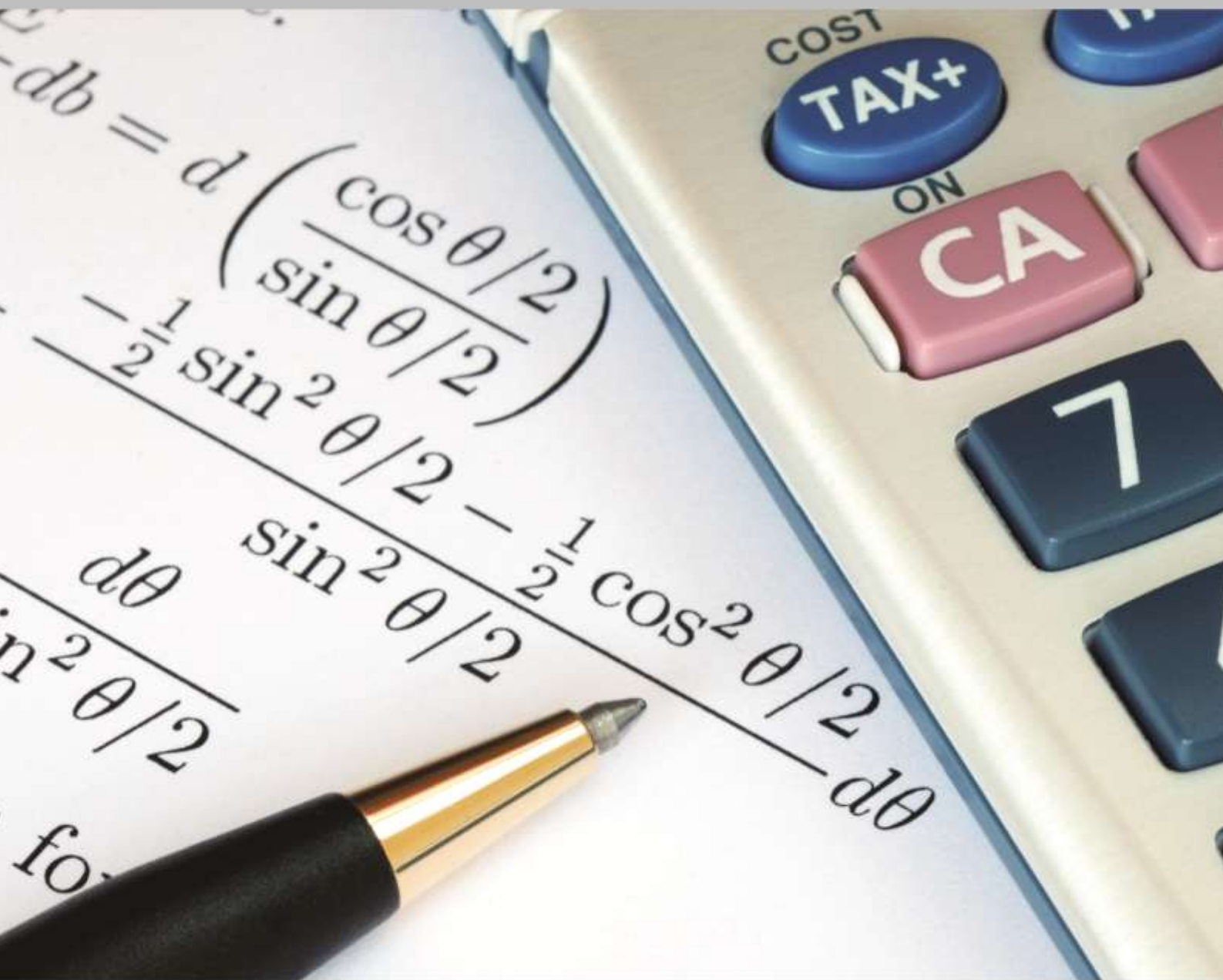


NASCA

MATHEMATICS

Curriculum Statement



higher education
& training

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Introduction

Mathematics is a language that makes use of symbols and notations for describing numerical, geometric and graphical relationships. It is a human activity that involves observing, representing and investigating patterns and qualitative relationships in physical and social phenomena and between mathematical objects themselves. It helps to develop mental processes that enhance logical and critical thinking, accuracy and problem-solving that will contribute in decision-making. Mathematical problem-solving enables us to understand the world (physical, social and economic) around us, and, most of all, to teach us to think creatively.

2. Aims

- 2.1. The Mathematics curriculum is to provide students with the fundamental mathematical knowledge and skills.
- 2.2. Develop fluency in computation skills without relying on the usage of calculators.
- 2.3. Develop the necessary process skills for the acquisition and application of mathematical concepts and skills.
- 2.4. Develop the mathematical thinking, problem-solving and modelling skills and apply these skills to formulate and solve problems.
- 2.5. Develop the abilities to reason logically, to communicate mathematically, and to learn cooperatively and independently.
- 2.6. Provide the opportunity to develop in students the ability to be methodical, to generalise, make conjectures and try to justify or prove them.
- 2.7. Be able to understand and work with the number system.
- 2.8. To promote accessibility of Mathematical content to all students. It could be achieved by catering for students with different needs.
- 2.9. Make effective use of calculators and other technology in the learning and application of Mathematics.
- 2.10. To prepare the students for further education and training as well as the world of work.

3. Exit-Level Outcomes

By the end of this course candidates should be able to:

1. Recognise, describe represent and work with numbers and their relationships to estimate, calculate and check in solving problems:
 - 1.1. Use laws of exponents, surds and logarithms to simplify expressions and solve equations;
 - 1.2. Investigate number patterns;
 - 1.3. Problems involving number patterns are identified and solved
Range: Includes arithmetic and geometric sequences and series;
 - 1.4. Prove and use formulae to calculate the sum of arithmetic and geometric sequences (series);
 - 1.5. Apply knowledge of geometric series to solve problems;
 - 1.6. Critically analyse investment and loan options to make informed decisions.

2. Investigate, analyse, describe and represent a wide range of functions and solve related problems:
 - 2.1. Manipulate, simplify and factorise algebraic expressions;
 - 2.2. Solve algebraic equations;
 - 2.3. Apply and use the formal definition for the function concept;
 - 2.4. Characteristics of functions are explained and graphs are sketched;
 - 2.5. Investigate the effect of parameters on the graphical representations of functions;
 - 2.6. Use a variety of techniques to sketch and interpret information for graphs of the inverse of a function;
 - 2.7. Use functions to solve real-life context problems via modelling.

3. Describe, represent, analyse and explain properties of shapes in 2-and 3-dimensional space with justification:
 - 3.1. Calculate, investigate and solve problems involving volume and surface area of geometric objects such as cubes, rectangular prisms, cylinders, triangular prisms, hexagonal prisms, right pyramids (with square, equilateral or hexagonal basis), right cones, spheres and a combination of such geometrical objects;
 - 3.2. Investigate, prove and apply theorems of the geometry of circles;

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- 3.3. Represent geometric figures in a Cartesian co-ordinate system, and derive and apply, for any two points $(x_1; y_1)$ and $(x_2; y_2)$, a formula for calculating:
- The distance between two points;
 - The gradient of the line segment joining the points;
 - Conditions for parallel and perpendicular lines;
 - The coordinates of the midpoints of the line segment joining the points.
- 3.4. A two dimensional Cartesian co-ordinate system is used to derive and apply:
- The equation of a line through two given points;
 - The equation of a line through one point and parallel or perpendicular to a given line;
 - The inclination of a line;
 - The equation of a circle (any centre);
 - The equation of a tangent to a circle at a given point on the circle.
- 3.5. Define the trigonometric ratios $\sin\theta$, $\cos\theta$ and $\tan\theta$ in right angled triangles and then extend the definitions of $\sin\theta$, $\cos\theta$ and $\tan\theta$ for $0^\circ \leq \theta \leq 360^\circ$;
- 3.6. Derive and use values of the trigonometric ratios for the special angles $\theta \in \{0^\circ; 30^\circ; 45^\circ; 60^\circ; 90^\circ\}$;
- 3.7. Derive and use the identities $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\sin^2\theta + \cos^2\theta = 1$;
- 3.8. Derive and use the reduction formulae to simplify trigonometric expressions and solve trigonometric equations;
- 3.9. Determine the general solution and/or specific solutions of trigonometric equations;
- 3.10. Establish and apply the sine, cosine and area rules;
- 3.11. Derive and apply the compound angle and double angle identities;
- 3.12. Problems in 2-and 3-dimensions are solved by constructing and interpreting geometrical and trigonometric models;
4. Collect and use data to establish statistical models to solve related problems:
- 4.1. Collect, organise and interpret univariate numerical ungrouped data by using measures of central tendency and dispersion;
 - 4.2. Collect, organise, represent and interpret univariate numerical grouped data by using measures of central tendency and dispersion.
 - 4.3. The use of variance and standard deviations as measures of dispersion of a set of ungrouped data is understood and demonstrated;
 - 4.4. Represent, analyse and interpret data using various techniques;
 - 4.5. Calculate, represent and interpret relationships between bivariate data.

Weighting of Levels of Cognitive Demand

4.1 The Four Cognitive Levels Used to Guide the Setting of the External Examination Papers are:

- Knowledge;
- Routine procedures;
- Complex procedures; and
- Problem-solving.

The suggested weightings for Paper 1 and Paper 2 are shown in Table 1.

Table 1: Weightings of Levels of Cognitive Demand

Knowledge	Routine Procedures	Complex Procedures	Problem-solving
20%	30%	35%	15%

4.2 Descriptors for Each Cognitive Level With Illustrative Examples

The four cognitive levels used to guide all assessment tasks are based on those suggested in the TIMSS study of 1999.

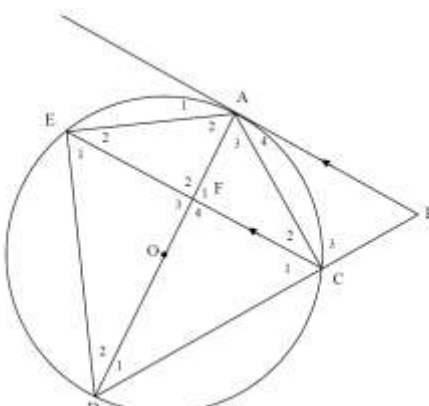
Descriptors for each level and the approximate percentages of tasks, tests and examinations which should be at each level are given below:

Cognitive Levels	Explanation of Skills to be Demonstrated	Examples
Knowledge (K) 20%	<ul style="list-style-type: none"> • Theorems • Straight recall • Identification of correct formula on the Formulae sheet (no changing of the subject) • Use of simple mathematical facts • Know and use of appropriate mathematical vocabulary • Knowledge and use of formulae (without changing of the subject) • Answer only: e.g. Definitions <p><i>All of the above will be based on known knowledge.</i></p>	1. Write down the domain and range of the function f where $f(x) = -x^2$. 2. The angle \hat{AOB} subtended by arc AB at the centre O of a circle is equal to 3. Write down the first five terms of the sequence with general term $T_k = \frac{1}{3k-1}$. 4. Find the asymptotes, axis of symmetry, x- and y intercepts: reading from the graph. 5. Simplify: $\sin x \cos y + \cos x \sin y$.
Routine Procedures (R) 30%	<ul style="list-style-type: none"> • Algorithms • Estimation and appropriate rounding of numbers • Application of prescribed theorems • Identification and use of correct formula on the Formulae sheet and manipulating of formulae 	1. Solve for x : $6x^2 + 10x - 56 = 0$. 2. Given: $f(x) = 4x-1$ Determine the inverse function f^{-1} . 3. Sketch the graphs of $f(x) = 4x-1$, f^{-1} and $y = x$ line on the same set of axes. What do you notice?

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	<p>(after changing of the subject)</p> <ul style="list-style-type: none"> • Problems can involve the integration of different LOs • Perform well-known procedures • Simple applications and calculations which might involve a few steps • Derivation and interpretation from given information may be involved • Generally similar to those encountered in class <p><i>All of the above will be based on known procedures.</i></p>	<ol style="list-style-type: none"> 4. Find the 8th term in the series 3; 5; 7;..... 5. Determine the largest value of n such that $\sum_{i=1}^n (3i - 2) < 2000$. 6. Determine the value of $\cos 75^\circ$, without using a calculator. 7. O is the centre of the circle and B is the midpoint of the chord AC. Prove using Euclidean geometry methods, the theorem that states OB is perpendicular to AC. (NB. The diagram will be provided). 8. Find the value of $\sin 3x$ in terms of $\sin x$.
<p>Complex Multi-Step Procedures (C)</p> <p>35%</p>	<ul style="list-style-type: none"> • Problems are mainly unfamiliar and students are expected to solve by integrating different LOs • Problems involve complex calculations and/or higher order reasoning • Problems do not always have an obvious route to the solution but involve: <ul style="list-style-type: none"> ➢ using higher level calculation skills and reasoning to solve problems ➢ mathematical reasoning processes • These problems are not necessarily based on real world contexts and may be abstract requiring fairly complex procedures in finding the solutions • Could involve making significant connections between different representations • Require conceptual understanding 	<ol style="list-style-type: none"> 1. Find the sample regression equation by using the method of least squares. 2. Drawing a box and whisker diagram when the outliers are outside the upper and or lower fence or both. 3. If $f(x) = -x^2 + 1$; Determine the equation of the inverse of $f(x) = -x^2 + 1$; $x \geq 0$. 4. Sketching of graphs with horizontal or vertical shift. 5. Determine the 5th term of the geometric sequence of which the 8th term is 6 and the 12th term is 14. 6. Prove that $\frac{1 + \sin 2x}{\cos 2x} = \frac{\cos x + \sin x}{\cos x - \sin x}$.

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<p>Solving Problems (P)</p> <p>15%</p>	<ul style="list-style-type: none"> • Non-routine problems (which are not necessarily difficult) • Using higher level cognitive skills, reasoning and processes to solve non-routine problems • Being able to break down a problem into its constituent parts – identifying what is required to be solved and then using appropriate methods in solving the problem 	<ol style="list-style-type: none"> 1. Determine the value of: $\left(1 + \frac{1}{2}\right)\left(1 + \frac{1}{3}\right)\left(1 + \frac{1}{4}\right)\left(1 + \frac{1}{5}\right) \dots$ up to 98 factors. 2. In the figure below, AB is a tangent to the circle with centre O. AC = AO and BA CE. DC produced, cuts tangent BA at B. Prove that AD = 4AF.  <ol style="list-style-type: none"> 3. Jeffrey invests R700 per month into an account earning interest at a rate of 8% per annum, compounded monthly. His friend also invests R700 per month and earns interest compounded half yearly at $r\%$ per annum. Jeffrey and his friends' investments are worth the same at the end of 12 months. Calculate r.
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Structure of Assessment

5.1 Scheme of Assessment

The Mathematics course will be examined by means of two three hour examination papers (Paper 1 and Paper 2). This is primarily done in order to prevent a student from having to cover a vast breadth of content for any examination.

Table 2: Examination Papers

Paper	Marks	Duration	Weighting
1	100	3hours	50%
2	100	3 hours	50%

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5.2 Content, Sub-Topics and Weightings per Examination Paper

The content topics and its weightings examined in each of Paper 1 and Paper 2 are indicated in Table 3.

Table 3: Content, Sub-Topics & Weightings Per Examination Paper

Description	Duration	Weightings (Marks)
Paper 1	3hrs	
Exponents, surds and logarithms		15 ± 3
Patterns, sequences and series		35 ± 3
Finance, growth and decay		30 ± 3
Functions, graphs and algebra		70 ± 3
Total		150
Paper 2:		
	3hrs	
Measurement		10 ± 3
Circle geometry		35 ± 3
Analytical geometry		30 ± 3
Trigonometry		45 ± 3
Statistics		30 ± 3
Total	150	

Content

6.1 Focus of Subject Content Areas

Mathematics in the NASCA curriculum covers the following four main content areas:

1. Number and number relationships;
2. Functions, Graphs and Algebra;
3. Measurement, Euclidean geometry, Analytical geometry and Trigonometry;
4. Statistics.

Each content area contributes towards the acquisition of the specific skills. The table below shows the main topics and sub-topics in the NASCA curriculum.

	Content Topic	Sub-Topics
1.	Number and number relationships	<ul style="list-style-type: none"> • Exponents, surds and logarithms • Number patterns • Formulae to calculate the sum of Arithmetic and Geometric Sequences and Series • Application of the knowledge of geometric series to solve problems • Investment and Loan options
2.	Functions, Graphs and Algebra	<ul style="list-style-type: none"> • Manipulation, simplification and factorisation of algebraic expressions • Solving of algebraic equations

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		<ul style="list-style-type: none"> • Function concept • Various types of functions: straight line; parabola; hyperbola; exponential and trigonometric graphs • Characteristics of functions • Effects of parameters on graphs (manipulation of representations) • Inverses of functions: straight line; parabola and exponential graphs • Use of functions to solve real-life problems
3.	Measurement, Euclidean Geometry, Analytical Geometry and Trigonometry	<ul style="list-style-type: none"> • Measurement: Volume and surface area of geometric objects • Euclidean geometry: Investigate prove and apply circle geometry theorems • Analytical Geometry: distance between two points, gradient of a line, parallel and perpendicular lines, mid-point of a line segment, inclination of a line, equation of a line, equation of a circle and equation of a tangent to a circle • Trigonometry: Trigonometric ratios $\sin\theta$, $\cos\theta$ and $\tan\theta$; special angles; reduction formulae; specific and general solutions to trigonometric equations; sine, cosine and area rules; compound and double angles; problems in 2- and 3- dimensions
4.	Statistics	<ul style="list-style-type: none"> • Univariate numerical ungrouped data: Measures of central tendency; five number summary; box and whisker diagrams; outliers; measures of dispersions • Univariate numerical grouped data: Frequency distribution table; ogive curve; quartile values; histograms; mean, median and mode • Ungrouped data: Variance and standard deviation • Bivariate data: Scatter plots; correlation; best fit function; linear regression line

6.2. Subject Content

The mathematical content for each topic/unit is detailed below. The order in which topics are listed is not intended to imply anything about the order in which they might be taught.

Topic 1: Number and Number Relationships

Overview of Number and Number Relationships

When solving problems, the learner is able to recognise, describe, represent and work confidently with numbers and their relationships to estimate, calculate and check solutions.

In this topic students should be able to:

- Expand their capacity to represent numbers in a variety of ways and move flexibly between representations;
- Use laws of exponents, surds and logarithms to simplify expressions and solve equations;
- Calculate confidently and competently with and without a calculator, guarding against over-dependence on the calculator;
- Develop the concept of simple and compound growth decay;
- Solve problems related to arithmetic, geometric and other sequences and series including contextual problems related to hire-purchase, bond repayments and annuities;
- Explore real-life and purely mathematical number patterns and problems which develop the ability to generalise, justify and prove.

Prior Knowledge of the Following Sub-Topics and Their Extensions are Required for this Topic:

- a. Working knowledge of numbers up to and including rational numbers;
- b. Classify real numbers as rational and irrational;
- c. Good calculator knowledge and how to use the memory functions on a basic calculator;
- d. Identifying the sign, coefficient, base and exponent of a power;
- e. Laws of exponents;
- f. Application of laws of exponents in simplifying algebraic expressions and equations;
- g. Scientific notation;
- h. Basic algebra, arithmetic and substitution;
- i. Ratio, percentage and decimals;
- j. Profit, loss, discount and inflation;
- k. Solving simultaneous equations;
- l. Add, subtract, multiply and divide simple surds;
- m. Solve simple equations involving surds;
- n. Simple and compound interest;
- o. Reading and interpreting basic tables and graphs.

Details of Content Coverage:

1.1. Use Laws of Exponents, Surds and Logarithms to Simplify Expressions and Solve Equations

Content:

- a. Exponents;
- b. Surds;
- c. Logarithms.

Learning Outcomes:

Students should be able to:

1.1.1 Simplify expressions and solve equations using the laws of exponents for rational exponents where $x^{\frac{p}{q}} = \sqrt[q]{x^p}$; $x > 0$; $q > 0$;

1.1.2 Add, subtract, multiply and divide simple surds;

1.1.3 Solve simple equations involving exponents and surds;

1.1.4 Use and understand the definition of a logarithm:

$$y = \log_b x \Leftrightarrow x = b^y; \text{ where } b > 0 \text{ and } b \neq 1;$$

1.1.5 Derive the laws of logarithms

$$\log_a(xy) = \log_a x + \log_a y$$

$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

$$\log_a x^n = n \log_a x$$

$$\log_a b = \frac{\log b}{\log a};$$

Note: Manipulation of logarithmic laws will not be examined.

1.1.6 Use the laws of logarithms to simplify expressions and solve equations;

Range: Simple expressions and equations.

1.2. Investigate Number Patterns

Content:

- a. Linear number patterns – first constant difference;
- b. Quadratic number patterns - second constant difference.

Learning Outcomes:

Students should be able to:

- 1.2.1 Investigate number patterns leading to those where there is a constant difference between consecutive terms, and the general term is therefore linear;
- 1.2.2 Investigate number patterns leading to those where there is a constant second difference between consecutive terms, and the general term is therefore quadratic.

1.3 Identify and Solve Problems Involving Number Patterns Including Arithmetic and Geometric Sequences and Series

Content:

- a. Arithmetic sequence;
- b. Geometric sequence;
- c. Neither arithmetic nor geometric sequences and series.

Learning Outcomes:

Students should be able to:

- 1.3.1 Identify and solve problems involving number patterns associated with arithmetic sequences and series;
- 1.3.2 Identify and solve problems involving number patterns associated with geometric sequences and series;
- 1.3.3 Identify and solve problems involving number patterns not limited to arithmetic and geometric sequences and series.

1.4 Use Sigma Notation, Derive, Prove and Apply Formulae to Calculate the Sum of Arithmetic and Geometric Sequences (Series)

Content:

- a. Sigma notation ;
- b. Formulae: arithmetic and the geometric sequence;

Learning Outcomes:

Students should be able to:

- 1.4.1 Use and apply Sigma notation;
- 1.4.2 Derive, prove and apply formulae to calculate the sum of an arithmetic sequence (series) and the sum of a geometric sequence (series)

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_n = \frac{n}{2}(a+l)$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; (r \neq 1; r > 1) \quad \text{or} \quad S_n = \frac{a(1 - r^n)}{1 - r}; (r \neq 1; r < 1)$$

$$S_\infty = \frac{a}{1 - r}; (-1 < r < 1), (r \neq 1).$$

1.5 Apply Knowledge of Geometric Series to Solve Problems

Content:

- a. Bond repayment problems;
- b. Sinking fund problems.

Learning Outcomes:

Students should be able to:

- 1.5.1 Apply knowledge of geometric series to solve bond repayment problems;
- 1.5.2 Apply knowledge of geometric series to solve sinking fund problems.

1.6 Critically Analyse Investment and Loan Options in Order to Make Informed Decisions

Content:

- a. Simple and compound growth/decay formulae;
 - i. Interest, hire purchase and inflation
 - ii. Population growth
 - iii. Straight line and reducing balance depreciation
- b. Annuity formulae: Present and future values;
 - i. Investment options
 - ii. Loan options
- c. Time period: Investments and loans.

Learning Outcomes:

Students should be able to:

- 1.6.1 Use simple and compound growth/decay formulae:

$A = P(1+in)$ and $A = P(1+i)^n$ to solve and analyse problems including interest, hire purchase, inflation and other real life problems;

- 1.6.2 Make use of logarithms to calculate the value of n , the time period, in the equations:

$$A = P(1+i)^n \text{ or } A = P(1-i)^n ;$$

- 1.6.3 Use the two annuity formulae to make decisions on which investment or loan option to choose:

$$F = \frac{x[(1+i)^n - 1]}{i} \quad \text{and} \quad P = \frac{x[1 - (1+i)^{-n}]}{i};$$

- 1.6.4 Use simple and compound decay formulae, $A = P(1-in)$ and $A = P(1-i)^n$ to solve and analyse problems including straight line depreciation and depreciation of a reducing balance.

Topic 2: Functions, Graphs and Algebra

Overview of Functions, Graphs and Algebra

In this section the learner should be able to investigate, analyse, describe and represent a wide range of functions and solve related problems.

A fundamental aspect of this outcome is that it provides students with versatile and powerful tools for understanding their world while giving them access to the strength and beauty of mathematical structure. The language of algebra will be used as a tool to study the relationship between specific variables in a situation.

In this topic students should be able to:

- a. Manipulate, simplify and factorise algebraic expressions;
- b. Solve linear, quadratic, exponential and simultaneous equations;
- c. Understand the concepts of a function and associated notations;
- d. Sketch and interpret various types of functions and relations, including linear, quadratic, exponential, rectangular hyperbola, trigonometric and some rational functions;
- e. Investigate the effect of changing parameters on the graphs of functions;
- f. Determine and sketch the inverse graphs of prescribed functions;
- g. Use functions to solve real-life context problems via modelling.

Prior Knowledge of the Following Topics and their Extensions are Required for this Topic

Functions and Graphs:

- a. Basic understanding of the Cartesian plane, coordinates and point plotting;
- b. Plotting a set of ordered number pairs;
- c. Functions and relationships;
- d. Read and interpreting basic tables and graphs;
- e. Basic understanding of exponents;
- f. Linear relationships between two variables (linear functions);
- g. Generate straight line graphs by means of point-by-point plotting, intercept method and gradient-intercept method;
- h. Determining the equation of the straight line graph;
- i. Finding the gradient, x and y-intercept of a straight line;
- j. Sketching the graph $y = x$;
- k. Parallel and perpendicular lines;
- l. Use of a basic scientific calculator.

Algebra:

- a. Understand that real numbers can be irrational or rational;
- b. Solid working knowledge of product, factors and fractions;
- c. Basic algebra: distribution, i.e eliminating brackets by multiplying binomial by binomial expressions;
- d. Solving basic linear equations by substitution and elimination;
- e. Simplifying adding and subtracting algebraic fractions up to and including monomial denominators;
- f. Exponential laws.

Details of Content Coverage:

2.1. Manipulation, Simplification and Factorisation of Algebraic Expressions

Content:

- a. Products of binomials with trinomials;
- b. Factorisation: Common factor, grouping, difference of two squares, trinomials;
- c. Simplification of algebraic expressions including algebraic fractions.

Learning Outcomes:

Students should be able to:

- 2.1.1 Find products of binomials with trinomials;
- 2.1.2 Factorise by identifying/taking out of common factor;
- 2.1.3 Factorise by grouping in pairs;
- 2.1.4 Factorise the difference of two squares;
- 2.1.5 Factorise trinomials;
- 2.1.6 Simplify algebraic expressions and algebraic fractions with binomial and trinomial denominators.

2.2. Solve Algebraic Equations

Content:

Solution of:

- a. Linear equations;
- b. Quadratic equations;
- c. Exponential equations;
- d. Equations with algebraic fractions;
- e. Simultaneous equations;
- f. Word problems involving linear, quadratic or simultaneous linear equations.

Learning Outcomes:

Students should be able to:

- 2.2.1 Solve linear equations;
- 2.2.2 Solve quadratic equations by;
 - a. Factorisation
 - b. Using quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
 - c. Completing the square
- 2.2.3 Solve exponential equations in the form ka^x (where x is an integer) by using the laws of exponents;

- 2.2.4 Solve equations involving algebraic fractions;
- 2.2.5 Solve simultaneous equations with two unknowns algebraically and graphically, where both equations are linear;
- 2.2.6 Solve simultaneous equations with two unknowns algebraically and graphically; where the one equation is linear and the other equation is quadratic;
- 2.2.7 Solve word problems involving linear, quadratic or simultaneous linear equations.

Note: This does not necessarily mean standardised problems with a standard algorithmic solution that can be learnt by heart. Rather students should learn to express a variety of different verbal descriptions of relationships in the language of mathematical symbols. Students should learn to respond to the wording of the descriptions to create equations expressing the relationship described rather than trying to recognise the ‘trigger’ phase that indicates a standard expression that needs to be used.

2.3 Apply and use the Formal Definition for the Function Concept

Content:

- a. Definition of function concept;
- b. Function notation.

Learning Outcomes:

Students should be able to:

- 2.3.1 Use and apply the formal definition of a function;
- 2.3.2 Use and apply different function notations, e.g. $f(x) = x^2 + 6$; $f : x \rightarrow y$.

2.4 Characteristics of Various Types of Functions and their Sketch Graphs

Content:

- a. Relationships between variables in terms of numerical, graphical, verbal and symbolic representations;
- b. Sketching graphs: straight line; parabola, hyperbola, exponential and trigonometric graphs;
- c. Characteristics of functions: straight line, parabola, hyperbola, exponential and trigonometric graphs.

Learning Outcomes:

Students should be able to:

- 2.4.1 Recognise relationships between variables in terms of numerical, graphical, verbal and symbolic representations and convert flexibly between these representations (tables, graphs, words and formulae);

2.4.2 Use a variety of techniques to sketch and interpret information from the following graphs of functions:

- a. $y = ax + q$;
- b. $y = ax^2 + q$; $y = a(x + p)^2 + q$; $y = ax^2 + bx + c$;
- c. $y = \frac{a}{x} + q$; $y = \frac{a}{(x + p)} + q$;
- d. $y = ab^x + q$; $y = ab^{x+p} + q$; where $b > 0$; $b \neq 1$;
- e. $y = a \sin kx$; $y = a \sin x + q$; $y = a \sin(x + p)$; $x \in [-360^\circ; 360^\circ]$;
- f. $y = a \cos kx$; $y = a \cos x + q$; $y = a \cos(x + p)$; $x \in [-360^\circ; 360^\circ]$;
- g. $y = a \tan kx$; $y = a \tan x + q$; $y = a \tan(x + p)$; $x \in [-360^\circ; 360^\circ]$.

2.4.3 Identify the following characteristics of the above functions:

- a. Shape;
- b. Domain (input values);
- c. Range (output values);
- d. Intercepts on the axes (where applicable);
- e. Asymptotes;
- f. Axes of symmetry;
- g. Turning points (minima and maxima);
- h. Periodicity and amplitude;
- i. Intervals in which a function increases/decreases.

2.5 Investigation of the Effect of Parameters on the Graphical Representations of Functions

Content:

- a. Effects of parameters on graphs: straight line, parabola, hyperbola, exponential and trigonometric graphs;
- b. Transformation of functions;
- c. Finding equations of graphs.

Learning Outcomes:

Students should be able to:

2.5.1 Investigate the effects of the parameters a and q as well as p , k , b and c on the graphs defined by:

- a. $y = ax + q$;
- b. $y = ax^2 + q$; $y = a(x + p)^2 + q$; $y = ax^2 + bx + c$;
- c. $y = \frac{a}{x} + q$; $y = \frac{a}{(x + p)} + q$;
- d. $y = ab^x + q$; $y = ab^{x+p} + q$; where $b > 0$; $b \neq 1$;
- e. $y = a \sin kx$; $y = a \sin x + q$; $y = a \sin(x + p)$; $x \in [-360^\circ; 360^\circ]$;
- f. $y = a \cos kx$; $y = a \cos x + q$; $y = a \cos(x + p)$; $x \in [-360^\circ; 360^\circ]$;
- g. $y = a \tan kx$; $y = a \tan x + q$; $y = a \tan(x + p)$; $x \in [-360^\circ; 360^\circ]$.

2.5.2 Work with transformations (translations, reflections and rotations) of functions and solve related problems;

2.5.3 Find the equations of the following types of graphs:

- a. $y = ax + q$;
- b. $y = ax^2 + q$; $y = a(x + p)^2 + q$; $y = ax^2 + bx + c$;
- c. $y = \frac{a}{x} + q$; $y = \frac{a}{(x + p)} + q$;
- d. $y = ab^x + q$; $y = ab^{x+p} + q$; where $b > 0$; $b \neq 1$;
- e. $y = a \sin kx$; $y = a \sin x + q$; $y = a \sin(x + p)$; $x \in [-360^\circ; 360^\circ]$;
- f. $y = a \cos kx$; $y = a \cos x + q$; $y = a \cos(x + p)$; $x \in [-360^\circ; 360^\circ]$;
- g. $y = a \tan kx$; $y = a \tan x + q$; $y = a \tan(x + p)$; $x \in [-360^\circ; 360^\circ]$.

2.6 Use a Variety of Techniques to Sketch and Interpret Information for Graphs of the Inverse of a Function

Content:

- a. Concept of inverse function;
- b. Inverse function notation;
- c. Sketching the inverses of functions: straight line; parabola and exponential graph;
- d. Characteristics of the inverse of functions: linear, quadratic and exponential;
- e. Equations of the inverse graphs: straight line; parabola and exponential graph.

Learning Outcomes:

Students should be able to:

- 2.6.1 Understand the general concept of an inverse of a function;
- 2.6.2 Use the inverse function notation;
- 2.6.3 Understand and use the relationship between a function and its inverse as reflection in the line $y = x$;
- 2.6.4 Determine the equations of the inverses of the functions: (the general concept of the inverse of a function):
 - a. $y = ax + q$
 - b. $y = ax^2$
 - c. $y = b^x$ where $b > 0$; $b \neq 1$;
- 2.6.5 Sketch and interpret information from graphs of the inverses of the functions:
 - a. $y = ax + q$
 - b. $y = ax^2$
 - c. $y = b^x$ where $b > 0$; $b \neq 1$;

2.6.6 Identify the following characteristics of the inverses of the functions:

$$y = ax + q; \quad y = ax^2; \quad y = b^x \text{ where } b > 0; b \neq 1:$$

- Shape;
- Domain (input values);
- Range (output values);
- Intercepts on the axes (where applicable);
- Asymptotes (horizontal and vertical);
- Axes of symmetry;
- Turning points (minima and maxima);
- Intervals in which a function increases / decreases.

2.6.7 Determine the equations of the inverse graphs of the following functions given as a sketch:

- $y = ax + q$;
- $y = ax^2$;
- $y = b^x$ where $b > 0$; $b \neq 1$.

2.6.8 Determine which inverses are functions and how the domain of a function may need to be restricted (in order to obtain a one-to-one function) to ensure that the inverse is a function.

2.7 Use functions to solve real-life context problems via modelling

Content:

- Interpretation of graphs that describe given real life situations;
- Draw graphs to describe given real-life situations.

Learning Outcomes:

Students should be able to:

- 2.7.1 Describe situations by interpreting graphs from the description of a situation, with special focus on trends and pertinent features;
- 2.7.2 Describe situations by drawing graphs from the description of a situation, with special focus on trends and pertinent features.

Note: Examples should include issues related to health, social, economic, cultural, political and environmental matters.

Topic 3: Measurement, Euclidean Geometry, Analytical Geometry and Trigonometry

Overview of Measurement, Euclidean Geometry, Analytical Geometry and Trigonometry

In this section we will study measurement, euclidean geometry, analytical geometry and trigonometry.

The important aspect of this topic is that it provides students an opportunity to develop necessary concepts, skills and knowledge to describe, represent and explain properties of shapes in two-dimensional and three dimensional space with justification.

Measurement and Euclidean Geometry

In this topic students should be able to:

- Solve problems involving volume and surface area of geometric objects such as cubes, rectangular prisms, cylinders, triangular prisms, hexagonal prisms, right pyramids (with square, equilateral or hexagonal basis), right cones, spheres and a combination of such geometrical objects;
- Investigate, conjecture and prove theorems of the geometry of circles assuming results from earlier grades and accepting that the tangent to a circle is perpendicular to the radius drawn to the point of contact;
- Solve circle geometry problems and prove theorems, using circle geometry theorems, their converses and corollaries (where they exist) as well as geometry results established in earlier grades.

Analytical Geometry

In this topic students should be able to:

- Represent geometric figures in a Cartesian co-ordinate system, and derive and apply, for any two points $(x_1; y_1)$ and $(x_2; y_2)$, a formula for calculating:
 - The distance between two points;
 - The gradient of the line segment joining the points;
 - Conditions for parallel and perpendicular lines;
 - The coordinates of the midpoint of the line segment joining the points.
- Use a two dimensional Cartesian co-ordinate system to derive and apply:
 - The equation of a line through two given points;
 - The equation of a line through one point and parallel or perpendicular to a given line;
 - The inclination of a line;
 - The equation of a circle (any centre);
 - The equation of a tangent to a circle at a given point on the circle.

Trigonometry

In this topic students should be able to:

- Define trigonometric ratios $\sin\theta$, $\cos\theta$ and $\tan\theta$ in right angled triangles;
- Extend the definitions of $\sin\theta$, $\cos\theta$ and $\tan\theta$ to $0^\circ \leq \theta \leq 360^\circ$;
- Derive and use values of the trigonometric ratios for the special angles $\theta \in \{0^\circ; 30^\circ; 45^\circ; 60^\circ; 90^\circ\}$;
- Derive and use values the identities $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\sin^2\theta + \cos^2\theta = 1$;
- Derive and use the trigonometric reduction formulae;
- Determine the general solution and/or specific solutions of trigonometric equations;
- Establish and apply the sine, cosine and area rules;
- Prove and use the compound angle and double angle identities;
- Solve problems in 2-and 3-dimensions by constructing and interpreting geometrical and trigonometric models.

Prior Knowledge of the Following Topics and their Extensions are Required for this Topic

Measurement:

- a. Basic understanding and working knowledge of conversions;
- b. All formulae for the areas of quadrilaterals, triangles and circles.

Circle Geometry:

- c. Set up basic equations and solve the variable;
- d. Work with angles related to lines and triangles;
- e. Recognise and classify different type of triangles and indicate their properties on a given sketch;
- f. Use the theorem of Pythagoras;
- g. Use geometry of straight lines and triangles to solve problems and to justify relationships in geometric figures;

Concepts to include are:

- i. Angles of a triangle;
 - ii. Exterior angles;
 - iii. Straight lines;
 - iv. Vertically opposite angles;
 - v. Corresponding angles;
 - vi. Co-interior angles and;
 - vii. Alternate angles.
- h. Solve problems related to shapes using congruence and similarity;
 - i. Use definitions, axioms and theorems associated with quadrilateral geometry to do connected calculations and solve geometry riders.

Analytical Geometry:

- a. Addition and simplification of surds;
- b. Basic algebraic manipulation and factorisation;
- c. Linear equations involving fractions;
- d. Quadratic equations;
- e. Equations of straight line and gradient;
- f. Theorem of Pythagoras to solve problems in right-angled triangles;
- g. Properties and classification of triangles;
- h. Properties and classification of quadrilaterals;
- i. Use the Cartesian coordinate system to plot points, lines and polygons.

Trigonometry:

- a. Simplify, add, subtract, multiply and divide fractions;
- b. Simplify, add, subtract, multiply and divide surds;
- c. Basic triangle geometry;
- d. Solve basic algebraic linear equations;
- e. Ratios;
- f. Cartesian plane and point plotting;
- g. State the theorem of Pythagoras in words;
- h. Use the theorem of Pythagoras to calculate the missing length in a right-angled triangle leaving the answers in the most appropriate form.

Details of Content Coverage:

3.1. Calculate, Investigate and Solve Problems Involving Volume and Surface Area of Geometric Objects

Content:

- a. Volume and surface areas of cubes, rectangular prisms, cylinders, triangular prisms and hexagonal prisms;
- b. The effect on the volume and surface area of right prisms and cylinders, when one or more dimensions are multiplied by a constant factor;
- c. Volume and surface areas of spheres, right pyramids and right cones;
- d. Volume and surface areas of a combination of geometrical objects: right prisms, cylinders, spheres, right pyramids, and right cones.

Learning Outcomes:

Students should be able to:

- 3.1.1 Calculate the volume and surface areas of cubes, rectangular prisms, cylinders, triangular prisms and hexagonal prisms;
- 3.1.2 Investigate the effect on volume and surface area of right prisms and cylinders, when one or more dimensions are multiplied by a constant factor k ;
- 3.1.3 Calculate volume and surface areas of spheres, right pyramids and right cones;
- 3.1.4 Calculate volume and surface areas of a combination of the above mentioned geometrical objects: right prisms, cylinders, spheres, right pyramids and right cones;
- 3.1.5 Solve problems involving volume and surface area of geometrical objects.

3.2 Investigate, Prove and Apply the Theorems of the Geometry of Circles

Content:

- a. Circles, perpendicular lines through the centre, chords and midpoints; Angle subtended at the centre of a circle; Angle subtended by diameter; Cyclic quadrilaterals; Tangents to circles;
- b. Solutions of geometric riders and problems.

Learning Outcomes:

Students should be able to:

- 3.2.1 Investigate, conjecture, prove the following theorems of the geometry of circles:
 - a. The line drawn from the centre of the circle perpendicular to the chord bisects the Chord;
 - b. The perpendicular bisector of a chord passes through the centre of the circle;
 - c. The angle subtended by an arc at the centre of the circle is double the size of the angle subtended by the same arc at the circle (on the same side as of the chord as the centre);
 - d. The angle subtended at the circle by a diameter is a right angle;
 - e. Angles subtended by a chord of the circle on the same side of the chord, are equal;
 - f. The opposite angles of a cyclic quadrilateral are supplementary;
 - g. An exterior angle of a cyclic quadrilateral is equal to the interior opposite angle;
 - h. Two tangents drawn to a circle from the same point outside the circle are equal in length;
 - i. The angle between a tangent to a circle and a chord drawn from the point of contact is equal to an angle in the alternate segment. (tan-chord theorem).

Note: Accept results established in earlier grades as axioms and also that a tangent to a circle is perpendicular to the radius, drawn to the point of contact.

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Range: The proof of the following theorems will be examined:

- a. The line drawn from the centre of a circle to the midpoint of a chord is perpendicular to the chord;
- b. The angle subtended by an arc at the centre of the circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre);
- c. The opposite angles of a cyclic quadrilateral are supplementary;
- d. The angle between a tangent to a circle and a chord drawn from the point of contact is equal to an angle in the alternate segment.

3.2.2 Use the above theorems, their converses and corollaries (where they exist) to solve and/prove riders:

- a. Theorems from section 3.2.1;
- b. Equal chords subtend equal angles at the circumference;
- c. Equal chords subtend equal angles at the centre;
- d. Equal chords of equal circles subtend equal angles at the circumference;
- e. If the angle subtended by a chord at point on the circle is a right angle, then the chord is a diameter;
- f. If a line is drawn perpendicular to a radius at the point where the radius meets the circle, then it is a tangent to the circle;
- g. If the exterior angle of a quadrilateral is equal to the interior opposite angle then the quadrilateral will be a cyclic quadrilateral;
- h. If two opposite angles of a quadrilateral are supplementary, then the quadrilateral is cyclic;
- i. If through one end of a chord of a circle, a chord is drawn making with the chord an angle equal to the angle in the alternate segment, then the line is a tangent to the circle.

3.3 Use the Cartesian Co-ordinate System to Derive and Apply Formulae

Content:

- a. The distance between two points;
- b. The gradient of the line segment joining two points;
- c. Parallel lines;
- d. Perpendicular lines;
- e. The co-ordinates of the midpoints of the line segment joining the points.

Learning Outcomes:

Students should be able to:

- 3.3.1 Use the Cartesian co-ordinate system to derive and apply a formula to calculate the distance between any two points $(x_1; y_1)$ and $(x_2; y_2)$;
- 3.3.2 Use the Cartesian co-ordinate system to derive and apply a formula to calculate the gradient of the line joining any two points $(x_1; y_1)$ and $(x_2; y_2)$;
- 3.3.3 Use the Cartesian co-ordinate system to derive and apply formula to calculate the gradient of a line that is parallel to another line;
- 3.3.4 Use the Cartesian co-ordinate system to derive and apply formula to calculate the gradient of a line that is perpendicular to another line;
- 3.3.5 Use the Cartesian co-ordinate system to derive and apply a formula to calculate the midpoint of the line segment joining any two points $(x_1; y_1)$ and $(x_2; y_2)$.

3.4 Use the Cartesian Co-ordinate System to Derive and Apply Equations

Content:

- a. The equation of a line through two given points;
- b. The equation of a line through one point and parallel or perpendicular to a given line;
- c. The inclination of a line;
- d. The equation of a circle (any centre);
- e. The equation of a tangent to a circle at a given point on the circle.

Learning Outcomes:

Students should be able to:

- 3.4.1 Use the Cartesian co-ordinate system to derive and apply the equation of a line between two given points;
- 3.4.2 Use the Cartesian co-ordinate system to derive and apply the equation of a line parallel or perpendicular to another given line;
- 3.4.3 Use the Cartesian co-ordinate system to derive and apply the angle of inclination of a line;
- 3.4.4 Use the Cartesian co-ordinate system to derive and apply the equation of a circle (any centre);
- 3.4.5 Use the Cartesian co-ordinate system to derive and apply the equation of a tangent to a circle given a point on the circle.

Note:

Straight lines to be written in the following forms only:

$$y = mx + c$$

and/or $ax + by + c = 0$ (general form)

Students are expected to know and be able to use as an axiom "the tangent to a circle is perpendicular to the radius drawn to the point of contact."

3.5 Define, Extend and Apply the Trigonometric Ratios of $\sin \theta$, $\cos \theta$ and $\tan \theta$

Content:

- a. Definitions of the trigonometric ratios $\sin \theta$, $\cos \theta$ and $\tan \theta$, using right angled triangles;
- b. Extension of definitions $\sin \theta$, $\cos \theta$ and $\tan \theta$ for $0^\circ \leq \theta \leq 360^\circ$;
- c. Use of diagrams to determine the numerical values of the trigonometric ratios.

Learning Outcomes:

Students should be able to:

- 3.5.1 Define and apply the trigonometric ratios $\sin \theta$, $\cos \theta$ and $\tan \theta$, using right angled triangles;
- 3.5.2 Extend the definitions of $\sin \theta$, $\cos \theta$ and $\tan \theta$ for $0^\circ \leq \theta \leq 360^\circ$;
- 3.5.3 Use diagrams to determine the numerical values of the trigonometric ratios for the angles from 0° to 360° .

3.6. Derive and use Values of Trigonometric Ratios for Special Angles

Content:

- a. Trigonometric ratios, $\sin \theta$, $\cos \theta$ and $\tan \theta$ for special angles (without using a calculator) for $\theta \in \{0^\circ; 30^\circ; 45^\circ; 60^\circ; 90^\circ\}$;
- b. Simplification of trigonometric expressions;
- c. Solutions of trigonometric equations for angles in the interval $[0^\circ; 360^\circ]$.

Learning Outcome:

Students should be able to:

- 3.6.1 Derive and apply values of trigonometric ratios, $\sin \theta$, $\cos \theta$ and $\tan \theta$ for the special angles (without using a calculator) for $\theta \in \{0^\circ; 30^\circ; 45^\circ; 60^\circ; 90^\circ\}$;
- 3.6.2 Simplify trigonometric expressions;
- 3.6.3 Solve trigonometric equations for angles in the interval $[0^\circ; 360^\circ]$.

3.7. Derive and Use Trigonometric Identities

Content:

- a. Derivation and use of identities: $\tan \theta = \frac{\sin \theta}{\cos \theta}$ and $\sin^2 \theta + \cos^2 \theta = 1$;
- b. Simplification of trigonometric expressions using the identities: $\tan \theta = \frac{\sin \theta}{\cos \theta}$ and $\sin^2 \theta + \cos^2 \theta = 1$ for angles in the interval $[0^\circ; 360^\circ]$;
- c. Solution of trigonometric equations using the identities: $\tan \theta = \frac{\sin \theta}{\cos \theta}$ and $\sin^2 \theta + \cos^2 \theta = 1$ for angles in the interval $[0^\circ; 360^\circ]$.

Learning Outcome:

Students should be able to:

- 3.7.1 Derive and use the identities $\tan \theta = \frac{\sin \theta}{\cos \theta}$ and $\sin^2 \theta + \cos^2 \theta = 1$;
- 3.7.2 Use the above trigonometric identities to simplify trigonometric expressions for angles in the interval $[0^\circ; 360^\circ]$;
- 3.7.3 Use the above trigonometric identities to solve trigonometric equations for angles in the interval $[0^\circ; 360^\circ]$;
- 3.7.4 Prove other identities by using the identities $\tan \theta = \frac{\sin \theta}{\cos \theta}$ and $\sin^2 \theta + \cos^2 \theta = 1$.

3.8 Derive and use Reduction Formulae to Simplify Trigonometric Expressions and Solve Trigonometric Equations

Content:

- a. Derivation and use of reduction formulae:
 - i. $\sin(90^\circ \pm \theta)$; $\cos(90^\circ \pm \theta)$;
 - ii. $\sin(180^\circ \pm \theta)$; $\cos(180^\circ \pm \theta)$; $\tan(180^\circ \pm \theta)$;
 - iii. $\sin(360^\circ \pm \theta)$; $\cos(360^\circ \pm \theta)$; $\tan(360^\circ \pm \theta)$;
 - iv. $\sin(-\theta)$; $\cos(-\theta)$; $\tan(-\theta)$.

Learning Outcome:

Students should be able to:

- 3.8.1 Derive reduction formulae for the following expressions:
 - a. $\sin(90^\circ \pm \theta)$; $\cos(90^\circ \pm \theta)$;
 - b. $\sin(180^\circ \pm \theta)$; $\cos(180^\circ \pm \theta)$; $\tan(180^\circ \pm \theta)$;
 - c. $\sin(360^\circ \pm \theta)$; $\cos(360^\circ \pm \theta)$; $\tan(360^\circ \pm \theta)$;
 - d. $\sin(-\theta)$; $\cos(-\theta)$; $\tan(-\theta)$.

3.8.2 Use reduction formulae to:

- Reduce a trigonometric ratio of any angle to the trigonometric ratio of an acute angle;
- Simplify trigonometric expressions;
- Solve trigonometric equations for angles in the interval $[-360^\circ; 360^\circ]$.

3.9 Determine the General Solution and / or Specific Solutions of Trigonometric Equations.

Content:

- General solutions of trigonometric equations;
- Specific solutions of trigonometric equations in specific intervals.

Learning Outcome:

Students should be able to:

- Use trigonometric ratios, $\sin\theta$, $\cos\theta$ and $\tan\theta$, reduction formulae and trigonometric identities to determine the general solutions of trigonometric equations;
- Use the general solution of a trigonometric equation to determine specific solutions in a specific interval.

3.10 Establish and Apply the Sine, Cosine and Area Rules

Content:

- Sine rule;
- Cosine rule;
- Area rule.

Learning Outcome:

Students should be able to:

- Derive and apply the sine rule;
- Derive and apply the cosine rule;
- Derive and apply the area rule.

3.11 Derive and Apply Compound Angle and Double Angle Identities

Content:

- Compound angle identities;
- Double angle identities.

Learning Outcomes:

Students should be able to:

- Derive the following compound angle identities;

$$\sin(\alpha \pm \beta) = \sin\alpha \cos\beta \pm \cos\alpha \sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha \cos\beta \mp \sin\alpha \sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha \cdot \tan\beta}$$

3.11.2 Use the compound angle identities to derive the following double angle identities;

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 2 \cos^2 \alpha - 1 \\ 1 - 2 \sin^2 \alpha \end{cases}$$

3.11.3 Determine the specific solutions of trigonometric expressions using compound and double angle identities without a calculator. (e.g. $\sin 120^\circ$, $\cos 75^\circ$, etc.);

3.11.4 Use compound angle identities to simplify trigonometric expressions and to prove trigonometric identities;

3.11.5 Determine the specific solutions of trigonometric equations by using knowledge of compound angles and identities;

Note:

a. Solutions: $[0; 360^\circ]$

b. Identities limited to:

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

c. Double and compound angle identities are included.

Note: Radians are excluded.

3.12 Solve Problems in 2-and 3-Dimensions by Constructing and Interpreting Geometrical and Trigonometric Models

Content:

- a. Solution of problems in two dimensions in right-angled triangles;
- b. Solution of problems in two dimensions using the sine, cosine and area rules;
- c. Solution of problems in three dimensions.

Learning Outcomes:

Students should be able to:

- 3.12.1 Solve problems in two dimensions by using the trigonometric functions ($\sin \theta$, $\cos \theta$ and $\tan \theta$) in right-angled triangles by constructing geometrical and trigonometric models;
- 3.12.2 Solve problems in two dimensions by using the sine, cosine and area rules, and by constructing and interpreting geometrical and trigonometric models;
- 3.12.3 Solve problems in three dimensions by constructing and interpreting geometrical and trigonometric models.

Note: Compound angle identities are excluded.

Topic 4: Statistics

Overview of Statistics

In this section the learner should be able to collect, organize, analyse and interpret data to establish and use statistical methods to solve related problems.

A fundamental aspect of this outcome is that it provides opportunities for students to be become aware and also participate in the completion of the 'statistical cycle' of formulating questions, collecting appropriate data, analysing and representing this data, and so arriving at conclusions about the questions raised.

In this topic students should be able to:

- a. Collect, organise and interpret univariate numerical ungrouped data by using measures of central tendency and dispersion;
- b. Collect, organise, represent and interpret univariate numerical grouped data by using measures of central tendency and dispersion;
- c. Understand the various concepts and representations relating to data handling and its uses, and used them correctly;
- d. Represent, analyse and interpret data using various techniques.

Prior Knowledge of the Following Topics and their Extensions are Required for this Topic:

- a. Use and apply the concepts of mean, median and mode to small sets of data;
- b. Data collecting experience;
- c. Draw and interpret the following graphs:
 - i. Bar and compound bar graphs;
 - ii. Histograms (grouped data);
 - iii. Frequency polygons;
 - iv. Pie charts;
 - v. Line and broken line graphs.
- d. Use of the basic calculator for computation purposes.

Details of Content Coverage

4.1 Collect, Organise and Interpret Univariate Numerical Ungrouped Data by Using Measures of Central Tendency and Dispersion

Content:

- a. Measures of central tendency: mean, median and mode;
- b. Five number summary:
 - Measures of dispersion: percentiles, quartiles, inter-quartile range and semi-interquartile range;
 - Box and whisker diagrams.
- c. Identification and interpretation of outliers in the box and whisker diagrams.

Learning Outcomes:

Students should be able to:

- 4.1.1 Calculate the measures of central tendency:
 - a. Mean;
 - b. Median;
 - c. Mode.

4.1.2 Work out the five number summary by:

- a. Calculating the maximum, minimum and quartiles;
- b. Determine the fences;
- c. Constructing the box and whisker diagram;
- d. Indicating any outliers.

4.1.3 Interpret the meaning of the representation of the box and whisker diagram with its outliers.

4.2 Collect, Organise, Represent and Interpret Univariate Numerical Grouped Data by Using Measures of Central Tendency and Dispersion

Content:

- a. Frequency distribution table;
- b. Histograms;
- c. Ogive curve;
- d. Quartile values;
- e. Mean, median and mode values.

Learning Outcomes:

Students should be able to:

- 4.2.1 Construct a frequency distribution table by grouping data into classes;
- 4.2.2 Construct histograms using tabulated grouped data;
- 4.2.3 Calculate the cumulative frequency and plot the Ogive curve;
- 4.2.4 Use the Ogive curve to estimate quartile values;
- 4.2.5 Calculate the mean, median and modal values of grouped data.

4.3 The Use of Variance and Standard Deviation as Measures of Dispersion of a Set of Ungrouped Data is Understood and Demonstrated

Content:

- a. Variance;
- b. Standard deviation.

Learning Outcomes:

Students should be able to:

- 4.3.1 Calculate the variance and standard deviation manually for small sets of data and use calculators for larger sets of data;
- 4.3.2 Interpret the meaning of the calculated variance and standard deviation of given numerical data.

4.4 Represent, Analyse and Interpret Data Using Various Techniques

Content:

- a. Statistical analysis;
- b. Appropriate and efficient methods to record, organise and interpret given data;
- c. Justification and application of statistics.

Learning Outcomes:

Students should be able to:

4.4.1 Identify situations or issues that can be dealt with through statistical analysis;

Range: data given should include problems relating to health, social, economic, cultural, political and environmental issues.

Note: not for examination purposes.

4.4.2 Discuss the use of appropriate and efficient methods to record, organise and interpret given data by making use of:

- a. Manageable data sample sizes: (less than or equal to 10) and which are representative of the population;
- b. Graphical representations and numerical summaries which are consistent with the data, and clear and appropriate to the situation and target audience;

Note: Discussion only, not expected to draw again.

- c. Compare different representations of given data.

4.4.3 Justify and apply statistics to answer questions about problems.

4.5 Calculate, Represent and Interpret Relationships Between Bivariate Data

Content:

- a. Scatter plots;
- b. Best fit function;
- c. Least square regression method;
- d. Regression line;
- e. Correlation coefficient.

Learning Outcomes:

Students should be able to:

4.5.1 Represent bivariate numerical data as a scatter plot and suggest intuitively and by simple investigation whether a linear, quadratic or exponential function would best fit the data;

4.5.2 Draw a line of best fit (by intuitive inspection)

Range:

- a. *Data given should include problems related to health, social, economic, cultural, political and environmental issues*
- b. *For small sets of data only (limited to 8);*

4.5.3 Use a calculator to determine the linear regression line which best fit a given set of bivariate numerical data;

4.5.4 Use the regression line to predict the outcome of a given problem;

4.5.5 Use a calculator to calculate the correlation coefficient of a set of bivariate numerical data and make relevant deductions.

7. Recommended Study Hours

Mathematics is a 30 credit course, which relates to 300 notional study hours.

It is envisaged that a typical one-year offering of the course will cover 30 weeks, excluding revision and examination time. Candidates should therefore spend 10 hours per week on mathematics. This should consist of 6 hours of face-to-face instruction and 4 hours of self-study.

A suggested time allocation for the course is shown on the Table 4:

Table 4: Recommended Study Hours

Topic	Face-to-Face Teaching Time	Self-Study Time
Number and number relationships	6 hours per week x 30 weeks	4 hours per week x 30 weeks
Functions, graphs and algebra		
Measurement, euclidean geometry, analytical geometry and trigonometry		
Statistics		
Total Course Hours	300 Notional Hours	

8. Approach to Teaching and Learning

The candidates envisaged in the NASCA curriculum are a varied group of adult and out-of-school students, and come from a wide range of backgrounds and socio-economic contexts. In addition, the modes of delivery are envisaged to be varied, ranging from face-to-face teaching in community colleges to distance learning in remote areas. The curriculum is therefore not prescriptive about learning activities or teaching methods, but allows for a range of teaching and learning styles. The sequence is not prescribed, although there is a logic behind the order in which topics are presented in each section.

The teaching and learning that takes place in Mathematics should include a wide variety of learning experiences and strategies that promote the development of mathematical skills, concepts, procedures, understanding and mathematical thinking that embraces problem-solving and problem posing, as well as values and attitudes that will enable candidates to be constructive citizens of South Africa. Lecturers and materials developers are encouraged to use a combination of strategies to allow for active participation and critical thinking. These strategies should include investigative and problem-solving skills, effective communication and reflection on the learning process.

Glossary

Amplitude – The maximum difference between the value of a periodic function and its mean.

Angle of Inclination – The angle that a line makes with positive direction of the x-axis.

Arc – A part of the **circumference** of a **circle**.

Asymptote – A straight line to which a curve continuously draws nearer without ever touching it.

Bivariate Data – Two sets of data values that both vary.

Bonds – Written promise by a company that borrows money usually with fixed interest payment until maturity (repayment time).

Centre of Circle – A point about which all the points on the **circumference** of a circle are **symmetrical**.

Chord – A straight line segment joining any two points on a **circumference** of a **circle**.

Circle – A curve that is the **locus** of a point which moves at a fixed distance from a fixed point (centre).

Circumference – A perimeter of a circle or any closed curve.

Collinear – Points that lie on the same line; the gradients between any two of these points will be the same.

Common Difference - If the differences between consecutive terms of a sequence are all equal then these are collectively known as the common difference.

Common Ratio – If the ratios between consecutive terms of a sequence are all equal then these are collectively known as the common ratio.

Compounding – Calculating the interest periodically over the life of the loan and adding it to the principal.

Compound Interest – The interest that is calculated periodically and then added to the principal. The next period the interest is calculated on the adjusted principal (old principal plus interest).

Conjecture – A tentative solution inferred from collected data.

Concentric – Circles that share the same centre.

Congruent – Identical in all respects.

Converse – Reversing the logic and proving a theorem in reverse.

Co-ordinates – A pair of numbers to indicate a point on a graph.

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Corresponding Angles – Angles in similar positions; angles which lie on the same side of a pair of **parallel lines** and the **transversal** (see **transversal**).

Corollary – A deduction based on the result of a theorem.

Cyclic Quadrilateral - A four sided figure with all four points lying on the circumference of a circle.

Diagonal – A straight line joining opposite vertices.

Diameter – Any straight line drawn from any point on a circumference of a circle, through the centre to a second point on the circumference.

Dependent Variable – The element of the **range** of a function which depends on the corresponding value(s) of the domain (e.g. in $y = f(x) = \pi x^2$ the area of a circle (y) depends on the radius, x if x is the **independent variable** and y the dependent variable.

Depreciation – The process of allocating the cost of an asset (less residual value) over the asset's estimated life.

Enlargement – A mapping that increases the distances between parallel lines by the same factor in all directions.

Effective Rate – True rate of interest. The more frequent the compounding, the higher the effective rate.

Exterior Angle - Outside angle of a polygon formed by one of the sides which has been extended.

Future Value (FV) – Final amount of the loan or investment at the end of the last period. Also called *compound amount*.

General Solution – the formula which lists all possible solutions to a trigonometric equation; takes into account the period of the trigonometric equation so an angle can be positive or negative.

Gradient – In general a slope i.e. an inclination to the horizontal expressed either as an angle the path makes with the horizontal or as the ratio between the vertical distance travelled and the horizontal distance travelled.

Hypotenuse – The side opposite the right angle in a right angled triangle.

Line Segment (1) – A straight line that has a beginning and an end.

Line Segment (2) – A portion of a straight line between two points; it has a finite length.

Logarithm – A quantity representing the power to which a fixed number (the base) must be raised to produce a given number.

Midpoint – The point in the middle of a line segment.

Normal – A line through a given point on a curve **perpendicular** to the **tangent** to the curve at that point.

Independent Variable – As used in dealing with functions: the value that determines the value of the **dependent variable** (e.g. in $y = f(x) = \pi x^2$ the area of a circle (y) depends on the radius, (x) if x is the independent variable and y the dependent variable.

Outstanding Balance on a Loan – Present value of a series a of payments still to be made.

Parallel Lines – Lines that are always the same distance apart and lie in the same direction.

Parameter – A constant whose value determines in part how interrelated variables are expressed and through which they may then be regarded as being **dependent** upon one another.

Periods – Number of years times the number of times compounded per year.

Perpendicular – At a right angles or at 90° ; lines are perpendicular when the angles between the lines is 90° .

Perpendicular Bisector – A line which is drawn at a right angles to a **line segment** and divides it in half.

Present Value Annuity – Amount of money needed today to receive a specified stream (annuity) of money in the future.

Present Value (PV)- How much money will have to be deposited today (or some date) to reach a specific amount of maturity (in the future).

Radius – The distance from a centre of a circle to any point on the **circumference** or the distance from the centre of a **sphere** to any point on its surface.

Rate of Interest – Percent of interest that is used to commute the interest charge on a loan for a specific time.

Reduction – A mapping that reduces the distances between parallel lines by the same factor in all directions.

Reduction Formulae – Trigonometric identities that express the trigonometric ratios of an angle of any size in terms of the trigonometric ratios of an acute angle.

Regression Line – The 'line of best fit' for a set of plotted points; also called the line of least squares.

Secant – A line that cuts a given curve.

Sector – A part of a circle lying between two radii (singular is radius) and either of the arcs that they cut off.

Sigma – The symbol Σ denoting the sum.

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Simple Interest - Interest is only calculated on the principal. In $I = P \times R \times T$, The interest plus original principal equals the maturity value of an interest-bearing note.

Simple Interest Formula - Interest = Principal \times Rate \times Time.

Specific Solutions- Solutions which satisfy a given trigonometric equation in a restricted interval.

Stretch - The shape of a graph or object undergoes a vertical or horizontal increase or decrease in scaling.

Supplementary Angles - Two angles that add up to 180° ; each angle is said to be the supplement of the other.

Symmetrical - A geometric figure is symmetric if it is identical with its own reflection in an axis of symmetry (a line through the centre of the figure).

Tangent - A line that touches a curve at only one point.

Time- Expressed as years or fractional years, used to calculate the simple interest.

Theorem - A formal proof of a geometric statement.

Transformation (1) - The change of one figure (transformation geometry) or one expression (algebra) to another.

Transformation (2) - A change in the position or size of an object or expression.

Translation - A **transformation** that moves or points the same distance in a common direction.

Transversal - Any line that intersects two or more lines.

Trigonometric Equation - An equation involving trigonometric ratios which is true only for certain values of the unknown variable.

Trigonometric Identity - An identity which is true for all values of an unknown variable, for which both sides of the identity are defined (so no zero denominators).

Turning Point - A maximum or minimum point on a curve where the y -value changes from increasing to decreasing or vice versa (and the tangent is horizontal).

Variance - The average of the squared differences from the mean.

Vertex - The point at which two straight lines meet to form an angle.

X-Intercept (root) - The point where the graph cuts the x -axis (where $y = 0$).

Y-Intercept - The point where the graph cuts the y -axis (where $x = 0$).

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