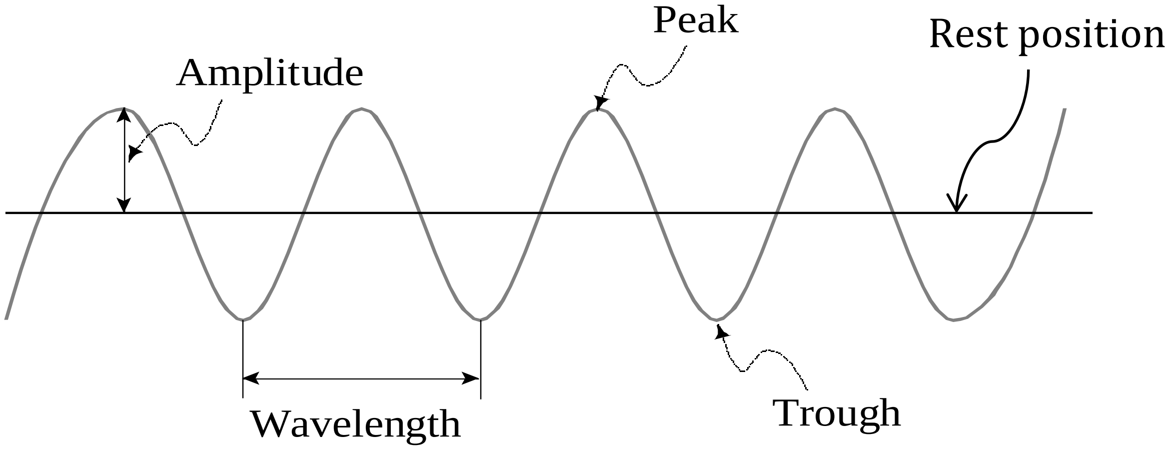
1. Put the spring or rope on a smooth floor or on a large desk top.
2. Two students should hold the ends of the spring so that the spring is slightly stretched. If you are using a rope, make sure that the rope is quite loose and free to move.
3. Move the end of the rope up and down quickly to make a wave on the slinky.
4. What do you observe?
5. Tie a piece of coloured wool somewhere in the middle of your spring or rope. This will represent a particle.
6. Now send a wave along the spring or rope and observe what happens to the coloured wool. Did the piece of wool move forwards with the wave?
7. Explain the movement of the piece of wool in your own words.

Go to https://www.youtube.com/watch?v=y66PSaiGH7Y to see a transverse wave on a slinky.

The wave that you have created on your rope or slinky is an example of a **transverse wave**, which is the type of wave where the particles of the medium move at right angles (transverse) to the movement of the wave through the medium. The movement of the piece of wool in Activity 1 shows how the particles of the medium move as the wave passes through the slinky or rope. We will now explore the characteristics of transverse waves.

1.2. Characteristics of transverse waves

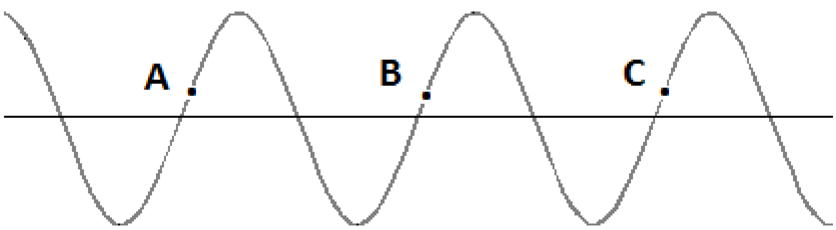
The wave shown in the diagram below is a **transverse wave**. (There is another type of wave, called a longitudinal wave, which you will learn about in Unit 2.)



From this diagram, you will notice that the wave has a regular pattern. Some of the key characteristics of a transverse wave are described in the following points:

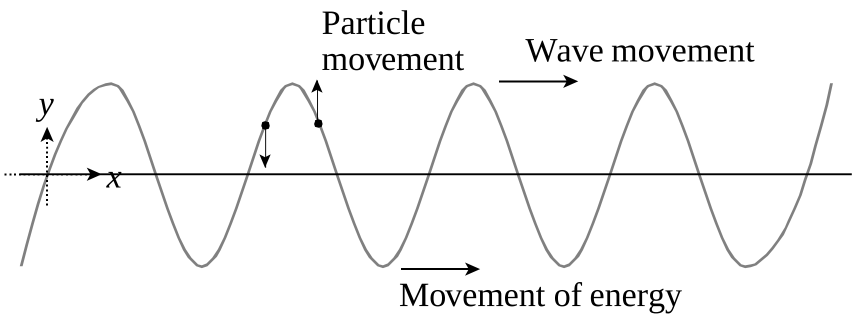
> **rest position** – the position that the particles of the medium would have had if there was no wave

- The sections of the wave that are at a maximum displacement above the rest position are called "**peaks**".
- The sections that are at a maximum displacement below the rest position are called "**troughs**".
- The **amplitude** of the wave is the distance between the peak of a wave and its rest position, or the distance between the trough and the rest position.
- Positions on a wave that have a whole number of wavelengths between them are **in phase** with each other. For example, the troughs of a wave are in phase with each other. In the diagram below, points A, B and C are in phase with each other.



Points A, B and C are in phase

- The **wavelength** is the horizontal distance between any two consecutive points on a wave that are in phase. For example, the wavelength is the horizontal distance between two consecutive troughs, or two consecutive peaks, or the distance from A to B, or B to C in the above diagram. In Physics the symbol λ (called "lambda") is used for the wavelength, measured in metres (m).
- The **particles** of the medium vibrate from **side-to-side**, at right angles (transverse) to the direction of the movement of the wave. They do not move forward with the wave, it is the energy that moves forward through the medium.



- Examples of transverse waves are water waves and light waves.

MAIN IDEAS:

- **Wavelength** (***λ***) is the distance between two parts of a wave that are in phase
- **Amplitude** is the distance between the peak of a wave and its rest position (or between the trough of a wave and its rest position)
- The **particles** of the medium move from side-to-side, **transverse** to the direction of the movement of the wave.

1.3. Frequency, period and speed of a wave

The ***frequency*** (f) of a wave is the number of full wavelengths that pass a fixed point **in one second** (per second). Frequency is measured in units of *per second* (s^{-1}), which is also be called *hertz* (Hz).

The ***period*** (T) of a wave is the time that it takes for a full wavelength to pass a fixed point, measured in *seconds* (s). The period is mathematically related to the frequency in the following way:

$$T = \frac{1}{f}$$

The **wave equation** links the speed of a wave with the frequency and wavelength:

$$v = f \times \lambda$$

where v is the speed of the wave, measured in metres per second ($m{\cdot}s^{-1}$)

f is the frequency, measured in hertz (Hz) or (s^{-1})

λ is the wavelength, measured in metres (m).

<u>Example 1</u> *(Try to solve this problem* ***on your own or with a fellow student*** *while covering the solution, and then check your work using the solution below).*

Six full wavelengths of a wave that has wavelength of 20 cm pass by a fixed point in a time of 3 seconds. Calculate the following for this wave:

a. frequency

b. period

c. speed.

<u>Solution</u>

Given: λ = 20 cm = 0,2 m

a. 6 full wavelengths pass a point in 3 seconds, so the number of wavelengths in 1 second are:

$$\text{frequency} = \frac{\text{number of wavelengths}}{\Delta t} = \frac{6}{3s} = 2\ s^{-1}$$

Therefore the frequency f = 2 Hz.

b. $T = \frac{1}{f} = \frac{1}{2\ s^{-1}} = 0{,}5$ seconds

c. $v = f \times \lambda = 2\ Hz \times 0{,}2\ m = 0{,}4\ m{\cdot}s^{-1}$

Activity 2: ***Test your understanding of the characteristics of a wave***

Answer the following questions.

Ntombi is sitting in a boat watching water waves. She observes that 4 wavelengths of a wave pass the front of her boat in 6 seconds, and that the distance between three consecutive peaks is 60 cm. Calculate the following for this wave:

a. The wavelength
b. The frequency
c. The period
d. The speed

MAIN IDEAS:

- The **frequency** (f) of a wave is the number of complete wavelengths (a peak and a trough) that pass an observer in one second. Frequency is measured in Hz.
- The **period** (T) of a wave is the time that it takes for a complete wavelength to pass by a fixed point. The period is measured in seconds.
- The period is the inverse of the frequency: $T = 1/f$
- The **speed** (v) of a wave can be found using the wave equation: $v = f \times \lambda$

Unit 2.	Longitudinal waves

Learning outcomes:

When you have completed this unit, you should be able to:

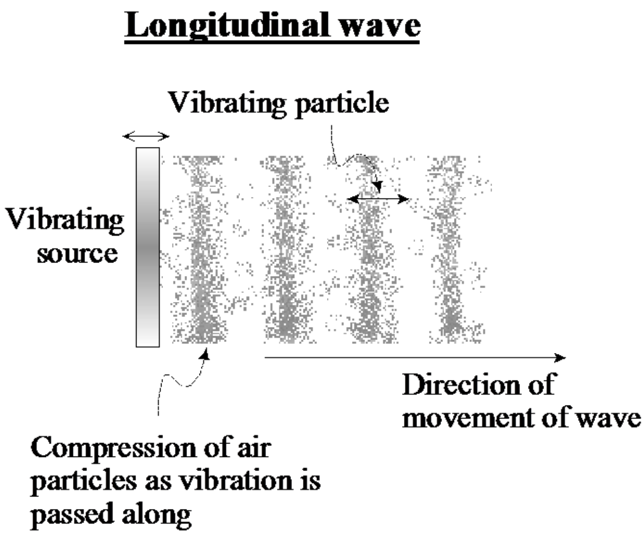
- differentiate between transverse and longitudinal waves, and give examples of longitudinal waves;
- define frequency, wavelength, period and amplitude for a longitudinal wave;
- apply the wave equation to solve problems involving longitudinal waves, in familiar and novel contexts.

Introduction

In the previous unit you learnt about transverse waves. In this unit you will learn about a different kind of wave, called a longitudinal wave.

2.1. Structure of a longitudinal wave

The diagram on the right shows a longitudinal wave that is created by a vibrating source. The particles of the medium vibrate forwards and backwards, in the plane of the direction that the longitudinal wave is traveling in. In a similar way to the transverse wave, the particles do not move forward in the direction of the wave, but just vibrate backwards and forwards around a fixed position. Energy is moved forward with the wave.



The positions where the particles are closer together are called *compressions*, and the positions where the particles are further apart are called *rarefactions*.

The ***wavelength*** of a longitudinal wave is the distance between two consecutive parts of the wave that are identical to one another. The wavelength is therefore the distance between two successive compressions, or two successive rarefactions.

successive – next to one another, or one after each other

rarefaction - a decrease in density and pressure caused by the passage of a sound wave

compression – a squeezing of the particles of a medium

Activity 1: Making a longitudinal wave

<u>You will need:</u>

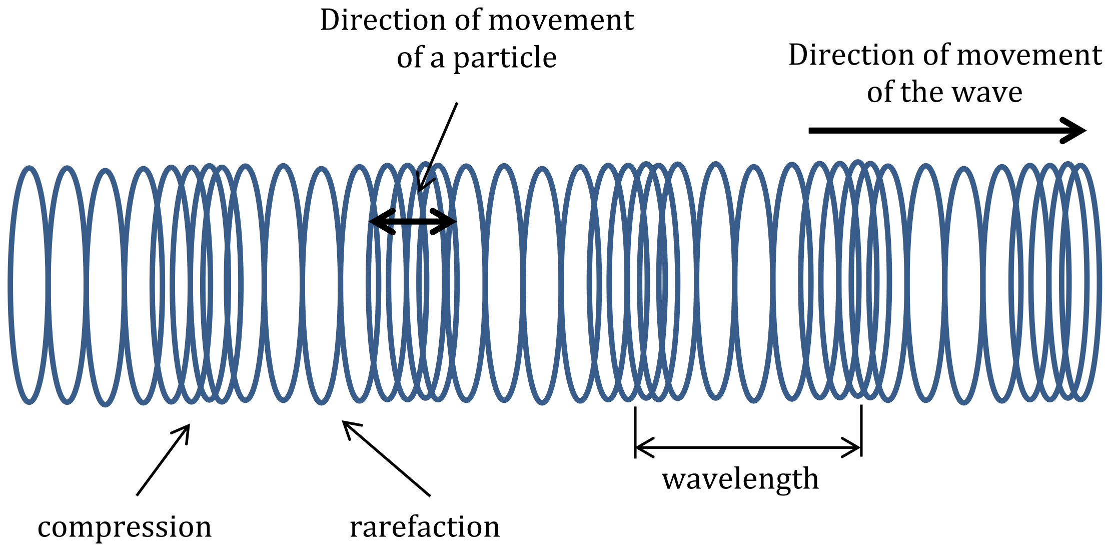
A long spring (slinky)
A piece of coloured cloth

1. Put the spring on a smooth floor or on a large desktop.
2. Two students should hold the ends of the spring so that the spring is slightly stretched.
3. One student should hold their end of the spring steady while the other moves theirs to create the wave. This student should move this end of the spring forward and then backward in a continual movement. What do you observe about the longitudinal wave? Write a description of this wave in your workbooks.
4. Can you identify the wavelength of the wave? Discuss your ideas in your group.
5. Tie a piece of coloured string to some point along the spring. This represents a particle on the spring. Create a longitudinal wave, and observe the movement of the string. What does this tell you about the movement of a particle due to a longitudinal wave? Discuss your observations in your group.
6. Recall that the amplitude of a wave is the maximum displacement of a particle from its rest position. By observing your piece of coloured string, can you identify the amplitude of this longitudinal wave?

Go to https://www.youtube.com/watch?v=GIkeGBXqWW0 to see a longitudinal wave on a slinky.

The diagram below shows what a longitudinal wave looks like on a long spring. This diagram shows the following:

- the compressions and rarefactions of the wave
- the wavelength of the wave
- the direction of motion of the wave
- the direction in which a particle of the medium moves



A longitudinal wave on a spring

MAIN IDEAS:

- In a longitudinal wave, the **particles** of the medium **vibrate** backwards and forwards around a fixed position.
- The direction of the movement of the wave is away from the source.
- The **wavelength** of a longitudinal wave is the distance between two successive compressions, or two successive rarefactions.

2.2. Properties of longitudinal waves

Similar to transverse waves, the **frequency (f)** of a longitudinal wave is the number of complete wavelengths that pass an observer in one second. The **period (T)** of a longitudinal wave is the time that it takes for one complete wavelength to pass by a fixed point. We can again express the period in terms of the frequency:

$$T = \frac{1}{f}$$

The **amplitude** of a longitudinal wave is the maximum displacement of a particles from their rest position.

The **speed** of a longitudinal wave is the distance travelled by a wave compression or rarefaction divided by the time taken. The wave equation can be applied to longitudinal waves:

$$v = f \times \lambda$$

where v is the speed of the wave, measured in metres per second ($m{\cdot}s^{-1}$)

f is the frequency, measured in hertz (Hz) or (s^{-1})

λ is the wavelength of the wave, measured in metres (m).

Example: *(Try to solve this problem* ***on your own or with a fellow student*** *while covering the solution, and then check your work using the solution below).*

A longitudinal wave is formed on a slinky so that it travels along the slinky with a speed of 15 $cm{\cdot}s^{-1}$. The distance between two consecutive rarefactions is 0,1 m. What is the frequency of this wave?

Solution

Given: λ = 0,1 m; v = 15 $cm{\cdot}s^{-1} \times 10^{-2}$ = 0,15 $m{\cdot}s^{-1}$

From the wave equation v = f x **λ** we can solve for the frequency:

$$f = \frac{v}{\lambda} = \frac{0{,}15\ m{\cdot}s^{-1}}{0{,}1\ m} = 1{,}5\ s^{-1} = 1{,}5\ Hz$$

Activity 2 – *Properties of a Longitudinal Wave*

Answer the following questions:

A longitudinal wave is formed on a slinky so that it travels along the slinky with a speed of 5 $cm{\cdot}s^{-1}$. A particle of the medium moves a total distance of 1×10^{-3} m between its maximum vibrations backwards and forwards. The distance between three successive compressions is $2{,}5 \times 10^{-2}$ m. Calculate the following for this wave:

a. The wavelength
b. The frequency
c. The period
d. The amplitude

MAIN IDEAS:

- The **frequency** of a longitudinal wave is the number of **complete wavelengths** (a compression and a rarefaction) that pass an observer **in one second**.
- The **period** of a longitudinal wave is the time that it takes for a complete wavelength to pass by a fixed point.
- The period is the **inverse** of the frequency: $T = 1/f$
- The **amplitude** of a longitudinal wave is the maximum displacement of a particle from its rest position
- The **wave speed** of a longitudinal wave is the distance travelled by a wave compression divided by the time taken.
- The speed (v) of a wave can be found using the **wave equation**: $v = f \times \lambda$

2.3. Sound waves as examples of longitudinal waves

Sound waves are longitudinal waves that are created by a vibrating source, which causes the air particles closest to the source to vibrate. These particles then bump the ones next to them, passing energy on to them. As a result, the vibrations are sent outwards from the source of the sound as a pressure wave.

Activity 3: Observing the effect of a sound wave

You will need:

A bowl
Some plastic cling film
Some table salt

1. Pull the cling film over the opening of the bowl so that it is tight.
2. Pour some salt onto the cling film.
3. Make a loud noise by clapping your hands, or playing loud music nearby. Watch what happens to the grains of salt on the cling film.

In this activity you should have observed the grains of salt vibrating with the sound. This shows that sound causes a change in the air pressure, which sends out vibrations in the air particles. The way that we hear sound is that the eardrums in our ears vibrate when they receive the pressure vibrations, and send a message to our brain.

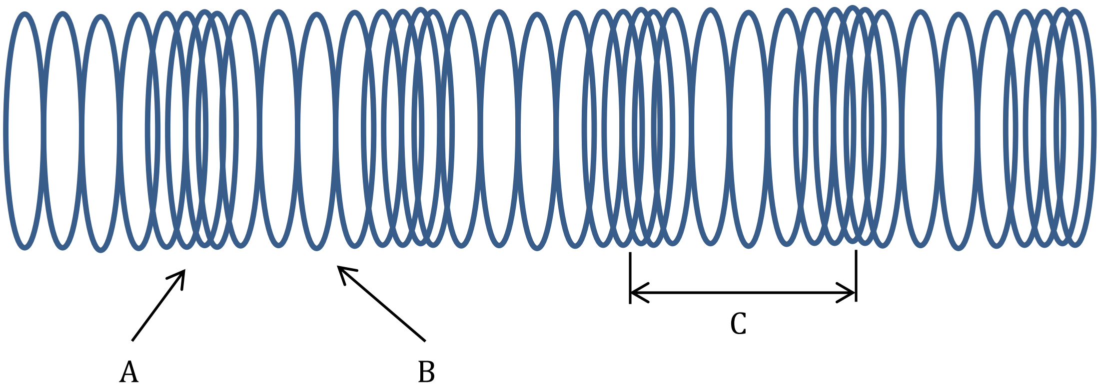
MAIN IDEAS:

- Sound waves are **longitudinal waves** that are created by vibrations in a medium in the direction of propagation.
- The vibrations cause a regular variation in the pressure of the medium. Sound waves are therefore **pressure waves**.

Assessment Activity: Transverse and longitudinal waves **Total marks = 50**

Answer the following questions to assess your understanding of waves:

1. Give an example of a transverse wave, and of a longitudinal wave. (2)
2. Draw a diagram of a transverse wave and label the following on your diagram (6)
   a. The rest position
   b. The wavelength
   c. The amplitude
   d. The peak
   e. The trough
   f. Use the letters P and Q to label two points on the wave that are in phase with each other.
3. Five full wavelengths of a wave that has a wavelength of 25 mm pass by a fixed point in a time of 2 seconds. Calculate the following for this wave:
   a. frequency (4)
   b. period (3)
   c. speed. (3)
4. The diagram below shows a longitudinal wave that is formed on a spring. The wave is moving from left to right.



a. Provide labels for the letters A to C. (3)
b. Describe the movement of the particles of the medium as the wave travels through it. (2)
c. Describe the movement of energy as the wave travels through the medium. (2)

5. Khaya and Thembi formed a longitudinal wave on a 2m long slinky. The wave took 1,6 seconds to move from one end of the slinky to the other. They tied a piece of string to the slinky, and observed that it moved a total distance of 5 mm backwards and forwards. They also observed that the distance between four successive compressions is 60 cm.
    a. Explain how they should move the slinky to create this wave. (2)
    b. Find the speed of the wave. (3)
    c. What is the wavelength of the wave? (2)
    d. Work out the number of complete wavelengths that pass the piece of coloured string in one second. (3)
    e. Find the time that it takes for one complete wavelength to pass a fixed point. (3)
    f. What is the amplitude of the wave? (2)

6. A sound wave with a frequency of 500 Hz travels 2,04 km through air in 6 seconds.
    a. Calculate the speed of sound in air. (3)
    b. Calculate the period of this sound wave. (3)
    c. Calculate the distance between two successive rarefactions of the sound wave. (4)

My Notes:

Use this space to write your own questions, comments or key points.

Summary of key learning:

- **Wavelength** (**λ**) is the distance between two parts of a wave that are in phase
- **Amplitude** is the distance between the peak of a wave and its rest position (or between the trough of a wave and its rest position)
- The **particles** of the medium move from side-to-side, **transverse** to the direction of the movement of the wave.
- The **frequency** (f) of a wave is the number of complete wavelengths (a peak and a trough) that pass an observer in one second. Frequency is measured in Hz.
- The **period** (T) of a wave is the time that it takes for a complete wavelength to pass by a fixed point. The period is measured in seconds.
- The period is the inverse of the frequency: $T = 1/f$
- The **speed** (v) of a wave is found using the wave equation: $v = f \times \lambda$
- In a longitudinal wave, the **particles** of the medium **vibrate** backwards and forwards around a fixed position.
- The direction of the movement of the wave is away from the source.
- The **wavelength** of a longitudinal wave is the distance between two successive compressions, or two successive rarefactions.
- The **frequency** of a longitudinal wave is the number of **complete wavelengths** (a compression and a rarefaction) that pass an observer **in one second**.
- The **period** of a longitudinal wave is the time that it takes for a complete wavelength to pass by a fixed point.
- The **amplitude** of a longitudinal wave is the maximum displacement of a particle from its rest position
- The **wave speed** of a longitudinal wave is the distance travelled by a wave compression divided by the time taken.
- Sound waves are **longitudinal waves** that are created by vibrations in a medium in the direction of propagation.
- The vibrations cause a regular variation in the pressure of the medium. Sound waves are therefore **pressure waves**.

Sub-topic 2. Geometrical optics

Content:

Unit 1: Reflection

Unit 2: Refraction

Unit 3: Total internal reflection

Unit 1. Reflection

Learning outcomes:

When you have completed this unit, you should be able to:

- describe reflection of light;
- define normal, angle of incidence and angle of reflection;
- state that, for reflection, the angle of incidence is equal to the angle of reflection;
- apply the concept of reflection in various familiar and novel contexts.

Introduction

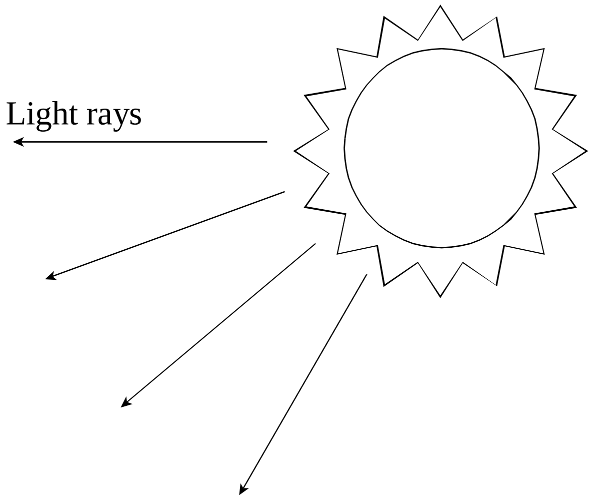
Light is everywhere around us. We would not be able to see anything if it was not for light. Nearly all of the energy on earth is caused by light that comes from the sun. In our modern society we have found many ways of making use of the properties of light, for example in microscopes, telescopes and eye-glasses.

In this unit you will start with learning about the properties of light, and then you will discover what happens to light to enable you to see your reflection in a mirror.

1.1. Properties of light

When you stand in the sun, you will notice that you have a shadow. This shadow is caused when we block the sun's rays. This shows us that light travels in straight lines. Sound waves can bend around corners, but light rays cannot. This is why you can hear somebody that is around the corner but not be able to see them.

When light is given off by the sun or by a light bulb, the light waves are sent out in all different directions. We can never actually see a single light wave. In science we use the term "light ray" to describe a group of light waves which are all travelling in the same direction. We use arrows to show the direction of these light rays.



Light travels with a constant speed as it travels through a medium. When light travels through a vacuum, it has a speed of 3×10^8 m·s^{-1}. This is the maximum speed that light can have. The speed of light in air is slightly less, as it is 2,998 $\times 10^8$ m·s^{-1}, but in this course we will round this off to 3×10^8 m·s^{-1}.

Medium – the substance that a wave travels through

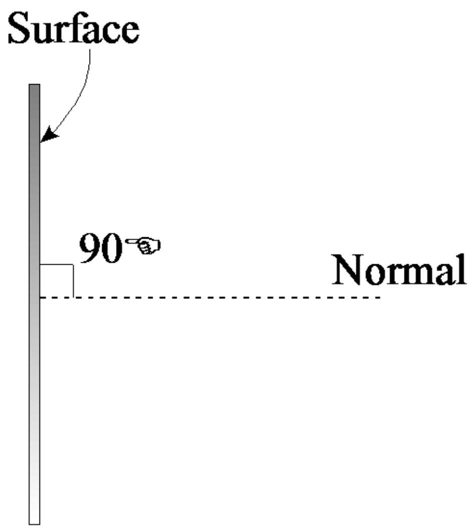
Vacuum - a space that is empty of matter

MAIN IDEA:

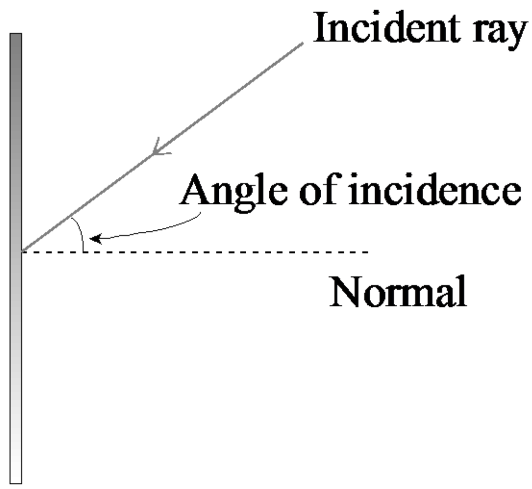
- Light travels in **straight lines**
- A group of light waves that travel in the same direction are called a **light ray**
- Light travels with a **constant speed** in a medium
- The speed of light in a vacuum is 3×10^8 m·s^{-1}.

1.2. Reflection of light

When we shine light rays onto a surface, we can measure the angle that these light rays make by constructing a line that is at right angles to a surface (or perpendicular to it). This line is called the **normal to the surface**. For an example of this, see the diagram on the right.



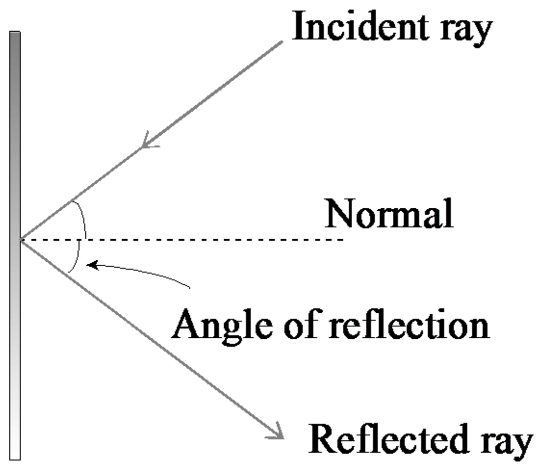
Normal – at right angles



The normal to the surface is important because when we observe the behaviour of light at surfaces, we always measure angles from the normal. In other words, we put the zero line of our protractor along the normal, and measure the angle of the light ray from this line.

Incident – falling on, or striking something

We call any ray that falls onto a surface an ***incident ray***. The angle that this ray makes with the normal is called the ***angle of incidence***.



We call any ray that is reflected off a surface the ***reflected ray***. The angle that this ray makes with the normal is called the ***angle of reflection***.

In the following activity you will explore the relationship between the angle of incidence and the angle of reflection.

Activity 1: Exploring the reflection of a ray of light

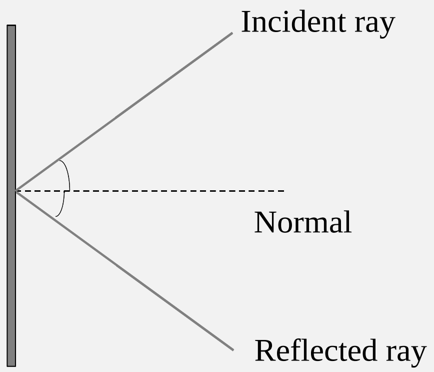
<u>You will need:</u>

a torch, a piece of cardboard, scissors
a piece of clean white paper, a mirror
a pencil, a protractor, a pin

(NOTE TO STUDENTS: If you need help with doing the diagrams in this activity, you will see some hints after the activity.)

1. Cut a narrow slit in the cardboard.
2. Place your piece of white paper on a table top, and shine your torch through the slit in the cardboard. Can you see the light ray coming through the slit?

3. Position your mirror so that the light ray shines onto it. This is your *incident ray*.
4. Now move the mirror from side to side. Can you see the reflection of the light ray? This ray that is reflected from the mirror is called the *reflected ray.*
5. Can you notice any relationship between the angle that the incident ray makes with the normal to the surface of the mirror, and the angle that the reflected ray makes with the normal? (This angle is called the *angle of reflection*). Write down your observations.
6. Fix your mirror in one position on the page where you see a clear incident and reflected ray. Draw a line along the surface of the mirror.
7. Remove your mirror, and construct a normal to the surface. Put the mirror back into place as accurately as possible, making sure that you can see the incident and reflected rays clearly. Adjust your torch and cardboard slightly so that the incident ray hits the surface of the mirror at the same point where the normal to the surface meets the surface. (Look at the diagram if you are unsure about this.)



Ray diagram showing angle of incidence and angle of reflection

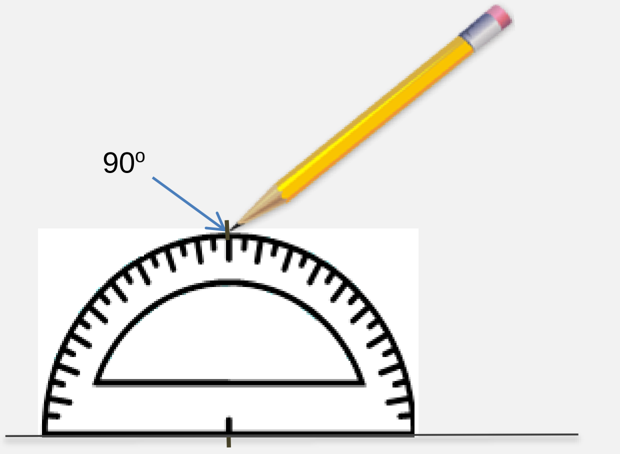
8. Use your pin to make three dots that lie along the path of the incident ray, and three dots that lie along the path of the reflected ray.
9. Remove your mirror again, and use your ruler to draw in the incident ray by connecting the three dots that you have made with your pin. Do the same for the reflected ray. You have just constructed what is called a "*ray diagram*".
10. Measure the angle of incidence and the angle of reflection using a protractor. (Remember to measure these angles from the normal). Record your readings.
11. What can you conclude from your measurements?
12. Repeat this experiment for 3 different angles of incidence, and record your results in a table.
13. Write a conclusion to this activity based on your observations.

Look at the video at https://www.youtube.com/watch?v=ETF2-Zz3J18 to see a practical on the reflection of light.

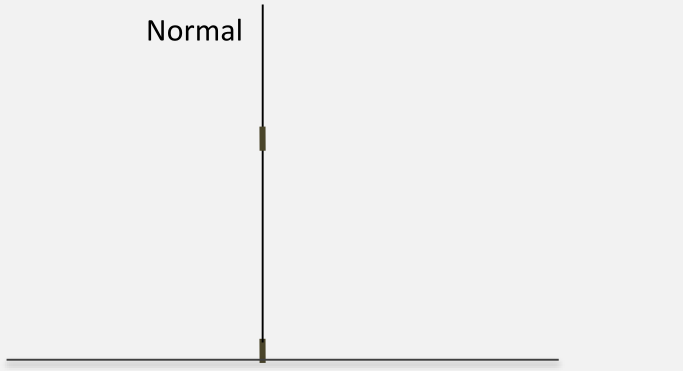
Guidance for doing ray diagrams:

To construct a line that is the **normal** to a surface, do the following:

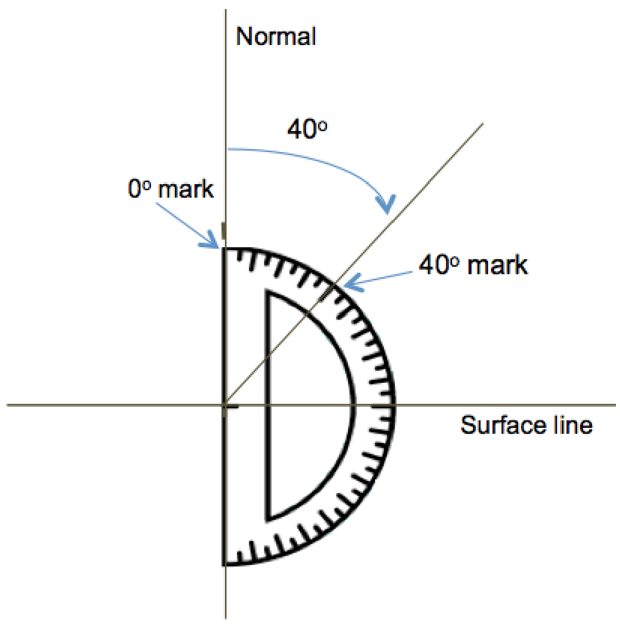
- Draw a straight line along the surface
- Make a mark on this line where the incident ray will strike the surface.
- Position a protractor so that its zero line is along the surface line, and its centre is on this mark that you have made on the surface.
- Make a mark on the page at the 90o line of the protractor (see the diagram).



- Remove the protractor and use a ruler to join the mark that is on the surface with the mark that you have made at the 90° line of the protractor.
- Extend this line a little bit. This line is the normal to the surface.



- When you are measuring the angles of incidence and reflection, make sure that you position your protractor with the zero line along the normal, and the centre of the protractor at the point where the ray strikes the surface.
- The diagram on the right shows how an angle of 40° is measured.

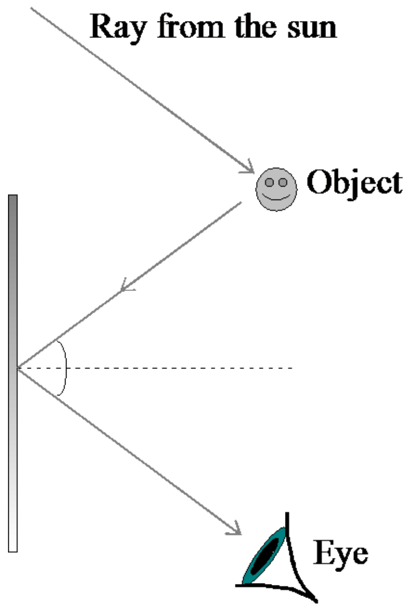


In Activity 1, you should have observed that, when light is reflected (bounced) off a surface, the angle of reflection is always equal to the angle of incidence. This is called the **law of reflection**.

MAIN IDEA:

- The normal to a surface is a line that is at right angle to the surface.
- The angle that an incident ray of light makes with the normal to the surface is called the ***angle of incidence***.
- The angle that a reflected ray of light makes with the normal to the surface is called the ***angle of reflection***.
- The law of reflection states that for a light ray that is reflected off a surface:
  angle of incidence = angle of reflection

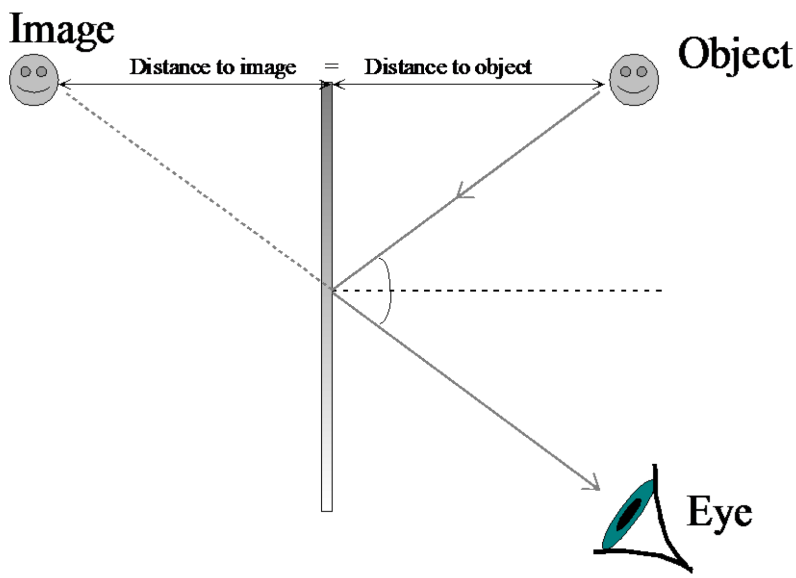
1.3. How the eye sees an image formed by reflection



Rays that are reflected do not necessarily need to come from a ray box or torch. If you stand an object in front of a mirror and look into the mirror, you will see a reflection of that object. Here light rays come from the sun and are reflected off the object into the mirror. They are reflected from the mirror into your eyes.

The image is formed at the **same distance** behind the mirror as the distance between the object and the mirror. The ray diagram shown below should help you to see this.

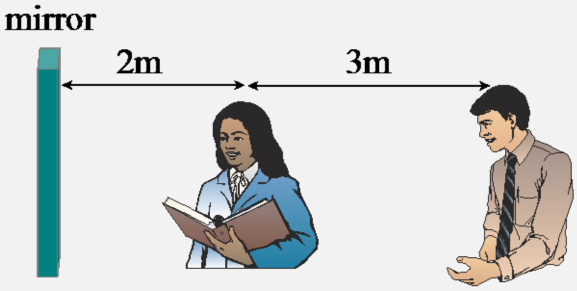
Light does not actually pass through the image that is formed. For this reason we call it a virtual image. This virtual image is upright, but is laterally inverted (in other words, the left and right hand sides are swapped around).



Activity 2: Reflection from a mirror

Answer the following questions:

1. Nosipho is standing 2m from a mirror, reading her book. Gareth is standing 3m behind Nosipho.



   a. If Nosipho looks at Gareth's reflection in the mirror, where will she see his image formed?

   b. If Gareth looks at Nosipho's image in the mirror, where will he see her image formed?

2. The front of an ambulance has the word written backwards, as shown in the diagram on the right. Can you explain this?

MAIN IDEA:

- When the eye sees a reflection in a mirror, the image is formed at the same distance behind the mirror as the distance between the object and the mirror, and is laterally inverted (the left and right hand sides are swapped around).

Unit 2.	Refraction

Learning outcomes:

When you have completed this unit, you should be able to:

- define refraction as a change of wave speed in different media, while the frequency remains constant;
- define angle of refraction, refractive index and optical density;
- draw ray diagrams to show the path of light as it travels between mediums with different optical densities;
- apply the concept of refraction in various familiar and novel contexts.

Introduction

Sometimes when you look at a straw in water, it looks like the straw is broken or bent. In this unit we will explore the scientific reason for this, and see some of the ways in which this phenomenon is applied in everyday life.

2.1. Refraction of light

When light moves from one medium into a different medium, its speed changes. This change of speed is called **refraction**. Each medium changes the speed of light by a different amount. The measure of the speed of light in a medium is called the **refractive index** of the medium. We can calculate the refractive index of a medium using the following equation:

$$n = \frac{c}{v}$$

where n is the refractive index of the medium (it has no units)

c is the speed of light in a vacuum = 3×10^8 m/s

v is the speed of light in the new medium, measured in units of m/s

This refractive index is linked with the **optical density** of the medium. The optical density is defined as "a measure of the extent to which a substance transmits light or other electromagnetic radiation". In other words, it is a measure of how much of the light is absorbed by the substance. The more light the substance absorbs, the greater its optical density. Also, the more optically dense a substance is, the slower a wave will move through the material. So a substance with a high optical density has a high refractive index, because it slows the light wave down more than a substance with a low optical density.

absorbed – taken in, or sucked in

As light enters a new medium and is refracted, although the speed of the light waves has changed, their **frequency is not changed**, and remains constant. The wave equation tells us that the relationship between wave speed, frequency and wavelength is $v = f \times$

λ. So if the speed changes, and the frequency is unchanged, this means that the **wavelength is changed** as the wave enters a new medium.

Example: *(Try to solve this problem **on your own or with a fellow student** while covering the solution, and then check your work using the solution below).*

The speed of light in Perspex is $2{,}04 \times 10^8\ m{\cdot}s^{-1}$. What is the refractive index of Perspex?

Solution

$$n_{Perspex} = \frac{c}{v} = \frac{3 \times 10^8\ m{\cdot}s^{-1}}{2{,}04 \times 10^8\ m{\cdot}s^{-1}} = 1{,}47$$

Therefore the refractive index of Perspex is 1,47.

Activity 1: *Refractive index*

Answer the following questions.

e. The speed of light in water is $2{,}25 \times 10^5\ km{\cdot}s^{-1}$. Calculate the refractive index of Perspex.

f. A light wave with a frequency of $7{,}5 \times 10^{14}$ Hz enters glass that has a refractive index of 1,5. Find the following:
   a. The speed of the light wave in glass.
   b. The frequency of the light wave in glass.
   c. The wavelength of the light wave in glass.

g. The speed of a ray of red light in Substance X is greater than the speed of a ray of red light in Substance Y. Answer the following questions, and explain your reasoning in each case.
   a. Which substance has a greater optical density, Substance X or Y?
   b. In which substance is the wavelength of the red light the longest, in Substance X or Y?
   c. In which substance is the frequency of the red light the highest, in Substance X or Y?

MAIN IDEAS:

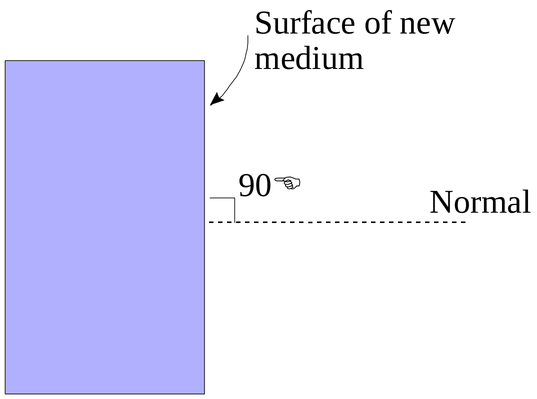
- When light enters a new medium, its speed changes. This is called **refraction**.
- The **refractive index** of a medium is the measure of the speed of light in the medium.
- The refractive index of a medium can be calculated using the equation:

$$n = \frac{c}{v}$$

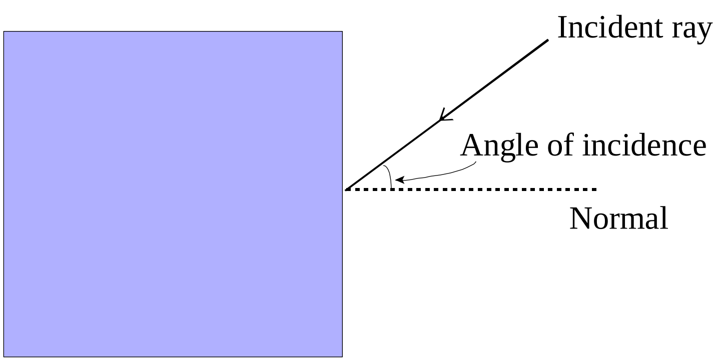
- The **optical density** is defined as a measure of the extent to which a substance transmits light or other electromagnetic radiation.
- When light enters a new medium its speed and wavelength changes, but its frequency stays the same.

2.2. Ray diagrams of refraction

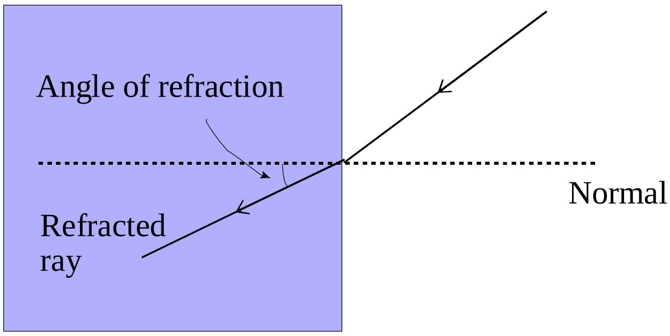
In a similar way to what you learnt in Unit 1 (Reflection), whenever we work with light rays shining onto a surface, we construct the normal to the surface. We therefore do this when light shines onto the surface between one medium and another different medium.



The angle that the incident ray makes with the normal is called the ***angle of incidence***.



The light ray that has entered the new medium is called the refracted ray. The angle that this ray makes with the normal is called the ***angle of refraction***.



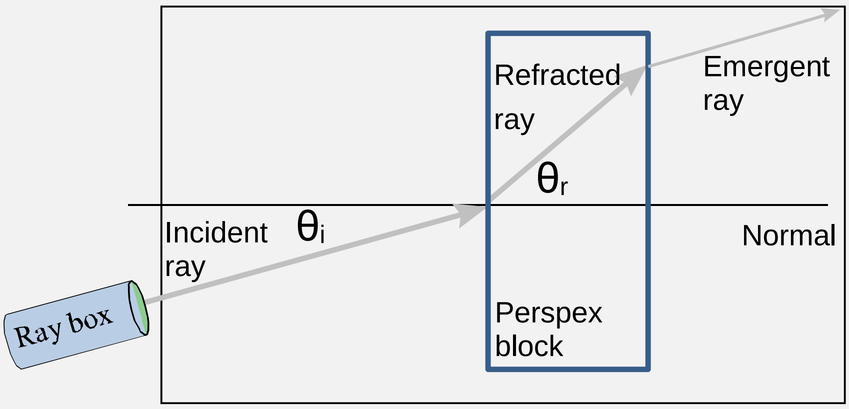
Activity 2: Constructing ray diagrams for refraction

You will need:

a rectangular block of Perspex or glass
a ray box (OR a torch shining through a slit in a piece of cardboard)
a clean sheet of A4 paper
a pencil
a ruler
a protractor

Experimental Procedure:

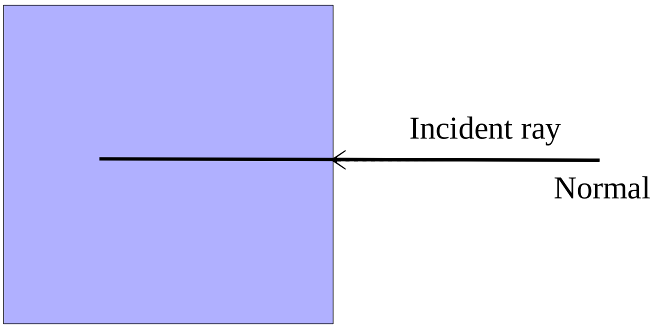
1. Place your Perspex block on the A4 paper, and position your ray box so that a light ray shines onto one of the surfaces of your Perspex block at an angle (as shown in the diagram). Draw a line around your rectangular block to mark its position on the paper.



2. What do you observe about the direction of the refracted ray inside the block? Is it bent towards or away from the normal to the surface? Write down your observations.
3. Now look carefully at the ray that comes out on the other side of the block, as it shines back into the air (this is called the emergent ray). Is it bent towards or away from the normal?
4. Use your pencil to make some marks on your piece of paper along the incident ray.
5. Make a mark at the position where the refracted ray comes out of the Perspex block. Remove the block and use your ruler to draw in the incident ray, the refracted ray, and the emergent ray on your piece of paper.
6. Construct the normal to the surface at the position where the incident ray strikes the surface, and at the position where the refracted ray strikes the opposite surface before emerging into the air again.
7. Use your protractor to measure the angle of incidence and the angle of refraction. Remember to measure your angles **from the normal**.
8. How does the angle of incidence compare with the angle of refraction?

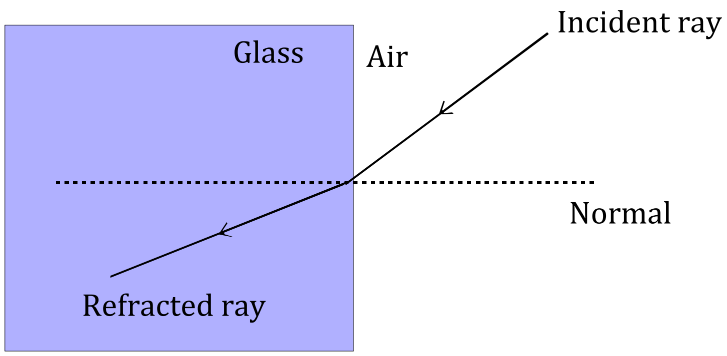
See the video at https://www.youtube.com/watch?v=qrUCxvpaRc8 to see a practical of light refracted through a glass block.

When light is shone onto a surface along the normal to that surface, it is transmitted into the new medium without bending, as the diagram below shows.



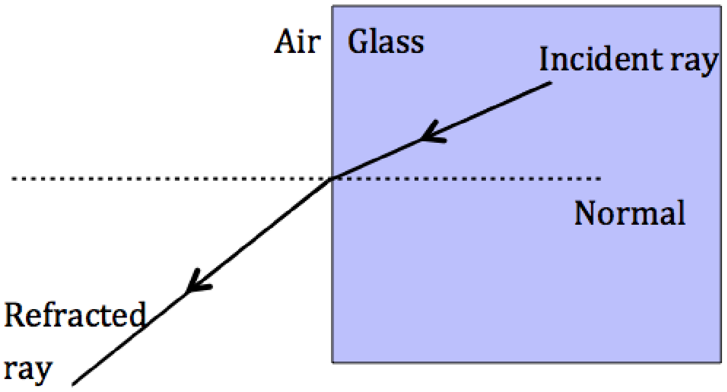
Light that is incident along the normal does not bend.

When the light ray hits the surface at an angle to the normal, this ray is bent as soon as it travels into the new medium. If the ray is moving from a medium with a low optical density (like air) to a medium with higher optical density (like glass), it is bent **towards the normal**. In other words, the angle of refraction is smaller than the angle of incidence.



Light ray from less dense to more dense medium bends towards the normal (e.g. from air to glass)

If the ray is moving from a medium with a high optical density (like glass) to a medium with lower optical density (like air), it is bent **away from the normal**. In other words, the angle of refraction is greater than the angle of incidence.



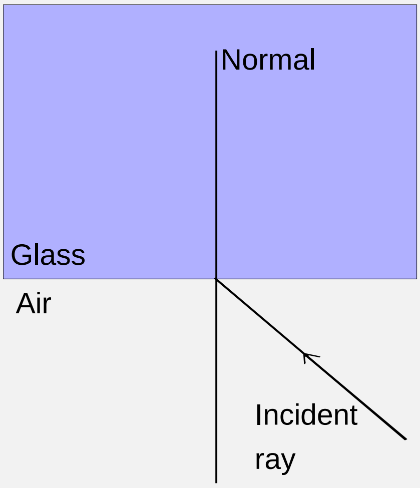
Light ray from more dense to less dense medium bends away from the normal (e.g. from glass to air)

Activity 3: Test your understanding of refraction

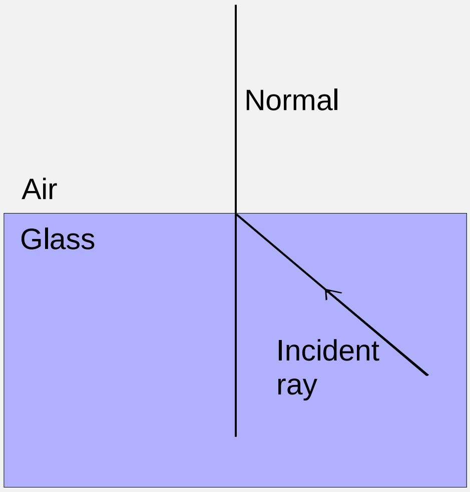
Answer the following question.

Complete the ray diagrams shown below to show the path of light as the ray enters the new medium.

(a)



(b)



MAIN IDEAS: When a light ray moves from one medium into a different medium:

- if it is incident at right angles to that surface (along the normal), it is transmitted into the new medium **without bending**
- if it is moving from a medium with a low optical density to a medium with higher optical density, it is bent **towards the normal**
- if it is moving from a medium with a high optical density to a medium with lower optical density, it is bent **away from the normal**

Unit 3. Total internal reflection

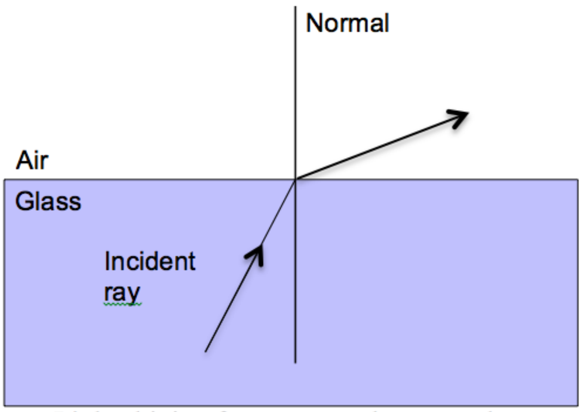
Learning outcomes:

When you have completed this unit, you should be able to:

- explain critical angle and total internal reflection;
- apply the concepts of critical angle and total internal reflection in various familiar and novel contexts.

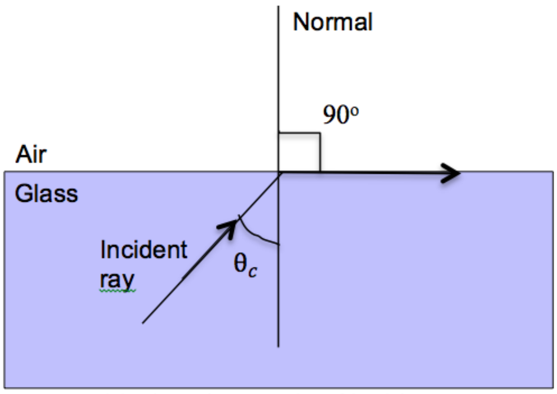
3.1. The critical angle

Recall from Unit 2 that a light ray that shines from a more dense to a less dense medium is bent away from the normal.



Light shining from a more dense to a less dense medium is bent away from the normal

If you keep increasing the angle of incidence, at some point the refracted ray that emerges into the less dense medium disappears. The angle of incidence for which the refracted ray just disappears (or shines along the surface of the incident medium) is called the ***critical angle*** (θ_c).



Critical angle = angle of incidence when angle of refraction = 90°

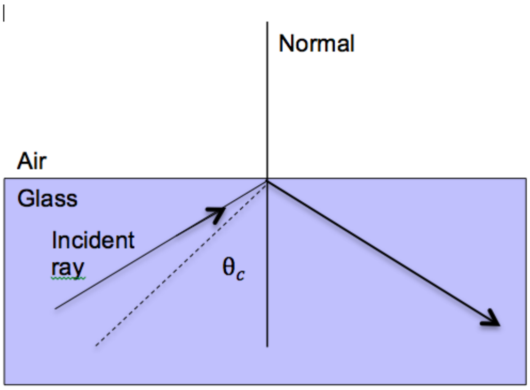
In other words, the critical angle is the minimum angle of incidence for which there is no emergent ray. This only happens for light shining from a more optically dense to a less optically dense medium. The greater the optical density of the medium, the smaller the critical angle in that medium.

MAIN IDEAS:

- The ***critical angle*** is the minimum angle of incidence for which there is no emergent ray. This only happens for light shining from a more optically dense to a less optically dense medium.
- The greater the optical density of the medium, the smaller the critical angle in that medium.

3.2. Total internal reflection

When the angle of incidence is bigger than the critical angle ($\theta_i > \theta_c$), there is no ray emerging into the air, and as a result there is *total internal reflection* inside the glass block. This means that all of the light that shines onto the boundary between two media is reflected, and none is refracted.

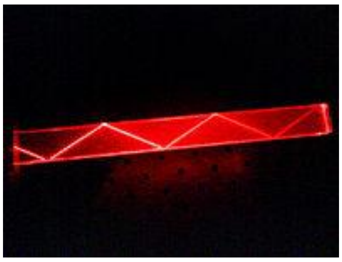


Total internal reflection when angle of incidence > critical angle

A video of the critical angle and total internal reflection can be seen at https://www.youtube.com/watch?v=8VZHym6HqVU

The conditions that we need for total internal reflection to take place are:

- light must be incident from the medium that has a higher optical density, and must be approaching the medium that has a lower optical density
- the angle of incidence must be bigger than the critical angle ($\theta_i > \theta_c$)



Total internal reflection of a light ray

Activity 1: Observing total internal reflection

You will need:

A 2-litre plastic bottle
A torch
Sellotape

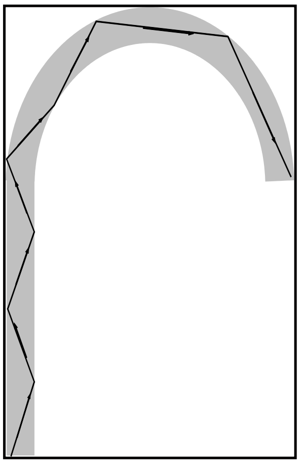
1. Make a small hole in the juice bottle, about a third of the way from the bottom of the bottle. Cover this hole with sellotape.
2. Fill the bottle with water.
3. In the darkest place you can find, shine the torch into the bottle so that the light from the torch falls onto the hole that you made.
4. Remove the sellotape and look at the stream of water that comes out of the bottle?

MAIN IDEAS:

- ***Total internal reflection*** occurs when the angle of incidence is larger than the critical angle ($\theta_i > \theta_c$).
- Total internal reflection can only occur when light is incident from the medium with the greater optical density, and is approaching the medium that has a lower optical density.

3.3. Applications of total internal reflection

An optic fibre is made of glass that can be bent to any shape. The light is totally internally reflected inside the optic fibre. In this way, light can be transmitted from one place to another without any loss in the strength of the light, since none of the light is absorbed or refracted at the edges.



Total internal reflection of a light ray inside an optic fibre.

This principle of total internal reflection in optic fibres is applied in medicine, in an instrument called an *endoscope* that is used by doctors to examine parts of the body that cannot be seen by the human eye.

See the video at https://www.youtube.com/watch?v=0MwMkBET_5I to find out more about how optic fibres work.

Assessment Activity: Geometrical optics **Total marks = 60**

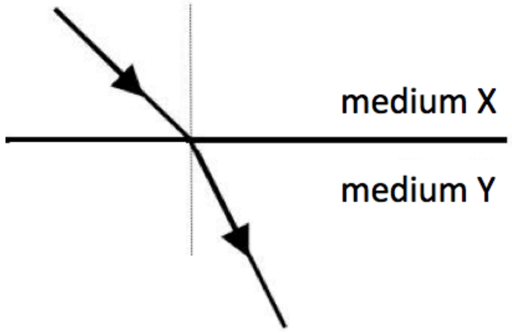
Answer the following questions to assess your understanding of geometrical optics:

Multiple choice questions:

Choose the correct answer for the questions below:

1. Kefilwe is running towards a mirror with a speed of 4 $m{\cdot}s^{-1}$. To Kefilwe it appears that her image in the mirror is moving: (3)
   A. towards her with a speed of 4 $m{\cdot}s^{-1}$
   B. towards her at a speed of 8 $m{\cdot}s^{-1}$
   C. away from her with a speed of 4 $m{\cdot}s^{-1}$
   D. away from her with a speed of 8 $m{\cdot}s^{-1}$

2. Which of the following is correct for a light ray moving from medium X into medium Y? (3)



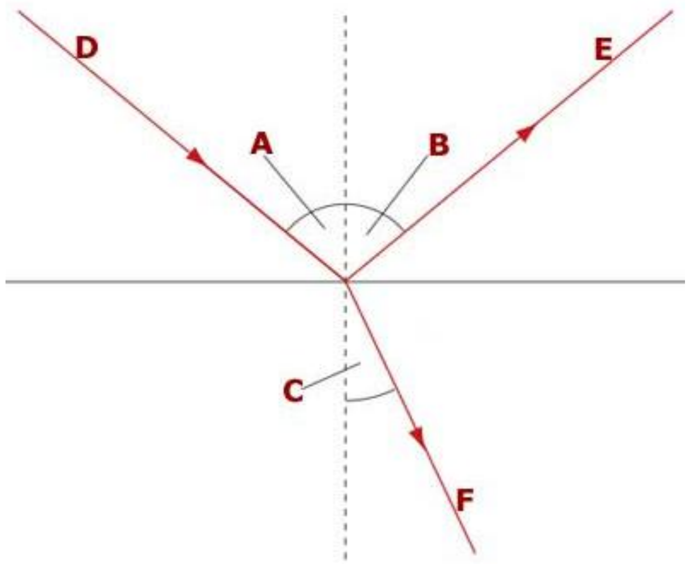
	Optical density	Speed of light
A.	medium X < medium Y	speed in X < speed in Y
B.	medium X > medium Y	speed in X < speed in Y
C.	medium X < medium Y	speed in X > speed in Y
D.	medium X > medium Y	speed in X > speed in Y

3. Which of the following statements is true for medium X and medium Y: (3)
   A. A light ray moving from medium X into medium Y will be totally internally reflected if the angle of incidence is greater than the critical angle.
   B. A light ray moving from medium X into medium Y will be totally internally reflected if the angle of incidence is less than the critical angle.
   C. A light ray moving from medium Y into medium X will be totally internally reflected if the angle of incidence is greater than the critical angle.
   D. A light ray moving from medium Y into medium X will be totally internally reflected if the angle of incidence is less than the critical angle.

Written response questions:

1. The diagram below shows reflection and refraction of a light ray. Choose the correct label from the phrases in the box for each letter A to F. (6)

Incident ray	Critical angle	Angle of refraction
Refracted ray	Angle of incidence	Reflected ray
Refractive index	Angle of reflection	Total internal reflection



A = ______________________________

B = ______________________________

C = ______________________________

D = ______________________________

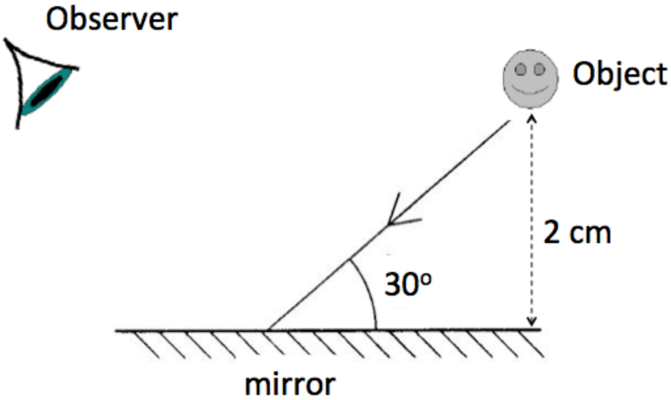
E = ______________________________

F = ______________________________

2. State the law of reflection. (2)

3. Write a sentence that describes what the following terms mean:
   a. Normal to the surface (2)
   b. Angle of incidence. (2)
   c. Angle of reflection. (2)
   d. Angle of refraction. (2)
   e. Critical angle. (2)
   f. Total internal reflection. (2)

4. The diagram shows a ray of light that is incident on a mirror.



a. What is the angle of incidence? (2)

b. What is the angle of reflection? (2)

c. What is the angle that the reflected ray makes with the surface of the mirror? (2)

d. If the object is 2 cm away from the mirror, describe how and where an image will be formed by an observer. (3)

5. Tumelo sits in a chair, looking into a mirror, which is 1,5 m away from him. There is a soccer poster on the wall, which is 60 cm behind him. How far away from his eyes does the poster appear to be? Show your working clearly. (4)

6. A light wave has a frequency of 5×10^{14} Hz and a speed of 3×10^{8} $m \cdot s^{-1}$ in air. When this wave enters diamond, its speed changes to $1{,}25 \times 10^{8}$ $m \cdot s^{-1}$. Find the following:

a. The refractive index of the diamond. (3)

b. The frequency of the light wave in diamond. (2)

c. The wavelength of the light wave in diamond. (3)

7. Study the information in the table and answer the questions that follow:

Medium	Refractive index
ice	1,31
ethanol	1,37
ruby	1,54

a. Which of these mediums has the smallest critical angle for a light ray traveling from the medium into air? (2)

b. In which of these mediums does light slow down the most? (2)

c. If a light ray travels from ethanol into ruby, does it bend towards or away from the normal? Explain your answer. (3)

d. Calculate the speed of light in ruby. (3)

My Notes:

Use this space to write your own questions, comments or key points.

Summary of key learning:

- Light travels in **straight lines**
- A group of light waves that travel in the same direction are called a **light ray**
- Light travels with a **constant speed** in a medium
- The speed of light in a vacuum is 3×10^8 m/s.
- The normal to a surface is a line that is at right angle to the surface.
- The angle that an incident ray of light makes with the normal to the surface is called the ***angle of incidence***.
- The angle that a reflected ray of light makes with the normal to the surface is called the ***angle of reflection***.
- The law of reflection states that for a light ray that is reflected off a surface: angle of incidence = angle of reflection
- When the eye sees a reflection in a mirror, the image is formed at the same distance behind the mirror as the distance between the object and the mirror, and is laterally inverted (the left and right hand sides are swapped around).
- When light enters a new medium, its speed changes. This is called **refraction**.
- The **refractive index** of a medium is the measure of the speed of light in the medium.
- The refractive index of a medium can be calculated using the equation:

$$n = \frac{c}{v}$$

- The **optical density** is defined as a measure of the extent to which a substance transmits light or other electromagnetic radiation.
- When light enters a new medium its speed and wavelength changes, but its frequency stays the same.
- When a light ray moves from one medium into a different medium:
  - if it is incident at right angles to that surface (along the normal), it is transmitted into the new medium **without bending**
  - if it is moving from a medium with a low optical density to a medium with higher optical density, it is bent **towards the normal**
  - if it is moving from a medium with a high optical density to a medium with lower optical density, it is bent **away from the normal**
- The ***critical angle*** is the minimum angle of incidence for which there is no emergent ray. This only happens for light shining from a more optically dense to a less optically dense medium.

- The greater the optical density of the medium, the smaller the critical angle in that medium.
- ***Total internal reflection*** occurs when the angle of incidence is larger than the critical angle ($\theta_i > \theta_c$).
- Total internal reflection can only occur when light is incident from the medium with the greater optical density, and is approaching the medium that has a lower optical density.

Topic 4. Electricity and Magnetism

Introduction

Electricity and magnetism are very important aspects of Physics, because they underlie many of the daily tools and instruments that we rely on. The ability to understand and harness electricity has resulted in the development of lights, televisions and other electrical devices. An understanding of the relationship between electricity and magnetism has led to the development of motorcars and other machines.

In this topic you will learn about electrostatics, which is the force between electrical charges that are not moving. You will also learn about electrical circuits, and how to solve problems with these. You will be introduced to magnets and the magnetic field, and then explore the relationship between electricity and magnetism (electromagnetism).

Sub-topic 1. Electrostatics

Content:

Unit 1: Coulomb's Law

Unit 2: The electric field

Unit 1. Coulomb's Law

Learning outcomes:

When you have completed this unit, you should be able to:

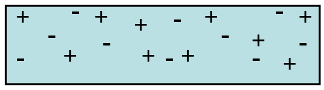
- describe charge as either positive or negative, and measured in coulombs;
- state that unlike charges attract and like charges repel;
- explain the attraction between a charged object and a neutral object (polarization);
- state Coulomb's Law in words and mathematically: $F = \frac{kQ_1Q_2}{r^2}$;
- solve problems using Coulomb's Law to calculate the force exerted on a charge by one or more charges in 1-dimension.

Introduction

Have you experienced a small shock when you touch a metal object? Or maybe you have noticed sparks when you take your jersey off. You experience these things because of static electricity, or electrostatics. In this unit you will explore the interactions between different kinds of charges, and you will learn about how this links with some of your everyday experiences.

1.1. Two types of charge

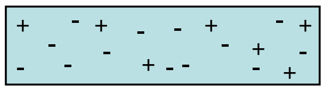
In the Chemistry component of this course you learn that all objects are made of atoms, which have got two different types of charge, positive charges called protons, and negative charges called electrons. Objects usually have an equal number of positive and negative charges, and so the object is ***neutral***.



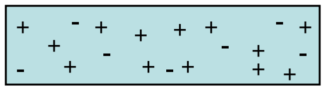
Neutral object

An object can be charged by friction, for example by rubbing the object. The process of rubbing causes electrons to be added to, or removed from, an object. This causes an unbalanced number of positive and negative charges, and as a result the object becomes *charged*.

- When there are more electrons than protons, the object becomes ***negatively charged***.
- When there are fewer electrons than protons, the object becomes ***positively charged***.



Negatively charged



Positively charged

1.2. Interactions between charges

When two charged objects are brought near to each other, there is a force between them. In the following activities, you will investigate the forces between charged objects.

Activity 1: Investigating charge

<u>You will need:</u>

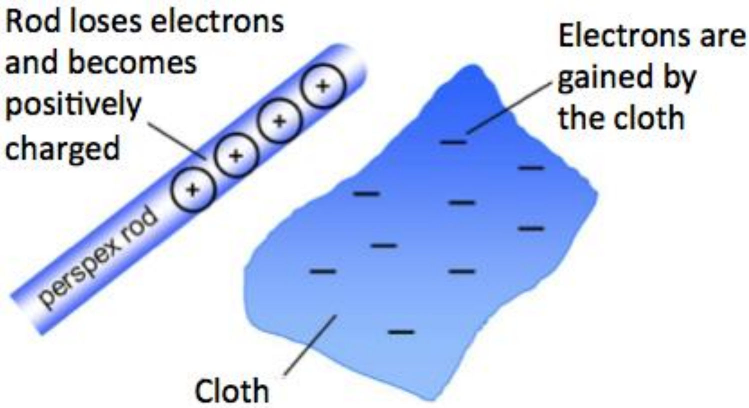
a plastic ruler or a perspex rod
a piece of paper that has been torn into small pieces

8. Rub the ruler or perspex rod with a piece of cloth. Bring this ruler near to your small pieces of torn paper. What do you observe?

9. Rub the other objects that you have collected in the same way, and hold each of them near to the pieces of paper. Do they all have the same effect on the pieces of paper?

When you rub a ruler with a piece of cloth, it becomes charged. The process of rubbing causes the small negatively charged electrons to be added to, or removed from, an object. These electrons are not created by the rubbing process, but already exist in all objects.

The diagram below shows what happens when a perspex rod is rubbed by a piece of cloth, causing the rod to be positively charged.



From your investigation you will notice that there is a force of **attraction** between the charged ruler and the bits of paper.

MAIN IDEA:

- There are **two types of charge**: positive charges, and negative charges.
- When there is an unbalanced number of positive and negative charges on an object, the object becomes ***charged***.
- When there are more electrons than protons, the object is ***negatively charged***.
- When there are fewer electrons than protons, the object is ***positively charged***.

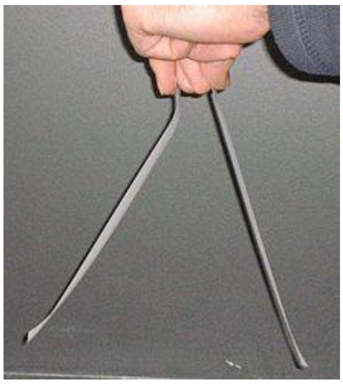
Activity 2: Investigating the interaction between two objects with the same charge

<u>You will need</u>:
A piece of paper that has been torn into small pieces
Some scotch magic tape

1. Stick a length of magic tape to your desktop, and pull it up very quickly. With the sticky side facing upwards, bring this magic tape close to the bits of paper. What do you notice?

2. Stick two lengths of magic tape to your desktop. Pull them up very quickly, and hold them close to one another, and look carefully to see how they move. What do you observe?

3. When two objects made from the same material are charged in the same way, they must have the same type of charge on them (either both positive, or both negative). We say that they have **like charges**. From your investigation, what can you infer about the effect that objects with like charges have on each other?



What is happening?

In this investigation, you would have observed that two objects that are charged in the same way push each other apart, or repel one another. We can therefore conclude that **like charges repel one another**.

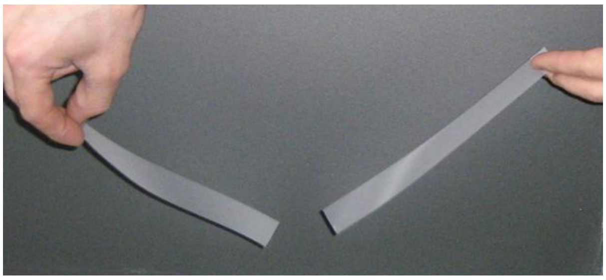
Activity 3: Investigating the interaction between two objects with opposite charge

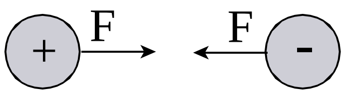
> You will need:
> A piece of paper that has been torn into small pieces
> Some scotch magic tape

1. Stick two lengths of tape on top of one another, so that the sticky side of one is placed onto the non-sticky side of the other. Remove any excess charges that might be on this combination of tape by rubbing the non-sticky side against your lips. Test that there is no charge on this combination of tape using your torn pieces of paper.
2. Now pull these two lengths of tape apart very quickly. Bring each of them near to your pieces of paper. You will notice that they are both charged by pulling them apart.
3. Stick your lengths of tape together again, rub them against your lips, and then pull them apart again. Bring them near to one another, and look carefully to see how they move. What do you observe?
4. When you pull two pieces of tape away from each other, you will notice that they each become charged. Recall that objects are charged by either having electrons added to them, or removed from them. Since the combination of tapes had no charge, pulling them apart must have caused one of them to gain electrons, and the other to lose electrons. They therefore have **opposite charges** to one another. From your observations, what can you infer about the effect that objects with opposite charges have on each other?

What is happening?

In this investigation you would have observed that two objects that have opposite charges are pulled towards each other, or attract one another. We can therefore conclude that **unlike charges attract one another**.



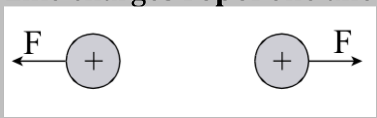


Activity 4: Test your knowledge of forces between charges

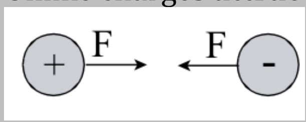
1. A ruler is negatively charged by rubbing it.
   a. Were electrons added to the ruler or removed from it when it was charged?
   b. If a charged piece of tape is **attracted** to the ruler, what is the charge on the tape?
   c. If a charged piece of tape is **repelled** by the ruler, what is the charge on the tape?
   d. What will you observe when you bring the two pieces of tape from (b) and (c) above close to each other? Explain your answer.
2. Object A is charged, and as a result it is attracted to Object B. When Object B is brought close to Object C there is a force of repulsion. If Object C is negatively charged, what is the charge on Object A? Use a diagram to show how you get your answer.

MAIN IDEA:

- Like charges **repel** one another.



- Unlike charges **attract** one another.



1.3. Polarisation

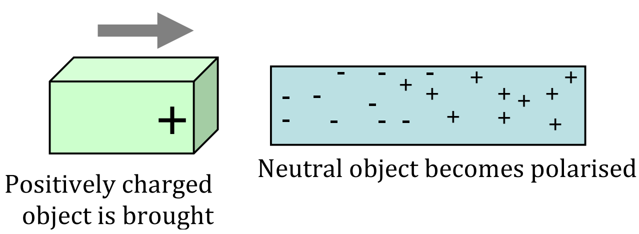
A charged object can attract an object that is uncharged. This is caused by a process called ***polarisation***. In the following activity you will investigate polarisation.

Activity 5: Investigating polarisation

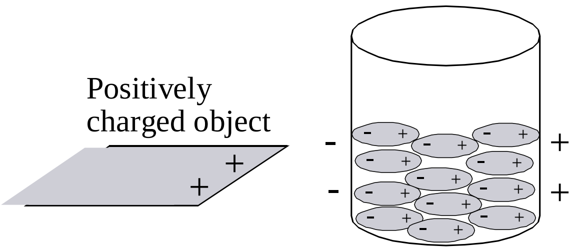
<u>You will need</u>:
A balloon
A stream of smooth flowing water (e.g. from a tap)

1. Blow up the balloon and tie it so that it stays inflated.
2. Rub the balloon against your hair or a piece of material. This will charge the balloon.
3. Open a tap so that a smooth stream of water is flowing.
4. Bring the charged balloon near to the stream of water. What do you observe?
5. Water is made up of polar molecules, which are positively charged on the one side, and negatively charged on the other. If the balloon is negatively charged, can you explain why it attracts the stream of water, which is neutral?

In the process of polarization, a positively charged object attracts the negative charges in a neutral object, causing the part of the neutral object that is nearby to be negatively charged, and the part that is further away to be positively charged.



Some substances, such as water, are made up of particles that have a positive end and a negative end. These are called ***polar*** substances. When a positively charged object is brought near to a polar substance, it causes the particles to turn around so that their negative end is nearer to the charged object, and their positive



end is further away. This results in a force of attraction between the positively charged object and the negative ends of the particles in the polar substance.

Activity 6: Test your knowledge of forces between charged and neutral objects

1. A neutral object experiences a force of attraction to a **negatively** charged object.
   a. Explain what happens to the charges in the neutral object when the negatively charged object is brought close to it.
   b. What is the name of this process?
2. A ruler is negatively charged by rubbing it.
   a. Describe what you will observe if you bring this ruler close to a continual stream of water.
   b. Explain your observations by describing what has happened to the water molecules as the ruler is brought close to them.

MAIN IDEA:
- A charged object attracts an uncharged object because of **polarisation**.
- In this process, a positively charged object attracts the negative charges in a neutral object, causing the part of the neutral object that is nearby to be negatively charged, and the part that is further away to be positively charged.

1.4. Coulomb's Law for calculating the force between charges

We can calculate the magnitude of the force between charged objects using ***Coulomb's law***. This law states:

"The magnitude of the force of interaction between two point charges is directly proportional to the product of the magnitudes of charges and inversely proportional to the square of the distances between them."

We can write this using a mathematical equation: $F = k\,\frac{Q_1 Q_2}{r^2}$

where F is the magnitude of the force between the charges, measured in newtons (N)

k is a proportionality constant, $k = 9{,}0 \times 10^{9}\ N{\cdot}m^2/C^2$

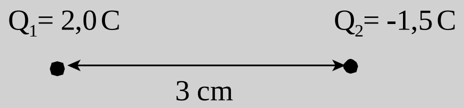
Q_1 and Q_2 are the magnitudes of the two point charges, measured in coulombs (C) r is the distance between the charges, measured in meters (m)

Notes on this equation:

- When we use this equation we do not include the + or – signs of the charges. We use this equation to calculate the size (magnitude) of the force.
- The direction of the force can be worked out from the attraction or repulsion of the + and – signs of the charges.

<u>Example</u>: *(Try to solve this problem* ***on your own or with a fellow student*** *while covering the solution, and then check your work using the solution below).*

Two charges Q_1 and Q_2 are placed a distance of 3 cm from each other.



a. What is the magnitude and direction of the force on charge Q_1?
b. What is the magnitude and direction of the force on charge Q_2?
c. Draw a free body diagram for each charge, showing the force on that charge.

<u>Solution</u>

Given: Q_1 = 2,0 C Q_2 = -1,5C r = 3 cm = 3×10^{-2} m

a. Magnitude of the force on charge Q_1 due to charge Q_2 is:

$$F_{21} = k\frac{Q_1Q_2}{r^2} = 9\times10^9\,\text{Nm}^2\text{C}^{-2}\times\frac{2{,}0\,\text{C}\times1{,}5\,\text{C}}{(3\times10^{-2}\,\text{m})^2} = 3\times10^{13}\,\text{N}$$

Unlike charges attract one another.

Therefore, the force on charge Q_1 is $\mathbf{F}_{21} = 3\times10^{13}\,\text{N}$ to the right

b. Force on charge Q_2 is equal and opposite to force on charge Q_1 (Newton's 3rd law). Therefore force on charge Q_2 is:

$\mathbf{F}_{12} = 3\times10^{13}\,\text{N}$ to the left

c. Free body diagram for charge Q_1:

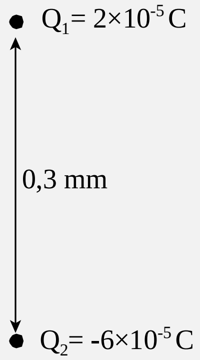
d. Free body diagram for charge Q_2:

MAIN IDEA: The magnitude of the force F between point charges Q_1 and Q_2 that are a distance r apart can be found using Coulomb's Law, which we can write mathematically as:

- $F = k\frac{Q_1 Q_2}{r^2}$ where $k = 9{,}0 \times 10^9\ N{\cdot}m^2/C^2$.

Activity 7: Test your knowledge of Coulomb's Law

Answer the following questions:



1. Two charges Q_1 and Q_2 are arranged as shown in the diagram:
   a. Find the magnitude and the direction of the force on charge Q_1.
   b. Draw a free body diagram showing the force acting on charge Q_1.
   c. Find the magnitude and the direction of the force on charge Q_2.
   d. Draw a free body diagram showing the force acting on charge Q_2.

2. The magnitude of the force between two unknown charges is 8 N. The distance between the charges is then halved. What is the new force between these charges? Explain your answer.

3. A 1,6 μC charge experiences a force of 7200 N to the right when it is placed 2 mm to the left of an unknown charge Q. What is the charge on Q?

Unit 2.	The electric field

Learning outcomes:

When you have completed this unit, you should be able to:

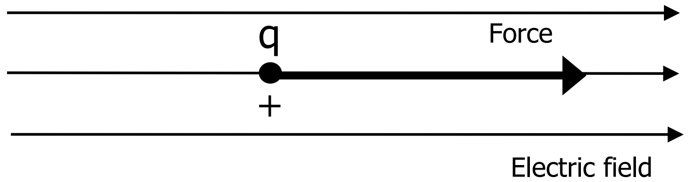
- describe an electric field as a region in which an electric charge experiences a force;
- draw the electric field of an isolated point charge and recall that the direction of the field lines gives the direction of the force acting on a positive test charge;
- draw the electric field pattern between two isolated point charges
- define the magnitude of the electric field at a point as the force per unit charge, $E = F/q$;
- calculate the electric field at a point due to a number of point charges in 1-dimension, using the equation $E = \frac{kQ}{r^2}$.

Introduction

The electric field is a region in which an electric charge experiences a force. In this unit you will learn how to show the electric field using electric field lines, and how to find the magnitude and direction of the electric field.

2.1. Electric field lines

When a charge experiences a force, this is because there is an electric field at the position where that charge is placed. The electric field is a vector, and so it has a direction. The direction of the electric field is the same as the direction of the force that a positive test charge q experiences when it is placed in the electric field.



We use electric field lines to show the direction of the field. The arrow on the end of an electric field line shows us the direction of the field. The more closely spaced the electric field lines are, the stronger the electric field is at that point.

2.2. The electric field strength

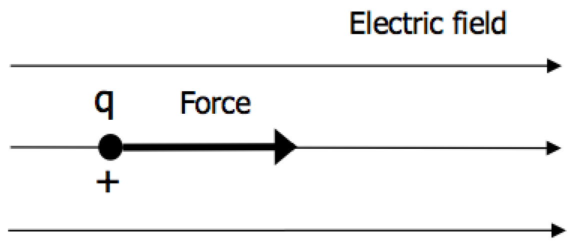
The strength of the field is the amount of force that a +1 C test charge would experience. In other words, we can calculate the magnitude of the electric field (E) by dividing the magnitude of the force (F) experienced by some test charge by the value of its charge (q).

Mathematically we write this as: $E = \frac{F}{q}$

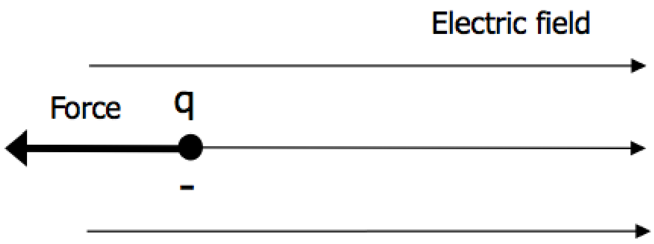
where E is the magnitude of the electric field strength, measured in units of N/C
F is the magnitude of the force on the test charge, measured in newtons (N)
q is the charge on the test charge, measured in coulombs (C)

<u>Notes on this equation:</u>

- When we use this equation we do not include the + or – signs of the test charge. We use this equation to calculate the magnitude of the electric field. So we just use the magnitude of the test charge.
- The electric field and the force are both vectors, and so they both have a magnitude and a direction. We have already seen that the direction of the force on a positive test charge is the same as the direction of the electric field at that point.



- When a negative test charge is brought into a space where there is an electric field, the direction of the force on the test charge is opposite to the direction of the electric field at that point.



MAIN IDEA:

- The magnitude of the **electric field** E on a test charge q that experiences a force F can be calculated using the equation:

$$E = \frac{F}{q}$$

- The direction of the force on a positive test charge is the same as the direction of the electric field at that point.
- The direction of the force on a negative test charge is opposite to the direction of the electric field at that point.

<u>**Example:**</u> *(Try to solve this problem* ***on your own or with a fellow student*** *while covering the solution, and then check your work using the solution below).*

a. A positive test charge q = 2 nC experiences a force of 6 N to the right. What is the electric field at this point?

b. What is the force on a negative test charge q = -3 μC that is brought into this same electric field?

<u>Solution</u>

a. **Given**: q = 2 nC = 2×10^{-9} C **F** = 6 N to the right
The electric field strength is:
$$E = \frac{F}{q} = \frac{6\ N}{2\times10^{-9} C} = 3 \times 10^9\ N/C$$
The electric field direction is the same as the direction of the force on the positive test charge, so **E** = 3×10^9 N/C to the right.

b. **Given**: q = -3 μC = -3×10^{-6} C **E** = 3×10^9 N/C to the right
To find the magnitude of the force we can change the equation $E = \frac{F}{q}$ to make F the subject of the formula:
$F = E{\cdot}q = 3 \times 10^9\ N{\cdot}C^{-1} \times 3 \times 10^{-6}\ C = 9 \times 10^3\ N$
The direction of the force is opposite to the direction of the electric field on a negative test charge, so **F** = 9×10^3 N to the left.

2.3. The electric field around a charge

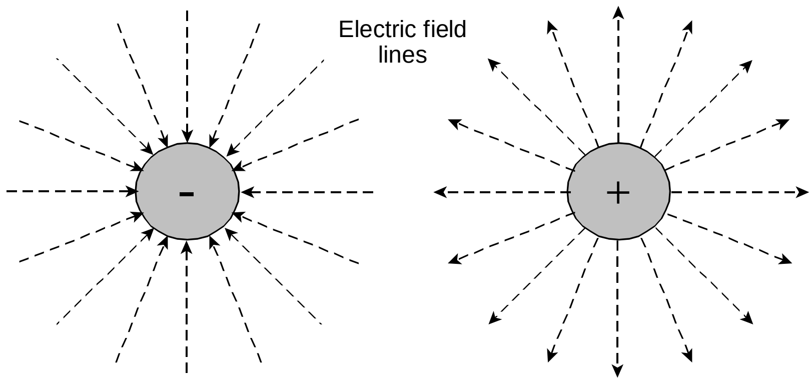
In the same way that a magnet has a magnetic field around it, there is an ***electric field*** that exists around a charged object.

If we want to find the strength of the electric field at a distance r from a charge Q, we can use the equation: $E = \frac{kQ}{r^2}$

where E is the magnitude of the electric field strength, measured in units of N/C
k is a proportionality constant, $k = 9{,}0\times10^{9}\ N{\cdot}m^2/C^2$
Q is the magnitude of the charge that creates the field, measured in coulombs (C)
r is the distance from the charge, measured in meters (m)

Notes on this equation:

- When we use this equation we do not include the + or – signs of the charge. We use this equation to calculate the magnitude of the electric field. So we just use the magnitude of the charge.
- The direction of the electric field can be found using the sign on the charge. The electric field points outward from a positive charge, and inward towards a negative charge.



<u>Example:</u> *(Try to solve this problem **on your own or with a fellow student** while covering the solution, and then check your work using the solution below).*

What is the electric field at a point that is 3 mm to the right of a charge Q = -2 nC?

<u>Solution</u>

Given: $r = 3\ mm = 3\times10^{-3}\ m$ $Q = -2\ nC$

The electric field strength is:

$$E = \frac{kQ}{r^2} = \frac{9\times10^{9}\times2\times10^{-9}}{(3\times10^{-3})^2} = 3\times10^{6}\ N{\cdot}C^{-1}$$

The electric field direction at this point is left, since the electric field points inwards toward a negative charge.

So the electric field at this point is **E** = $3\times10^{6}\ N{\cdot}C^{-1}$ to the left.

MAIN IDEA:

- The magnitude of the electric field E at a distance r from a charge Q that is creating the field can be calculated using the equation:

$$E = \frac{kQ}{r^2}$$

- The direction of the electric field points outward from a positive charge, and inward towards a negative charge.

Activity 1: Test your knowledge of the electric field

Answer the following questions:

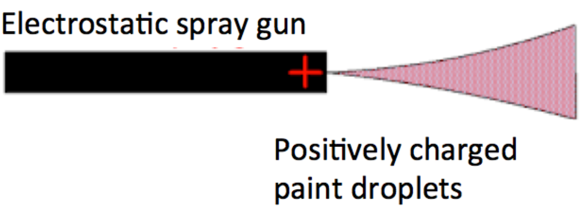
1. A positive test charge q = 2 C experiences a force of 0,8 N downward. What is the electric field at this point?
2. A negative test charge q = -3 μC experiences a force of 90 N to the left. What is the electric field at this point?
3. The electric field at point P is 5×10^4 $N{\cdot}C^{-1}$ to the left.
    a. What is the force on a positive test charge q_1 = 2 nC that is placed at this point P?
    b. Charge q_1 is removed, and an unknown charge q_2 is placed in its place at point P. This charge experiences a force of 500 kN to the right. What is the charge on q_2?
4. What is the electric field strength and direction at a point that is 3 mm to the left of a charge Q = 1,8 μC?

2.4. Applications of electrostatics

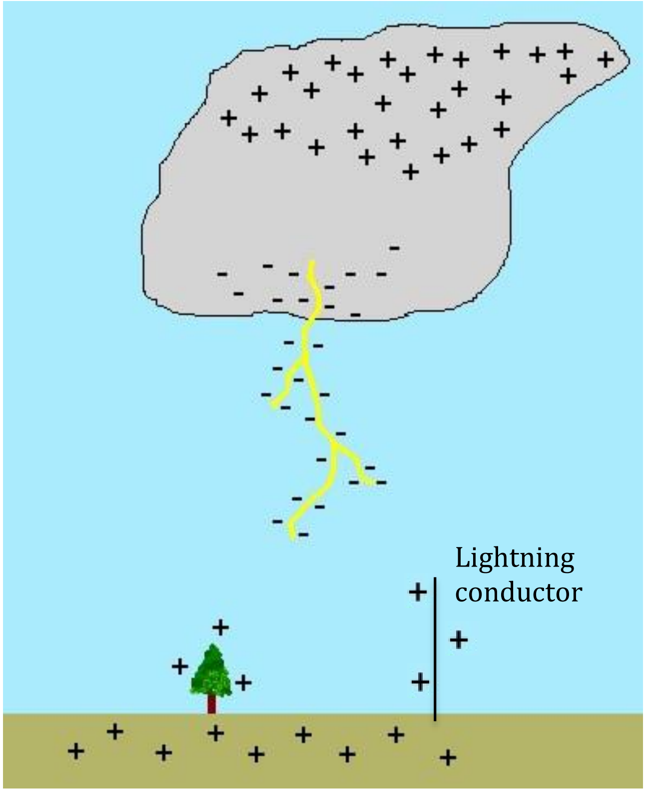
Photocopying machines and printers use the principle of electrostatics to make photocopies. Light is reflected off the original page onto a rotating drum. The drum becomes positively charged in the places where light does **not** shine onto it (in other words, the black parts of the page), and a negatively charged black powder is attracted to these positively charged parts of the drum. This is then transferred to the blank paper. This makes it look similar to the original page.

Photocopying machine **(image from http://ecx.images-amazon.com/images/I/413fZwmNESL.jpg)**

Electrostatics is also applied in **spray painting**, where particles of paint are given a positive charge as they leave the spray gun. The object to be painted is earthed so that there is an electric field between the spray gun and the object. The charged paint droplets follow the electric field lines, and are deposited evenly over the object's surface.



Lightning takes place when a thunder cloud becomes charged by the rubbing together of air and water particles moving past each other. This creates an electric field between the cloud and the ground. A lightning strike occurs when there is a massive release of charge between the cloud and the ground. **Lightning conductors** are tall metal poles that are attached to the earth by a conducting wire. This creates a safe path for lightning to pass through.



Reflection Activity

Have you noticed that large trucks often have a small chain that hangs down and touches the ground. Can you think of a reason for this? Why do you think this is especially important for petrol tankers? Do some investigation to find out why this chain is necessary.

Assessment Activity: *Electrostatics* **Total marks = 40**

Answer the following questions to assess your understanding of electrostatics:

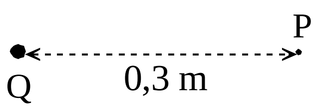
1. A positively charged object is brought close to a negatively charged object. What do you observe? (1)

2. Object A is a positively charged.
    a. Were electrons added to Object A or removed from Object A when it was charged? (1)
    b. Object B is a charged object that is **repelled** by Object A. What is the charge on Object B? (1)
    c. If Object C is a charged object that is **attracted** to Object A, what is the charge on Object C? (1)
    d. Will Object B and Object C attract each other, or repel each other? (1)

3. Object X has an unknown charge, and is attracted to both a positively charged object and a negatively charged object.
    a. What is the charge on Object X to make this possible? (positive / negative / neutral) (1)
    b. Explain the process that happens in Object X to allow it to experience a force of attraction to the negatively charged object. (3)
    c. What is the name given to this process? (1)

4. The force between two charged objects is F. Which of the following changes will increase the force between the charges to 4F? (Just write YES or NO)
    a. Double the charge on one of the objects. (1)
    b. Increase the charge on one of the objects by a factor of 4. (1)
    c. Double the charge on both of the objects. (1)
    d. Increase the charge on both of the objects by a factor of 4. (1)
    e. Halve the distance between the charges. (1)
    f. Double the distance between the charges. (1)
    g. Reduce the distance between the charges by a factor of 4. (1)
    h. Increase the distance between the charges by a factor of 4. (1)

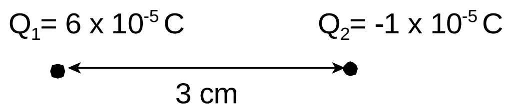
5. The electric field at point P in the diagram is 600 $N \cdot C^{-1}$ to the left.



a. What is the charge on Q? (4)

b. If a test charge q = 4nC is placed at point P, what is the magnitude and direction of the force it would experience? (4)

6. Two charges Q_1 and Q_2 are arranged as shown in the diagram below.



a. Calculate the magnitude and direction of the force that charge Q_1 experiences. (4)

b. If charge Q_1 is removed, find the magnitude and direction of the electric field at this position. Calculate this in **two different ways**. (7)

7. Describe one of the ways that electrostatics is applied in everyday life. (3)

My Notes:

Use this space to write your own questions, comments or key points.

Summary of key learning:

- There are **two types of charge**: positive charges, and negative charges.
- When there is an unbalanced number of positive and negative charges on an object, the object becomes **charged**.
- When there are more electrons than protons, the object is **negatively charged**.
- When there are fewer electrons than protons, the object is **positively charged**.
- Like charges **repel** one another, and unlike charges **attract** one another.
- A charged object attracts an uncharged object because of **polarisation**.
- In polarisation, a positively charged object attracts the negative charges in a neutral object, causing the part of the neutral object that is nearby to be negatively charged, and the part that is further away to be positively charged.
- Coulomb's Law states: *"The magnitude of the force of interaction between two point charges is directly proportional to the product of the magnitudes of charges and inversely proportional to the square of the distances between them."*
- The magnitude of the force F between point charges Q_1 and Q_2 that are a distance r apart can be found using Coulomb's Law, which we can write mathematically as: $F = k\frac{Q_1Q_2}{r^2}$ where k = $9,0\times10^9$ $N{\cdot}m^2/C^2$.
- The magnitude of the electric field E on a test charge q that experiences a force F can be calculated using the equation:
  $$E = \frac{F}{q}$$
- The direction of the force on a positive test charge is the same as the direction of the electric field at that point.
- The direction of the force on a negative test charge is opposite to the direction of the electric field at that point.
- The magnitude of the electric field E at a distance r from a charge Q that is creating the field can be calculated using the equation:
  $$E = \frac{kQ}{r^2}$$
- The direction of the electric field points outward from a positive charge, and inward towards a negative charge.

Sub-topic 2. Electric circuits

Content:

Unit 1: Current, resistance and potential difference in electric circuits

Unit 2: Energy transfer in electrical circuits

Unit 1. Current, resistance and potential difference in electric circuits

Learning outcomes:

When you have completed this unit, you should be able to:

- state that current (I) is a rate of flow of charge, measured in amperes;
- apply the equation $I = Q/\Delta t$;
- explain conventional current;
- define electromotive force (emf) as the work done in moving a unit charge around a complete circuit, measured in volts;
- define the potential difference (V) across an element in a circuit as the work done to move a unit charge through the element, measured in volts;
- draw diagrams to show how to connect an ammeter to measure current through a circuit element, and a voltmeter to measure voltage across a circuit element;
- define resistance;
- apply the formulae for the equivalent resistance of a number of resistors in series and in parallel;
- state that the current is constant through each element in a series circuit;
- state that the total potential difference is equal to the sum of the potential differences across the individual elements in a series circuit;
- calculate the equivalent resistance of resistors connected in series: $R_s = R_1 + R_2 + \ldots$;
- state that the potential difference is constant across circuit elements that are connected in parallel;
- state that the current from the source is the sum of the currents in the separate branches of a parallel circuit;
- calculate the equivalent resistance of resistors connected in parallel: $1/R_p = 1/R_1 + 1/R_2 + \ldots$;

- state that the potential difference across the separate branches of a parallel circuit is the same and apply the principle to new situations or to solve related problems;
- state Ohm's Law in words and mathematically: R = V/I ;
- solve problems for circuits involving resistors connected in series and parallel.

1.1. Current

When charges move from one atom to another in an electrical circuit, this forms a flow of charges around the circuit, which is called an electrical ***current***. When these charges move through a light bulb, they rub against the metal of the bulb's filament, causing friction, and the bulb lights up.

> ***Filament:*** thin wire inside a light bulb that glows when there is electrical current in the bulb

The word "current" describes how fast the charges move in a circuit. In other words, we can define current as the rate of the flow of charges. We can express this in an equation:

$$I = \frac{Q}{\Delta t}$$

where I is the current, measured in amperes (A),

Q is the amount of charge, measured in coulombs (C), and

Δt is the time interval, measured in seconds (s).

From this equation we can see that 1 A =1 C/s. In other words, the current tells us how many coulombs of charge are passing a point in the circuit in one second. The value of the current affects the physical effect that this current has in a circuit. For example, the greater the current that flows through a light bulb, the brighter the bulb will be.

Example: *(Try to solve this problem* ***on your own or with a fellow student*** *while covering the solution, and then check your work using the solution below).*

1. 16 C of charge passes a point in a circuit in the time of 4 seconds. What is the value of the current that passes this point in the circuit?
2. A current of 2 A flows through a resistor. How much charge flows through this resistor in 30 seconds?

Solution

1. **Given**: Q = 16 C; Δt = 4 s

$$I = \frac{Q}{\Delta t} = \frac{16\ C}{4\ s} = 4\ A$$

2. **Given**: I = 2 A; Δt = 30 s

From the equation $I = \frac{Q}{\Delta t}$ we can solve for Q:

$Q = I \times \Delta t = 2\ A \times 30\ s = 60\ C$

MAIN IDEAS: Current (measured in amperes) is a measure of the rate of flow of charges in a circuit:

$$I = \frac{Q}{\Delta t}$$

1 A = 1 C/s

1.2. What is needed for current to flow in a circuit

The *convention* that has been chosen by scientists is that current flows from the positive terminal of the battery to the negative terminal. The term "conventional current" is sometimes used to describe this direction for the current flow.

Convention: an accepted practice that people have all agreed on

In the following activity you will investigate what is needed for an electrical current to flow in a circuit.

Activity 1: Investigating closed circuits

You will need:

1 battery (a 1.5V torch battery is best)
1 torch bulb
2 electric leads (any pieces of wire which have both ends not covered in plastic)

10. Using your torch battery, bulb and leads, try to find all of the ways in which these can be arranged so that your bulb lights up. (You should be able to find four different arrangements.)
11. Write a list of the conditions that need to be met in order for a bulb to light up in a circuit. (You can see when a current is flowing if the bulb lights up).

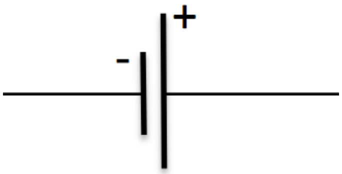
For current to flow in a circuit, the circuit elements need to be connected in a closed loop, in other words there should be no gaps in the circuit. This is called a **closed circuit**. We also need an electrical energy source, which is the battery. Resistors are the elements of a circuit that use the electrical energy for some purpose, for example to provide heat, light or sound.

Circuit elements: parts of a circuit, such as batteries, bulbs and switches

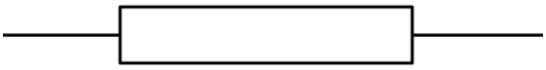
1.3. Circuit Diagrams

When we draw diagrams of electrical circuits, we do not show the physical shapes or connections of the circuit elements. Rather, we use symbols to show the circuit elements, and lines to show the electrical connections. This way of representing an electrical circuit is called a **circuit diagram**. From a circuit diagram, we cannot tell how close together the circuit elements are. This is because circuit diagrams do not show the physical layout of the circuit. They show us the electrical connections.

The symbol for a battery, a resistor, a switch and a light bulb are shown in the diagrams below.

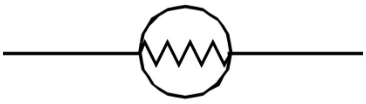


Symbol for a battery



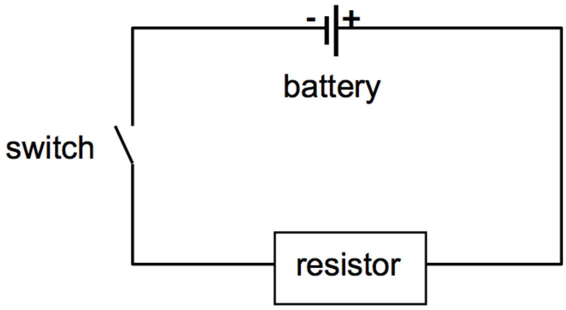
Symbol for a resistor

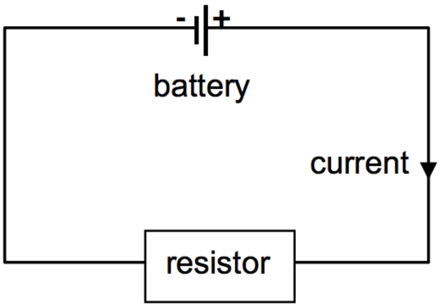
Symbol for a switch



Symbol for a light bulb

The circuit diagram on the left below represents an open circuit with a battery, a resistor and a switch. The circuit diagram on the right shows a closed circuit, which is formed when the switch is closed, allowing the current to flow. When circuit elements are connected together in a single loop this is called a **series circuit**.





Activity 2: Test your understanding of current

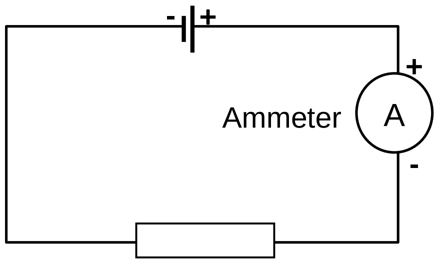
Answer the following questions:

1. Draw a circuit diagram for a circuit containing a battery, a switch, a resistor and a light bulb connected in a closed loop.
2. Explain in your own words what the term *current* means.
3. 6 C of charge passes a point in a circuit in the time of 3 seconds. What is the value of the current at this point in the circuit?
4. A current of 0,2 A flows through a bulb. How much charge flows through this bulb in 5 minutes?
5. In Circuit A half a coulomb of charge passes through a bulb in each second. In a Circuit B 10 C of charge passes through a bulb in half a minute. In which circuit will the bulb will be brightest, Circuit A or Circuit B?

MAIN IDEAS:

- Current flows from the positive terminal of the battery to the negative terminal
- For current to flow we need a **closed circuit** with a power supply (battery)
- When circuit elements are connected together in a single loop this is called a **series circuit**.

1.4. Measuring current



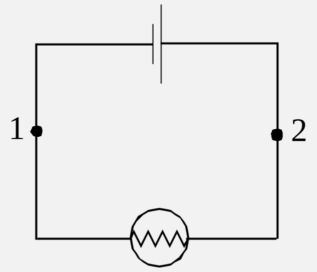
When we want to measure the amount of current that is flowing in a circuit, we use an instrument called an ***ammeter***. The ammeter measures the current that flows through it, so we connect the ammeter in **series** with a circuit element.

The positive (red) terminal of the ammeter must always be connected to the positive terminal of the battery, and the negative (black) terminal of the ammeter must be connected to the negative terminal of the battery.

Activity 3: Measuring current in a 1-bulb circuit

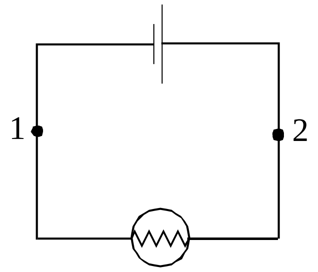
<u>You will need</u>:

1 battery
1 torch bulb
An ammeter
Electric leads



1. Create a circuit where the battery is connected in a loop with a light bulb, as shown on the right.
2. Connect your ammeter into the position marked 1 in this circuit. Write down the current reading. (NOTE: If the ammeter has more than one scale, always connect to the largest scale first so that the ammeter will not be damaged by having to measure values that are too large).
3. Connect your ammeter into the position marked 2 in this circuit. How does this current reading compare to the one at position 1?
4. Write a summary of your findings from this experiment.

When we measure the current in a series circuit, we find that the current is the ***same at different points*** in the circuit. For example, at points 1 and 2 in the circuit on the right the current is the same.



MAIN IDEAS:

- We use an **ammeter** to measure electrical current.
- The ammeter must be connected in **series** in the circuit.
- The current is the **same** at all points in a **series circuit**.

1.5. Potential difference

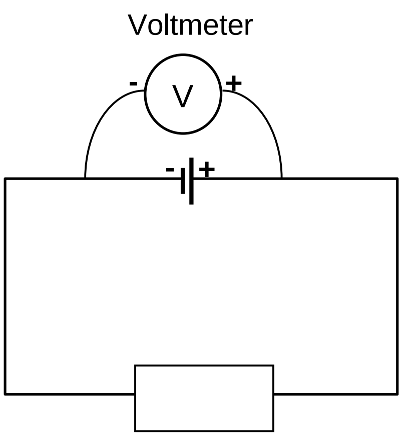
In a circuit the battery is the source of the electrical energy. There is no flow of charges without a battery, since it gives a push which sends the charges around the wires in the circuit. This push of the charges is called the ***electrical potential difference***. Electrical potetial difference is measured in units of ***volts*** (V).

The definition of potential difference across a circuit element is the work done to move a unit charge through the element.

In a circuit, the **battery** creates an **increase** in the potential difference, and the **resistors** cause a **decrease** in the potential difference. In other words, the resistors use up all of the potential difference created by the battery. The potential difference that is measured across the battery in a circuit is called the **electromotive force** (**emf**) of the battery. We therefore define emf as the work done in moving a unit charge around a complete circuit, measured in volts. We define the potential difference (V) across an element in a circuit as the work done to move a unit charge through the element, measured in volts.

MAIN IDEAS:
- The **electrical potential difference** across a circuit element is the work done to move a unit charge through the element.
- Electrical potential difference is measured in **volts** (V).
- The **electromotive force** (**emf**) is the potential difference across the battery and is defined as the work done in moving a unit charge around a complete circuit.

1.6. Measuring potential difference



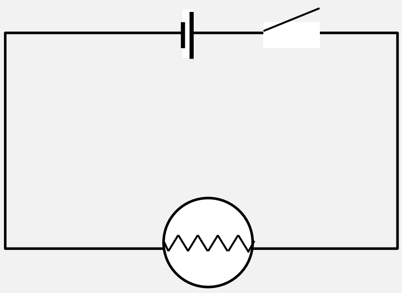
When we measure the potential difference we use an instrument called a ***voltmeter*** . The voltmeter measures the potential difference across its terminals, so we connect the voltmeter in ***parallel*** to the circuit element. In the diagram on the left the voltmeter is connected in parallel to the battery.

The positive (red) terminal of the voltmeter must be connected to the positive terminal of the battery, and the negative (black) terminal of the voltmeter to the negative terminal of the battery.

Activity 4: Measuring potential difference

<u>You will need</u>:

1 battery
1 torch bulb
1 voltmeter
Electric leads



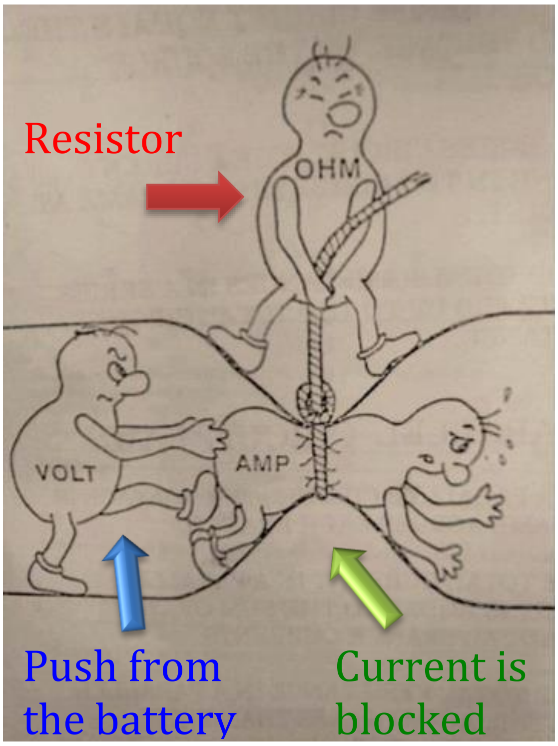
1. Create a circuit where the battery is connected in series with a switch and a torch bulb, as shown in the circuit diagram.
2. Connect your voltmeter to measure the potential difference across the battery. Record the voltage reading when you have closed the switch in your circuit.
3. Connect your voltmeter to measure the potential difference across the bulb when the switch is closed. Write down the voltage reading.
4. How does the potential difference across the resistor compare with the potential difference measured across the battery?
5. Connect your voltmeter to measure the potential difference across the switch when it is closed. Can you explain this reading?

MAIN IDEAS:

- We use a **voltmeter** to measure electrical potential difference.
- The voltmeter must be connected in **parallel** to the circuit element.

1.7. Resistance

A resistor makes it more difficult for charges to pass through it than through ordinary conductors. Resistors oppose, or resist, the path of the current. The greater the value of its resistance, the more a resistor will resist the flow of current in the circuit. Resistance is measured in ohms (Ω).



A higher value for the resistance in a circuit causes the current flowing in that circuit to decrease. This is because resistors cause an obstacle to the flow of current in a circuit. Therefore when the resistance increases, the current in the circuit decreases. The picture on the right gives you an idea of how a resistor works in a circuit, by causing a blockage to the flow of current.

MAIN IDEAS:

- A **resistor** opposes the path of the current.
- **Resistance** is measured in ohms (Ω).

1.8. Ohm's Law

In the following activity you will investigate the mathematical relationship between current, voltage and resistance.

Activity 5: ***Investigating the relationship between current, voltage and resistance***

<u>You will need</u>:

4 batteries and a battery holder,
2 resistors, one that has double the resistance of the other
1 ammeter and 1 voltmeter
Some connecting leads
1 switch

1. Design and build a circuit consisting of one battery connected in series with the smaller resistor and a switch.
2. Connect the ammeter and voltmeter in your circuit so that you can measure the current through the resistor, and the voltage across it.
3. Measure the current through the resistor, and the voltage across it, for circuits consisting of two, three and four batteries. (NOTE: only close your circuit for a short time when taking your readings, because the temperature must be kept constant.)
4. Record your measurements in a table like the one below:

		Reading 1 (One battery)	**Reading 2** (Two batteries)	**Reading 3** (Three batteries)	**Reading 4** (Four batteries)
Small resistor	Voltage (V)				
	Current (A)				
Large resistor	Voltage (V)				
	Current (A)				

5. Using a piece of graph paper, plot a graph of voltage on the y-axis against current on the x-axis. What do you notice about the shape of this graph? What does this tell you about the relationship between voltage and current?
6. Draw a best-fit line through your points.
7. Calculate the gradient of the best-fit line.
8. Repeat steps two to four with the larger resistor.
9. On the same set of axes as your previous graph, plot the graph of voltage against current for this larger resistor. Answer the following questions:
   a. How do your two graphs compare with one another?
   b. Which graph has the higher value for its gradient?
   c. What would the graph look like for a resistor that has half the resistance of the smaller resistor?
   d. What can you infer about the physical quantity that the gradient of this graph tells us about?
10. Write a clear and precise summary of your conclusions.

Suggestion for alternative equipment

1. If you do not have two resistors with different resistances, you can use different lengths of pencil lead, since pencil lead is made of graphite, which is a resistor. The longer the length of pencil lead, the greater its resistance.
2. If you do not have any of the equipment to do this experiment yourself, we have provided some possible values for you to work with. These are shown in the table below. Use these values to try to complete all of the steps from point 5 onwards.

		Reading 1 (One battery)	**Reading 2** (Two batteries)	**Reading 3** (Three batteries)	**Reading 4** (Four batteries)
Small resistor	Voltage (V)	0,9 V	1,4 V	2,6 V	3,2 V
	Current (A)	1,2 A	2,5 A	4,1 A	5,8 A
Large resistor	Voltage (V)	1,5 V	2,5 V	4,5 V	5,9 V
	Current (A)	1,3 A	2,6 A	4,5 A	5,7 A

From this activity, you would have observed that, as the voltage in the circuit is increased (by adding batteries to the circuit), the current also increases. If you only closed your switch for a short time when taking your readings, the temperature would have remained constant, and so your graph should have been a straight line. This tells us that, at constant temperature, the **current is directly proportional to the voltage**. We can summarize this relationship in the following mathematical way: $V \propto I$.

This relationship between voltage and current is called ***Ohm's Law***, which states that the current through a resistor is directly proportional to the voltage across the resistor at constant temperature.

The gradient of your graph for the smaller resistance should have been lower than the gradient for the higher resistance. So the gradient of the graph of voltage against current tells us about the size of the resistance. The greater the resistance, the higher the value that we find for the gradient.

When we calculate the gradient of this graph, we are dividing the voltage by the current, and this tells us about the resistance.

We can express Ohm's Law using a mathematical equation as: $R = \frac{V}{I}$

where R is the resistance of a resistor, measured in ohms (Ω)

V is the voltage (potential difference) across the resistor, measured in volts (V)

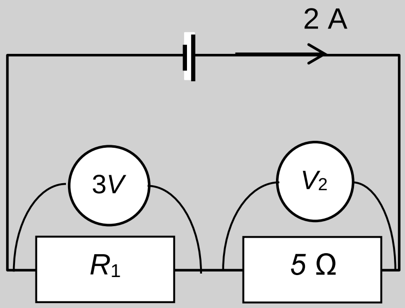
I is the current through the resistor, measured in amperes (A)

From this equation you can see that 1 ohm is equal to one volt per ampere. We can write this mathematically as 1 Ω = 1 V/A.

<u>Example</u>: *(Try to solve this problem* ***on your own or with a fellow student*** *while covering the solution, and then check your work using the solution below).*

For the circuit shown, use Ohm's Law to calculate:

a. the resistance R_1
b. the voltage V_2



<u>Solution:</u>

Given: For resistor 1 $V_1 = 3\ V$ $I = 2\ A$

For resistor 2 $R_2 = 5\ \Omega$ $I = 2\ A$

a. From Ohm's Law:

$$R_1 = \frac{V}{I} = \frac{3\ V}{2\ A} = 1{,}5\ \Omega$$

b. We can rewrite Ohm's Law as:

$$V_2 = R \cdot I = 5\ \Omega \times 2\ A = 10\ V$$

Activity 6: Calculations with Ohm's Law

Use Ohm's law to answer the following questions:

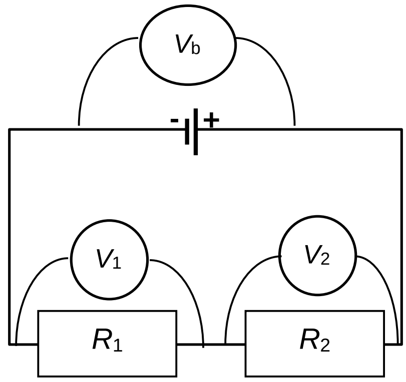
1. The voltage across a resistor is measured to be 2 V. The current through the resistor is 0,5 A. Calculate the resistance of this resistor.
2. Calculate the voltage across a 3,5 Ω resistor that has a current of 2 A flowing through it.
3. The current in a circuit is 3 A when a resistor that has a resistance of ***x*** Ω is connected in a closed loop with a battery that has a voltage V. What is the current measured in the circuit when the **same battery** is connected in a closed loop with a resistor that has a resistance of:
   a. ½ ***x***
   b. 2 ***x***
   c. 5 ***x***

1.9. Resistors in series

In a circuit that has two or more resistors that are connected in series (next to each other), the current in the resistors is equal. This is because current in a series circuit only has one path to move through.

The potential difference across the battery in a circuit is equal to the total potential difference across the resistors in a series circuit. For example, in a series circuit with two resistors, R_1 and R_2, with potential differences of V_1 and V_2, the potential difference across the battery can be expressed as:

$V_{battery} = V_1 + V_2$



$V_b = V_1 + V_2$

For any circuit with two or more resistors connected in series, we can write the battery potential difference in a circuit as:

$$V_{battery} = V_1 + V_2 + \ldots$$

Resistors that are connected in series are called ***potential dividers*** because they share the battery's potential difference. The larger resistance will get the greater share of the potential difference. In other words, the potential difference across a resistor is proportional to the value of its resistance.

MAIN IDEAS:

- The **current** in series resistors is equal.
- The terminal potential difference across the battery in a circuit is always equal to the **sum of the potential difference** across the resistors in a series circuit: $V_{battery} = V_1 + V_2 + \ldots$.
- Series resistors are **potential dividers**
- The potential difference across a resistor is **proportional** to its resistance

The total resistance is equal to the sum of the resistances that are connected in series. For example, in a series circuit with two resistors, R_1 and R_2, with total resistance can be expressed as: $R_s = R_1 + R_2$

For any circuit with two or more resistors connected in series, we can write the total equivalent resistance of resistors in series as:

$$R_s = R_1 + R_2 + \ldots$$

When resistors are connected in series, they together oppose the flow of charge more than when they are on their own. This has the effect of decreasing the current through the circuit. Therefore, the higher the resistance in a circuit, the lower the current. Mathematically we say that the current is inversely proportional to the resistance.

We can prove this mathematically from the equation for Ohm's Law, which is $R = \frac{V}{I}$.
From this equation we can see that

$$R \propto \frac{1}{I}$$

Mathematically, this tells us that R is inversely proportional to I.

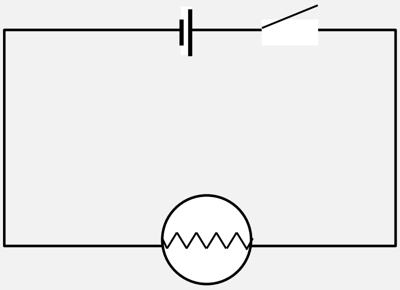
MAIN IDEAS:

- The **total equivalent resistance** of series resistors is equal to the sum of the individual resistances: $R_s = R_1 + R_2 + \ldots$
- The current in a circuit is **inversely proportional** to the total resistance.

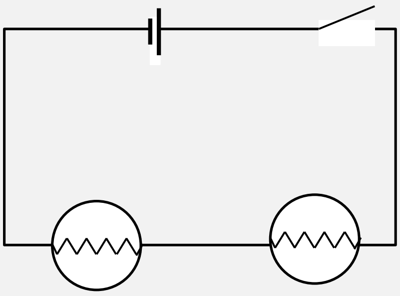
Activity 7: Resistors in series

You will need:

2 batteries
3 torch bulbs
Electric leads
2 switches



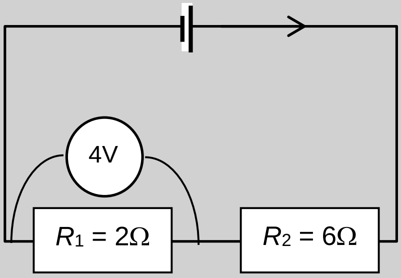
Circuit 1



Circuit 2

1. Connect the two circuits shown in the diagrams on the right.
2. Close the switch in Circuit 1 and observe the brightness of the bulb.
3. Close the switch in Circuit 2 and observe the brightness of the bulbs.
4. Compare the brightness of the bulbs in Circuit 2 with Circuit 1. Discuss the following questions:
   a. What can you conclude about the current in the bulb in Circuit 1 compared to Circuit 2?
   b. What can you conclude about the current through the battery in Circuit 1 compared to Circuit 2?
5. Use your ammeter to measure the current through the battery in Circuit 1 and Circuit 2. Do your current readings agree with the conclusions that you made from your discussions?
6. Use your voltmeter to measure the potential difference across the bulb and the battery in Circuit 1 **with the switch closed**. How do these voltage readings compare?
7. Use your voltmeter to measure the potential difference across the bulbs and the battery in Circuit 2 **with the switch closed**. How do these voltage readings compare?
8. What can you conclude about the relationship between the battery terminal potential difference and the potential difference across the resistors in a series circuit?

Example: *(Try to solve this problem* ***on your own or with a fellow student*** *while covering the solution, and then check your work using the solution below).*



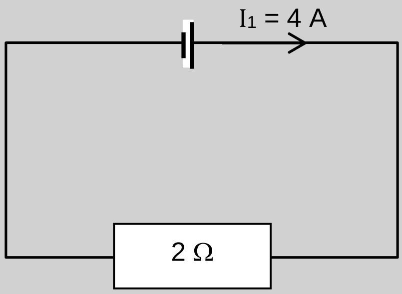
1. What is the total resistance in the circuit on the right?
2. What is the potential difference across the 6 Ω resistor?
3. What is the battery voltage?
4. Calculate the current in the 2 Ω resistor.

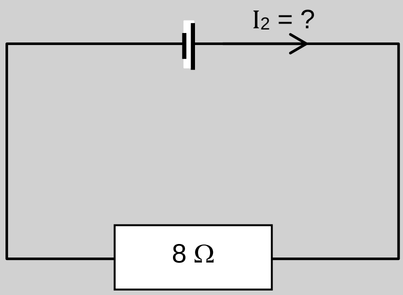
Solution

1. $R_s = R_1 + R_2 = 2\ \Omega + 6\ \Omega = 8\ \Omega$
2. If V_1 = 4 V, then since R_2 has 3 times the resistance of R_1, its voltage must be 3 times V_1. So $V_2 = 3 \times 4$ V = 12 V.
3. $V_{battery} = V_1 + V_2 = 4\ V + 12\ V = 16\ V$
4. From Ohm's Law $R = \frac{V}{I}$, so $I = \frac{V}{R} = \frac{4\ V}{2\ \Omega} = 2$ A.

Example *(Try to solve this problem* ***on your own or with a fellow student*** *while covering the solution, and then check your work using the solution below).*

Find the current I_2 in the second circuit below. (All of the batteries are the same, and ignore internal resistance.)





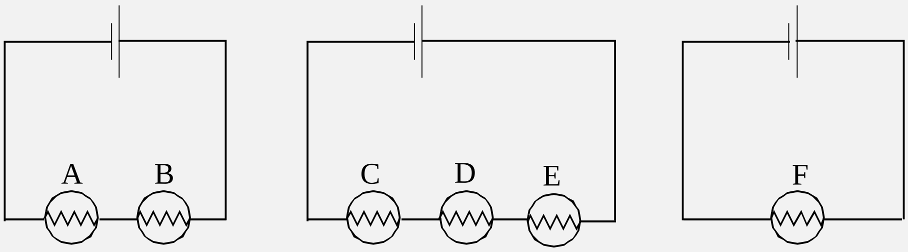
Solution

The current is inversely proportional to the resistance. The resistance in the second circuit is 4 times the resistance in the first circuit, so its current must be a quarter of that in the first circuit.

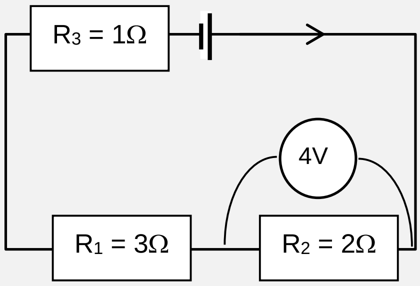
So $I_2 = \frac{1}{4} I_1 = \frac{1}{4} \times 4\ A = 1\ A$

Activity 8: Test your understanding of resistors in series

1. List the bulbs in the circuits shown in the diagram in order from brightest to dimmest. Explain your reasons for your answer.



2. The potential difference across the 2 Ω resistor in the circuit on the right is 4 V. (Ignore the effects of the battery's internal resistance.)
   a. Find the total equivalent resistance in the circuit.
   b. Find the current through resistors R_1 and R_3.
   c. Find the potential difference across resistors R_1 and R_3.
   d. Find the potential difference of the battery.



3. Resistor R_1 is removed from the circuit so that you now have a closed circuit with just R_2 and R_3 connected in series with the battery.
   a. Find the total equivalent resistance in the circuit.
   b. Find the current through the battery.
   c. Find the potential difference of the battery.

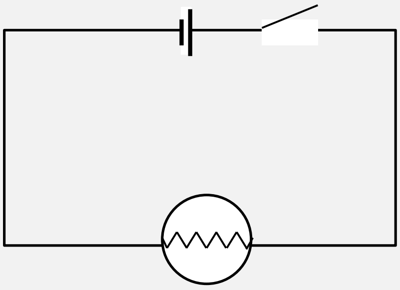
1.10. Resistors in parallel

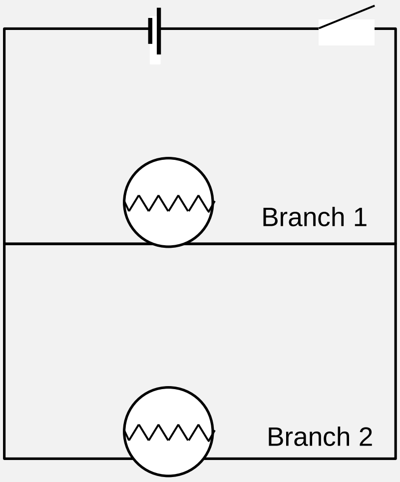
In the following activity you will investigate the effect on the current and resistance in a circuit when two resistors are connected in parallel.

Activity 9: Resistors in parallel

You will need:

2 batteries
3 torch bulbs
Electric leads
2 switches





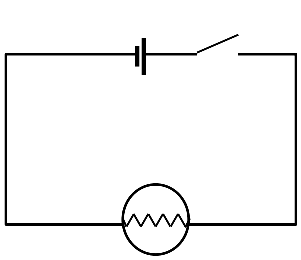
1. Connect the two circuits shown in the diagrams on the right.
2. Close the switch in Circuit 1 and observe the brightness of the bulb.
3. Close the switch in Circuit 2 and observe the brightness of the bulbs.
4. Compare the brightness of the bulbs in Circuit 2 with Circuit 1. Discuss the following questions:
   a. What can you conclude about the current in the bulb in Circuit 1 compared to Circuit 2?
   b. What can you conclude about the current through the battery in Circuit 1 compared to Circuit 2?
5. Use your ammeter to measure the current through the battery in Circuit 1 and Circuit 2. Do your current readings agree with the conclusions that you made from your discussions?
6. Use your ammeter to measure the current in Branch 1 and Branch 2 of Circuit 2. How do these current readings compare with the current through the battery?
7. Use your voltmeter to measure the potential difference across the bulb and the battery in Circuit 1 **with the switch closed**. How do these voltage readings compare?
8. Use your voltmeter to measure the potential difference across the bulbs and the battery in Circuit 2 **with the switch closed**. How do these voltage readings compare?
9. What can you conclude about the relationship between the battery potential difference and the potential difference across the resistors in a parallel circuit?

See the video at https://www.youtube.com/watch?v=vIicY0Y491Q for an experiment showing light bulbs connected in parallel.

You can also find videos on series and parallel circuits at:

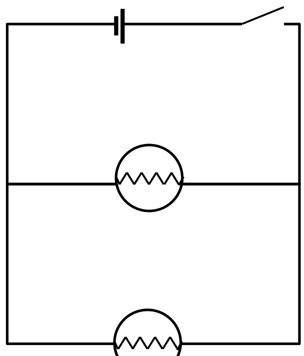
- https://www.youtube.com/watch?v=tZOoBr4ghrw
- https://www.youtube.com/watch?v=XSukRnxGy5c
- https://www.youtube.com/watch?v=x2EuYqj_0Uk

When two resistors are connected in parallel, their combined equivalent resistance is less than for one of the resistors on its own. This is because the parallel connection opens more pathways for the current to move through, so there is less total resistance to the flow of the current.



Circuit 1

Circuit 2 has more paths for the current, so R_{total} is **less** than for Circuit 1.



Circuit 2

We can find the equivalent resistance for parallel resistors using the equation:

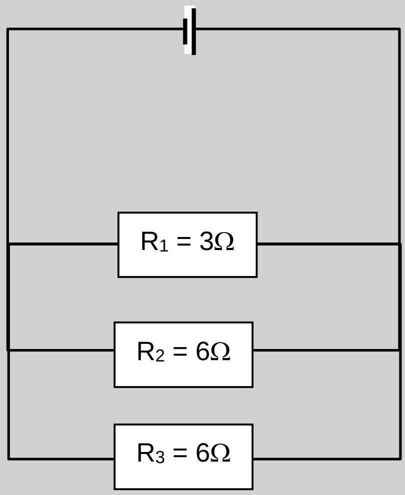
$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots$$

where R_p is the total resistance of the parallel resistors, and

R_1, R_2 etc are the resistances of the parallel resistors.

Example *(Try to solve this problem **on your own or with a fellow student** while covering the solution, and then check your work using the solution below).*

What is the total resistance in the circuit on the right?



Solution

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$= \frac{1}{3\,\Omega} + \frac{1}{6\,\Omega} + \frac{1}{6\,\Omega}$$

$$= \frac{2}{6\,\Omega} + \frac{1}{6\,\Omega} + \frac{1}{6\,\Omega}$$

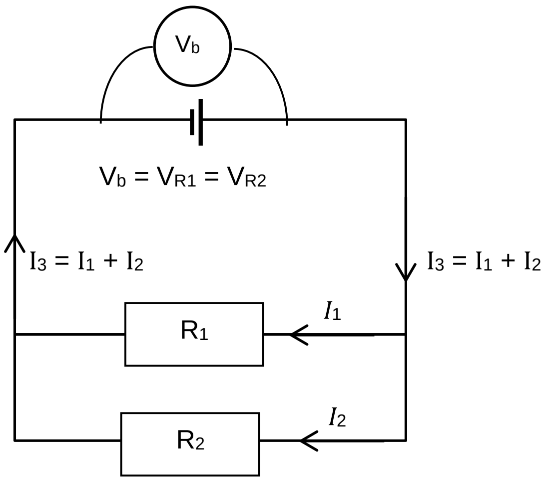
$$= \frac{4}{6\,\Omega}$$

Therefore $R_p = \frac{6\,\Omega}{4} = 1{,}5\ \Omega$.

MAIN IDEAS:

- The equivalent resistance of **parallel resistors** is less than for one resistor on its own.
- The **total equivalent resistance** can be calculated using the equation:
$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots$$

When **current** enters a parallel branch in a circuit, it is divided. The branch with the lower resistance receives a higher current. For this reason, parallel resistors are called ***current dividers***.

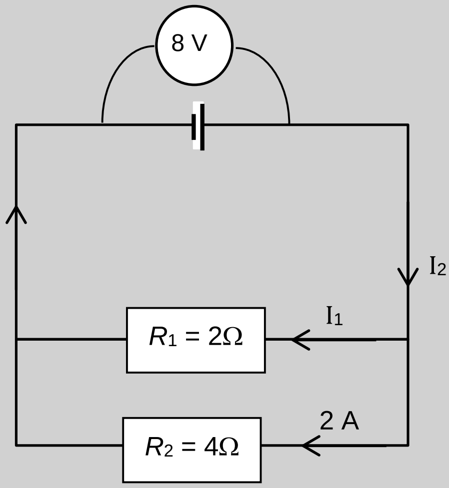


The **potential difference** across parallel branches is **equal**, even if the branches have different resistances. When parallel branches are connected directly across the battery, the potential difference across each branch is equal to the potential difference of the battery.

__**Example**__ *(Try to solve this problem* ***on your own or with a fellow student*** *while covering the solution, and then check your work using the solution below).*

The battery in the circuit has a potential difference of 8 V. (Ignore the effects of the internal resistance of the battery.)

a. Find the unknown currents I_1 and I_2
b. Find the potential difference across the 4 Ω and 2 Ω resistors.



Solution

a. R_1 has half the resistance of R_2, so the current divides so that the R_1 branch will receive double the current of the R_2 branch. So $I_1 = 2 \times 2$ A = 4 A.
$I_2 = I_1 + 1$ A = 4 A + 2 A = 6 A
b. The potential difference across each of the parallel branches is equal to the potential difference across the battery.
Therefore $V_{6\Omega} = V_{3\Omega} = V_b = 8$ V.

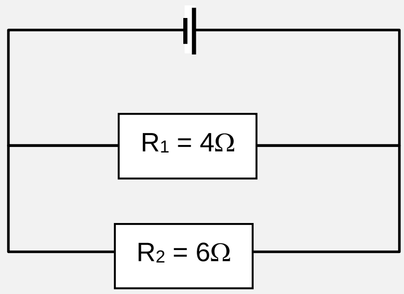
Activity 10: Test your understanding of resistors in parallel

Answer the following questions. Ignore the effects of internal resistance of the batteries.

1.

a. Calculate the total resistance in Circuit 1 on the right.

b. What is the direction of the current in resistor R_1 of Circuit 1?



Circuit 1

2.

a. Calculate the total resistance in the circuit on the right.

b. Find currents I_1, I_2 and I_3.

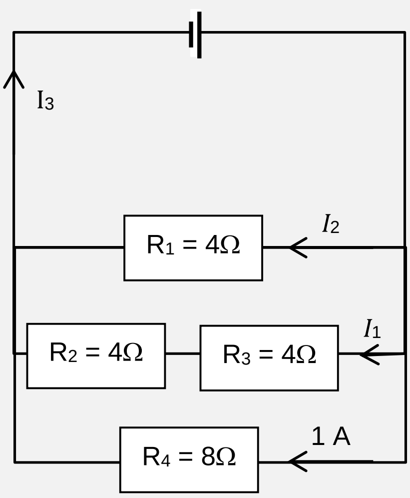
c. The potential difference of the battery is 8 V. Find the potential difference across resistors R_1, R_2 and R_4.

d. If the branch with the resistor R_4 is removed from the circuit, explain what effect this will have on:

i. the total resistance in the circuit,

ii. the current through the battery, and

iii. the potential difference across resistor R_1.



Circuit 2

MAIN IDEAS:

- **Current is divided** as it enters parallel branches.
- Greater current will go to the branch with lower resistance.
- The **potential difference** for resistors connected in parallel is **equal**.
- When parallel branches are connected directly across the battery, the potential difference across each branch is equal to the terminal potential difference of the battery.

Unit 2. Energy transfer in electrical circuits

Learning outcomes:

When you have completed this unit, you should be able to:

- define power as the rate at which electrical energy is converted in an electric circuit, measured in watts;
- state that electrical power dissipated in a device is equal to the product of the potential difference across the device and current flowing through it: $P = IV$;
- apply the concepts of electrical energy and power to solve related problems, in familiar and novel contexts;
- apply knowledge of electrical circuits, energy and power to everyday electrical appliances, for example the torch, kettle etc.

2.1. Electrical Energy

Energy is transferred or converted in an electric circuit when a current passes through a circuit element that has resistance. Energy is transferred as the work that is done in overcoming the resistance. The work done in a resistor can be calculated using the equation:

$W = V\,I\,t$

where W is the work done (or energy transferred) in the resistor, measured in joules (J)
V is the voltage across the resistor, measured in volts (V)
I is the current through the resistor, measured in amperes (A)
t is the amount of time for which the current flows, measured in seconds (s).

<u>Example:</u> *(Try to solve this problem **on your own or with a fellow student** while covering the solution, and then check your work using the solution below).*

A resistor with a resistance of 5 Ω is connected to a 2 V battery. How much work is done in the light bulb in 2 minutes?

<u>Solution:</u>

Given: $R = 5\ \Omega$ $V = 2\ V$ $t = 2\ min \times 60\ s/min = 120\ s$

$I = V / R = 2\ V / 5\ \Omega = 0{,}4\ A$

So $W = V\,I\,t = 2\ V \times 0{,}4\ A \times 120\ s = 96\ J$

MAIN IDEAS:

- The work done in a resistor with a current I and a voltage V for time t can be calculated using the equation: W = V I t
- Electrical energy is measured in joules (J).

2.2. Power

When we talk about the power that is used by electrical appliances, we are usually interested in the rate of energy transfer, in other words, how much energy is transferred to the appliance per unit time. The name of this quantity is "power". So we define power as the rate at which electrical energy is transferred or converted in an electric circuit. We say that power is dissipated by the resistor. We can write this mathematically as:

dissipated **–** used or lost

$$P = \frac{W}{t}$$

where P is the power dissipated by the resistor, measured in watts (W)

W is the energy converted in the resistor, measured in joules (J)

t is the amount of time for which the current flows, measured in seconds (s).

From this equation we can see that 1 watt is equal to 1 joule per second. (1 W = J/s)

Recall that the energy converted by a resistor can be expressed as:

W = V I t

If we substitute this equation into the equation for power, we get:

$$P = \frac{W}{t} = \frac{V\,I\,t}{t} = V\,I$$

This gives us another way of calculating the power dissipated in a resistor:

P = V I

where P is the power dissipated by the resistor, measured in watts (W)

V is the voltage across the resistor, measured in volts (V)

I is the current flowing through the resistor, measured in amperes (A).

From this equation we can see that 1 W is equivalent to 1 V·A. Recall that current is the rate at which charge flows, so 1 A = 1 C/s. Voltage is the potential energy per unit charge, so 1 V = 1 J/C. If we combine these we again find that 1 W = J/s.

MAIN IDEAS:

- Power is defined as the rate at which electrical energy is converted in an electric circuit: $P = \frac{W}{t}$
- Power is measured in watts (W).
- Power can also be calculated using the equation: $P = V\ I$

Example: *(Try to solve this problem* ***on your own or with a fellow student*** *while covering the solution, and then check your work using the solution below).*

A torch bulb is labelled with 4 V and 0,8 A.

a. Calculate the power dissipated by this bulb.
b. If this bulb is connected to a battery for 10 minutes, what is the energy that is transferred to the bulb in this time?

Solution:

Given: V = 4 V I = 0,8 A t = 10 min × 60 s/min = 600 s

a. $P = V\ I = 4\ V \times 0{,}8\ A = 3{,}2\ W$
b. From the equation $P = \frac{W}{t}$ we can make W the subject of the formula:
   $W = P\ t = 3{,}2\ W \times 600\ s = 1920\ J$

OR $W = V\,I\,t = 4\ V \times 0{,}8\ A \times 600\ s = 1920\ J$

If we combine Ohm's Law with the equation for calculating power, we can derive two other mathematical expressions for calculating power.

- If we know the resistance (R) and the current (I), we can substitute the equation $V = I\ R$ into the equation $P = V\ I$, and we will get the equation:
  $P = I^2\ R$

- If we know the resistance (R) and the voltage (V), we can substitute the equation $I = V / R$ into the equation $P = V\ I$, and we will get the equation:
  $P = \frac{V^2}{R}$

Example: *(Try to solve this problem* ***on your own or with a fellow student*** *while covering the solution, and then check your work using the solution below).*

A 12,1 Ω heater is connected to a 110 V power supply.

a. Calculate the power dissipated by this heater.

b. Calculate the current in the heater.

Solution:

Given: R = 12,1 Ω V = 110 V

a. $P = \frac{V^2}{R} = \frac{(110\ V)^2}{12{,}1\ \Omega} = 1\ 000\ W$

b. From the equation $P = I^2 \cdot R$ we solve for I:

$$I = \sqrt{\frac{P}{R}} = \sqrt{\frac{1000\ W}{12{,}1\ \Omega}} = 9{,}09\ A$$

OR $I = \frac{V}{R} = \frac{110\ V}{12{,}1\ \Omega} = 9{,}09\ A$

Activity 1: Test your understanding of electrical energy and power

Answer the following questions:

1. The voltage across a lightbulb in a circuit is measured to be 2,5 V, and the current through it is 0,6 A.
   a. Calculate the resistance of the lightbulb.
   b. Calculate the power dissipated by this lightbulb.
   c. If the lightbulb is operated for half an hour, what is the total energy transferred to this lightbulb?

2. A 200 W mini-heater is connected across a voltage of 220 V. Find the value of the resistance of the heater.

3. A 1760 W kettle is plugged into a 220 V socket.
   a. Calculate the current drawn by the kettle.
   b. If the kettle is used for an average of 2 hours per day, what is the total energy that is used in 1 week?

MAIN IDEAS:

- Electrical power can also be calculated using the following equations:

$$P = I^2 R$$

$$P = \frac{V^2}{R}$$

Assessment Activity: Electric circuits **Total marks = 90**

Answer each of these questions on your own to assess your understanding of Electric Circuits.

1. Choose the correct word or phrase from the box that matches each of the following descriptions. (10)

emf	**resistor**	**current**
ohm	**volt**	**electrical potential difference**
voltmeter	**ampere**	**power**
work	**watt**	**ammeter**

A. Instrument that is used to measure potential difference

B. Instrument that is used to measure current

C. Unit that is equivalent to C/s

D. Unit that is equivalent to V/A

E. Unit that is equivalent to J/s

F. The work done to move a unit charge through a circuit element

G. Circuit element that opposes the path of the current

H. The rate of flow of charges in a circuit

I. The work done in moving a unit charge around a complete circuit

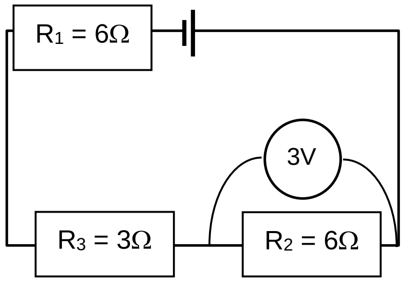
J. The rate at which electrical energy is transferred or converted in an electric circuit

2. A current of 6 mA flows through a resistor. How much charge flows through this resistor in 15 seconds? (3)

3. 2 coulombs of charge passes a point in a circuit in the time of 5 minutes. What is the value of the current at this point in the circuit? (3)

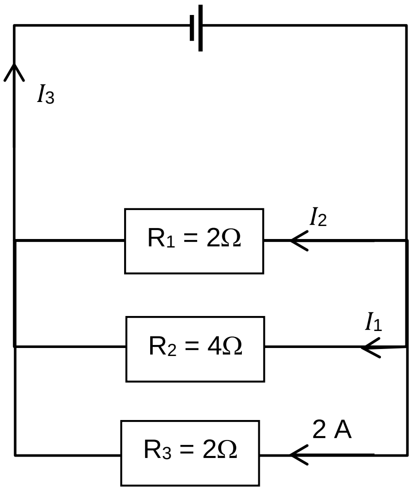
4. Explain the difference between series and parallel resistors. (4)

5. Study the circuit on the right and answer the questions:



   a. What is the total resistance in the circuit? (2)
   b. What is the potential difference across resistors R_1 and R_3? (2)
   c. What is the battery voltage? (3)
   d. Calculate the current flowing in the circuit. (3)

6. Study the circuit on the right and answer the following questions:

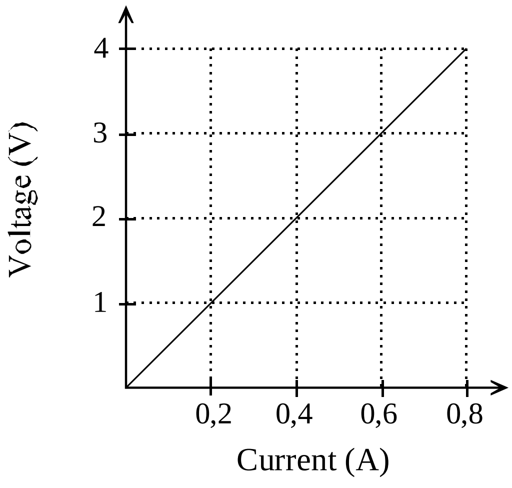


   a. Calculate the total resistance in the circuit.(4)
   b. Find currents I_1, I_2 and I_3. (6)
   c. Find the potential difference across R_2. (2)
   d. What is the battery potential difference? (2)

   Another resistor is added in parallel to resistor R_3. Explain what effect this will have on:

   e. the total resistance in the circuit, (2)
   f. the current through the battery, and (2)
   g. the potential difference across resistor R_1. (2)

7. The diagram shows a graph of voltage across a resistor against current through that resistor. From this graph calculate:



   a. the current from a 2,5V battery (2)
   b. the number of 1,5V batteries that would be needed to give a current of 0,9A (3)
   c. the size of the resistance. (4)

8. For the circuit shown on the right, calculate:

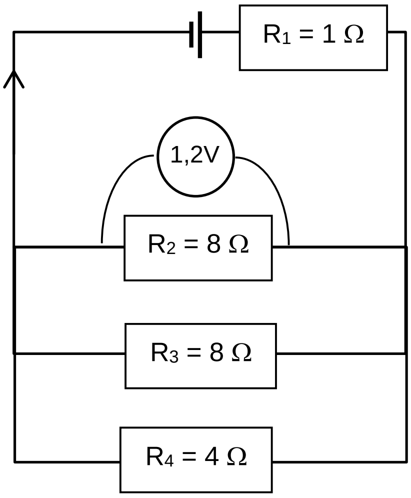
a. the total resistance (4)

b. the current through R_2 (3)

c. the current through R_4 (2)

d. the current through R_1 (3)

e. the battery potential difference (4)



9. A 9 V battery and a 3 Ω resistor are connected in a closed loop.

a. Draw a circuit diagram for this circuit. (3)

b. How much energy does the resistor use in 5 minutes? (3)

c. What is the power dissipated in the resistor? (3)

d. If another 3 Ω resistor is connected in ***series*** with the first, will the total power dissipated in the circuit increase or decrease? Explain your answer. (3)

e. If another 3 Ω resistor is connected in ***parallel*** (instead of in series) with the first, will the total power dissipated in this circuit increase or decrease, compared with the circuit with just the 3 Ω resistor? Explain your answer. (3)

My Notes:

Use this space to write your own questions, comments or key points.

Summary of key learning:

- **Current** (measured in amperes) is a measure of the rate of flow of charges in a circuit: $I = \frac{Q}{\Delta t}$ where1 A = 1 C/s.
- Current flows from the positive terminal of the battery to the negative terminal
- For current to flow we need a **closed circuit** with a power supply (battery)
- When circuit elements are connected together in a single loop this is called a **series circuit**.
- We use an **ammeter** to measure electrical current.
- The ammeter must be connected in **series** in the circuit.
- The current is the **same** at all points in a **series circuit**.
- The **electrical potential difference** across a circuit element is the work done to move a unit charge through the element.
- Electrical potential difference is measured in **volts** (V).
- The **electromotive force** (**emf**) is the potential difference across the battery and is defined as the work done in moving a unit charge around a complete circuit.
- We use a **voltmeter** to measure electrical potential difference.
- The voltmeter must be connected in **parallel** to the circuit element.
- A **resistor** opposes the path of the current.
- **Resistance** is measured in ohms (Ω).
- **Ohm's Law** states that the current through a resistor is directly proportional to the voltage across the resistor at constant temperature.
- Ohm's Law can be written mathematically as: $\mathrm{R} = \frac{\mathrm{V}}{\mathrm{I}}$
- The **current** in series resistors is equal.
- The terminal potential difference across the battery in a circuit is always equal to the **sum of the potential difference** across the resistors in a series circuit: $V_{\text{battery}} = V_1 + V_2 + \ldots.$
- Series resistors are **potential dividers.**
- The potential difference across a resistor is **proportional** to its resistance.
- The **total equivalent resistance** of series resistors is equal to the sum of the individual resistances: $R_s = R_1 + R_2 + \ldots$

- The current in a circuit is **inversely proportional** to the total resistance.
- The equivalent resistance of **parallel resistors** is less than for one resistor on its own:
  $$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots$$
- **Current is divided** as it enters parallel branches.
- Greater current will go to the branch with lower resistance.
- The **potential difference** for resistors connected in parallel is **equal**.
- When parallel branches are connected directly across the battery, the potential difference across each branch is equal to the terminal potential difference of the battery.
- The **energy** (E) converted in a resistor with a current I and a voltage V for time t can be calculated using the equation: E = V I t
- Electrical energy is measured in joules (J).
- Power is defined as the rate at which electrical energy is converted in an electric circuit: $P = \frac{E}{t}$
- Power is measured in watts (W).
- Power can also be calculated using the equation: P = V I
- Electrical power can also be calculated using the following equations: $P = I^2 R$

Sub-topic 3. Magnetism

Content:

Unit 1: Magnetism

Unit 2: The magnetic field

Unit 1. Magnetism

Learning outcomes:

When you have completed this unit, you should be able to:

- describe a permanent magnet as having a North Pole and a South Pole;
- state that like poles repel and unlike poles attract.

Introduction

Magnets form an important part of our everyday lives. You rely on magnetism every time you drive in a car, listen to a radio, turn on your light, or close a fridge door. In this unit you will investigate the concept of magnetism. You will learn about the interactions between magnets, and about the way magnets are used in our everyday lives.

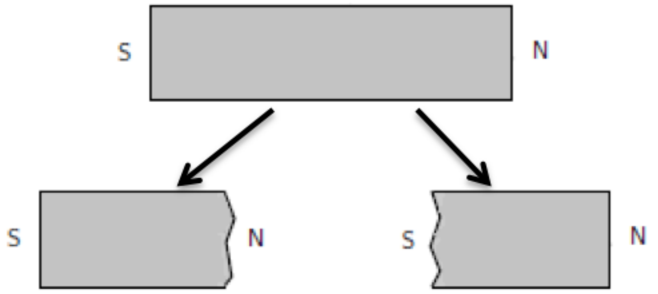
1.1. Magnets

Magnets are able to attract other magnets, or other objects that are made from magnetic materials. Many metals are attracted to magnets, but not all metals are. For example, aluminium is a metal that is not magnetic.

A magnet has a **North Pole** at one end, and a **South Pole** at the other end. The North Pole of a bar magnet is usually painted blue, and the South Pole is usually painted red. (Not all magnets are painted with these colours.)

A bar magnet

It is never possible to have an object that only has a North Pole, or only a South Pole. If a magnet is broken into smaller pieces, each of these small pieces will have a North Pole at one end and a South Pole at the other.



Activity 1: Exploring magnets

<u>You will need:</u>

A magnet
Various objects made from different materials

1. Find a range of objects made from different materials such as plastic, aluminium, steel, brass, cloth and wood.
2. Bring your magnet close to each object. What do you observe?
3. Make a table like the one below. In the first column, fill in the list of objects that are not attracted to the magnet, and in the right hand column fill in those that are attracted to the magnet.

Objects that are not attracted to the magnet	Objects that are attracted to the magnet

4. Do you notice any trends about the objects that are attracted to the magnet?
5. Are all metals able to be attracted by a magnet?

1.2. Interactions between magnets

There is a force between magnets that are close to one another. In the following activity you will explore the interactions between magnets.

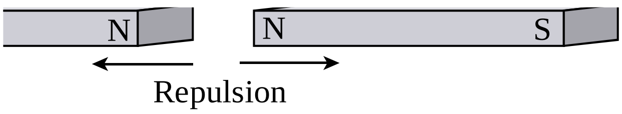
Activity 2: Exploring the interactions between magnets

You will need:

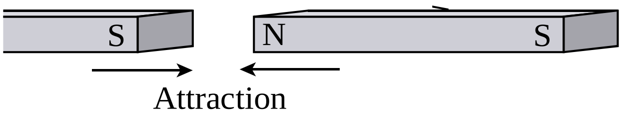
2 bar magnets
Some string
A compass

1. Hang one of the magnets from a string so that it is free to move around. Make sure that there are no other magnets or ferromagnetic objects nearby.
2. Bring the one end of the other magnet near to one of the ends of the hanging magnet. What do you observe?
3. Turn the magnet that you are holding around, and observe the effect that it has on the hanging magnet.
4. Bring a compass close to the North Pole of the magnet. What do you observe?
5. Now move this compass around to the South Pole of the magnet. What do you observe?
6. What can you infer from your observations?

If we bring two **like** poles close to each other, such as two North Poles, they **repel** one another.



If we bring two **unlike** poles close to each other, such as a North Pole and a South Pole, they **attract** one another.



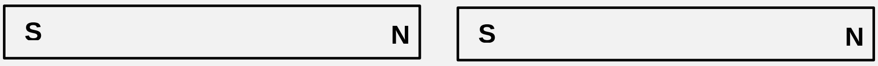
When a compass is brought close to a magnet, the compass turns to line up with the magnet. This is because the compass is a small magnet that is free to turn. The North Pole of the compass turns to point towards the South Pole of the magnet.

A compass (from http://walkinginsunlight.com/wp-content/uploads/2014/05)

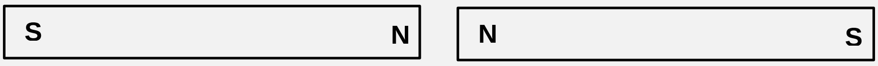
Activity 3: Test your knowledge of interactions between magnets

1. Explain what you will observe in each of the following arrangements of magnets:

   a.

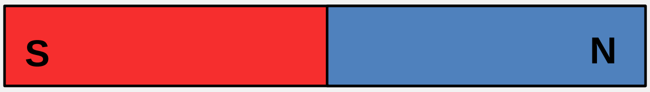


   b.



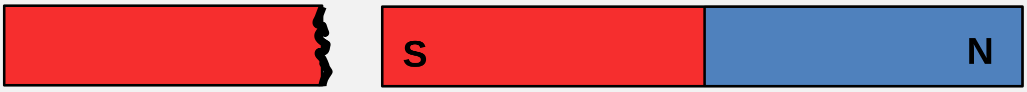
2. You bring one side of a magnet close to a compass and you find that the **South Pole** of the compass is attracted to this side of the magnet.
   a. Is this side of the magnet a north or a South Pole?
   b. If you bring a compass close to the **opposite side** of the magnet, what will you observe?

3. You have a bar magnet like the one shown in the picture below.



   a. If you break the magnet in half, and you bring the broken part of the blue half near to the South Pole of another magnet, what will you observe?

   b. If you bring the broken part of the red half near to the South Pole of another magnet, what will you observe?



MAIN IDEAS:

- A magnet has a North Pole at one end, and a South Pole at the other end.
- Unlike poles attract each other, and like poles repel each other.
- A compass is a small magnet that is free to spin around.

Unit 2. The magnetic field

Learning outcomes:

When you have completed this unit, you should be able to:

- define the magnetic field;
- describe the magnetic field around a bar magnet and a pair of bar magnets placed close together;
- explain how a compass indicates the direction of a magnetic field;
- describe the Earth's magnetic field.

2.1. The magnetic field around a magnet

In the space around a magnet, there is a **magnetic field**. This is a region in space where another magnet, or an object made from a magnetic material, will experience a force. In the following activity you will explore the shape of the magnetic field around a bar magnet.

Activity 1: Exploring the magnetic field around a magnet

<u>You will need:</u>

1 bar magnet
A sheet of clear plastic or an overhead transparency
Iron filings
A piece of paper
A compass

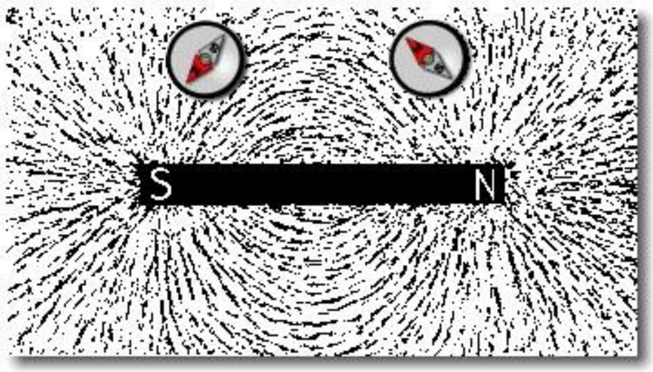
1. Place the bar magnet in the middle of your piece of paper.
2. Cover the bar magnet with your sheet of clear plastic or transparency.
3. Scatter your iron filings onto the sheet of plastic and lightly tap the plastic until the filings form into a pattern. Observe the shape of the patter that is formed. This tells you the shape of the magnetic field around the magnet.
4. Remove your sheet of plastic and carefully pour your iron filings back into their container.
5. Place the compass near to the magnet.
6. Draw a small arrow on your piece of paper to show the direction of the compass needle.

7. Move your compass to a new position on your piece of paper, and draw in the needle direction.
8. Continue moving your compass to as many positions on the paper as you can, drawing in the needle direction.
9. The arrows that you have drawn are the magnetic field lines which show the direction of the magnetic field around the bar magnet. What conclusions can you draw about this shape?

See a video of a practical experiment showing the magnetic field at https://www.youtube.com/watch?v=j8XNHlV6Qxg.

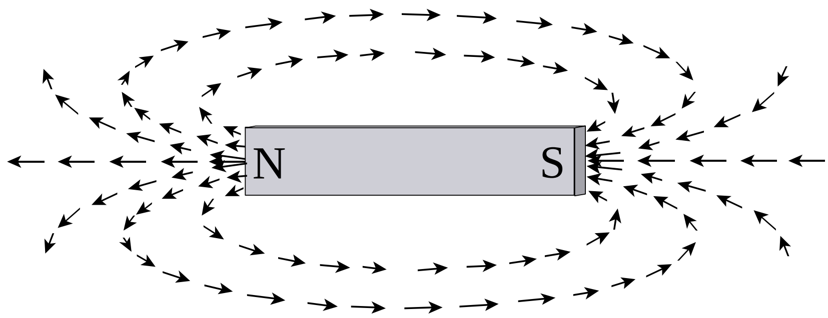
The shape of the magnetic field around a bar magnet is shown in the photograph on the right. We can use a compass to show the direction of the magnetic field.

If we want to draw the shape and direction of a magnetic field, we use little arrows called **magnetic field lines**. These lines don't cross each other, and are further apart where the field is weaker, and closer to each other where the field is stronger.



The magnetic field (from http://resources.yesican-science.ca/magnets/lines2.jpg)

The magnetic field around a straight bar magnet has the following shape:



MAIN IDEAS:

- There is a **magnetic field** around a magnet.
- Any other magnet or magnetic object that is brought into this magnetic field will experience a **magnetic force**.
- The **magnetic field lines** are used to show the magnetic field, and point outward from the North Pole, and inward toward the South Pole.

2.2. The magnetic field around a pair of magnets

When magnets are brought close to each other, their magnetic fields interact with one another. In the following activity you will explore the shape of the magnetic field around a pair of bar magnets.

Activity 2: Exploring the magnetic field around a group of magnets

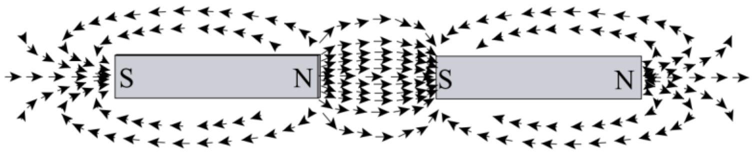
<u>You will need:</u>

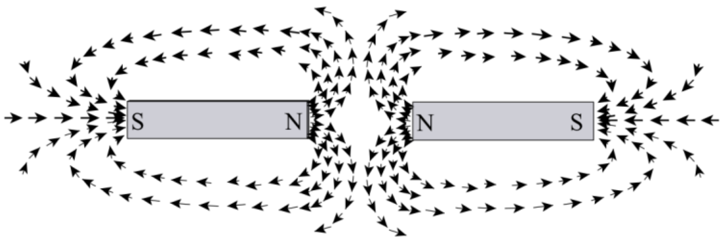
2 bar magnets
A sheet of clear plastic or an overhead transparency
Iron filings
2 pieces of paper
A compass

Design and conduct experiments that use this equipment to test the magnetic field around the following arrangements of the magnets:

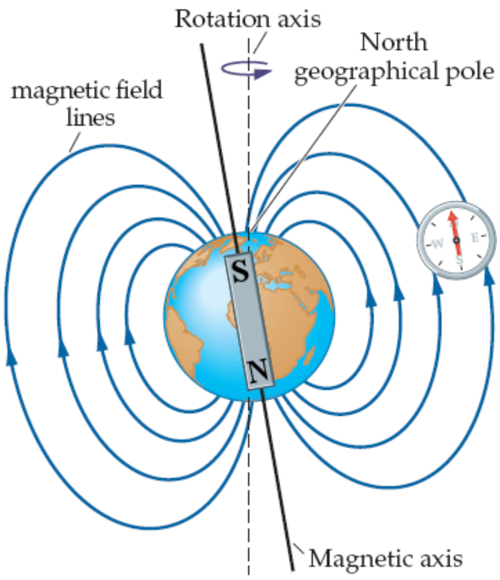
- Two magnets with their north poles close to each other
- Two magnets with opposite poles close to each other

The magnetic field lines for various arrangements are shown in the diagram below:





2.3. The Earth's magnetic field



The Earth's magnetic field (from http://www4.uwsp.edu/physastr/kmenning/images/Earth_field_in-class_answer.png)

There is a large magnetic field around the Earth. This is why compasses help us to find our direction if we are lost.

The North Pole of a compass points to the Earth's magnetic North Pole, and so the earth's magnetic North Pole is actually a South Pole, as the diagram shows.

MAIN IDEAS:

- The **Earth** has a **magnetic field** surrounding it, which can be observed using a compass.

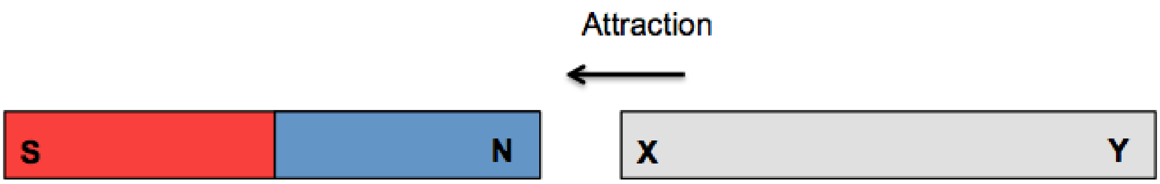
Assessment Activity: Magnetism **Total marks = 20**

Answer the following questions to assess your understanding of magnetism:

1. Give one word for each of the following: (3)
   a. An object that attracts magnetic materials.
   b. An instrument consisting of a small, rotating magnet that shows the direction of a magnetic field.
   c. The area around a magnet where a magnetic force would be experienced by another magnet or an object made of ferromagnetic material.

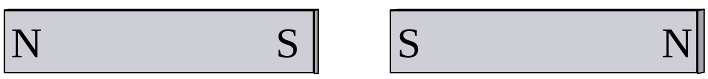
2. Describe what you would observe if you brought the South Pole of a bar magnet close to the following:
   a. The North Pole of another magnet (2)
   b. The South Pole of another magnet (2)
   c. An object made from a magnetic material (2)
   d. A piece of aluminium (2)

3. Side X of a bar magnet is brought close to the North Pole of another magnet, and you observe a force of attraction between them.



   a. Is side Y of the bar magnet a North or a South Pole? (1)
   b. What will you observe if you bring side X of the bar magnet close to the South Pole of the other magnet? (2)

4. Draw the magnetic field lines around the arrangement of bar magnets shown in the diagram below. (3)



5. Draw a circle to represent the Earth, and draw the magnetic field lines around the Earth. Show the directions of the magnetic field lines on your diagram. (3)

My Notes:

Use this space to write your own questions, comments or key points.

Summary of key learning:

- A magnet has a **North Pole** at one end, and a **South Pole** at the other end.
- **Unlike** poles **attract** each other, and **like** poles **repel** each other.
- A **compass** is a small magnet that is free to spin around.
- There is a **magnetic field** around a magnet.
- Any other magnet or magnetic object that is brought into this magnetic field will experience a **magnetic force**.
- The **magnetic field lines** are used to show the magnetic field, and point outward from the North Pole, and inward toward the South Pole.
- The **Earth** has a **magnetic field** surrounding it, which can be observed using a compass.

Sub-topic 4. Electromagnetism

Content:

Unit 1: The magnetic effect of a current

Unit 2: Force on a current-carrying conductor

Unit 1. The magnetic effect of a current

Learning outcomes:

When you have completed this unit, you should be able to:

- draw the direction of the magnetic field near a current-carrying wire and a current-carrying loop;
- state the effect on the magnetic field of changing the magnitude and / or direction of the current;
- explain how an electromagnet works.

1.1. The magnetic field near a current-carrying wire

In the following activity, you will investigate the magnetic field near a wire that has a current flowing through it.

Activity 1: Investigating the magnetic field around a current carrying conductor

<u>You will need:</u>

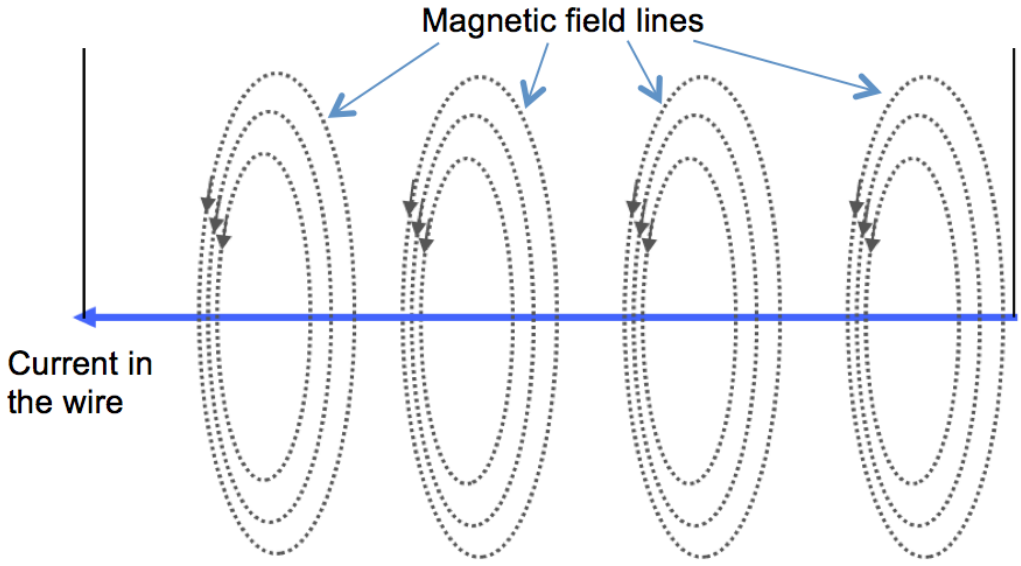
2 torch batteries (1,5V each)

1 electric lead (about 30cm in length)

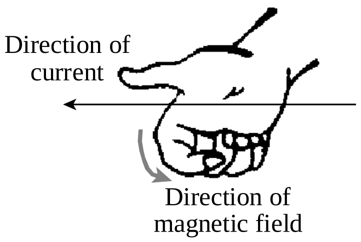
A compass

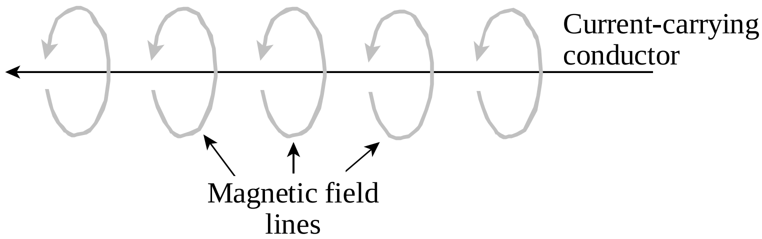
1. Lay your compass on a table, facing upwards. Wait until it points north.
2. Lay the middle of your electric lead above the compass needle, so that it lines up with the needle.
3. Connect one end of the wire to one end of the battery. Very briefly connect the other end to the other terminal of the battery. What do you observe on your compass? (**NOTE**: only connect the leads to the battery for a **very short amount of time**, as you will drain the battery if you leave it connected for too long!)
4. What do you observe about the needle as you disconnect the leads from the battery?
5. Reverse the direction of your battery, and repeat this procedure. What do you observe on your compass?
6. Move your compass so that it is on top of the lead. (One learner in your group will need to hold it in place). Repeat the above procedures with the battery in both positions, and note the effect on your compass needle.
7. What can you infer about the direction of the magnetic field, compared with the direction of the current?
8. Draw a diagram of the wire, and show the direction of the current, and the direction of the magnetic field around it.

As your investigation has shown you, the relationship between electricity and magnetism is **circular**. Any wire or conductor that has a current flowing through it has a magnetic field that circles it. The greater the value of the current in the wire, the stronger the magnetic field that will be produced around the wire.



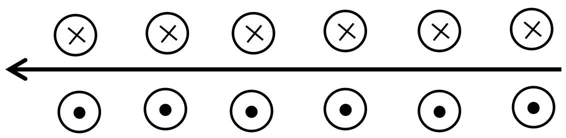
A simple rule to help one to work out the direction of the magnetic field that is induced by the current is called the “Right Hand Rule”. Curl the fingers of your right hand around, and point your thumb out straight. With your thumb pointing in the direction of the current flow, your curled fingers will be pointing in the direction of the magnetic field.





We can show the direction of the magnetic field coming out of the page using a dot ⊙. A cross ⊗ can be used to show that the direction of the magnetic field is into the page. These symbols are used because, if you picture an arrow coming out of the page, you would see its sharp tip, which looks like a dot. If the arrow is going into the page, you would see the back of the arrow, which looks like a cross.

We can use these symbols show the direction of the magnetic field around a current-carrying conductor. This is shown in the diagram below.



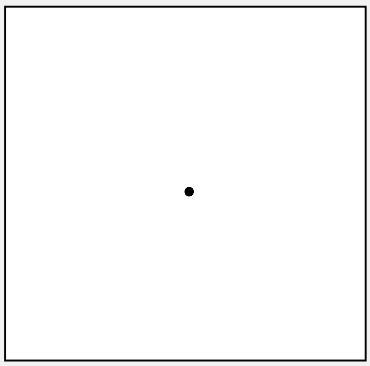
Activity 2: Test your understanding of the magnetic field around a current-carrying wire

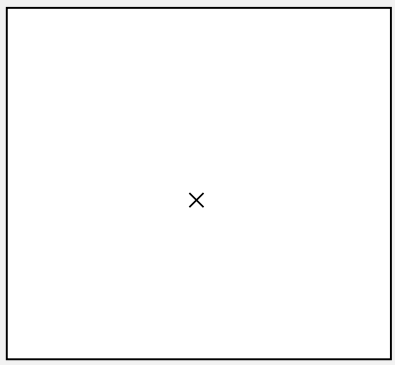
6. Use the Right Hand Rule to draw the direction of the magnetic field around the wires shown below, where the current direction is shown by the arrows. (You can use curved arrows to show the direction of the magnetic field, or you can use dots and crosses).

    a.

    b.

7. We can also use dots to show that the direction of the current is flowing out of the page, or crosses to show that current is flowing into the page. Draw the direction of the magnetic field around the current represented by the dot and the cross shown below:



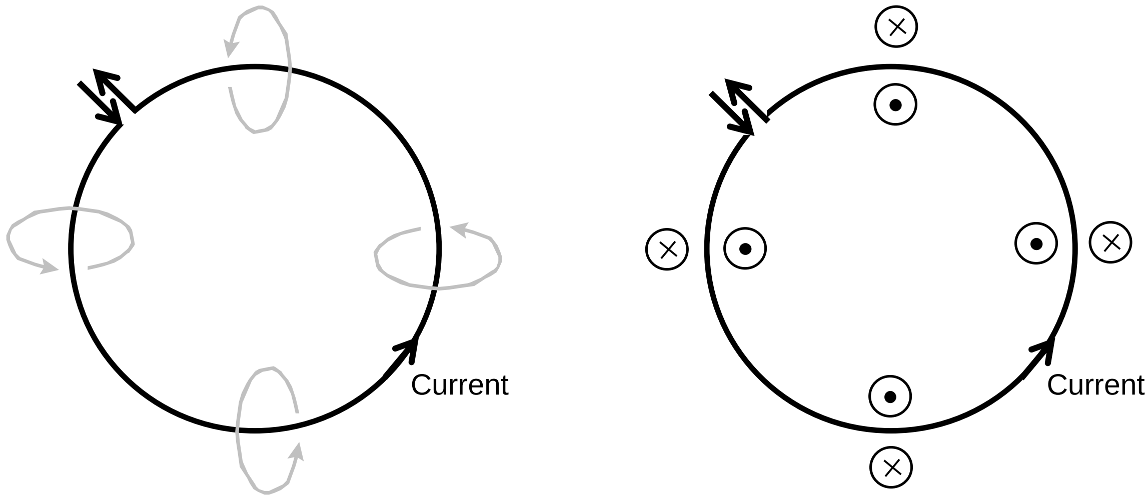


MAIN IDEA:

- When a current flows in a conducting wire, it creates a **circular magnetic field** around the wire.
- The direction of the magnetic field lines can be found using the **Right Hand Rule.**

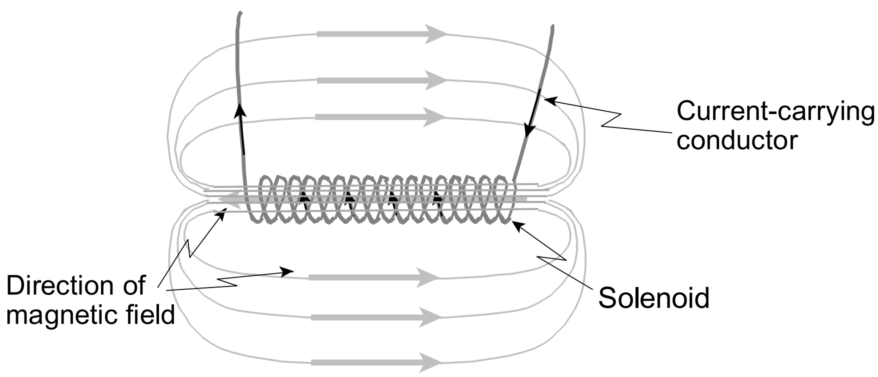
1.2. The magnetic field near looped wires

If we take a current-carrying wire and make a single loop with it, we can work out the pattern of the magnetic field near the wire. The magnetic field around the loop is shown on the left using curved arrows, and in the diagram on the right using dots where the magnetic field is pointing out of the page, and crosses where it is pointing into the page.



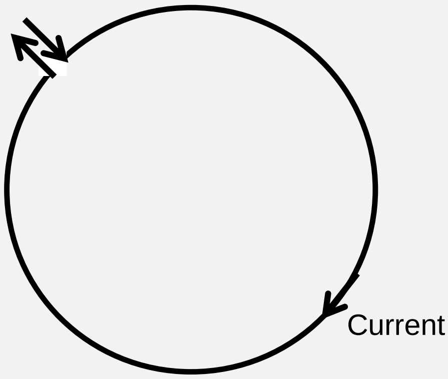
From this diagram you can see that the magnetic field at the center of the loop points outward from the page, and on the outside of the loop it points into the page. Again we can use the Right Hand Rule to work this out. If you curl your fingers in the direction of the current in a current-carrying loop, your thumb will point in the direction of the magnetic field at the center of the loop.

If we have a number of current-carrying loops, all wound in the same direction, they form a **solenoid**. The magnetic fields from these loops have the same direction, so they add together through superposition to create a stronger magnetic field. The diagram below shows what a magnetic field associated with a solenoid looks like.



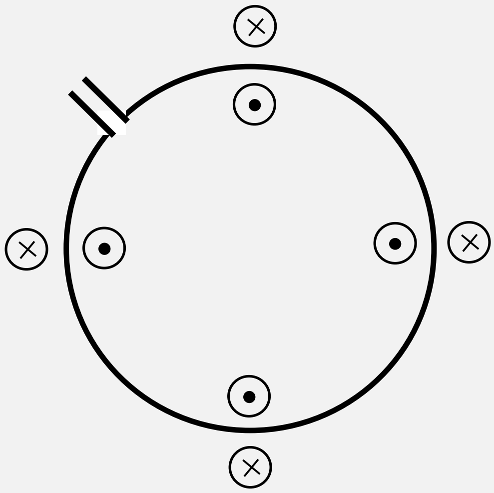
Activity 3: The magnetic field near looped wires

Answer the following questions:



1. The diagram on the right shows the direction of the current in a single looped wire.
    a. What is the direction of the magnetic field at the center of the loop?
    b. What can you do to change the direction of the magnetic field so that it has the opposite direction?

2. The diagram below shows the direction of the magnetic field around a current-carrying looped conductor. Draw in the direction of the current in the loop.



MAIN IDEA:

- We can use the Right Hand Rule to work out the direction of the magnetic field created by a current-carrying loop.
- A number of current-carrying loops, all wound in the same direction, form a coil that is called a **solenoid.**

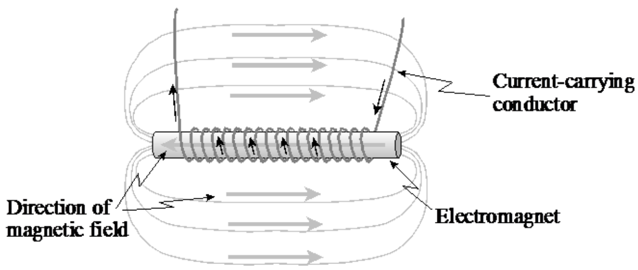
Activity 4: *Build an electromagnet*

You will need:

4 torch batteries (1,5V each) , taped together to create one large 9V battery
About 1.5m to 2m of plastic-coated wire
A large nail
Some metal paper clips or pins
A compass

1. Wind the wire around the nail, and connect one of the bare ends of the wire to a terminal of the battery. Do not connect the other end of the wire to the battery yet.
2. Bring the tip of the nail near to the pins or paper clips. Close your circuit so that there is current flowing through the wire. What do you observe?
3. Disconnect the circuit by removing one of the ends of the wire from the battery. What do you observe?
4. Design an experiment in which you use your compass to test which end of your electromagnet is north.
5. Draw a diagram of your electromagnet, showing the direction of the current in the wire. Draw in the magnetic field lines.

In this activity, you should have observed that the nail acted like a magnet when there was current flowing through the coil, since the nail exerted a force of attraction on the paper clips. As soon as the current was stopped, the nail no longer attracted the paper clips. The nail with the wire wrapped around it is called an **electromagnet**. The direction of the magnetic field set up in the nail is shown in the diagram on the right.



MAIN IDEA:

- An **electromagnet** can be made by winding a conducting wire around a magnetic material. When a current flows in the wire, a magnetic field is created in the magnetic material.

Unit 2. Force on a current-carrying conductor

Learning outcomes:

When you have completed this unit, you should be able to:

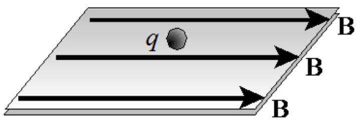
- determine the direction of the force on a current-carrying conductor in a magnetic field.

2.1. Charged particles moving through a magnetic field

Up to now you have noticed that a current that flows creates a magnetic field around it. What would happen if a current flows through a magnetic field that already exists?

We'll investigate this question by first looking at three possibilities for a single positive charge in a magnetic field. In the diagram below, the symbol B is used to label the magnetic field lines. The velocity of the charge is shown with the symbol *v*.

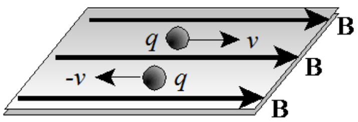
Possibility 1:



A positive charge *q* is **not moving** in a magnetic field.

Result: Scientists have found that there is **no force** on this charge

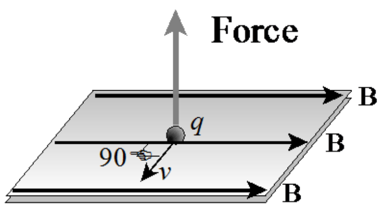
Possibility 2:



A positive charge *q* moves **parallel** to the direction of a magnetic field (either in the same direction, or in the opposite direction to the magnetic field lines).

Result: Scientists have found that there is **no force** on this charge

Possibility 3:

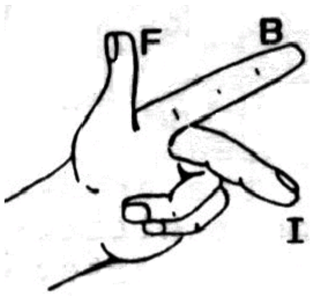


A positive charge *q* moves **at right angles** to the direction of a magnetic field.

Result: Scientists have found that there is an **upward force** on this charge.

In other words, if a charge moves at right angles to a magnetic field, it will experience a force. The direction of this force is at right angles to *both* the magnetic field and the direction of movement of the charge.

We can apply these results to a current, since a current is a continuous movement of charges. Therefore, if a current is flowing at **right angles to a magnetic field**, then there will be a **force** on that current which will be perpendicular to both the current direction, and the magnetic field direction.

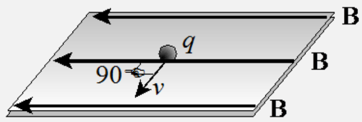


To determine the direction of the force, you can use your *left hand* to help you. With the index finger of your hand pointing in the direction of the magnetic field (**B**), and your middle finger pointing in the direction of the velocity of the charge (or in the direction of the current **I**), your thumb will be pointing in the direction of the force (**F**). (This can also be called the **FBI** rule).

Activity 1: Reflection questions

Discuss these questions with a fellow student, or reflect on these on your own:

1. What is the direction of the force on the positive charge shown in the diagram on the right?
2. What do you think the direction of the force would be on a *negative* charge moving in this same direction?



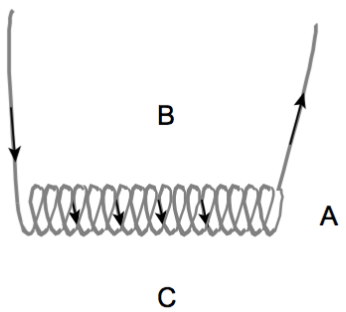
MAIN IDEA:

- When a charge moves at right angles to a magnetic field, it will experience a **force**.
- The direction of this force can be found using the **left-hand rule** (or the **FBI** rule).

Assessment Activity: Electromagnetism **Total marks = 30**

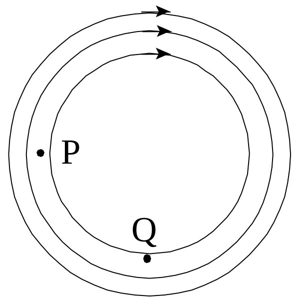
Test your understanding of electromagnetism by answering the following questions:

1. Describe how you would test for the presence of a magnetic field near a current-carrying conductor. (3)
2. The diagram below shows the direction of the current in a solenoid. What is the direction of the magnetic field at points A, B and C? (3)



3. An electromagnet is made by winding a length of current-carrying wire around a nail. The magnetic north is at the sharp tip of the nail. Draw a diagram of this set-up, showing how the wire should be connected to a torch battery to create a magnetic field in this direction. (3)

4. The diagram shows the direction of the field lines of a circular magnetic field.



   a. What direction of current would create this magnetic field? (1)
   b. How could the direction of the magnetic field lines be reversed? (1)
   c. If a positive charge moves to the left at point P, what is the direction of the force on it? (2)
   d. If a positive charge moves to the right at point Q, what is the force on it? (2)
   e. If a compass is placed at point Q, what would happen to its needle? (1)
   f. If the current is suddenly switched off, what would happen to the compass needle? (2)

5. In which of the following will a force of magnetic attraction be observed? Explain your answer in each case.
   a. A magnet is brought close to a paper clip that is made from iron. (3)
   b. An iron rod is brought close to an iron paper clip. (3)
   c. Some wire that is attached to a battery is wound around an iron rod, and the rod is brought close to an iron paper clip. (3)

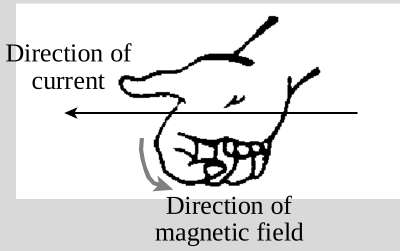
d. Some wire that is attached it to a battery is wound around an aluminium rod, and the rod is brought close to an iron paper clip. (3)

My Notes:

Use this space to write your own questions, comments or key points.

Summary of key learning:

- When a current flows in a conducting wire, it creates a **circular magnetic field** around the wire.
- The direction of the magnetic field lines can be found using the **Right Hand Rule.**



- We can use the Right Hand Rule to work out the direction of the magnetic field created by a current-carrying loop.
- A number of current-carrying loops, all wound in the same direction, form a coil that is called a **solenoid.**
- An **electromagnet** can be made by winding a conducting wire around a magnetic material. When a current flows in the wire, a magnetic field is created in the magnetic material.
- When a charge moves at right angles to a magnetic field, it will experience a **force**.
- The direction of this force can be found using the **left-hand rule** (or the **FBI** rule).