Mathematical Literacy

Unit 1

Number and Measurement

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This Study Unit is the property of the learner to whom it is given.

CONTENTS

Introduction

Welcome to Unit 1 of the Mathematical Literacy course. We use maths in everyday contexts and mathematical literacy helps make sense of realistic problems in real-life contexts. We have tried to put together an enjoyable course, which is easy to follow, interesting to read, and applicable to everyday situations.

Attitudes to Maths

One of the first things to get right when you start a maths course is your attitude to mathematics. People tend to have certain beliefs about maths.

Belief No 1. Only those people who are gifted at maths can do maths; everyone else is destined not to understand maths and to get low marks. True or false?

This is false. Everyone has to understand and learn maths and if you can read and understand what they read, can do maths.

Belief No 2. Only professionals such as engineers and astronomers need maths. It has very little to do with our everyday experiences. True or false.

This is false. Maths is used both professionally and in our everyday experiences.

In this course, we show you how maths is used in everyday contexts.

A distance education course

Education happens both in the classroom and outside the classroom. You can learn by reading, by discussing things with a friend or by forming groups to discuss information.

This is a distance education course, which means you will mainly be studying on your own. You are responsible for your own learning. You are the person who decided to do the course and you are the person who will benefit from your studies.

Of course, don't think you have to do it all alone. Try to find a friend to work with or work with someone in your community who enjoys maths. You will also attend classes and your tutor will help you.

Difficulties in teaching and learning maths

Finding the purpose

“Why do we have to do maths? What's the point? Where will it get me?” These are good questions. There are only so many hours in each day, so why spend time on something you don't think will benefit you? In fact, the way our minds work makes it difficult to concentrate on something we don't think is useful.

Look around you. Mathematics may come up in surprising places: in your salary packet; in the household shopping budget; in shelf building; when watching cricket; in judging whether a bit of information is true or not. Maths becomes much easier if you look for the maths around you.

Mpho says, ‘I wonder if I should buy a weekly train ticket? Some weeks I use the train everyday, and some weeks I get a lift on Monday mornings.’

The weekly train ticket to Cape Town costs R32,00. The daily return costs R8,40. How many train journeys make up the cost of a weekly ticket? What do you think?

It is worth Mpho buying a weekly ticket if he makes four journeys into town.

These are the kinds of problems which maths can help you with.

Lack of real objects

Another big problem when you study mathematics is the lack of real things to look at or handle. You are expected to work out the size of a room when there is no room; and work out how many litres in a fish tank when there is no fish tank. This works well for some people. They are able to imagine all sorts of things in their heads. But for others it is more difficult. If you have difficulty imagining things, try drawing or even making things out of cardboard for yourself.

Is neatness important?

Some people think the most important thing in mathematics is to be neat. If you see a maths book that is perfect with nothing crossed out, you may think the learner is good, but if you see scribbles you might think, this person does not know what he is doing. But this perception is not necessarily correct.

A teacher friend said to us, “If you want people to think, give them a rough piece of paper and a pencil.”

Give a mathematician a problem and what does he or she do? First she probably sees four or five ways of doing the problem. (There is usually more than one way.) Then she takes a rough piece of paper and a pencil and scribbles away.

He or she crosses out what doesn't work and then takes the right solution and writes it out neatly to show to other people. What we see in textbooks looks all very neat and tidy, but there was a lot of scratching out and re-working before the final product went to print. The mathematician may have a rubbish bin full of scrunched up paper. Feel free to work this way if it helps you. Paper can be recycled!

Use a calculator

If it is going to make it easier and quicker, use a calculator. But be aware that some questions may state, ‘Without the use of a calculator’, so you need to be sure you can master those questions without using your calculator. For the purpose of this course, we suggest you use a scientific calculator. The particular model we have used in our calculator lessons is the CASIO *fx-82ES PLUS* model.

How to tackle problems

A teacher tells us how to solve a problem, and we remember some things and forget others. When asked to solve the same problem in an exam, sometimes we remember and get it right. However, sometimes we forget and look blankly at the paper. This is not what maths is all about. Maths is about thinking things out for yourself. Get into the habit of doing this with the things you know.

The first thing to do is to find out the meaning of the question. If there are things you aren't clear about, ask somebody or look in a book. Try to picture the problem. Look at all the different parts to the question.

In this way maths is no different from the rest of your life. If you got a job as a cook in a restaurant you would want to know the layout of the kitchen. You would need to know where the pots are and where all the ingredients are kept. You would also want to know how the stove works. In fact you would probably like to have a good look around the day before you started cooking. Well, it's the same with a mathematical problem. Get to know the problem well.

Here are some steps to take when solving the problem:

- Understand the problem
- Simplify
- Experiment with possible answers
- Apply your own knowledge to the problems.

This is the method mathematicians use when making a new discovery. It may be slow, but it is better than staring at a blank page. If you try to memorise a method without understanding the mathematical process it will not better equip you in the long term to be successful at maths. Understand the method and you are well on the way to becoming a real mathematician. If you have a chance, watch how little children experiment and find out about their environment. You'll find they follow these steps, more or less. They are natural mathematicians and so are you.

Units, lessons and activities

Each unit is divided into lessons. Work through them at a pace that suits you. We have tried to involve you as much as possible in the learning process and the activities are designed to help with the learning.

Activities are important because they set you thinking about the problem. Try to do them on your own or with a friend. When you have gone as far as you can look at the feedback. The feedback gives you a detailed answer explaining what the activity is about.

The self-check exercises at the end of the lesson are for you to assess yourselves. “How much do I know? Is there a section I need to look at again?”

Perhaps there is something you don't understand. Don't worry about it. It will slot into place later. Or when you next meet a tutor, ask them to explain it to you. Get as far as you can before looking at the feedback to the self-check exercises.

Finally each unit concludes with a revision and consolidation lesson where you can revise and apply the skills you have learnt in that unit. These lessons are good preparation for your examinations. The answers to these lessons are contained in a separate booklet.

The following topics are included in the Mathematical Literacy course:

- numbers and calculations with numbers
- patterns, relationships and representations
- finance
- measurement
- maps, plans and other representations of the physical world
- data handling
- likelihood

We do not cover these topics in this particular order, but rather use an integrated approach. You will need the basic skills of numbers, calculations, patterns, relationships and representations as you explore and solve problems relating to contexts and situations involving finance, measurement, maps, plans and other representations of the physical world, data handling and likelihood. So you will notice that the lessons in the six units cover these topics, but that topics are usually presented as revision of basic skills and knowledge in the first few units and then elaborated with further application in the later units.

Enjoy the challenge!

1. The meaning of numbers

Introduction

'Numbers are easy ... I've been doing numbers all my life.' This is true, but many people make mistakes with numbers. Other people are frightened of 'sums' and will avoid them if at all possible.

People often make mistakes if they don't understand how numbers are organised. There is no reason to be frightened of sums. Perhaps an explanation of how numbers are organised will get rid of those fears.

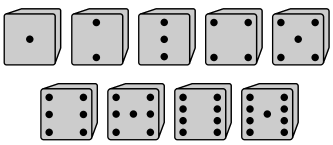
In this lesson you will:

- effectively use the groupings of place value in our number system
- give examples and work with other number groupings (conventions)
- write and understand the meaning of large numbers
- round off large numbers to the nearest appropriate place value
- solve problems involving large numbers.

Small numbers and number symbols

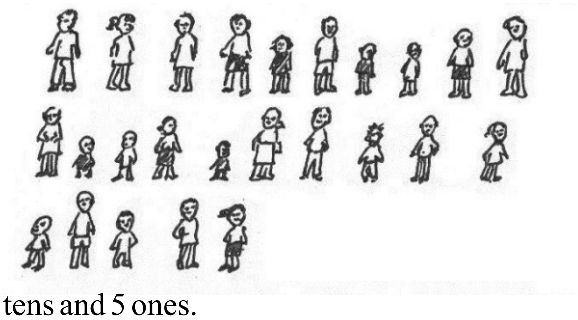
Most of us know the meaning of small numbers. We can see there are six children in the garden and we use the number symbol 6 to show this.

Similarly, we can use these symbols to show the numbers
1 2 3 4 5 6 7 8 9



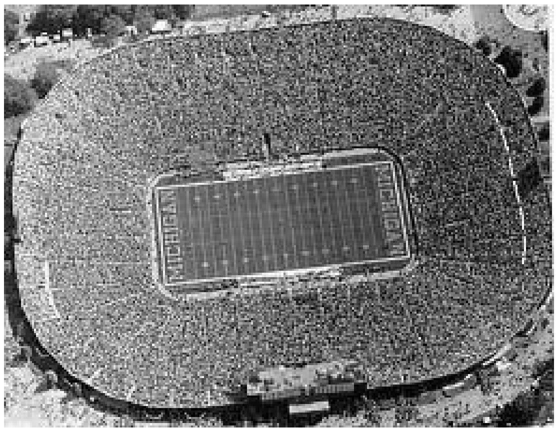
It is not quite so easy to count twenty-five children in a playground.

However, if the children stand in rows of ten it is easier to see. We write the number symbol 25 to show the number of children.



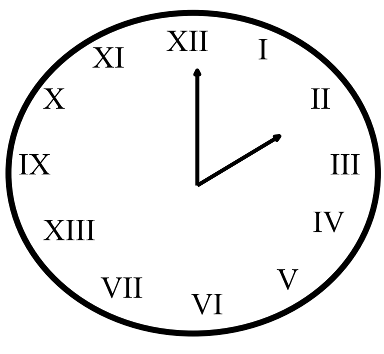
This means 2 tens and 5 ones.

It is quite difficult to tell how many people fill a soccer stadium.



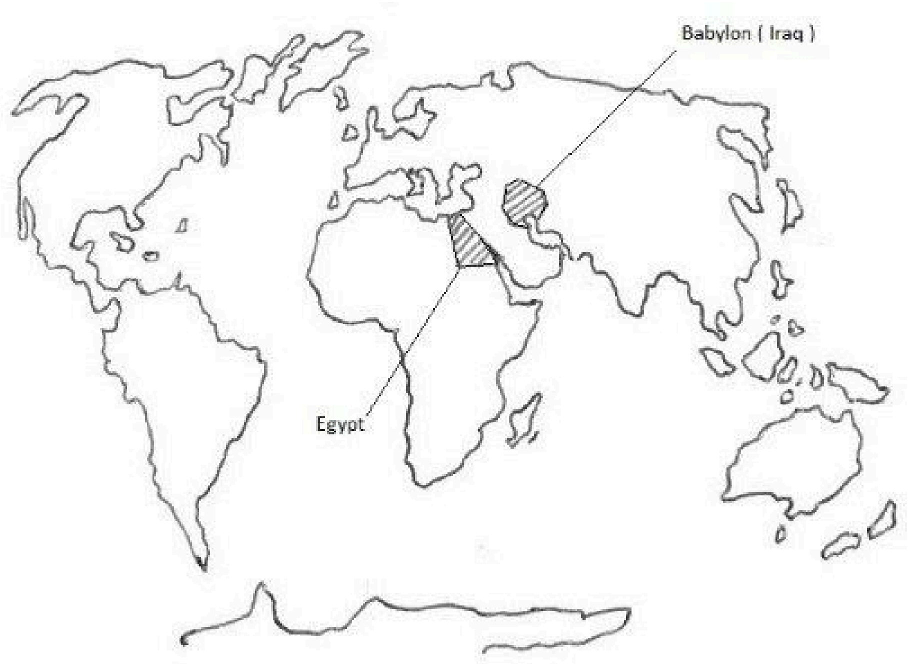
But if we know that there are one hundred seats in a block, then we can say there are six hundred and thirty-five people in the stadium.

We use the number symbol 635 which means 6 hundreds, 3 tens and 5 ones.



Match our common number symbols to the Roman numerals on the clock. Can you see logic in the Roman number system? Interesting information on the history of maths can be found in books in your local library or over the internet.

For a brief look at the history, read the information that follows.



Early number symbols

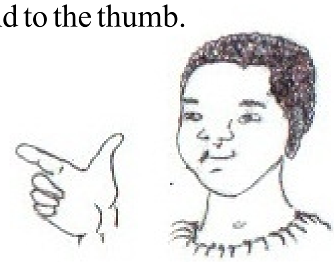
The development of a number system would probably start with the ideas of **more** and **less.** From this develops the 'tally' idea where, for example, a stone may be placed into a bag for every head of cattle counted. Any written counting system would start with a spoken system of counting. A spoken counting system would use convenient groupings of number. Many of the counting systems that have developed in different parts of the world have been based on groups of ten. This is most likely because we use our fingers to count. Some of the words used in counting mention fingers.

Example: The Zulu words for one to ten are:

Number	Word	Meaning
1	nye	being alone
2	bili	raise a separate finger
3	thathu	to pick up
4	ne	in company
5	hlanu	all fingers together
6	sithupa	take the right thumb
7	ikhombisa	point with the index finger of the right hand
8	shiya 'ngalombile	leave out two fingers
9	shiya 'ngalolunye	leave out one finger
10	shumi	make all fingers stand

Although some people may argue that these words do not refer to fingers but only to numbers, nobody would disagree that the word for seven, and the word for six, clearly refer to pointing, and to the thumb.

The use of finger gestures was very common in Africa. Finger gestures were a way of trading with people who spoke a different language.



Written number symbols

Historians say that the first people to start writing lived in Babylon (the area which is called Iraq today). They used clay tablets and wedge-shaped sticks.

These are the Babylonians symbols used to show numbers many thousands of years ago.

V	VV	VVV	VVVV	VVVVV	VVV	VVVV	VVVV	VVVVV
					VVV	VVV	VVVV	VVVV

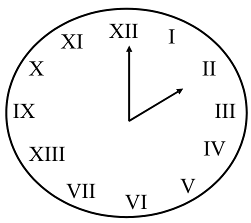
The Egyptians who lived not far from the Babylonians, wrote on papyrus with a sharp stick. The papyrus plant grew on the banks of the Nile. Let's look at their number symbols.

I	II	III	IIII	III	III	IIII	IIII	IIIII	X
				II	III	III	IIII	IIIII	

The Egyptians, like the Babylonians, used a different symbol for ten, in this case X.

You may have seen Roman numerals. They are often used on clocks and watches.

I II III IV V VI VII VIII IX X



The Hindu-Arabic number symbols

The number symbols used by the Egyptians, the Babylonians and the Romans were clear but very clumsy. Can you imagine multiplying using Roman numerals?

The number symbols we use today were brought to us from India by Arab traders. They took this knowledge into Europe. The advantage of these number symbols is that there is only one symbol for each number.

0 1 2 3 4 5 6 7 8 9. Numbers above 9 are made up by combining these, e.g. 10.

ACTIVITY 1

1. Complete the table:

	Egyptian symbols	Babylonian symbols	Roman symbols
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			

2. Write the following numbers using our number symbols:

a) X^{III}_{III}
b) <VVV
c) XX
d) XV
e) <

ANSWERS ON PAGE 94

Large numbers and numbers groupings (conventions)

It is easier to handle large numbers if we divide them up into manageable groups. In our number system we use groups of ten. The word **decimal** comes from the Latin ‘deci’ which means ten.

Different cultures throughout the world used to use different groupings. If you're interested, there are many books on the subject in libraries or on the internet. However, here are a few interesting points.

The Babylonians

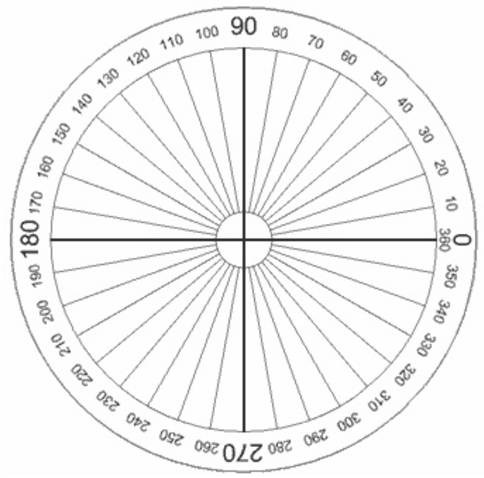
The Babylonians, thousands of years ago, grouped their numbers in units of 60.

<<<<<< = V	which was 60
V<	was 60 + 10
VV	was 60 + 60

The Babylonians were excellent astronomers. They worked out that a year was 360 days long. They might have extended their numbers base to 60 for greater accuracy. Many numbers divide into 60. 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60. This means that 60 can be divided into many little parts without any remainder.

Compare this with the number 10. The only numbers that divide into ten without a remainder are 1, 2,5 and 10.

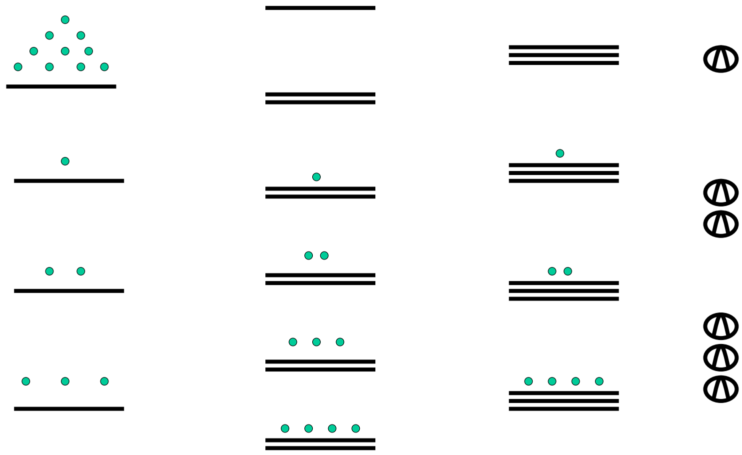
They also described a revolution as 360 degrees. They thought the sun revolved around the earth. They measured its path around the earth as 360 degrees. Each degree was divided into 60 minutes, and each minute was divided into 60 seconds.



It is from the Babylonians that we get 12 hours in a day, 60 minutes in an hour, and 60 seconds in a minute.

The Mayan people from Central America

The Mayan people in Central America grouped their numbers in units of 20.



The Yoruban people from Nigeria

The Yoruban people in Nigeria also based their number system on 20. A larger grouping has certain advantages. As in many places in Africa, cowry shells were used as currency. The Yoruban people had to count great numbers. It made sense to count in larger groups.

ACTIVITY 2

1. The game of cricket has its own numbering system. Each 'over' in cricket consists of 6 balls. This means that a bowler bowls six times to a batsman in one over. In 'limited overs' cricket, there are either 20 or 50 overs. For 20 overs it is called T20 cricket and 50 overs is known as ODI cricket (One Day International) cricket.

   Use this information to answer the following:

   a) How many balls are bowled in a T20 match if all the overs are completed?
   b) How many balls are bowled in an ODI match if all the overs are completed?
   c) If 72 balls remain to be bowled, how many overs is this?

2. In cricket the number of the over and the number of the ball being bowled in that over are shown as follows:

   4.3 indicates that it is the **4^{th} over** and the **third ball** of that over.

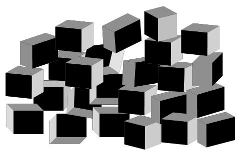
   a) How would you indicate the 2^{nd} ball of the 11^{th} over?
   b) How would you indicate that the game is now in its 30^{th} over with five balls already bowled?

ANSWERS ON PAGE 94

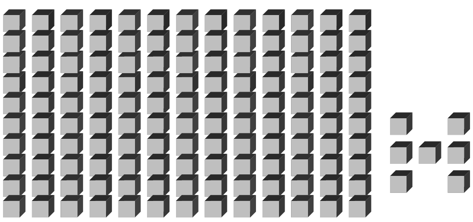
The decimal system

Units, tens and hundreds

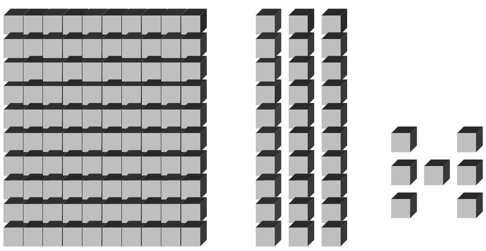
In the decimal system, grouping is always done in tens. Suppose we had a pile of cubes like this, how would we begin counting?



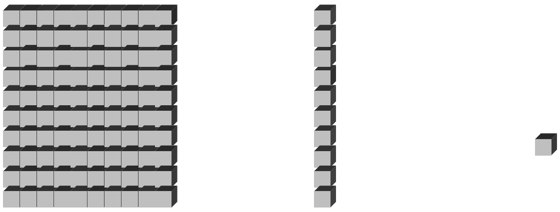
We could begin by arranging them in tens.



Let's arrange the tall piles into groups of ten.

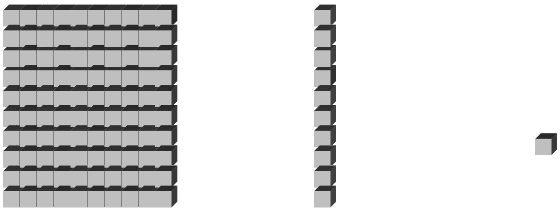


We can now describe the number as:



One of these ***Three of these*** ***Seven of these***

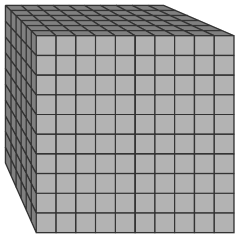
Now these groups have names.



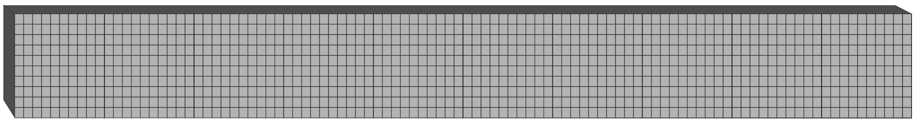
This is a hundred. ***This is a ten.*** ***This is a unit.***

Thousands, ten thousands, hundred thousands and a million

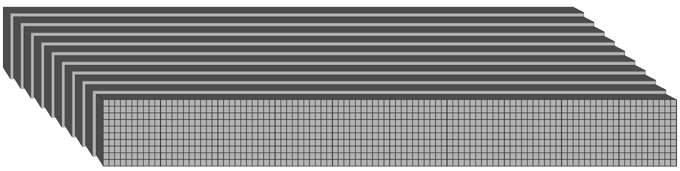
Ten hundreds together make a thousand.



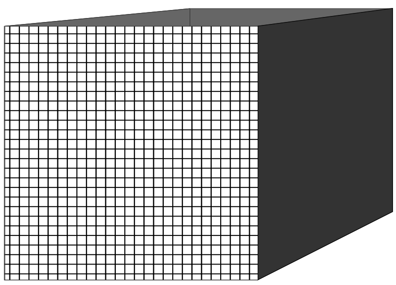
We can make a group of ten thousand.



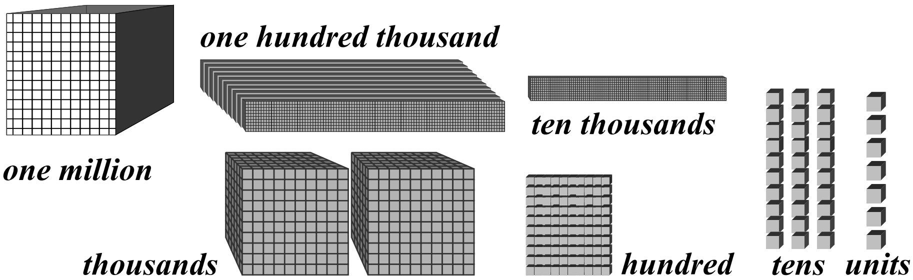
Ten of these make a group of a hundred thousand.



And ten of these make a million.



We can use these blocks to picture the meaning of a very large number.

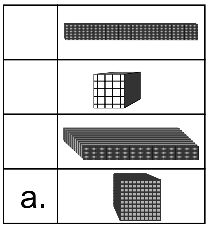


This shows the number: 1 112 137. In words, one million, one hundred and twelve thousand, one hundred and thirty-seven.

ACTIVITY 3

Match these numbers to the pictures. The first one is done for you.

a. One thousand (1 000)
b. Ten thousand (10 000)
c. One hundred thousand (100 000)
d. One million (1 000 000)

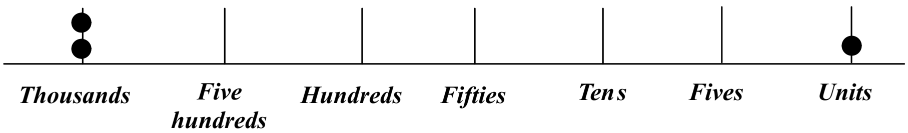


ANSWERS ON PAGE 94

Place value and the significance of zero

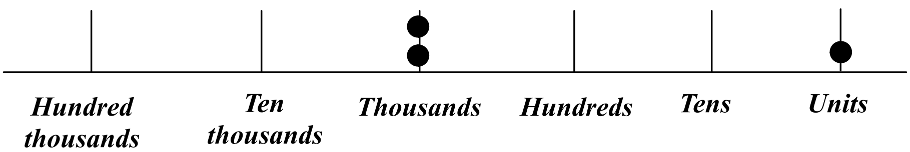
What the Roman, and the Babylonian, and many other number systems had in common was that they used different letters or symbols to represent particular number groupings. In the Roman system, a single unit is indicated with I, a thousand with M, and so on. To work with ever increasing numbers one would need many different symbols.

Example:
To show two thousand and one, you would write MMI.
The two Ms are like two beads on the thousands column of a counting frame:



The clever part of the Hindu-Arabic number system is that the *position* of the number symbol indicates the particular number grouping. This is only possible if there is a symbol to indicate where there is none of a particular number symbol.

Example:
Let's look again at the number two thousand and one.
This counting frame differs from the previous one, because we're using the decimal system. This is how it looks on the counting frame:



In the decimal number system, we use zero (0) to indicate that there are no hundreds or tens:

Ambiguous *- having more than one meaning*

Mathematicians talk about the ***place-value*** *of a digit in a number.*

Example: 2001

Instead of using a different **symbol** for a number grouping, the **position** shows the number grouping. Zero is the invention that allows us to place digits in their correct position. Without zeroes, we would not be able to tell the difference between 21, 201, 210, 2100, 2010 and many more numbers. Our notation would be **ambiguous.**

Example:

In the number 3071

- the 3 has the place value of thousands i.e. 3 thousands
- the 7 has the place value of tens i.e. 7 tens
- the 1 has the place value of units i.e. 1 unit

Activity 4

1. In the number 7509, write down the place value of the 9, of the 0, of the 5, of the 7.

2. In the number 231 810, write down the place value of the 8, of the 2, of the 0.

3. In the number 5, what is the place value of the 5?

ANSWERS ON PAGE 95

Rounding off very large numbers

One way of understanding very large numbers is to picture them as we did in the previous section.
Another way is to round off the numbers.

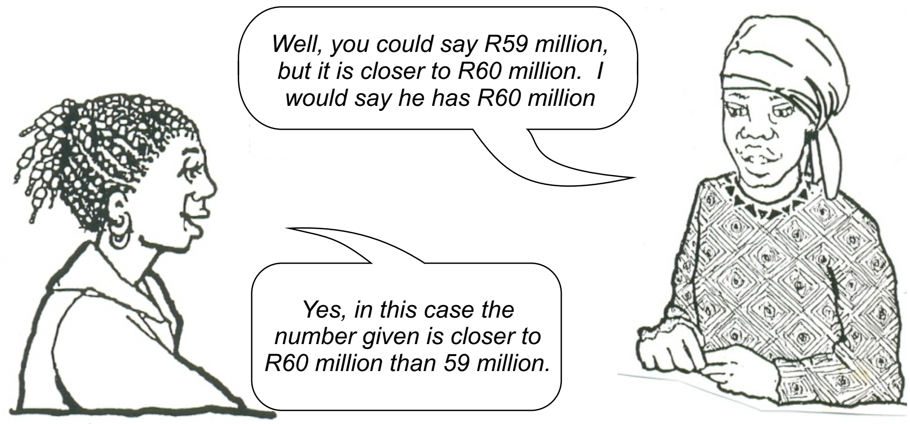
Example 1: Let's say Joe has R1121,53 in the bank.

It is easier to work with very large numbers if you round off. Let's take another example:

Example 2: Mr Nguni is a multi-millionaire. He has R57 156 387,56 in the bank. If you had to report on the wealth of this man, what answer would you give?

The taxman might want to know to the nearest thousand. A good answer would be R57 156 000.

Example 3: Mr Nguni has R59 989 987,99 in the bank. Round off to the nearest million.



When you see a figure like R5,7 million in the newspaper, simply change it to R6 million. When someone says a new house will cost you R351 678, take it as R352 000.

ACTIVITY 5

Round off the following amounts to what you think is the nearest appropriate amount:

1. In 2010 the government expenditure on correctional services was: R2 893 992,23

2. After 10 years of service Mr Bopape's pension paid out: R306 108,40

3. A new bed at "Dream beds" costs R1 058. How much money will you draw at the ATM to pay for the bed?

ANSWERS ON PAGE 95

Summary

A number symbol stands for quantity. 5 stands for IIIII. A number symbol can also stand for an amount of time. It is 5 o'clock. It is the 5th hour after midday. We use the Hindu-Arabic number symbols 0 1 2 3 4 5 6 7 8 9.

We saw in the lesson that there are other number symbols like the Roman numerals that are sometimes used on clocks. There are also finger gestures, which stand for a number.

Our number system is based on tens. We have tens, then hundreds (ten × ten), then thousands (ten × ten × ten) and so on.

The Mayan culture from Central America, and the Yoruban culture from Nigeria **base** their number system on twenties.

They had twenty, then 400 (20 × 20), then 8 000 (20 × 20 × 20) and so on. Large numbers have many more 0s. Otherwise they do obey the same laws as small numbers. Look back to the explanation of thousands, ten thousands and millions.

I hope you have gained some of the skills and confidence needed to make numbers, even large numbers, fun.

Self-assessment checklist:

Are you able to:

- use number symbols to stand for quantities;
- give examples of number systems that have been used in the past;
- compare the groupings of different numbering conventions;
- round off large numbers.

SELF-CHECK EXERCISE

1. Give two examples of number symbols we come across in our everyday lives.
2. The Babylonians used 60 as an important number grouping. Give two examples where we use groups of 60.
3. The decimal system we use is based on tens. Explain briefly how our number system is organised. You could start like this
   10 units = 1 ten
   10 tens = 1 hundred ... and so on
4. The government estimated revenue for the year 2010 to be R3 265 000 000. The estimate was wrong by R19 000 000. Is this error:
   a) very large or b) reasonable or (c) very small.

ANSWERS ON PAGE 106

2. More about numbers

Introduction

In lesson 1 we looked at how our decimal number system is constructed. We saw how the decimal system suited the needs of the developing world.

At school you may have heard about integers, rational numbers and counting numbers, and many other kinds of numbers. Well, these number sets were invented because people needed more complex number systems.

In this lesson you will see that these number sets became necessary. Three groups of people are used to illustrate the story: herders, fishermen and carpenters.

We know a bit about how herders counted their sheep. We count things every day. But have you ever wondered how fishermen know how deep to put their fishing lines? What about carpenters? How do they make sure their measurements are accurate?

In this lesson we will look at how more and more complex sets developed within our decimal system.

By the end of this lesson you will be able to:

- write a few lines about the early use of mathematics
- list three number sets
- solve a few mathematical problems from everyday life
- share your understanding of mathematics with a friend.

The mathematics of herders, fishermen and carpenters

ACTIVITY 1

1. The herder owns some sheep and cattle. She knows how many sheep and cattle she has. She sometimes gives sheep away and gets grain or clothing in return. List two ways she might use mathematics in her everyday life.

2. Fishermen make their living by catching fish and selling them. They know that different fish live at different depths. Their fishing lines are different lengths. When they get back to shore they sell their fish to the people in the village. List two ways fishermen might use mathematics to do their job.

3. The carpenter or builder requires many skills when building a house. In what ways would a carpenter or builder use mathematics?

ANSWERS ON PAGE 95

As societies began to produce more food, they began trading with other groups. They then had to develop te mathematics they needed to help them with trading.

Even children, as they grow older, need to know greater numbers and therefore need a greater understanding of maths.

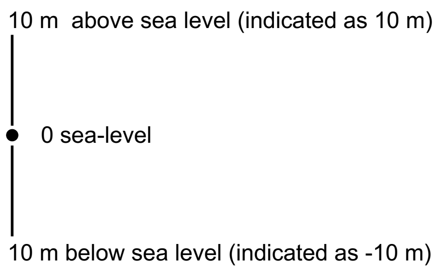
In the next section we look at how number sets developed within the decimal system to support the practical needs of people. These will be called number sets so as not to confuse them with other number systems.

Number sets

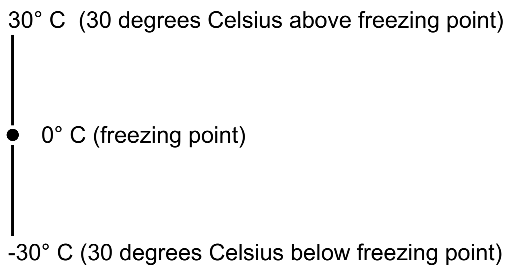
The herder needed a number set consisting of 1 2 3 4 5 6 7 up to the number of sheep or cattle she owned. Herders would use this number set to count. Mathematicians call these natural numbers the **natural number set**.

The Yoruba people of Western Nigeria could count up to 1 million. They would empty a bag of 20 000 cowry shells (their money) on to the ground and then count them very quickly. In the decimal number system you can go on counting forever.

The fishermen in *Activity 1* needed a set of numbers to describe distance above the water and distance below the water.



Weather reporters also need a set of numbers to describe the temperature above and below freezing point. (We'll look more at the measurement of temperature in lesson 5.)

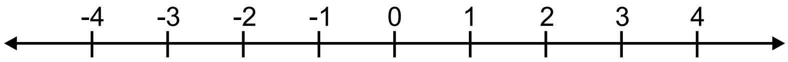


This number set has numbers in both directions, above and below 0. Mathematicians call these numbers **integers**.

The word integer is the Latin word meaning 'whole'.

The numbers above 0 (i.e. the counting numbers) are called **positive integers** and the numbers below 0 are called **negative integers**. (We'll look at integers in more detail in Lesson 4.)

These numbers can also be drawn on a number line like this:



Arrows mean the number line can be extended indefinitely.

Terms

Terminating means 'to end'. Examples of *terminating* decimals are:

0,5; 0,876; 0,25

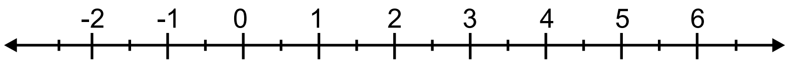
Recurring means 'to occur again' with a pattern. Examples of *recurring* decimals are:

0,333….

0,121212…

8,345634563456…

The carpenter needs to do a calculation like this: divide 3 metres into two equal parts. What would the answer be? It would be one and a half. A carpenter needs a number set which includes small parts of whole numbers. They can be shown on a number line like this.

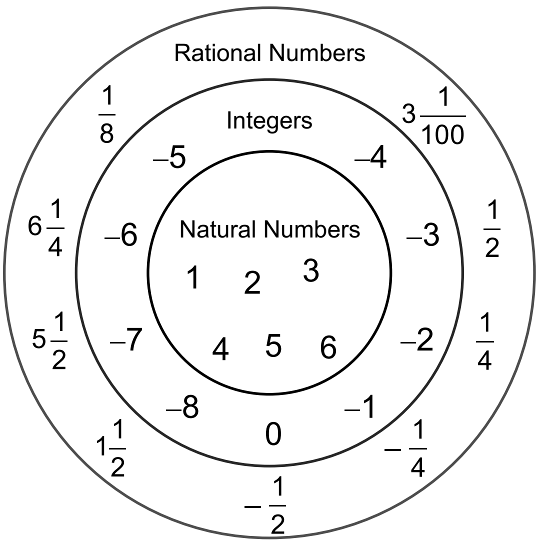


If we take a point half-way between 0 and 1, that point is a half. A half can be written $\frac{1}{2}$. It can also be written as a decimal number 0,5. Any number that can be written as a terminating or recurring decimal is called a rational number. (See decimal numbers in Lesson 3.)

This number set is called the **rational number set** because the numbers in this set can be written as a fraction or as a ratio. (We'll look at fractions in detail in Lesson *3*.)

Did you notice that rational numbers include integers, and integers include natural numbers?

The above number sets can be shown on a diagram like this:

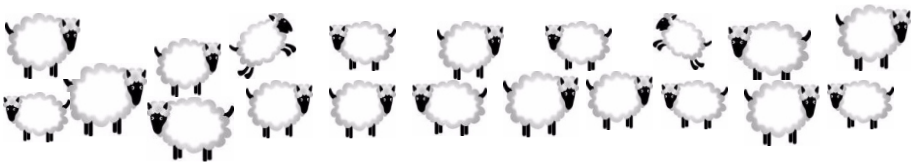


ACTIVITY 2

1. A herder had twenty sheep. His brother gave him four more sheep. Half of all his sheep were ewes. Each ewe had a lamb in spring. How many sheep does the herder have now?
   If you have difficulty with this problem try the following steps.

   Step 1: Read the question through slowly.
   Step 2: Ask yourself what the question means.
   Step 3: Read sentence by sentence. Try drawing pictures as you go.

He had twenty sheep.

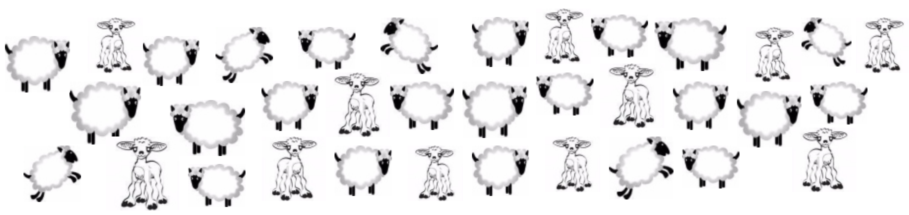


Brother gave four more.

Half the sheep were ewes (females).

Each ewe had a lamb.

Total number



   Step 4: Check the question again.

> ***Depth gauge:***
> *An instrument for measuring the depth of the ocean.*

2. A fisherman told his granddaughter, 'Red Roman like to feed 2 m from the bottom of the ocean floor.' The depth gauge on the fishing boat showed –5 metres.

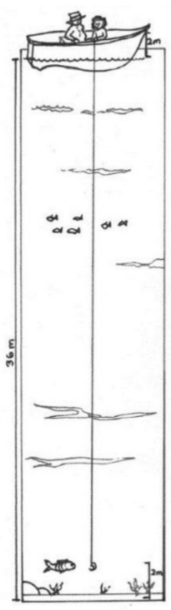
   How far below sea level should the young fisher-girl drop her line to catch a Red Roman?

If her rod was two metres above sea level, how long was her fishing line?

The steps outlined above will help:

Step 1: Read the problem through slowly.
Step 2: Ask yourself what the question means.
Step 3: Read sentence by sentence. Draw pictures as you go.

- Red Roman feed approximately 2 m above the ocean floor.
- The depth gauge measured the ocean floor as –5 m
- Her rod was two metres above sea level.



ANSWERS ON PAGE 96

ACTIVITY 3

Try these! Remember if you get stuck, use the steps you used in the previous examples:

1. One of the herders was a wealthy woman. She had one hundred sheep. One evening thirty of her sheep were killed by a pack of wolves. A few days later three sheep were killed for a feast. After the feast, 15 people from another community each brought her a sheep. How many sheep does she have now?

2. The fish in the ocean can be described as surface water feeders, middle water feeders and bottom feeders. The surface water is warmer and forms a layer of twenty metres. If a fisherman wants to catch a middle water feeder, how many metres deep must he drop his line?

3. A builder places skirting board around a room. The room is 3 metres long and 4 metres wide. There is a door 40 cm wide. (1 metre = 100 cm). How much skirting board must he buy?

ANSWERS ON PAGE 96

Mathematics for everyday life

When you solve a problem, you must be clear about the problem. Draw a picture and follow the steps in Activity 2 to help you.

ACTIVITY 4

1. Jennifer is baking biscuits for a school fete. She wants to make three times as many biscuits as the recipe asks for. The recipe asksfor 250 ml (1 cup) flour. How much flour does she need?

2. Faldielah has to pay a deposit for her college fees. The deposit is R500. She has R400 in the bank.

She went to see the bank manager. He said the bank would lend her the money she needed. How much money does she owe the bank?

3. Claudia has a length of wood 3 metres long. She wants to make shelves 60 cm long. How many shelves can she make from the piece of wood?

ANSWERS ON PAGE 97

Summary

Mathematics has developed over thousands of years to meet people's needs.

Mathematics deals with ideas and properties as well as concrete examples. Instead of drawing cups and fishing boats, we can represent numbers on a number line.

Mathematics generalizes from the concrete to the abstract. The herder knows that four sheep plus four sheep makes eight sheep.

The fisherman knows that four fish plus four fish makes eight fish.

The mathematician knows $4 + 4 = 8$.
She also knows that $40 + 40 = 80$
(4 tens + 4 tens = 8 tens).

She also knows that $400 + 400 = 800$
(4 hundreds + 4 hundreds = 8 hundreds).

Mathematics works with symbols. The mathematician can represent $4 + 4 = 8$ on paper. She doesn't need to use sheep.

Self-assessment checklist:

Are you able to:

- give examples of different ways in which people use mathematics in their everyday lives
- use numbers to solve simple problems
- identify the sets of counting numbers, integers and rational numbers.

SELF-CHECK EXERCISE

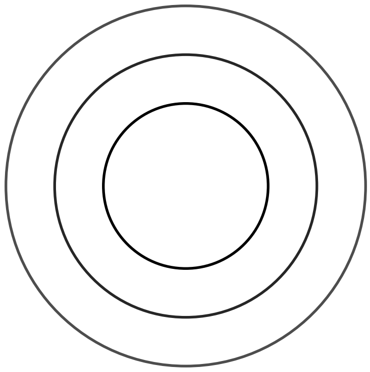
1. Name two ways in which herders may use mathematics.

2. Herders used a simple number set mathematicians call the set of _________________ numbers.

3. List two ways in which fishermen can use mathematics.

4. Fishermen need to use numbers in both directions of 0. Mathematicians call this number set ____________________

5. The three number sets you have learned about in this lesson can be represented in a drawing like this. Fill in the names of the sets.



Look again at the steps in solving problems then try the following:

6. At the shop Trevor bought:

bread	R9,50
2 litres milk	R12,50
oil	R7,60

He paid the cashier with a R50 note. When he got home, he checked the change. He had R20 in his pocket. Was that right?

7. Nomhle had a loan from the bank for R10 000. If she paid R1 000 each month, how many months would it take for her to pay back the bank? (No interest at this bank.)

8. Peter wanted to make 5 shelves each 80 cm in length. How many metres of wood would he need?

ANSWERS ON PAGE 106

3. Fractions, decimal fractions, and percentages

Introduction

Fractions aren't nearly as worrying in the real world. Also, now that we have decimal fractions, common fractions are not so important.

The mistake people usually make when working with decimal fractions is where to put the decimal point. When you understand how our number system is organised (and this includes decimals), this problem should clear up.

There are ways of making fractions easy. When you can picture a fraction in your mind, it is easier to work with it. Let's see what fractions are all about.

By the end of this lesson you will be able to:

- explain what a fractions is
- represent a fraction pictorially and on a number line
- change fractions into equivalent forms
- compare fractions, and order them on a number line
- define proper, improper and mixed fractions
- convert between improper and mixed fractions
- explain a decimal fraction
- tell the value of each place in a decimal fraction
- write common fractions as decimal fractions
- relate decimals to our monetary system
- explain what a percentage is
- understand percentage as used in everyday life, e.g. a 10% discount.

Fractions in early history

When people needed to use numbers to count sheep or to keep a record of time they used natural numbers. But when traders started measuring lengths and weighing quantities, they found it necessary to work with parts of numbers (or fractions).

The word fraction is like the word fracture, which means ‘to break’. In Arabic the word for fractions is ‘Kastra’ which means ‘broken numbers’. The important principle to remember is that fractions are “broken up” into EQUAL parts.

Therefore if we have a cake and cut it in half (into 2 pieces), these pieces will each be the same size. One half represents one of 2 equally divided parts. A half of my cake will not necessarily be the same amount or size as half of your pizza (because the pizza and the cake are different sizes) but we each get one slice of two equally sized slices.

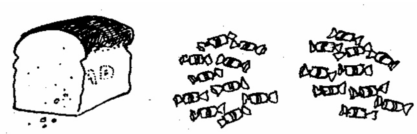
Common fractions

One of the ways of looking at fractions is as parts of a whole. Let's look at this more closely.

Example:
Seca goes to the shop. 'Please may I have half a loaf of bread and a packet of sweets?'

When he gets home, his two children run to greet him. 'Daddy, did you buy us some sweets?'

'Yes. Now you must share them. Each of you can have half the sweets.'

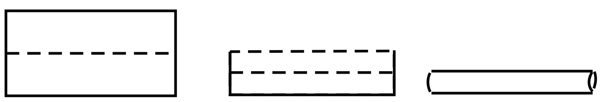


In the packet there are 20 sweets. Each child gets 10. They received a fraction of the whole packet.

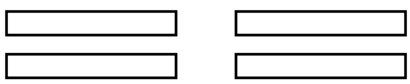
ACTIVITY 1

Try this activity.

1. Fold a strip of A4 paper (the size of this page) along its length. Then fold it again.
   It should look something like this:

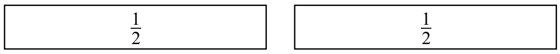


   Then tear it into 4 strips like this:

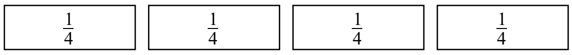


   Take the length of each strip to be 1.

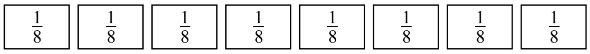
2. Fold one of the strips and tear it into 2 pieces.
Mark them as follows:



3. Fold another of the strips and tear it into 4 pieces.
Mark them as follows:



4. Fold another of the strips and tear it into 8 pieces.
Mark them as follows:

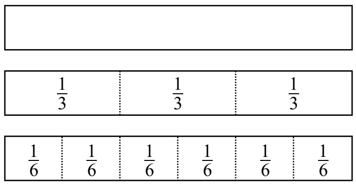


5. Use some of the pieces to show that $\frac{6}{8} = \frac{3}{4}$

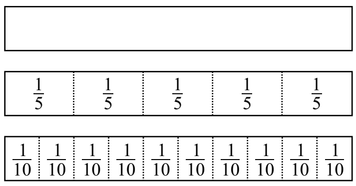
6. Use some of the pieces to show that $\frac{2}{8} = \frac{1}{4}$

7. Show $\frac{1}{2}$ at least two other ways.

8. You can use the same method for showing $\frac{1}{3}$, $\frac{1}{6}$, etc.



9. You can use the same method for showing $\frac{1}{5}$, $\frac{1}{10}$, etc.

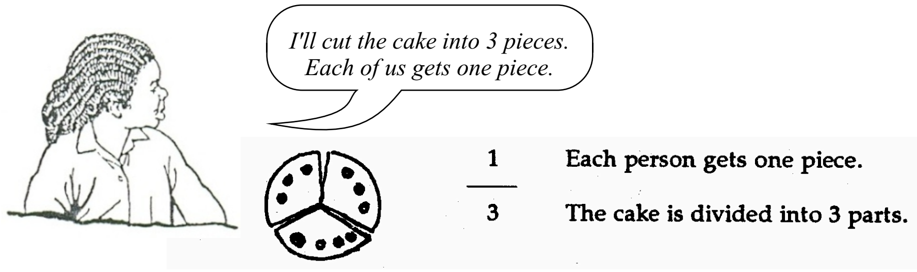


ANSWERS ON PAGE 98

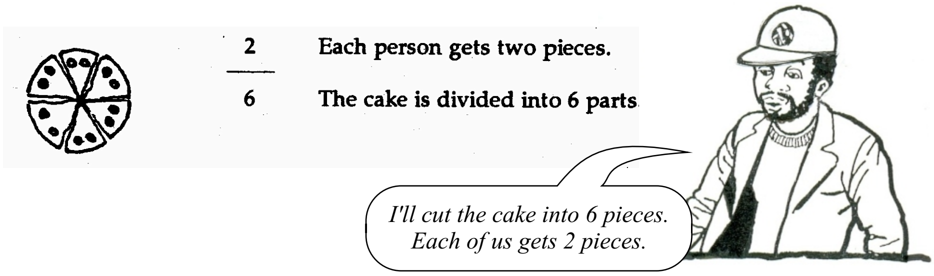
Let's look at another example where fractions are used.

Joe, Kennedy and Nunu want to share a cake so that each one gets the same amount.

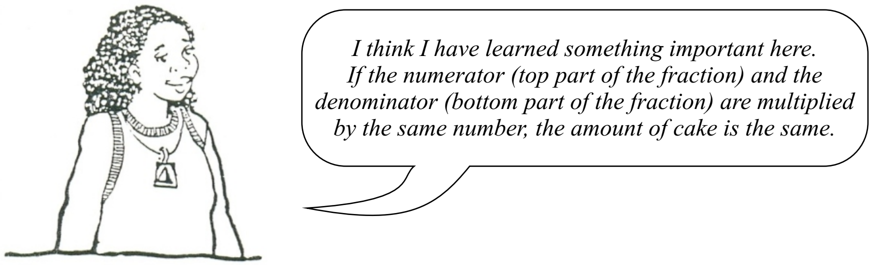
Joe’s solution: We write the part each person gets as a fraction like this:



Kennedy's solution: Now write the part each person gets as a fraction



Do they get the same amount of cake? Is $\frac{1}{3}$ of the cake the same size as $\frac{2}{6}$? Try drawing the cakes. Nunu’s solution:



The value of a fraction stays the same if you multiply the numerator and the denominator by the same number. This is because you are in fact multiplying the fraction by a value of 1. When you multiply any number by 1 the value of that number stays the same. So while we may have changed the numerator and the denominator of the fraction, the value of the fraction remains the same.

This is an extremely important fact that we will use throughout our work on fractions.

Early fractions

Here are some interesting facts from books on the history of maths. They are not important for the general understanding of maths on this course, but they may interest you.

The Egyptians had a method of writing fractions, called unit fractions, where every fraction was written with a numerator of 1, for example:

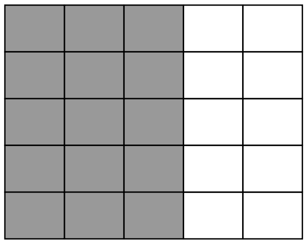
$\frac{1}{2}$, $\frac{1}{7}$, $\frac{1}{10}$, $\frac{1}{100}$.

The Babylonians wrote their fractions as parts of 60. These were called sexagesimal fractions. (Sexagesimal is another way of saying 60.)

Their denominators were always 60 or 3 600 (60 × 60). Because the denominators were always the same, they didn't have to write them every time.

They would write 3 24 36. This meant 3 wholes, $\frac{24}{60}$ and $\frac{36}{3600}$.

Here is another illustration that the value of a fraction stays the same if you multiply the numerator and the denominator by the same number.



3 columns out of 5 are shaded. That is, $\frac{3}{5}$ of the figure is shaded.
Also, 15 blocks out of 25 are shaded. That is, $\frac{15}{25}$ of the figure is shaded.
So we have shown that $\frac{3}{5} = \frac{15}{25}$.

That means that the value of $\frac{3}{5}$ does not change when we multiply the numerator and the denominator both by 5.

We have shown that a fraction stays the same when we multiply the numerator and denominator by the same number. We also know that the fraction stays the same when we divide the numerator and the denominator by the same number.

$\frac{12}{20} \div \frac{4}{4} = \frac{3}{5}$
$\frac{6}{8} \div \frac{2}{2} = \frac{3}{4}$

Remember that the value of the fraction remains the same because we are dividing the fraction by a value of 1. If we multiply or divide any number by 1, the value of that number remains the same.

Equi - valent means 'equal value'

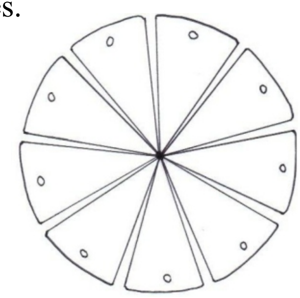
Equivalent fractions

Fractions which have the same value, such as $\frac{1}{3}$ and $\frac{2}{6}$; and $\frac{3}{4}$ and $\frac{6}{8}$ and $\frac{15}{25}$ and $\frac{3}{5}$, are called equivalent fractions.

ACTIVITY 2

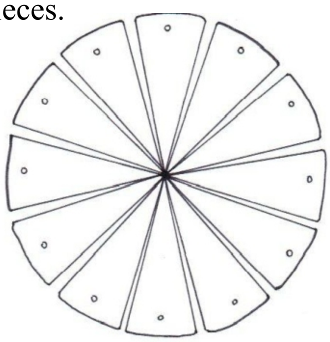
Let's look at the following fractions.

1. Cut the cake into 9 pieces. Share 9 pieces between 3 boys. Each boy gets 3 pieces.



Each gets $\frac{3}{9}$.

2. Cut the cake into 12 pieces. Share 12 pieces between three boys. Each boy gets 4 pieces.



Each gets $\frac{4}{12}$.

3. Cut the cake into 15 pieces. Share 15 pieces between three boys. Each boy gets ....

4. Cut the cake into 18 pieces. Share 18 pieces between three boys. Each boy gets ....

5. Fill in the missing numbers.

$$\frac{1}{3}=\frac{2}{6}=\frac{\square}{9}=\frac{\square}{12}=\frac{\square}{15}=\frac{\square}{18}=\frac{\square}{21}=\frac{\square}{24}=\frac{\square}{27}=\frac{\square}{30}$$

Note: These fractions all have the same value. If John gets 3 of the 9 slices, or 6 of the 18 slices, it is the same amount of cake (provided it is the same cake!)

ANSWERS ON PAGE 98

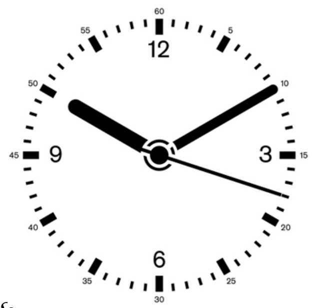
ACTIVITY 3

1. Four women share a loaf of bread equally.
   a) How much does each one get? Write the answer as a fraction.
   b) Divide another loaf of the same size into 8 pieces. How much would each person get? Write the answer as a fraction.
   c) If the loaf were cut into twelve pieces, how much would each person get? Write the answer as a fraction.
   d) What if the loaf of bread was cut into 20 slices? How much would each person get? Write the answer as a fraction.

2. Fill in the missing numbers.

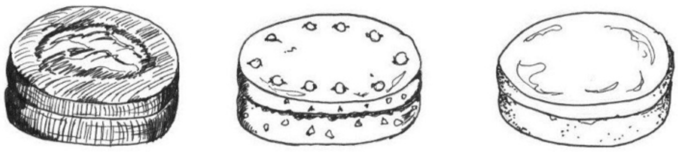
$$\frac{1}{4}=\frac{2}{8}=\frac{3}{12}=\frac{\square}{16}=\frac{\square}{20}=\frac{\square}{24}=\frac{\square}{32}=\frac{\square}{48}=\frac{\square}{64}$$

3. Look at this clock face. It is divided into 60 minutes. The minute hand completes a full circle every hour.



   a) How many minutes in $\frac{1}{2}$ an hour?
   b) How many minutes in $\frac{1}{4}$ of an hour?

4. Joe works an 8-hour day. On Saturdays he works half-day. How many hours does he work on Saturday?
5. There are 24 hours in 1 day. There are 12 hours of sunlight. What part of the day is dark? Write the answer as a fraction.
6. There are three cakes on the table.



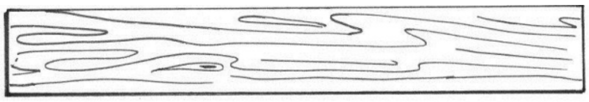
Cut the chocolate cake into 3 pieces.
Cut the cherry cake into 5 pieces.
Cut the sponge cake into 10 pieces.

a) Which cake has the biggest slices?
b) Which cake has the smallest slices?
c) A slice of chocolate cake written as a fraction is ....
d) A slice of cherry cake written as a fraction is ....
e) A slice of sponge cake written as a fraction is ....

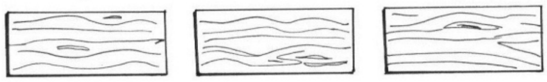
ANSWERS ON PAGE 99

Fractions on a number line

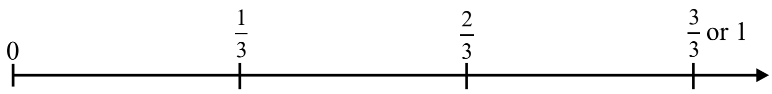
Let's look at a piece of wood 1 m in length.



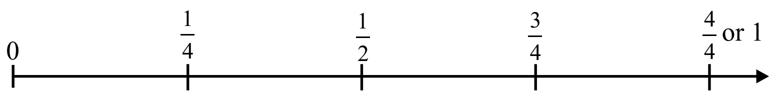
Divide the piece of wood into three equal pieces.



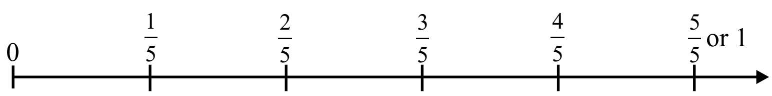
Each part written as a fraction is $\frac{1}{3}$.
Let's divide a number line into thirds.



Now divide the number line into halves and quarters.



Finally, divide the number line into fifths.



Let's compare three fractions $\frac{1}{3}$, $\frac{2}{5}$, and $\frac{1}{2}$. On the thirds number line, mark $\frac{1}{3}$ with an X. On the halves and quarters number line, mark $\frac{1}{2}$ with an X. On the fifths number line mark $\frac{2}{5}$ with an X.

Which is the biggest fraction? Which is the smallest fraction?
Write the three fractions from biggest to smallest.

How did you work it out? Did you make a good guess? Did you measure each length to see which was longest?

All these methods are ways of getting the right answer. But there is another method. This method was discovered by a mathematician, called al Khwarizmi, who lived in Baghdad in 700 A.D.

His method was to change all the fractions to equivalent fractions with the same denominator.

Let's practice writing the same fractions using different denominators.

ACTIVITY 4

Copy and complete:

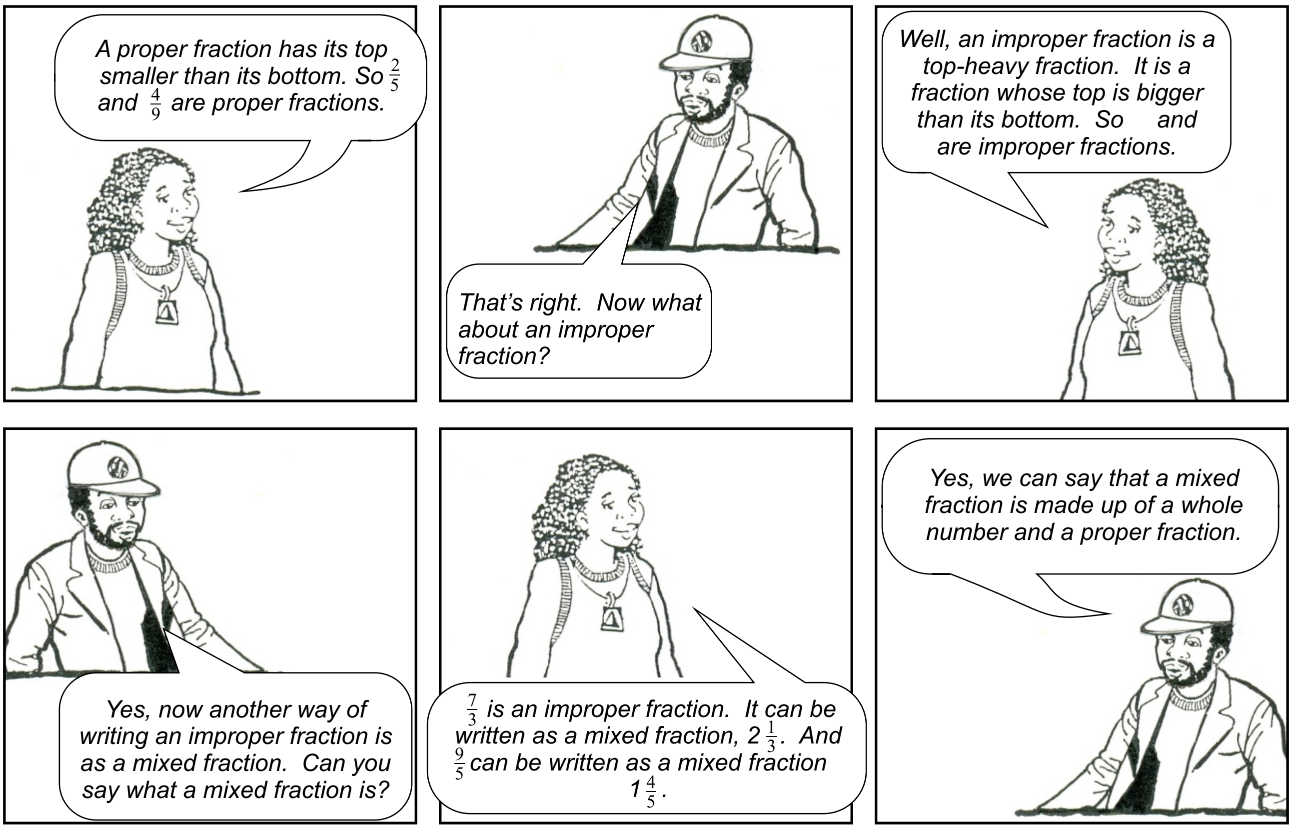
1. $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{\square} = \frac{\square}{12} = \frac{10}{\square} = \frac{\square}{100}$

2. $\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{\square}{12} = \frac{5}{\square} = \frac{6}{18}$

3. $\frac{3}{4} = \frac{6}{8} = \frac{\square}{16} = \frac{15}{\square} = \frac{\square}{32}$

ANSWERS ON PAGE 99

Proper, improper and mixed fractions



ACTIVITY 5

1. $\frac{5}{4}$ is an improper fraction
   a) How many quarters are there in $\frac{5}{4}$?
   b) How many quarters are there in one whole number?
   c) How many whole numbers in $\frac{5}{4}$ and how many quarters are left over from the whole number?

2. $\frac{17}{5}$ is an improper fraction.
   a) How many fifths are there in $\frac{17}{5}$?
   b) How many fifths are there in one whole number?
   c) How many whole numbers are there in $\frac{17}{5}$ and how many fifths are left over from the whole numbers?

3. $4\frac{1}{3}$ is a mixed number.
   a) How many thirds in one whole number?
   b) How many thirds in 4 whole numbers?
   c) How many thirds in $4\frac{1}{3}$?

ANSWERS ON PAGE 99

Fractions with denominators of 10, 100, or 1000

Joe hated working with fractions. 'All those denominators and all those numerators, and hundreds of equivalent fractions all meaning the same thing. I think it is a waste of time! There must be an easier way. What about always having the same denominator like the Babylonians did many years ago?'

What do you think? Do you agree with Joe?
Our number system is based on tens, why not extend it to tenths, hundredths and thousandths?

Long ago, in 1585, Simon Steven, who lived in Belgium. He thought, 'There must be a better way to work with fractions.' He worked out the system of **decimal fractions**. Thank heavens! All the denominators are 10's or 100's or 1 000's.

Decimal fractions

When we count, we always group things together in tens. We group ten ones together to make ten. Then we group ten tens together to make a hundred. Then we group ten hundreds together to make a thousand.

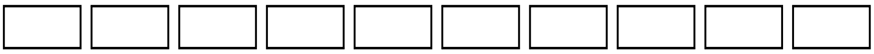
When we write 3 245 we mean 3 groups of a thousand, 2 groups of a hundred, 4 groups of ten, and 5 units (ones).

We might measure the distance to the station and find it to be 3 245 metres long. In this way, by grouping in tens and hundreds, we can measure things much larger than a metre.

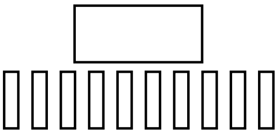
Decimal fractions are designed to measure things much smaller than a metre. We don't group metres together. We break them down into smaller parts.

A. 

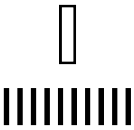
one metre or unit

B. 

is broken into ten pieces.
Then we take one of the pieces from B and break it up into ten smaller pieces.

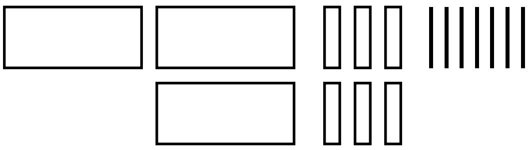
C. 

Then we take one of the pieces from C and break it up into ten still smaller pieces.

D. 

And this process can go on for as long as we like.

Suppose we took 3 of size B, and 6 of size C, and 7 of size D, we could represent it like this:



However, in fact it is written like this: 0,367
We understand from the position of each number, what size each number is.

A decimal fraction is a part of 10ths, 100ths, 1 000ths - or any multiple of 10. There are no denominators. The position of the number tells what its value is.

Decimal fractions	**Thousands 1 000s**	**Hundreds 100s**	**Tens 10s**	**Units 1s**	**Tenths** $\frac{1}{10}$	**Hundredths** $\frac{1}{100}$	**Thousandths** $\frac{1}{1000}$
0,367				0	3	6	7
1,25				1	2	5	
3,42				3	4	2	

Place value

The decimal comma or point shows where the whole part of the number ends. Everything to the right of the decimal comma is worth less than one whole.
The first digit to the right is the number of tenths. The first two digits are the number of hundredths. The first three digits are the number of thousandths.

When we write 3,4 we mean three and four tenths, or $\frac{34}{10}$ or $3\frac{4}{10}$
When we write 3,42 we mean three and forty-two hundredths, or $3\frac{42}{100}$

Writing common fractions as decimal fractions

Example 1: Write $\frac{4}{10}$ and $\frac{35}{100}$ as decimal numbers.

To write common fractions as decimal fractions, use your calculator. Divide the numerator (number above the line) by the denominator (number below the line).

Example 2: Write $1\frac{2}{5}$ and $5\frac{3}{4}$ as decimal numbers.

$1\frac{2}{5} = 1 + \frac{2}{5}$ Divide 2 by 5. Answer 0,4.

$1\frac{2}{5} = 1{,}4$ Add 1

$5\frac{3}{4} = 5 + \frac{3}{4}$ Divide 3 by 4. Answer 0,75, Add 5.

$5\frac{3}{4} = 5{,}75$

ACTIVITY 6

Use your calculator. Write the following fractions as decimal fractions. In Lesson 8 we will look at calculator keys that can help us do these conversions.

1. $\frac{1}{2}$
2. $\frac{1}{4}$
3. $\frac{3}{4}$
4. $\frac{9}{25}$
5. $\frac{7}{40}$
6. $\frac{3}{5}$

ANSWERS ON PAGE 100

Recurring decimals

One problem with decimal fractions is that some of them go on forever. For example, divide 1 by 3 on your calculator. What is your answer? It is 0,333333333. There are only 10 spaces on your calculator. If there were 100 spaces, your answer would be: 0,3333333333333333333333333 ...... (100 times).

The answer is $\frac{1}{3}$ but you cannot write $\frac{1}{3}$ exactly in decimal form. How many 3's do you need to write?

Generally, we only use two decimal places. So we round off to two decimal places.

Example 1: Write 0,3333333 ... to two decimal places. Look at the third place after the decimal point. 0,33 ... This number is less than 5. So we round off to 0,33. The calculator often displays this as $0,\dot{3}$ indicating that the 3 is recurring in the decimal.

Example 2: Write 0,666666 ... to two decimal places. Look at the third place. 0,666 ... This number is greater than 5. We round off to the next number, which is 7. Our answer is 0,67.

Remember if the next number is 5 or more, round off to the next number up (see example 2). If the next number is less than 5, leave it as it is.

ACTIVITY 7

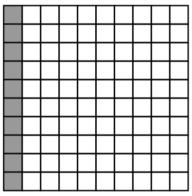
1. Round off the following decimal fractions to 2 decimal places. (Remember, look at the third place. If 5 or more, round up. If less than 5, leave it as it is.)
   a) 1,4444.... b) 3,88888.... c) 0,797979....

ANSWERS ON PAGE 100

Percentages

A percentage fraction is a fraction out of 100.

10% means
10 blocks out of 100 blocks
or
R10 out of R100



Changing fractions into percentages

$\frac{1}{2}$ of 100 is 50 $\frac{1}{2} = \frac{50}{100}$ these are equivalent fractions

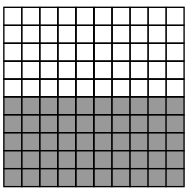
So $\frac{1}{2}$ is the same as '50 out of 100' That means $\frac{1}{2} = 50\%$

The calculations can be shown like this:

Example 1:

Write $\frac{1}{2}$ as a percentage.

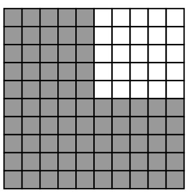
$\frac{1}{2}$ converts to $\frac{1}{2} \times \frac{100}{1} = \frac{100}{2} = 50\%$



Example 2:

Write $\frac{3}{4}$ as a percentage.

$\frac{3}{4}$ converts to $\frac{3}{4} \times \frac{100}{1} = \frac{300}{4} = 75\%$



When multiplying fractions, multiply the numerators together, and multiply the denominators together.

ACTIVITY 8

1. Change these fractions into percentages:

a) $\frac{1}{10}$ b) $\frac{3}{5}$ c) $\frac{1}{20}$ d) $\frac{4}{5}$

Sometimes the percentages are mixed numbers, e.g.
$\frac{1}{3}$ converts to $\frac{1}{3} \times \frac{100}{1} = \frac{100}{3} = 33\frac{1}{3}\ \%$

ANSWERS ON PAGE 100

2. Change these fractions into percentages:
a) $\frac{2}{3}$ b) $\frac{1}{8}$ c) $\frac{1}{6}$ d) $\frac{3}{8}$

Common, decimal and percentage fractions

$1 = \frac{1}{1} = 1{,}0$	$1 = \frac{100}{100} = 100\%$
$\frac{1}{2} = \frac{5}{10} = 0{,}5$	$\frac{1}{2} = \frac{50}{100} = 50\%$
$\frac{1}{4} = \frac{25}{100} = 0{,}25$	$\frac{1}{4} = \frac{25}{100} = 25\%$

ACTIVITY 9

1. Copy and complete:

Common Fractions	Decimal Fractions	Percentage
1	1,0	100%
$\frac{1}{5}$		
$\frac{1}{4}$		
$\frac{1}{3}$		
$\frac{1}{2}$		
$\frac{3}{4}$		
$\frac{9}{10}$		

ANSWERS ON PAGE 100

Changing a percentage into a simple fraction

Example: $45\% = \frac{45}{100}$. This can be simplified even further to $\frac{9}{20}$
Divide numerator and denominator by 5

ACTIVITY 10

Change these percentages into simple fractions

ANSWERS ON PAGE 100

a) 70% b) 55% c) 24% d) 50%

Money

We use the decimal every time we use money. Our unit for money is a rand. The rand is divided into 100 parts called cents.

Money and percentages

Percentages are often used in connection with money. You will see many examples in advertisements.

10% means '10 per hundred'
R10 per R100

To work out 10% of an amount we need to find out the number of 100's that are contained in the amount: that many 10's makes 10% of the number.

Example:
What is the price of a coat which is usually R85?

Usual price: R85

10% reduction: $\frac{85}{100} \times \frac{10}{1} = 8{,}50$

Sale price: R76,50

ACTIVITY 11

1. Find the sale prices.

   a) Dress: R75
   b) Jacket: R150

ANSWERS ON PAGE 100

Percentages are often used in connection with taxes.

VALUE ADDED TAX (VAT)

VAT in our country is 14 per cent. This means that if the value of something is R100, then R14 must be added to the price.

That R14 must be paid to the government.

Value of article decided by seller	Value Added Tax (sent to government)	Total amount paid by the buyer
R 1 (100 cents)	14c	R 1,14
R 10 (10 × 100 cents)	R 1,40 (140 cents)	R 11,40
R 100	R 14	R 114
R 1 000	R 140	R 1 140
R 10 000	R 1 400	R 11 400
R100 000	R14 000	R 114 000

ACTIVITY 12

You are a shop keeper. You have to add VAT to some articles in your shop. The items are:

a loaf of bread	R10
a packet of flour	R24
a block of cheese	R35
a pocket of potatoes	R30
2 kilograms of beef	R150
an electric hot plate	R300

1. Write the VAT next to the article.
2. Then add the VAT to the value of the article.
3. Finally add up the totals.

ANSWERS ON PAGE 100

Value of article decided by seller	**Value Added Tax (sent to government)**	**Total amount paid by the buyer**
loaf of bread R10		
packet of flour R24		
block of cheese R35		
pocket of potatoes R30		
2 kilograms of beef R100		
an electric hot plate R300		
Total cost before VAT:	Total VAT:	Total cost after VAT:

Summary

In this lesson we have looked at three ways of representing fractions. The first was to use common fractions, like $\frac{1}{2}$ or $\frac{50}{100}$.

These fractions have advantages. You can represent a part of a number exactly. There are also disadvantages to this system. It is sometimes difficult to work with a lot of different denominators.

The second way was using decimal fractions, such as 5,25.

These fractions are based on a number base of ten. This makes them easier to work with. A disadvantage that is easily overcome is that some fractions cannot be written exactly with ten as a denominator.

The third way of showing fractions was percentages, such as 20%. These fractions are parts of hundreds. They are easy to compare. It is easy to see that 20% is less than 50%.

VAT in our country is 14 per cent. We drew up a table of the VAT to be added to various amounts. A shopkeeper needs to add VAT to the product. The VAT is then sent to the government.

Self-assessment checklist:

Are you able to:

- represent fractions on a number line
- write down equivalent fractions
- compare fractions and arrange in increasing/decreasing order
- use common fractions
- convert mixed fractions to improper fractions and vice versa
- use decimal fractions
- use percentages e.g. in discounts and calculating VAT
- convert between common fractions, decimal fractions and percentages.

SELF-CHECK EXERCISE

1. Five children share a loaf of bread equally.
   a) How much does each one get? Write it as a fraction.
   b) Divide the loaf into 10 equal pieces. How much would each person get? Write the answer as a fraction.
   c) If the loaf were cut into 15 equal pieces, how much would each person get?
   d) What if the loaf of bread was cut into 20 slices? How much would each person get?

2. Fill in the numbers: $\frac{1}{5} = \frac{2}{10} = \frac{3}{15} = \frac{\square}{20} = \frac{\square}{25} = \frac{\square}{30} = \frac{\square}{60}$

3. In one day there are 24 hours.
   a) If there is sunlight for half the time, how many hours do we have sunlight?
   b) If we work for one-third of the day, how many hours do we work?

4. Fill in the numbers: $\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{\square}{12} = \frac{\square}{15} = \frac{\square}{30} = \frac{\square}{60}$

5. SALE 20% OFF
   Find the sale prices:
   a) Trousers: R300
   b) Shoes: R450

6. Calculate the Value Added Tax that should be paid on the following items (complete the table):

Value of article decided by seller	**Value Added Tax (sent to government)**	**Total amount paid by the buyer**
watch R400		
soccer boots R280		
dress R360		
basket R175		
jacket R420		
Total cost before VAT:	Total VAT:	Total cost after VAT:

7. a) Write the following common fractions as decimal fractions, and as percentages (complete the table):

Common fraction	**Decimal fraction**	**Percentage**
$\frac{12}{25}$		
$\frac{2}{5}$		
$\frac{18}{40}$		
$\frac{5}{8}$		
$\frac{3}{20}$		

b) Write the above fractions in order from smallest to biggest.

8. a) Write the following improper fractions as mixed numbers:

i) $\frac{12}{7}$

ii) $\frac{8}{5}$

iii) $\frac{23}{2}$

iv) $\frac{19}{5}$

v) $\frac{13}{3}$

ANSWERS ON PAGE 107

b) Write the following mixed numbers as improper fractions:

i) $1\frac{2}{3}$

ii) $2\frac{3}{4}$

iii) $5\frac{1}{7}$

iv) $6\frac{1}{2}$

v) $3\frac{1}{5}$

4. Integers

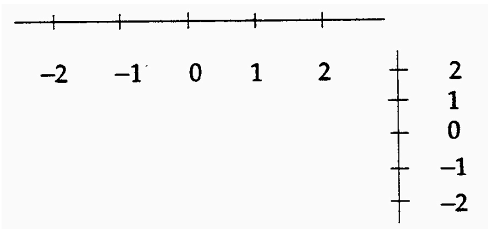
Introduction

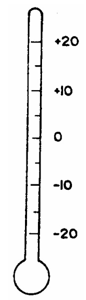
Do you remember learning about the position of integers (numbers written with a + or a − sign) on a number line? If you don't, look back at lesson 1. The set of positive and negative whole numbers is called the set of integers and mathematicians use the symbol ≥ to show that we are working with integers.

In this lesson you will:
- learn about the position of negative numbers on a number line
- add, subtract, multiply and divide integers
- learn how to use integer numbers
- understand how negative numbers can be useful in everyday life
- be able to use negative numbers to do calculations.

Revisiting the number line

Number lines are useful in mathematics when we learn how to add and subtract integer numbers. We also use them when we draw axes of a graph.
Look at the horizontal and vertical number lines.





Look at the thermometer on the right.

0°C are degrees of measurement on the Celsius (centigrade) scale of measuring temperature.

The numbers with (+) signs are called **positive** numbers. The numbers with (−) signs are called **negative** numbers. The temperatures that are greater than 0°C are positive. The numbers that are below (or less than) 0°C are negative.

If we now write our thermometer in the form of a vertical number line, we have

Below zero is negative.
+30°C
+20°C
+10°C
0°C
−10°C
−20°C
Above zero is positive.

We usually leave out the units (°C) when we draw a number line.
You can also have a horizontal number line.

If you put the thermometer in a horizontal position: To the left of zero you have negative numbers, and to the right of zero you have positive numbers.

–20 –10 0 +10 +20

In a vertical number line negative numbers are below zero. In a horizontal number line the negative numbers are to the left of zero.

Zero separates positive and negative numbers. Zero itself is neither positive or negative. We refer to 0 as a neutral number although it still belongs to the set of integers.

We have many other numbers between any two integers on a number line. The numbers between any two integers are fractions and decimal fractions known as rational numbers.

Integers

Positive numbers can be written with a positive sign (+) or with no sign at all. Negative numbers are **always** written with a negative sign (–) in front of the number.

For example:
Positive 5 can be written as +5 or 5.
Negative 5 is written as –5.
Positive $\frac{1}{3}$ can be written as $+\frac{1}{3}$ or $\frac{1}{3}$.
Negative $\frac{1}{3}$ is written as $-\frac{1}{3}$.

A number written without a sign is always assumed to be a positive number.

ACTIVITY 1

Write each number as an integer with a (+) sign or with a (–) sign.

1. a) positive $\frac{2}{5}$ = b) negative 3 = c) negative $\frac{1}{6}$ =

2. Say whether each number is positive or negative.
   $+\frac{3}{5}$, 2, 0, 6, 2, –0,6

ANSWERS ON PAGE 101

When, how and where do you use integers?

We often use integers to represent opposite quantities. For example, positive numbers represent temperatures above 0°C, distances above sea-level, and payments and credits in your bank account. Negative numbers represent temperatures below 0°C, distances below sea-level, and charges or withdrawals in your bank account.

Let us look at the following example.

Example

At night the temperature was −3°C, but it rose to 20°C in the morning. By how many degrees did the temperature rise?

The change in temperature is given by
20°C − (−3°C) = 20°C + 3°C = 23°C

Therefore, the temperature rose by 23°C.

ACTIVITY 2

1. Tsepo challenges Lerato to a round of mini-golf at the seaside course. Read the rules and look at their score card.

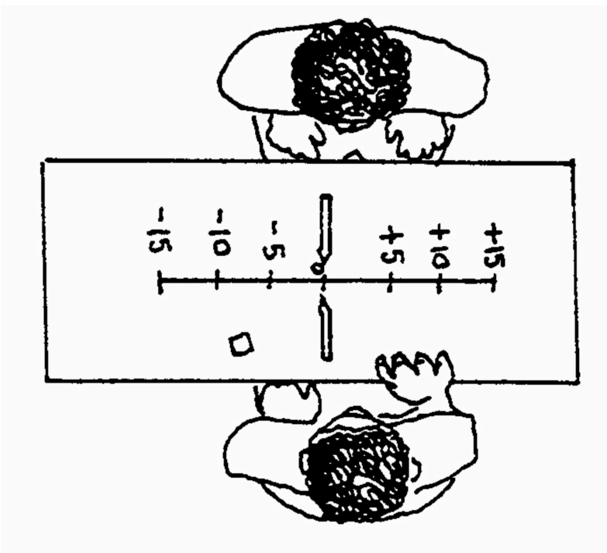
Hole	*1*	*2*	*3*	*4*	*5*	*6*
Number of strokes (Tsepo)	4	6	2	3	1	3
Score for Tsepo	+1	+3				
Number of strokes (Lerato)	3	6	1	3	2	5
Score for Lerato	0		−2			

Complete the table to see who won the game. The player with the lowest score is the winner as they took the least number of shots overall to get the ball in each hole.

Rules of mini-golf (putt-putt)

- *At each of the six holes you are supposed to get the ball into the hole in 3 shots or less.*
- *Your score is based on the number of shots. If you take 5 shots on hole 1 to get the ball into the hole, your score will be +2 because you took 2 more shots than the 3 allowed (which we call par).*
- *If you take 3 shots to get the ball into the hole, your score will be 0 (you were no shots under or over).*
- *If you take 2 shots to get the ball into the hole, your score on that hole is -1 as you took one shot less than the recommended shots.*

2. Here is a game for two players. Both you and your partner should have pencils. Each of you draw a number line from −15 to +15, spacing the points 5mm apart. Each place your number line in front of you with positive numbers to your right and negative numbers to your left.



Place your pencil points on zero then roll a dice.
If you score 2, 4 or 6, move your pencil 2, 4 or 6 to the right (adding to your total).
If you score 1, 3 or 5, move your pencil that number to the left (subtracting from your total).
Take turns. The first player to reach +15 is the winner. The player who reaches –15 loses the game.

ANSWERS ON PAGE 101

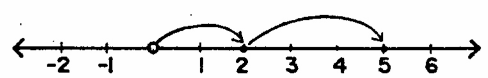
Adding integers

You learned from doing arithmetic that a (+) sign means addition and a (–) sign means subtraction. You now know that (+) can also indicate a positive number and (–) a negative number.

Integers are sometimes put in brackets.
(+2) + (+3) means positive 2 plus positive 3.
4 + (–2) means positive 4 plus negative 2.
(–7) + (–1) means negative 7 plus negative 1.
You can use the arrow representation on a number line to add integers.
Let us look at the following examples:

Examples

1. (+2) + (+3) =



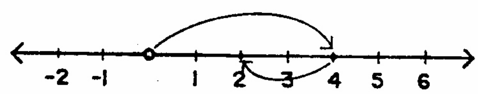
The sum of two positive numbers is positive.

Start from 0 and go 2 steps to the right. The arrow stops at 2.
2 is now our new starting point. We now go 3 steps to the right, starting from 2. The arrow stops at 5.

This means that:
(+2) + (+3) = (+5). This is another way of showing 2 + 3 = 5; a familiar addition.

Let us try another one.

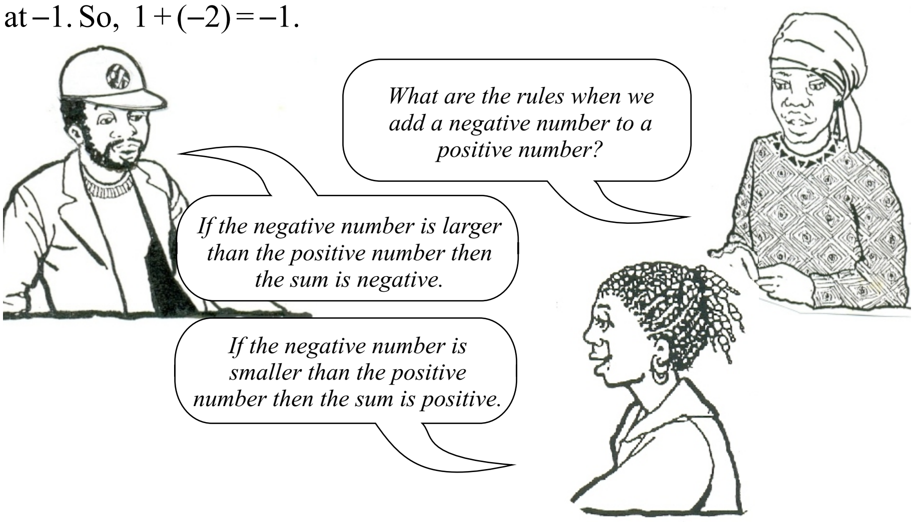
2. 4 + (–2) =



We start from 0 and go 4 steps forward. The new starting point is 4. We now need to go from the starting point 2 steps to the left. The arrow stops at 2. So, 4 + (–2) = 2.

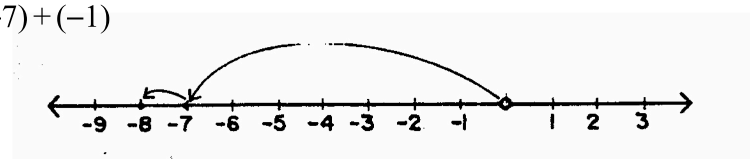
3. Let us try this example:
   1 + (–2)

Start from 0 and go 1 step to the right. The arrow stops at 1. 1 is now the new starting point. We now need to go 2 steps to the left. The arrow stops at -1. So, $1+(-2)=-1$.



Although these rules can be useful, it is always better in mathematics to understand what you are doing rather than just following rules. As you learn more and more about mathematics you will realise that there are more and more rules and it becomes impossible to memorise and recall them all. The more you understand the mathematical reasoning behind the rules, the less you will need to memorise them. Try this example:

4. $(-7)+(-1)$



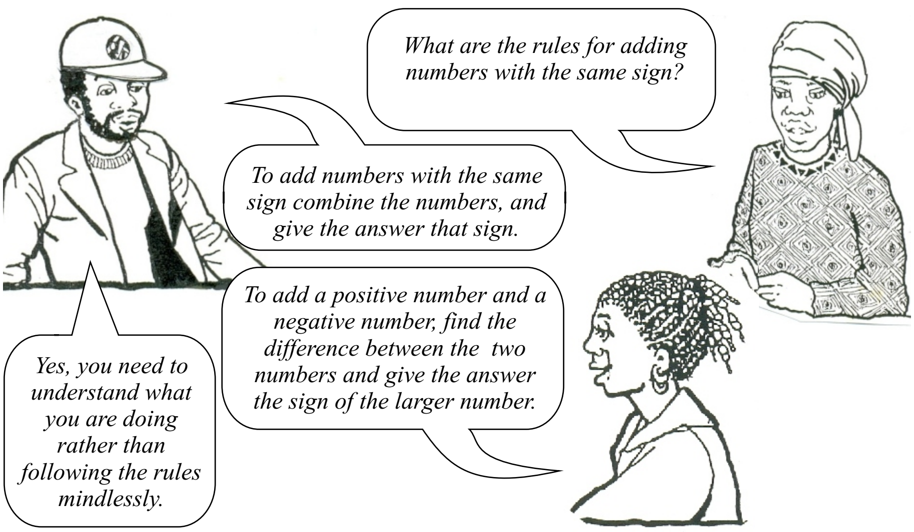
Start from 0 and go 7 steps to the left. The arrow stops at (-7). So (-7) is the new starting point. Start from (-7) and go 1 step to the left. The arrow stops at (-8). Therefore: $(-7) + (-1) = (-8)$ The sum of two negative numbers is negative.

ACTIVITY 3

Use either the number line, or the rules to do the following:

1. $9+6=$
2. $(-9)+6=$

ANSWERS ON PAGE 101

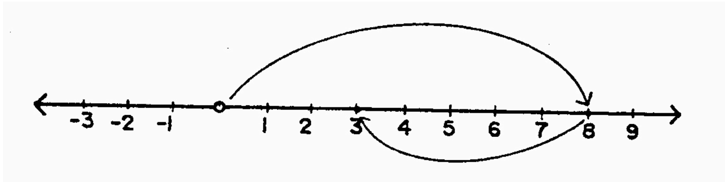


Subtracting integers

You have learned in arithmetic that a (–) sign means subtraction. You have also learned in this lesson that a (–) sign indicates a negative number. In this section you will subtract a big number from a small number. Look at the subtraction of integers.

Examples

1. We can write (+8) – (+5) as 8 – 5 using the number line.



Start from 0 and go 8 steps to the right to get (+8). The new starting point is now 8.

Then go 5 steps left from 8, and end at 3. So 8 – 5 = 3. (This is like adding a (+) number and a (–) number.)

Remember, when adding a (+) number to a (–) number, you need to find the difference between the two numbers and then give the answer the sign of the larger number.

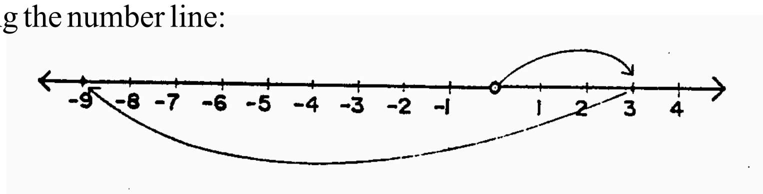
So the difference between 8 and 5 is 3. The sign of the larger number is (+) so we have the answer +3. So (+8) – (+5) = (+3).

Can you see that subtracting a positive number is the same as adding a negative number?

Let us look at the following example:

2. (+3) – (+12)

Using the number line:



Start from 0 and go 3 steps to the right to get to (+3).

Then go 12 steps to the left. The arrow ends at (–9).
So (+3) – (+12) = (–9)
Is it true that subtracting a positive number is like adding a negative number? Let us check.

Is (+3) – (+12) like (+3) + (–12)? Use the rule and take the difference of the two numbers. The difference is 9. Now give the answer the sign of the larger of the numbers. 12 is larger and its sign is (–) so we have –9.

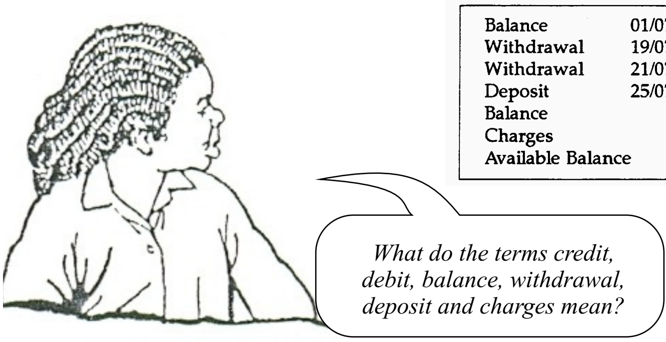
Therefore subtracting a positive number is like adding a negative number.

Have you noticed that you are now able to subtract a big number from a small number? Before you studied integers you were not taught to do this.

Subtracting a big number from a small number happens often in banking. If you want to withdraw more money than you have in your account, then the bank will be subtracting a big number from a small number. You will have overdrawn your account, so you will have a negative balance. That is, the money in your account will be less than R0,00. If your business runs at a loss then your expenses are more than your income and there is no profit.

Thabo's bank statement looks like this:

Balance	01/07	200,00	credit
Withdrawal	19/07	150,00	debit
Withdrawal	21/07	300,00	debit
Deposit	25/07	200,00	credit
Balance		50,00	debit
Charges		6,00	debit
Available Balance		56,00	debit



- Credit means the money in the bank
- Debit means money that is taken out of the bank
- Withdrawal means taking out the money from the bank
- Deposit means putting money into the bank
- Balance means the money you have left in the bank
- Charges are what you pay the bank for its services.

Look at how a new balance is calculated. Remember credits are represented by positive numbers.

Debit is the opposite of credit, so the debit is represented by negative numbers.

Transaction	*Date*	*Amount*	*Balance*
Opening deposit	1/10	+140,00	R140,00
Withdrawal	7/10	-40,00	R100,00
Deposit	10/10	+350,00	R450,00
Withdrawal	13/10	-250,00	R200,00
Withdrawal	15/10	-200	R0,00
Charges	28/10	-30,00	-R30,00

Thabo has overdrawn his account by R30,00. Thabo owes the bank R30,00

ACTIVITY 4

Use either the number line or the rules, to do the following calculations:

1. $(+7) - (+5) =$
2. $5 - (+9) =$

3. $(-2) - (+4) =$

4. Antifreeze is used in a car radiator so that it will not freeze unless the temperature drops to −18°C. One evening the temperature drop to −5°C.
   By how many degrees can the temperature fall before the radiator freezes?

5. Answer the following questions about Thabo's bank statement:

Transaction	*Date*	*Amount*	*Balance*
Opening deposit	1/02	+140,00	R140,00
Withdrawal	7/02	-90,00	R50,00
Deposit	10/02	+350,00	R400,00
Withdrawal	13/02	-250,00	R150,00
Charges	28/02	-30,00	R120,00

   a) What is the balance *before* paying bank charges?
      Is this a debit or credit?
   b) What is the available balance *after* paying bank charges? Is this a debit or credit?

ANSWERS ON PAGE 102

Subtracting a positive number is the same as adding a negative number and integers allow us to subtract larger numbers from smaller numbers.

Multiplying integers

In arithmetic multiplication, we use × or brackets to show that we are multiplying. So:

$+4 \times +3$ means positive 4 times positive 3.

$-2(4)$ means negative 2 times positive 4.

$(-9)(-2)$ means negative 9 times negative 2.

You can choose any of these methods when you multiply.

When we multiply two positive numbers such as:

$+2 \times +3$ we mean $(+3) + (+3) = 6$

So $2 \times 3 = 6$

We multiply the two numbers and give the answer the positive sign.

Let us look at what happens when we multiply two numbers that have different signs, say
-4×7

Have a look at the pattern below:
We are subtracting 1 each time from the column on the left.
$4 \times 7 \quad = 28$
$3 \times 7 \quad = 21$
$2 \times 7 \quad = 14$

$1 \times 7 \quad = 7$
$0 \times 7 \quad = 7$
$-1 \times 7 \quad = -7$
$-2 \times 7 \quad = -14$
$-3 \times 7 \quad = -21$
$-4 \times 7 \quad = -28$

The last column decreases by 7 each time.

Can you guess the answer by looking at the pattern?
From the pattern we have seen that $-4 \times 7 = -28$.
What about $4 \times (-7)$?
We know that $4 \times (-7)$ means $-7 + -7 + -7 + -7 = -28$
so, $-4 \times (-7) = -28$

We can now make a rule:
If the signs of the numbers are different, multiply the numbers and give the answer a negative sign.

What about: $-4 \times -7 = ?$
Let us look at the pattern as before.
We are subtracting 1 each time from the column of numbers on the left.
$4 \times (-7) \quad = -28$
$3 \times (-7) \quad = -21$
$2 \times (-7) \quad = -14$
$1 \times (-7) \quad = -7$
$0 \times (-7) \quad = 0$
$-1 \times (-7) \quad = 7$
$-2 \times (-7) \quad = 14$
$-3 \times (-7) \quad = 21$
$-4 \times (-7) \quad = 28$

The values of the numbers in the column on the right decrease by 7 each time.

Can you guess the answer by looking at the pattern?
So: $-4 \times (-7) = 28$

We can now make another rule.
If the signs of the two numbers you are multiplying are alike, then multiply the numbers and give the answer a positive sign.

ACTIVITY 5

Calculate the following:

1. $+4 \times 3 =$
2. $7 \times (-5) =$
3. $(-8)(-5) =$

4. In a game of Krazy Darts, even scores are positive, and odd scores are negative.

For example:
10, 5, 7 give $10 + (-5) + (-7) = (-2)$
Treble $9 = 3 \times (-9) = -27$

Calculate the Krazy Darts scores for:

ANSWERS ON PAGE 102

a) double 8   b) treble 11

Dividing integers

We will use ÷ and / to indicate the division sign.
$-15 \div 3$ means negative 15 divided by positive 3
$(+6)/(-3)$ means positive 6 divided by negative 3
$(-12) \div (-6)$ means negative 12 divided by negative 6
$\frac{14}{2}$ means positive 14 divided by positive 2
You can use any of the two signs of division, that is ÷ or /.
In division of integers we use the same rules as those for the multiplication of integers.

When we divide a positive number by a positive number, we do the division and give the answer a positive sign.
Therefore: $\frac{14}{2} = 7$

When we divide a negative number by a negative number, say, $(-12) \div (-6)$, we do the division and give the answer a positive sign.
Therefore: $(-12) \div (-6) = +2$

If the signs of the numbers are alike, do the division and give the answer a positive sign.

When we divide a negative number by a positive number, say, $(-6) \div 3$, we do the division and give the answers a negative sign.
Thus: $(-6) \div 3 = -2$

Also, when we divide a positive number by a negative number, say, $6 \div (-3)$ we do the division and give the answer a negative sign.
Therefore, $6 \div (-3) = -2$.

If the signs of the numbers are different, we do the division and give the answer a negative sign. So conditions of working with positive or negative signs also hold for division of integers.

ACTIVITY 6

Calculate the following:

1. $+12/+6$
2. $(-18) \div (-6) =$
3. $(-18) \div 6 =$

4. The minimum temperatures (°C) in towns in Canada in December were: –10, –8, 1, –5, 3, 0, –2 and –15. Calculate the average temperature in Canada.
(Hint: to find the average divide the total sum by the number of temperatures.)

ANSWERS ON PAGE 102

Summary

You have learned how to add two integers. You saw that:

- When you add two positive numbers you get a positive number.
- When you add a negative number to a positive number
  - if the negative number is larger than the positive number, then the sum is positive.
  - if the negative number is smaller than the positive number then the sum is positive.
- The sum of two negative numbers is negative.
- You saw that subtracting a positive number is the same as adding a negative number. You also learned that integers allow us to subtract larger numbers from smaller numbers.

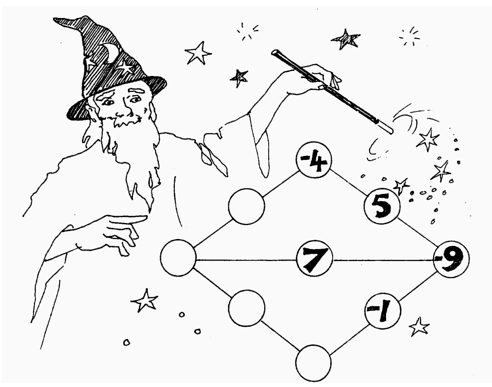
Self-assessment checklist:

Are you able to:

- add two integers
- understand that if a negative is larger than a positive number their sum is negative
- their sum is negative if a negative number is smaller than the positive number then the sum is positive
- see that subtracting a positive number is the same as adding a negative number
- understand that integers enable one to subtract larger numbers from smaller numbers
- apply rules for the multiplication and division of integers.

SELF-CHECK EXERCISE

1. Calculate the following:
   a) (–8) + (–9) =
   b) (–13) + (5) =
   c) Copy and complete the magic diamond.



Each line of three circles adds up to the same number.

2. Calculate the following:

a) $(-24) \div (+3) =$
b) $7 \times 8 =$
c) $-8 \times (9) =$
d) $-32 + (-8) =$
e) $(4)(5) =$
f) $16 \div 8 =$

3. At 07:00 a.m in Johannesburg, the temperature was −1°C. By 04:00 p.m., the temperature had risen to +21°C. By how many degrees did the temperature rise?

4. Here are some temperatures from places around the world during December:

Paris	Rome	Lesotho	Windhoek	New York	Accra	London
3°C	–2°C	25°C	31°C	–3°C	18°C	5°C

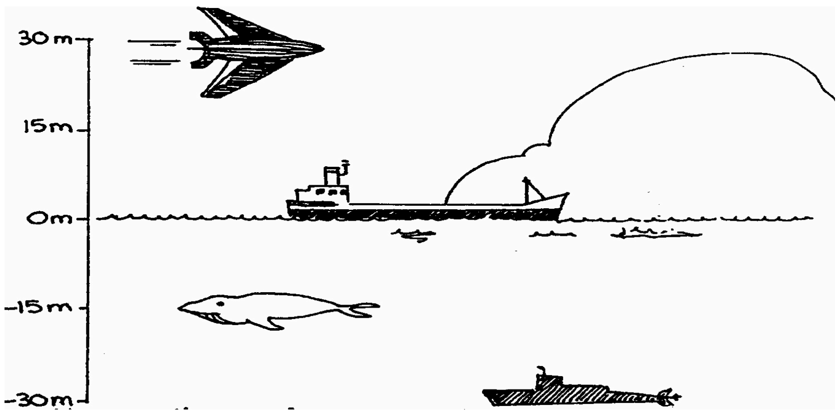
a) Which is the warmest place? Which is the coldest place?
b) What is the difference between the warmest and coldest temperatures?
c) Write down the change in temperature going from place to place, working from left to right. for example, from Paris to Rome: 3°C to –2°C = –5°C in temperature change

5. Pentsho's bank statement looks like this:

Balance	01/07	200,00	credit
Withdrawal	19/07	50,00	debit
Withdrawal	21/07	40,00	debit
Deposit	25/07	200,00	credit
Balance		310,00	
Charges		4,00	debit
Available Balance		306,00	

How is Pentsho's available balance calculated?

6.



a) Use negative numbers to give the positions of the whale and the submarine from sea-level.
b) The whale goes down 10 m, and the submarine rises 20 m. Describe their new positions, using negative numbers again.

ANSWERS ON PAGE 108

5. Measurement

Introduction

In this lesson we will focus on the measurement of length and quantity. We will look at the decimal units used to measure length and quantity.

People have always needed to measure things. The building of the pyramids in Egypt required great skill in measuring. So did the building of the Temple of Zimbabwe. When people began trading their products, measuring quantities became important.

Measurement was an important part of people's lives in the past and is part of people's lives today.

We measure quantity everyday. A housekeeper measures the Jungle Oats when he makes porridge in the morning. He measures out quantities of flour and sugar when making a cake.

We also measure length and distance. We say it is about two kilometres to the station. A carpenter may say, "I need 1,5 metres of wood".

By the end of this lesson you will:

- communicate some interesting historical facts about measurement
- estimate measurements quite accurately
- select appropriate units of measurement
- do simple calculations using standard measures
- convert from one unit of measurement to another
- measure accurately.

Measurement

Why do we measure?

We measure to find out something about an object.

- A carpenter needs to know the length of a piece of wood. It has to be the right size.
- A housekeeper needs to know how much flour she is buying. If she is baking a lot she needs a greater quantity.
- If Joe works in the city, he needs to know how long the train journey takes. He doesn't want to be late for work!
- If Joe wants to make a table and buy wood and catch a train, he needs to measure length, quantity and time.

ACTIVITY 1

Sometimes when you measure, you make an estimate which is a rough measurement. Write a list of the things you measure in your daily life and next to them, estimates.

How do we measure?

Joe takes a piece of wood. He stretches out his arms.

He compares the length of wood with the length of his arm. He compares something he doesn't know with something he knows. The shop assistant takes a piece of wood. She stretches her tape-measure along the wood. She compares the length of wood with her tape-measure. She compares something she doesn't know with something she does know. How do we measure? We compare an unknown quantity with a known quantity.

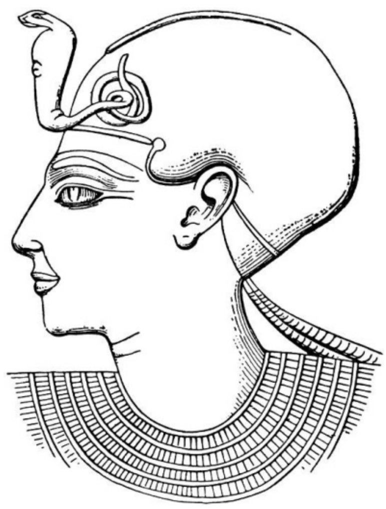
Length: an example from early Egypt

Let's take ourselves back to a time when Egyptian farmers built houses along the Nile.

Abdul wanted to build a house. His first step was to estimate the size of the house. The house should be slightly higher than his head, so he wouldn't bump his head on the roof. The rooms should be wide enough for all his family to sleep comfortably. And the door should be a bit wider than he is, so that he can walk through comfortably.

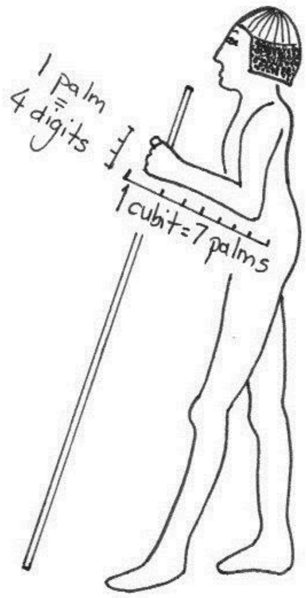
His second step was to choose a unit of measurement. He chose his full armspan to measure length and width, and he used his height to measure height.

There were many people building very fine houses. But they were all using different measuring units. Abdul may have been very tall. His neighbour may have been very short. So ‘a man's height’ is different for each person. It didn't matter that all the houses were different sizes.



But it did matter to the Egyptian architects who were building the pyramids. There were thousands of builders. Each had a different arm-span. Each used a different measuring unit. The architects had to develop standard measures, in other words the same unit of measurement for everyone.

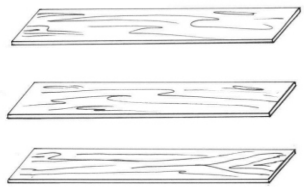
How did they find a standard measure? They took the tallest prince. They measured the length of his arm from the elbow to the tip of his fingers. They made that a standard measure. It was called a cubit. They made measuring ropes. They tied knots in the ropes to show the standard length of a cubit. They used other standard measures based on the human body.



Estimating in everyday life

Joe went to a hardware shop.He told the assistant that he wanted a piece of wook as long as his arms stretched out.
'I want to make a table. I need a piece of wood this long.' Jakes stretches his arms out.

Attendant: That is about 1,5 metres. Is that right?
She shows Joe 1,5 metres on the measuring tape.
Joe: No, that is a bit long. Let me have 1,2 metres.
Attendant: How wide do you want the table?"
Joe: About this wide ...
(placing his hand three times on the table)
Attendant: That is about 60 cm. Our planks are 20 cm wide. I'll give you three planks 1,2 m long and 20 cm wide.



Well, Joe got the right amount of wood. The next time he made a table, he would know the exact measurements.

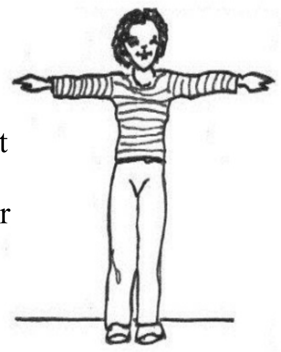
ACTIVITY 2

Here is a table for your dimensions (measurements). Use the information on the left to complete the table.

	Estimate	Measurement
Height in m and cm		
Full span in m and cm		
Handspan in cm		
Cubit in cm		
Length of foot in cm		
Digit in cm or mm		

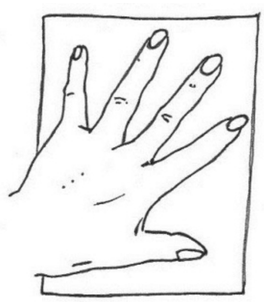
1. *Height*
   a) In the first column of the table write what you estimate your height to be.
   b) Then measure your height as accurately as possible using metres and centimetres.
   Ask a friend to mark your height on a wall.
   Enter this in the second column.

2. *Full span*
   a) Stand with your back to the wall and stretch out your arms as far as you can. The distance from finger tip to finger tip is a full span. Estimate what this is and record it on the table.
   b) Ask a friend to help and measure your full span as accurately as possible. Record it on the table.



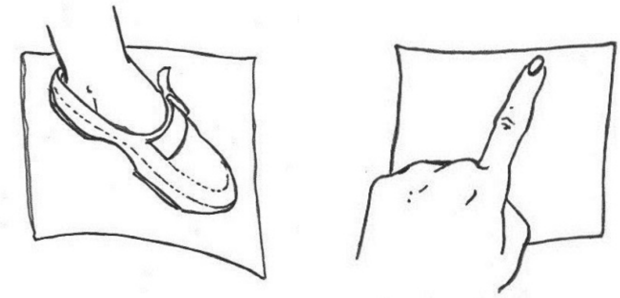
3. *Hand span*
   This is the distance from the tip of the little finger to the end of the thumb when the fingers are spread as widely as possible.

   a) Estimate your hand span and record it.
   b) Place your hand on a large piece of paper. Stretch your fingers as wide as possible. Mark the end of the thumb and the tip of the little finger. Measure the length between the two points.
   c) Use this knowledge to estimate the length of your table and the width of your window.



4. *Cubit*
   The cubit is a very ancient measurement. It is the distance from the elbow to the tip of the middle finger.
   a) Estimate how long your cubit is and record it on your table.
   b) Work with a friend and measure accurately.
   c) In the Bible (Genesis ch. 6 v 15) it tells us that Noah's Ark was 300 cubits long, 50 cubits wide, and 30 cubits high. If a cubit is $\frac{1}{2}$ a metre, how long was Noah's ark in metres?

5. *Length of foot*
Put your foot on a piece of paper and draw the outline of your shoe.
a) Estimate its length and record it on the table.
b) Measure and record the length as accurately as possible.
c) Use this to estimate the width of your room.



6. *Digit*
This is the width of your forefinger.
a) Estimate the width then record it.
b) Put your finger down on a piece of paper. Make a pencil mark on either side. Measure as accurately as possible and record your measurement. This is a useful measure for estimating the length of shorter objects.

Standard units

In the past many different units of measurement were used. At the time of Noah's Ark, they used cubits. More recently people used yards, feet and inches. In South Africa today we use the metric system (kilometres, metres, centimetres, etc.) developed in France many years ago. Some countries, however, such as North America and England, do not use the metric system.

Some older people in South Africa may still use yards, feet and inches. Yards are a little less than a metre. There are three feet in a yard, and 12 inches in a foot.

The fact that people all over the world used different units of measurement sometimes caused problems! Read this story!

Use the right unit!

In the 19th century, England was known for beautiful cathedrals. A congregation from a small village in Finland, called Kerimaki, wanted to build a new church. They sent to England for the plans of a beautiful little church. When the church was built it was enormous; about three times the size they wanted. What do you think happened? The plans, which came from England, were in feet. The Finnish builders worked in metres!

Today the standard units used in most of the world for measuring length are kilometres, metres, centimetres and millimetres.

How long is a metre? Scientists measured the distance from the Equator to the North Pole. They divided this distance into 10 000 units of 1km. Each of these was divided into metres.

A metre is divided into 100 parts called centimetres. Each centimetre is divided into 10 parts called millimetres. So, a metre is divided into 1 000 tiny parts called millimetres.

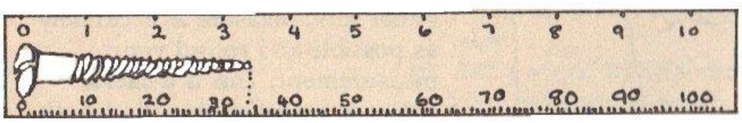
1 kilometre	=	1 000 metres
1 metre	=	100 cm
	=	1 000 mm
1 cm	=	10 mm

Measuring accurately

Sometimes our measurements have to be very accurate. For example, window panes need to fit the frames exactly.

Can you think of other situations where great accuracy is needed?

Measuring in millimetres gives a high degree of accuracy. The width of your forefinger is about 20 mm. Look at this ruler. It is marked in cm and mm.

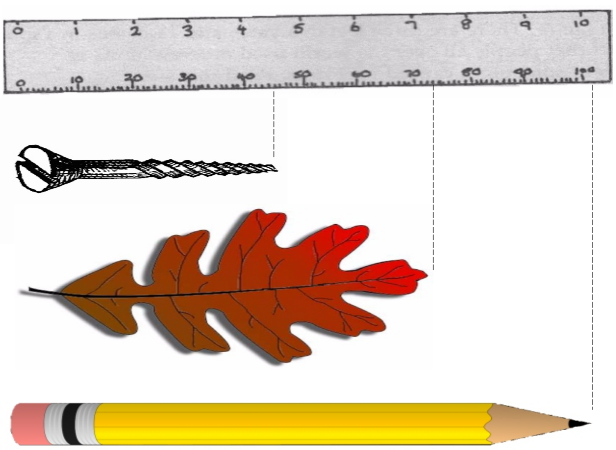


1 cm is divided into 10 mm. A mm is $\frac{1}{10}$ cm or 0,1 cm.

Look at the screw. It is longer than 3 cm and shorter than 4 cm. The screw is 34 mm long or 3,4 cm.

ACTIVITY 3

1.



a)

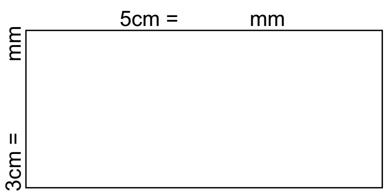
b)

c)

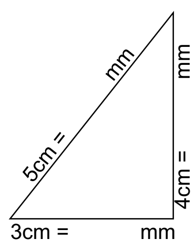
Write down the lengths of these objects first in centimetres then in millimetres.

2. The following shapes are measured in centimetres. Re-draw the shapes and convert the centimetres to millimetres. Some people prefer measurements in millimetres.

a)



b)



ANSWERS ON PAGE 102

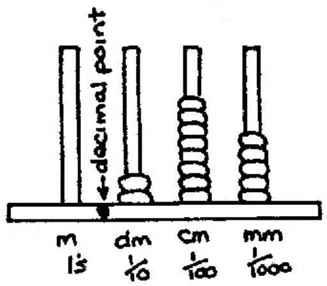
ACTIVITY 4

1. Draw abacus pictures of these measurements and record them in metres.

Example:

285 mm = 200 mm + 80 mm + 5 mm
= 20 cm + 8 cm + 0,5 cm
= 0,2 m + 0,08 m + 0,005 m
= 0,285 m

We can draw this number on an abacus as follows:



10 millimetres (mm) is 1 centimetre (cm)

100 centimetres = 1 metre
∴ 1 000 millimetres = 1 metre

"milli" comes from the Latin word meaning a thousand.

"centi" comes from the Latin word meaning a hundred.

1 mm = 0,001 m

a)	346 cm	c)	25 mm	e)	1 000 mm
b)	520 mm	d)	6 mm	f)	1 234 mm

ANSWERS ON PAGE 102

Quantity

How do we measure quantity? When people first started to sell crops they had to measure amounts. They did this by using special containers for each kind of grain. Corn was sold in a corn bushel.

They also used special containers for the wine and ale they made. These containers were always the same size. In Africa beer was sold in calabashes. In Europe fruit was sold in a punnet. Strawberries are still sold in punnets. In Mesopotamia (now called Iraq), the traders had to measure their goods. They used talents (about 20 kilograms) to measure heavy things and shekels (about 100 grams) to measure smaller things.

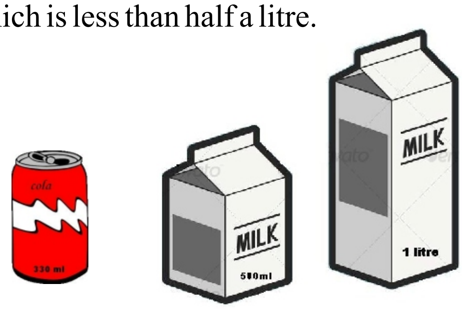
Many different units of measurement have been used throughout the world. In Britain they use pounds and stone. Another measurement they used was a hundredweight. This sometimes caused confusion in trade between countries who were using different units of measurement.

Standard units for mass and capacity

Today the metric units of measurement are used universally. Quantity is sometimes measured by mass (what we commonly call weight) and sometimes by capacity.

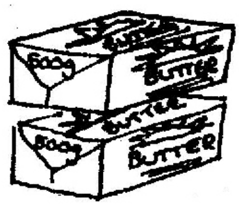
The standard measure for capacity is a litre. Scientists use cubic centimetres, but for our purposes we are going to look at litres for the measurement of capacity.

How much is a litre? Think of a litre of milk, or a litre of coke. You may buy half a litre of milk, which is 500 ml. When you buy a can of coke, this may be 340 ml, which is less than half a litre.



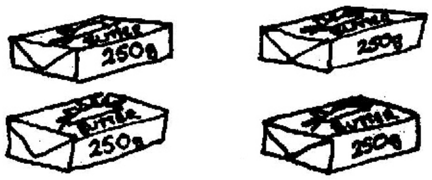
How much is a kilogram?

A kilogram of potatoes is about 8 medium-sized potatoes. Think about a kilogram of butter. In the supermarket there are big blocks and smaller blocks of butter.



Two of these make a kilogram. (Two 500g blocks)

Four of these make a kilogram. (Four 250g blocks)



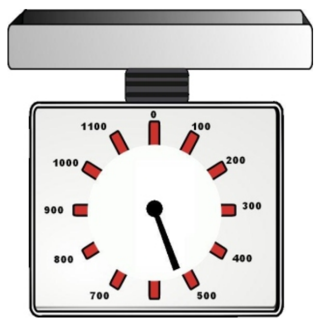
Many people have the following items in their kitchen cupboards:

coffee	125 g
flour	1 kg
tomato sauce	375 ml
lemonade	1 litre
long life milk	500 ml
honey	500 g
salt	500 g

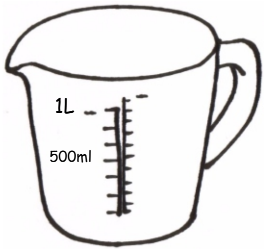
Coffee, flour, honey and salt are all measured in units of mass; that is kilograms (kg) and grams (g).

1 kilogram = 1 000 grams

Mass can be measured on a kitchen scale. The scale is marked in grams and kilograms.



Tomato sauce, lemonade and milk are all measured in units of capacity: that is litres (l) and millilitres (ml).
1 litre = 1 000 millilitres



Why is honey measured in grams (unit of mass) while yoghurt is measured in millilitres (unit of capacity)?

We use a measuring jug to measure capacity. The jug may contain a litre and be marked in millilitres.

ACTIVITY 5

1. List 10 items that you have in your kitchen. Next to each item write the quantity.
   Example
   milk 1 litre
   flour 500 grams

2. List the units of mass (kg and g) for each of the following items in order from greatest to smallest.
   500 g salt
   1 kg flour
   400 g honey
   250 g coffee
   750 g beans

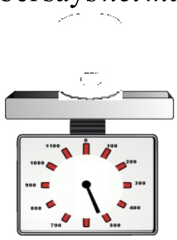
3. List the units of capacity (l and ml) for each of the following items in order from greatest to smallest.
   500 ml yoghurt
   1 litre milk
   375 ml tomato sauce
   750 ml orange juice
   125 ml cream

4. You like milk. A friend asks you if you want a litre of milk or a kilogram of milk. Which would you choose? Say why.

ANSWERS ON PAGE 103

Net mass and gross mass

The products we buy from a supermarket usually have the mass written on the label. Sometimes the label says *net mass*. What does this mean?



You bought a tin of beans. On the label it says 410 g. When you measured the mass of the tin it was 510 g. This means that the gross mass, i.e. the mass of the beans and the tin together was 510 g. The net mass of the beans is 410 g. So the tin has a mass of 100 g.

ACTIVITY 6

For this activity you will need a kitchen scale.

1. From the kitchen cupboard select five grocery items that have a mass of less than one kilogram. Look at the mass shown on the labels. Measure the item to find out the gross mass.
Copy and complete the table below.

Item	**Net mass**	**Gross mass**

What is the total gross mass for all five products?

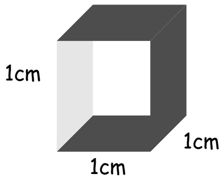
Volume and capacity

What is volume? The volume is the amount of space taken up inside the container.

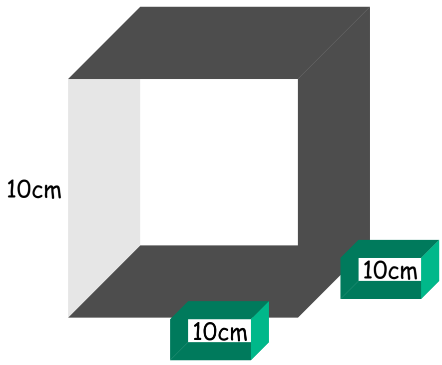
The capacity of a container is the amount the container will hold.

An empty cube with sides 1 centimetre long holds 1 ml.

$1 \text{ cm} \times 1 \text{ cm} \times 1 \text{ cm} = 1 \text{ cm}^3 = 1 \text{ ml}$



This hollow cube has a capacity of $1\,000 \text{ cm}^3$ or 1 litre:



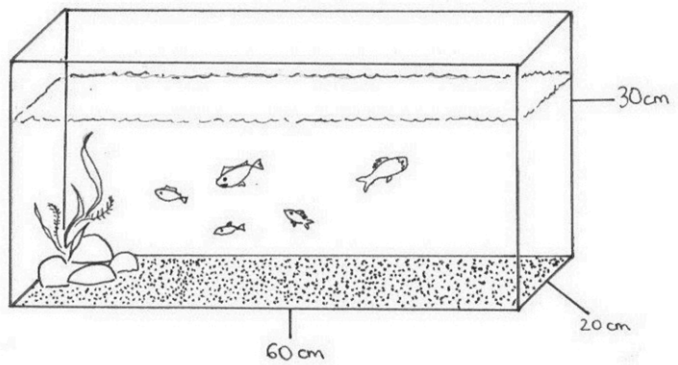
$10 \text{ cm} \times 10 \text{ cm} \times 10 \text{ cm} = 1\,000 \text{ cm}^3$

Units of Capacity and Volume		
Capacity		*Volume*
kilolitre	=	1 m^3
	=	1 000 000 cm^3
1 litre	=	1 000 cm^3
1 millilitre	=	1 cm^3

Liquids are usually measured in litres or millilitres.
1 000 ml = 1 litre
1 ml of liquid takes up 1 cm^3 of space.

ACTIVITY 7

1. The base area of this fish tank is 60 cm × 20 cm = 1 200 cm^2
   The fish tank's capacity is 60 cm × 20 cm × 30 cm = 36 000 cm^3
   What is this capacity in litres?
   (Note: You will learn more about capacity in Unit 4.)



2. If the water in the tank is 11 cm deep, the volume of water is 60 cm × 20 cm × 11 cm = 13 200 cm^3
   How many litres is this?

3. a) How many more litres can be poured into the fish tank?
   b) How many cm^3 is this?

4. a) Thandi uses 4 578 cm^3 of water in one day for all her drinking and cooking needs. How many litres is this?
   b) Thandi's neighbour uses 5 litres of water on the same day for all his drinking and cooking needs. Is this more or less than Thandi uses? By how much? (in litres and cm^3)

ANSWERS ON PAGE 103

Summary

In this lesson we have looked briefly at the measurement of length and quantity. Quantity included both capacity and mass. We looked briefly at the relationship between capacity and volume. Later in the course you will cover volume in greater detail.

Self-assessment checklist:

Are you able to:

- estimate measurements quite accurately
- select appropriate units of measurement
- use standard measures in simple calculations
- convert from one unit of measurement to another
- measure accurately.

SELF-CHECK EXERCISE

1. Estimate the distance from your front door to your gate.

2. Make yourself a metre measure. Use a piece of string (or a dowel stick). Use a ruler to mark off 100 cm.

3. Estimate the distance from your door to your gate. Now use the metre measure to check your estimate.

4. The next time you unpack your grocery shopping, look at the first ten items. How many are measured in units of mass (kg and g) and how many are measured in units of capacity (litres and ml)?

5. On a kitchen scale (borrow one if you haven't got one), find the gross mass of 5 tins. Add them together.

   Look at the labels. Add the net mass of the 5 tins.
   What is the difference between the net mass and the gross mass of the 5 tins?

ANSWERS ON PAGE 109

6. Measurement of time

Introduction

In Lesson 5 we looked at the measurement of length and quantity.

These measurements are easy in that they stand still. Time is a measurement we use every day. It is constantly moving forward. We will look at how people measured time in the past and how we measure time today.

The sun is our natural time-keeper. It rises in the east, follows a path across the sky, and sets in the west. Natural time is different in different parts of the world. To overcome this problem, the measurement of time needed to be standardised.

By the end of this lesson you will:

- explain what natural time is
- give an example of where natural time is used
- explain standard time, and Greenwich mean time and world time zones
- convert time from one time zone to another
- compare the 12 hour clock and the 24 hour clock
- convert displayed time from the 12 hour clock to the 24 hour clock.

Natural time

An example from East Africa

Mr Zarembka lives in Kenya. Kenya lies on the Equator. Sunrise occurs at the same time all year round. He knows it is midday because the sun is directly overhead. He also knows the length of the shadows at mid-morning and mid-afternoon.

He is a cattle-herder. He names the times of the day according to his activities.

- 6 a.m. he calls *akasheshe*, which means milking time
- 12 noon he calls *bari omubirago*, which means rest time for cattle and people
- 1 p.m. he calls *baaza ahamaziba*, which means time to draw water before the cattle drink
- 2 p.m. he calls *amasiyo niganywa*, which is time for the cattle to drink
- 7 p.m. is time for the cattle to drink again.

His month lasts roughly 30 days from new moon to new moon. In the month are 4 weeks of seven days. He also tells the weeks from the phases of the moon. He starts his year when the first rains fall.

An example from northern Sweden

Sven lives in Jokmok. Jokmok is on the Arctic Circle. On the 22nd June it is midsummer in the Northern Hemisphere. At this time of the year the sun never sets. The sun travels around the sky. It is at its highest at midday and its lowest at midnight. The Arctic is called the land of the midnight sun. On the 22nd December it is midwinter in the Northern Hemisphere. At this time of the year the people of Jokmok don't see the sun. It is lighter at midday, and darker at night but there is no sun to tell them the time.

Sven is a reindeer herder. He knows what time he needs to milk the reindeer. It is necessary for him to have a clock. His months follow the moon cycle and his year follows the cycle of the sun.

The following activity has no right or wrong answers. Write down what you know. Share your knowledge with a friend who has grown up in a different community. Note the similarities and differences.

ACTIVITY 1

1. Ask an older person, or someone who has lived in a rural area, how they tell the time of the day.

2. Write down the words for the different times of the day in any other language you know.

The Babylonian astronomers

Years, months, weeks and days

In all parts of the world a day is the time it takes for the earth to turn around its own axis once.

A month usually follows the cycle of the moon. Our calendar months vary from 28 days to 31 days to fit in with the year.

The four weeks of seven days follow the phases of the moon. A seven-day week is accepted in many parts of the world. However there are some cultures that have a four-day week. In parts of West Africa the beginning of a week is market day. The people there produced enough goods to have a market every four days. Their week was four days long.

The Babylonian astronomers, in 2 000 B.C., thought that the sun travelled around the earth. They worked out that this took 360 days, which they called a year. This was adjusted to 365 days. The Babylonians also had 12 months in the year. As we saw in earlier lessons, the Babylonians used base 60.

Hours, minutes, and seconds

Why are there 12 hours in a day? Why not 10? Or 20? This was chosen 4 000 years ago by the Babylonians. They divided each hour into 60 minutes and each minute into 60 seconds.

ACTIVITY 2

Copy and complete this table of units of time.

1 century	=	100 __________
1 decade	=	10 __________
1 __________	=	12 months
1 __________	=	about 365 days
1 day	=	hours
1 hour	=	60 __________
1 __________	=	60 seconds

ANSWERS ON PAGE 103

Standard time

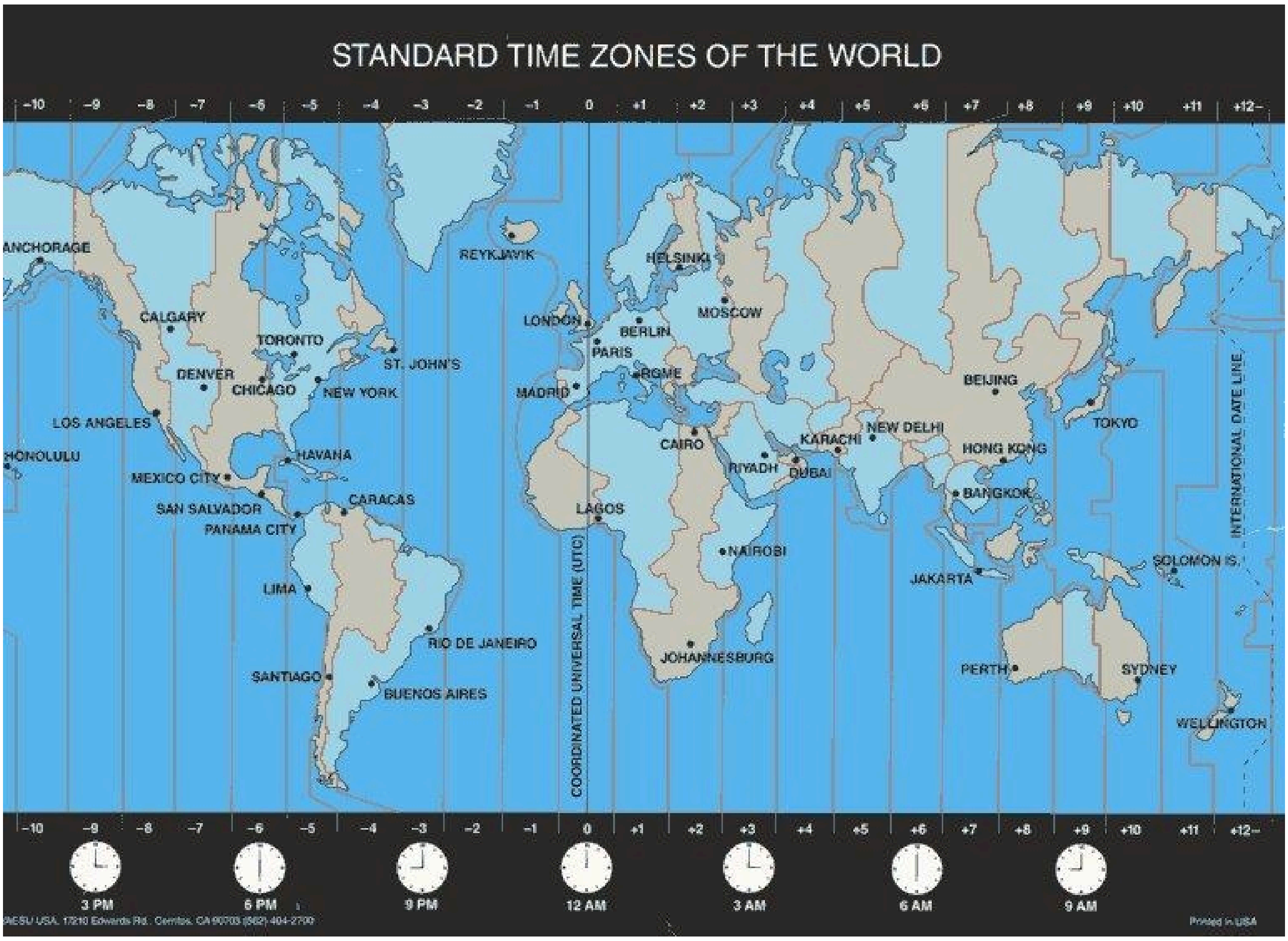
World time zones

Each day the earth turns from west to east. All the time, part of the earth is turning out of the sunlight into the shadow and part is turning out of the shadow into sunlight.

When it is midday in Johannesburg, it is later than midday in Kenya, which is slightly east of us.

When it is midday in Johannesburg, it is earlier than midday in London, which is slightly west of us.

The earth is divided into time zones.



Geographers divide the earth into 360 degrees of longitude.

Lines of longitude are lines that run from north to south. You cannot see them on the earth. But you will see these lines on all maps. They divide the earth into time zones.

For example, Durban lies on the line of longitude, 30 degrees east. London lies on the line of longitude 0 degrees. Another name for this line is the Greenwich (pronounced Gren-itch) Meridian. This is because it passes through a place near London called Greenwich.

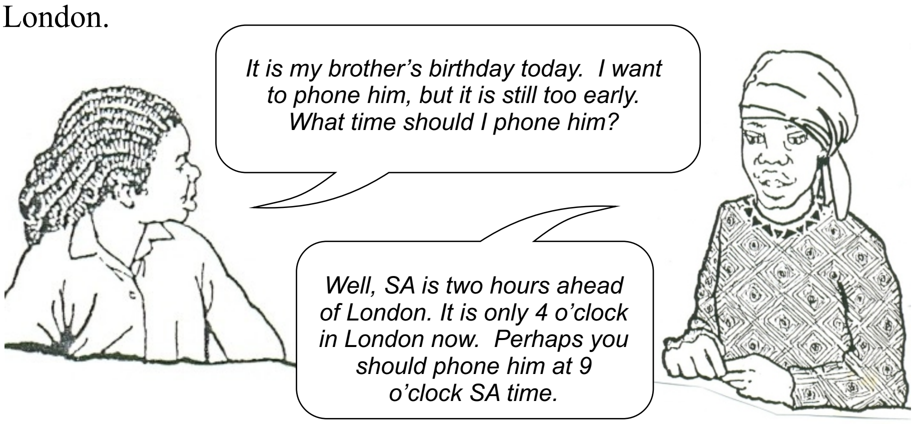
A British map maker introduced these lines. He put the 0 line running through Greenwich. From there, the lines go east and west.
We talk about Greenwich Mean Time. This is the time on the 0 degree line of longitude.

There are 24 hours in a day.
There are 1 440 (24 × 60) minutes in a day.
The difference for one degree of longitude is 4 minutes.
The difference for 15 degrees is 1 hour.

For each place in the world there is what we call a solar time. This is established by the line of longitude. But the solar time for Durban, at 30 degrees east, and the solar time for Cape Town at 25 degrees east are very different.

How is this problem solved?
Well, each country chooses a standard time from a line of longitude that runs through the country. In South Africa, we chose 30 degrees east. This means there is a two-hour time difference between our time and the time in London. Remember that for each 15 degrees, there is a 1 hour difference.

It is 6'0 clock in the morning in South Africa. Joe's brother is studying in London.



Changing time zones

If you travel 15 degrees east, add 1 hour.
If you travel 30 degrees east, add 2 hours.

If it is 1 p.m. in London, it is 3 p.m. in Johannesburg.

For every 15 degrees east add 1 hour.

Every 15 degrees is a different time zone.

If you travel 15 degrees west, subtract 1 hour.
If you travel 30 degrees west, subtract 2 hours.

For every 15 degrees west subtract 1 hour.

From New York to San Francisco you pass through 5 time zones going west. It is 8 a.m. in New York and you are eating your breakfast. Don't phone your friend in San Francisco. She won't be pleased. It is 3 a.m. and she is still asleep.

ACTIVITY 3

Look at the map of the world time zones on page 69 to help you with these problems.

1. Queen Elizabeth travels from London to meet Bill Clinton in Washington.
   How many time zones does she travel through?
   In which direction does she travel?
   How does she change her watch?

2. Queen Elizabeth travels from London to meet Empress Michiko in Tokyo.
   How many time zones does she pass through?
   In which direction does she travel?
   How doe she change her watch?

3. If you travelled from Johannesburg to London, how would you change your watch?

4. If it is 11 a.m. in Johannesburg, what is the time in Tokyo?

ANSWERS ON PAGE 104

The 12 and 24 hour clock

The first clocks worked mechanically. You may have seen an old grandfather clock, which you have to wind up once a week.

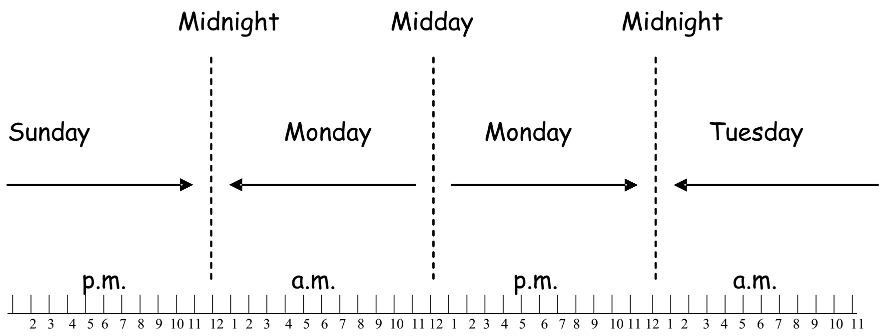
Clocks with a face like this:

are called analogue clocks.

A lot of clocks (and watches) that you see nowadays are digital clocks. They look like this:

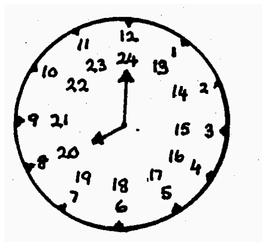
10:30

One way our day is divided like this:



Each day is split into two halves.
From midnight to midday the time is called a.m.
From midday to midnight the time is called p.m.
a.m. is short for ante meridian, which means "before noon".
p.m. is short for post meridian, which means "after noon".

But we also use the 24 hour clock.
Aeroplanes and trains use the 24 hour clock. On a train time table you read 15h00 instead of 3 p.m.



The outer ring is a.m. and the inner ring is p.m.

Notice that each number on the inner ring is twelve more than the numbers on the outer ring.

8 a.m. is 8h00, but 8 p.m. is (8 + 12 hours) 20h00.
10.30 a.m. is 10h30, but 10.30 p.m. is (10.30 + 12 hours) 22h30.

ACTIVITY 4

1. Write these times using the 24 hour system. Use two figures for the hours and two figures for the minutes, for example, 7.30 a.m. would be 07h30.
   a) 6 a.m.
   b) 8.30 a.m.
   c) midday
   d) 5 p.m.
   e) 10.55 p.m.

2. Here is a section of the Cape town suburban line train timetable.

Cape Town	06h38	17h07
Woodstock	06h41	17h10
Salt River	06h43	17h12
Observatory	06h46	17h15
Mowbray	06h48	17h17
Rosebank	06h50	17h19
Rondebosch	06h51	... doesn't stop
Claremont	06h54	17h24
Wynberg	06h58	17h28
Retreat	07h12	17h40
Simonstown	07h41	18h08

a) The train leaves Cape Town at 06h38.
How long does it take to get to Wynberg?
How long does it take to get to Simonstown?

b) In the evening the train leaves Cape Town at 17h07.
Is the journey to Wynberg longer or shorter in the evening?
By how much?
Is the journey to Simonstown longer or shorter in the evening?

ANSWERS ON PAGE 104

Summary

We have a standard time in our country. It is later than countries to the east of us and earlier than countries to the west of us. But if we know the time zones, we can work out the times all over the world.

All countries have the same calendar, and 24 hours in the day. The 12 hour clock and the 24 hour clock are used everywhere.

But what if people don't have clock? They often have their own way of telling when certain activities need to happen.

Self-assessment checklist:

Are you able to:

- explain natural time
- convert between different units of time
- calculate changes of time between different time zones
- represent given times on the 12 or 24 hour clocks,

SELF-CHECK EXERCISE

1. Copy and complete this table of units of time.

1 century	100	years
1 decade	10	
1 year		months
	365	days
1 day		hours
1 month		days
1 hour	60	
1 minute	60	seconds

2. If you travelled from Johannesburg to New York, how would you change your watch?

3. If it is 5 a.m. in Johannesburg, what is the time in Nairobi?

4. Ndu leaves home at half past twelve. She takes 2 hours to do shopping, $\frac{1}{2}$ an hour to go to the Post Office and 20 minutes to fetch children.
   a) What is the total time taken to do these tasks?
   b) What time does Ndu get back home?
   c) How much time does Ndu have for other activities at home before 17h00, when she must leave to go to evening class?

ANSWERS ON PAGE 109

7. Calculator skills

Introduction

If the calculator is going to make your life easier, then use it. A calculator can help with calculations. You can also use a calculator to check whether something you have done is correct.

There are many different calculators on the market. There are simple calculators for children, to help them to learn their numbers. There are calculators especially designed for people who work with money. There are calculators especially designed for engineers.

The calculator we recommend is the CASIO Scientific Calculator *fx* 82ES plus. If you learn how to use it properly, it will help you with many calculations throughout the course.

There are other very good calculators. There are other Casio calculators and Sharp calculators. There are slight variations between the different calculators. But all the scientific calculators have similar functions.

The aim of this lesson is to help you to understand your calculator and learn how to use it to add, subtract, multiply and divide. You will also learn how to use it to do calculations with fractions and negative numbers. These skills will make calculating throughout the course more efficient.

By the end of this lesson you will:

- look after a calculator properly
- use the keys +, –, ×, ÷, and =
- estimate the answers to calculations more easily
- correct an error, using the DEL key
- feed negative numbers in to your calculator
- use the key to put in fractions.

Looking after your calculator

A calculator is a sensitive electronic instrument. It is made of plastic and thin metal wires. These wires are sensitive to heat and can easily be damaged.

Don't leave your calculator in a hot car, or in direct sunlight.

The mechanical structure isn't designed to take weight. Be careful not to put heavy objects on top of it. Don't put it in your back pocket.

Dirt and moisture may also clog up the workings of your calculator. Don't let children play with it while they're eating their ice-creams.

Some cleaning materials include strong chemicals. These may damage the plastic and the inside workings of your calculator. If you want to clean it, use a soft dry cloth - do not use water.

Checking your calculator to see how it works

It is important for you to learn about the calculator. It is also important for you to spend time doing your own experiments.

Can you switch your calculator ON and OFF? On the CASIO fx-82ES there is a blue key in the far right corner which says ON. That is the key to press then you want to switch the calculator ON. It is also the key to press if you want to clear the screen. Try this!

What you do	**What you see**
switch on	0
press 123	123
+	+
press 321	123 + 321
=	444

Now press the ON key. This will clear the screen and you can go on to your next calculation.

Also on the right side of the calculator, around the middle part, there is an orange key with  on it and the word OFF written in yellow just above the key. If you want to switch your calculator off, press the blue SHIFT key in the left top corner , then press the orange AC key.

The calculator will also switch off automatically if you leave it on for too long without pressing a key.

The blue key that says SHIFT in yellow is the 2nd function key. This key enables you to use all the functions written in yellow above the keys. You will learn more about the 2nd function keys later.
On other calculators you may find a separate key to switch off. Try these using your calculator!

ACTIVITY 1

1. a) R6,75 – R6,25
   b) R0,50 + R1,05
   c) Now try this ,5 + 1,05
   Do you get the same answer in b) and c)?

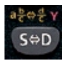
2. a) $3 + 4 + 5$
   b) $3 - 4 - 5$
   c) $3 \times 4 \times 5$
   d) $3 \div 4 \div 5$
   Note: You only need to use the equals key = at the end.

ANSWERS ON PAGE 104

Try the following activity which involves using the +, –, ×, and ÷ signs.

ACTIVITY 2

Here are some examples to work through together, showing keys pressed and what you see on display.

		Keys pressed	What you see
1.	12 × 8 = 96		12 × 8 96
2.	7,2 ÷ 0,9 = 8	7 . 2 ; ÷ ; 0. 9 ; =	7 . 2 ÷ 0. 9 8
3.	5,28 – 1,5 = 3,68	5 . 2 8; – ; 1.5; = 	5.28 – 1.5 $\frac{189}{50}$ 3.78
4.	5,4 + 5,6 + 19,1 = 30,1	5 . 4; + ; 5.6 ; + ; 1.91; = 	5.4 + 5.6 + 1.91 $\frac{1291}{100}$ 12.91
5.	25,3 – 47,8 = 22,5	25.3 ; – ; 47,8 ; = 	25.3 – 47.8 $-\frac{45}{2}$ –22.5

The value of estimating

It is also important to check the answers you get on your calculator. It is important to guess (estimate) what your answer will be beforehand.

What did Jakes do? He estimated the cost of the groceries. He didn't know the correct answer, but he knew the cashier was wrong!

People estimate all the time. 'I spend about R800 a week on groceries for my family,' says Mrs Ngoma.

‘This garage will need about 10 000 litres of petrol a day for the next month,’ says the garage manager.

Estimating should be quick and fairly accurate.

Here are some tips. When we estimate we don't need to be exact. It is easier for us to work with 800 than with 834. It is easier to multiply by 3 000 than by 2 789. We call this rounding off. Do you remember rounding off from *l*esson 2?

Round off to the nearest **unit**.
4, 6 is closer to 5 than 4.
Change 4, 6 to 5.

Round off to the nearest **ten**.
34 is closer to 30 than 40.
Change 34 to 30.

Round off to the nearest **hundred**.
165 is closer to 200 than 100.
Change 165 to 200.

Round off to the nearest **thousand**.
3 450 is closer to 3 000 than 4 000.
Change 3 450 to 3 000.

ACTIVITY 3

Which is the best estimate?

Example: $4{,}71 \times 38{,}5$
Ask yourself, ‘Is 4,71 nearer to 4 or to 5?’
Ask yourself, ‘Is 38,5 closer to 40 or to 30?’
Now look at the following.

Which is the best estimate, A, B, or C?

A: 5×30
B: 5×40
C: 4×40

Jakes thinks B is best. Nomhi checks:

A: $5 \times 30 = 150$
B: $5 \times 40 = 200$
C: $4 \times 40 = 160$
$4{,}71 \times 38{,}5 = 181{,}335$

So B is best.
181 is closest to 200.

Try these: First choose the estimate you think is closest, then use your calculator to check.

1.	$5{,}98 \times 37{,}8$	Estimates A: 6×40 B: 5×40 C: 6×30	2.	$88{,}6 - 23$	Estimates A: $90 - 30$ B: $80 - 20$ C: $100 - 25$ D: $90 - 20$
3.	$121{,}72 + 99{,}49$	Estimates A: $100 + 100$ B: $120 + 100$ C: $120 + 90$	4.	$90{,}24 \div 18{,}8$	Estimates A: $90 \div 20$ B: $90 \div 20$ C: $90 \div 18$

Which is closest?
Decide which answer is closest, A, B, or C.
Then check on your calculator.

		A	B	C
5.	$3\,016 \div 29$	150	100	50
6.	$8\,046 \div 27$	500	400	300

ANSWERS ON PAGE 104

What to do when you press the wrong key

The clear key  clears your previous calculation from the calculator.

If you press 4 + 3, then press , the calculator will only wipe out 3.

But if you press  everything you have put into the calculator is deleted.

Jakes uses his calculator to add a list of numbers. He makes a mistake and starts again.

On some very simple calculators it may be necessary to start again. But most calculators have a way of correcting the last digit you entered into your calculator. What happens when you press the wrong key? A 4 instead of a 3? Do you clear and start again?

Correcting errors

Here is a shopping list from a supermarket. Add up the list on your calculator to see if the total is correct.

Grocery	3,38
Grocery	3,65
	3,48
	7,79
etc.	6,38
	14,48
	6,99
	7,67
	5,55
	9,99
Total	69,36

Did you get it right first time? If you did, well done!

Or did you make a mistake halfway through and have to start again? On most calculators you can correct a mistake. It is possible to wipe out only the last entry.

This is what Jakes learned. Follow these steps on your calculator.

Jakes pressed:	The display showed:
3.38	3.38
+	3.38 +
3.65	3.38 + 3.65
+	3.38 + 3.65 +
Then he pressed:	
4.48 (by mistake)	3.38 + 3.65 + 4.48
Instead of starting again, he pressed:	The display showed:
	3.38 + 3.65 + 4.4
	3.38 + 3.65 + 4.
	3.38 + 3.65 + 4
	3.38 + 3.65 +
Then he pressed the correct amount of:	
3.48	3.38 + 3.65 + 3.48

He had pressed  to delete each digit of the incorrect number. He was back on track and could continue.

Of course there are many other ways to correct an error. Can you think of other ways to solve this problem?

Jakes pressed 4.48 instead of 3.48. He added an extra rand. Why not subtract a rand from the next item?

Try experimenting with easy numbers like 1 + 2 + 3, etc. Make a mistake and try correcting it. Once you are familiar with your calculator, you will find a lot of short cuts.

Working with negative numbers

If you haven't worked with negative numbers yet, this section may seem strange, but try it anyway!
To work with a negative number, we simply press the  before the number.

Example 1
Use your calculator to feed in −8.
Press  , which gives you −8.

Example 2
Work out -2×-3.

Answer: 6

ACTIVITY 4

Work out the following on your calculator.

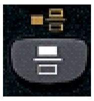
1. -2×-2
2. -2×-1
3. -2×0
4. -2×1
5. -2×2

What answers did you expect? Has the calculator taught you something? Try these.

6. -200×-200
7. -200×0
8. -200×200
9. -365×-365

What is the sign of the answer when you multiply a negative number by a negative number?

Fractions

The fraction keys  and  save time

We can do calculations with and simplify fractions, change mixed numbers into improper fractions, improper fractions into mixed numbers and fractions into decimals or decimals into fractions. Let's play around with some numbers to become better acquainted with these useful keys
.

Simplifying fractions and changing the format of the fraction

Example 1: Simplify and then write it as a mixed number.

You press:	Calculator displays:
25 ;  ; 15	$\frac{25}{15}$
=	$\frac{5}{3}$

The fraction is now in its simplest form but it is still written as an improper fraction (where the value of the numerator is larger than the value of the denominator). We can then also write it as a mixed number.

You press:	Calculator displays:
	$1\frac{2}{3}$

You will notice that the 2nd function of the  key (written in yellow above the key) is to change mixed numbers into improper fractions or vice-versa (shown by the ⇔).

If we now want to change this fraction into a decimal, we can press the again. This time we do so without pressing the SHIFT key as we are accessing the primary (rather than 2nd function of the key).

The display on your calculator should be: 1.666666667
The calculator is restricted to 10 digits so it rounds the last one off.

Example 2: Write $3\frac{5}{8}$ as: a) an improper fraction
b) a decimal fraction

You press:	Calculator displays:
3 ;  ;  ; 5	$3\frac{5}{\square}$

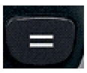
In order to enter the denominator of 8, you need to press the "down" (∇) arrow on the REPLAY key:

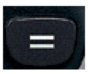
You press:	Calculator displays:
 8 ; 	$\frac{29}{8}$ (which is the improper fraction)
Now press: 	3.625 (which is the decimal fraction)

To get the number back to an improper fraction, press the  key again.

Converting a decimal fraction into a fraction

Example:
Convert 0,5 to a common fraction.

Press:	Calculator displays:
0.5 ; 	0.5 $\frac{1}{2}$

Note: if you forget to press the  key, the format of the number will not change.

Doing calculations with fractions on a calculator

Although it is important to also be able to do calculations involving fractions without a calculator, you can use the calculator to check your solutions where necessary and work at a faster pace.

Example 1:
Work out: $\frac{3}{4} + \frac{2}{3}$.

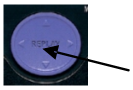
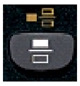
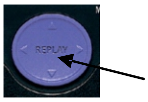
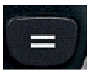
You press:	Calculator displays:
3 ;  ; 4 ;  (the right arrow)	$\frac{3}{4}$
 ; 2 ;  ; 3 ; 	$\frac{17}{12}$

It is possible to leave the answer as an improper fraction. But if you want write the number as a mixed number instead, press the:  ;  keys and the calculator will display: $1\frac{5}{12}$

Example 2:

Calculate: $\frac{3}{5}\times\left(\frac{7}{9}-\frac{2}{3}\right)$

Because this is a scientific calculator, it is already programmed to follow the correct order of operations of calculations. So you can enter the calculation just as it appears above. This is not possible with a normal calculator that is not scientifically programmed though.

You press:	Calculator displays:
3 ;  ; 5 ;  (right arrow)	$\frac{3}{5}$
 ;  ; 7 ;  ; 9 ;  (right arrow)	$\frac{3}{5}\times\left(\frac{7}{9}\right.$
 ; 2 ;  ; 3 ;  (right arrow) ; 	$\frac{3}{5}\times\left(\frac{7}{9}-\frac{2}{3}\right)$
	$\frac{1}{15}$

ACTIVITY 5

1. Calculate: $\frac{1}{4} + \frac{1}{3}$
2. Calculate: $\frac{5}{8} - \frac{4}{5}$
3. Convert: $\frac{1}{4}$, $\frac{1}{3}$, $\frac{5}{8}$, and $\frac{4}{5}$ to decimal fractions.
4. Convert $2\frac{5}{8}$ to: i) an improper fraction ii) a decimal fraction
5. Convert $\frac{30}{7}$ to: i) a mixed number ii) a decimal fraction

ANSWERS ON PAGE 105

Summary

We have covered some calculator skills in this lesson. As you progress from lesson to lesson, you will be shown more calculator skills. Keep your calculator by your side. Use it to check anything you are unsure of. But remember you are the intelligent operator behind the calculator.

Self-assessment checklist:

Are you able to:

- use a calculator to do simple calculations
- estimate the answers to calculations by rounding off
- correct an error using the  key
- feed negative numbers into the calculator
- use the calculator for fractions using the fractions keys:

   and  and the replay key  to move the cursor on the calculator around.

SELF-CHECK EXERCISE

Do these calculations on your calculator:

1. 15×6
2. $9{,}6 - 8$
3. $9{,}4 + 8{,}6 + 16{,}1$
4. $25{,}2 - 57{,}8$
5. $\frac{5}{6} \times \frac{4}{5}$
6. Work out $(4\frac{1}{4} + \frac{2}{3}) - \frac{1}{2}$
7. Rewrite $\frac{1}{4}$ and $\frac{2}{3}$ as decimal fractions.
8. Rewrite $\frac{12}{11}$ as a decimal fraction and then as a mixed number.
9. -8×-4
10. 8×-4
11. -8×4

12. Convert the following to improper fractions:
    a) $3\frac{5}{12}$
    b) $1\frac{3}{8}$

13. Convert the following to mixed numbers:
    a) $\frac{25}{8}$
    b) $\frac{12}{7}$
    c) $\frac{18}{12}$
    d) $\frac{21}{5}$

14. Choose the best estimate of the following (which answer is the closest to the real answer). Use your calculator to check.
    a) $799 \times 6{,}09$ A. 700×6 B. 800×6 C. 800×7
    b) $4{,}89 \times 12{,}9$ A. 5×13 B. 5×12 C. 4×13
    c) $16{,}72 \times 8{,}9$ A. 17×8 B. 16×9 C. 17×9

ANSWERS ON PAGE 109

8. Revision and consolidation

Introduction

In this final lesson we want you to revise and consolidate the skills you learnt in this unit and to build your confidence when you prepare for exams. This is a "test-yourself" lesson and the answers are not in this unit but are in a separate booklet called "Revision and consolidation answer booklet". Section A revises the whole unit to ensure that you understand and integrate the various topics dealt with in the unit. Section B is in an examination form and the questions are taken from previous Mathematical Literacy tests and exams.

Summary of unit

In this unit we covered the following knowledge and skills:

- Concepts related to numbers and calculations with numbers including:
  - The meaning of number
  - Number sets
  - Fractions, decimals and percentages
  - Integers
  - Using your calculator
- Measurement

Section A

1. The following numbers are in words. Write down the numbers.
   E.g. thirty five: 35

   a) ninety two
   b) one hundred and sixty five
   c) nine hundred and three
   d) five thousand, two hundred and twenty
   e) eighteen thousand and seven
   f) sixty four thousand, nine hundred and seventy eight
   g) two hundred and ninety thousand, four hundred and two
   h) nine hundred and five thousand and six
   i) two million, four hundred and seven thousand and eighty one
   j) ten million and nine

2. A cricket match indicates that the game is in 3.2 over.
   a) How many balls have been played?
   b) How many balls remain in a 20 over match?
   c) How many overs remain in a 50 over match?
   d) How many balls remain in a 50 over match?

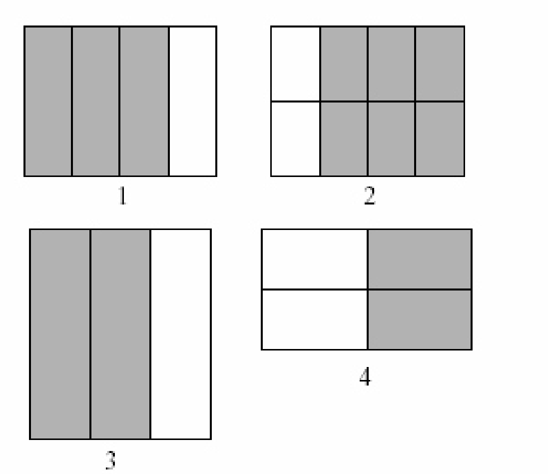
3. Complete the following table:

Roman numeral	Number
	3
IX	
VI	
	10
	8

4. Re-write the following numbers from **smallest** to **biggest**:
1 987 005; 145 300; 11 200; 9 009; 10 230; 900 345; 10 000 000

5. Re-write the following numbers from **biggest** to **smallest**:
2; –90; –5; 7; 0; –101; 1 002; –400; –1; 45

6. Each figure represents a fraction:



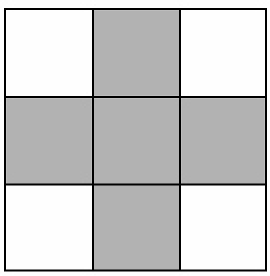
Which two figures represent the same fraction?
a) 1 and 2
b) 1 and 4
c) 2 and 3
d) 3 and 4

7. Which of these is largest?
a) 1 kilogram
b) 1 milligram
c) 1 gram
d) 100 grams

8. How many millimetres are in one metre?

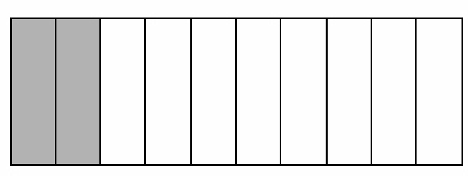
9. Part of the figure is shaded.

What fraction of the figure is shaded?



a) $\frac{5}{4}$
b) $\frac{4}{5}$
c) $\frac{6}{9}$
d) $\frac{5}{9}$

10. Which number represents the shaded part of the figure below?

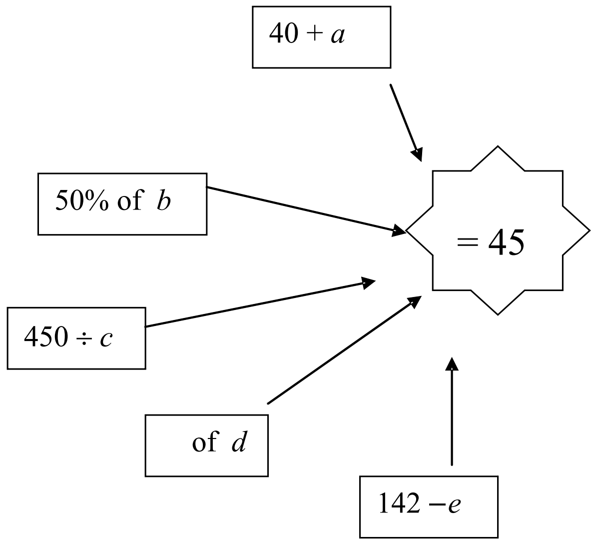


a) 2,8
b) 0,5
c) 0,2
d) 0,02

11. Write down a number to replace each letter so that the answer is **always 45.**

The first one has been done for you.

$a = 5$
$b =$
$c =$
$d =$
$e =$



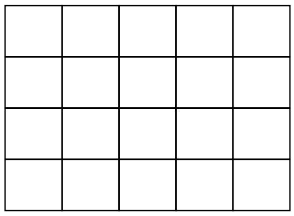
12. Calculate (without a calculator):
a) $-1 \times (-5)$
b) $6 + (-1)$
c) $-4 - 7 + 6$
d) $9 - (-5) - 2$
e) $-3 \text{ x } 7$

13. Calculate (using a calculator):
a) $-1\ 023 - (-248)$
b) $-902 + (-65)$
c) 389×241 (First give the exact answer and then the answer rounded off to the nearest thousand)
d) $123 \times (-6)$ (First give the exact answer and then the answer rounded off to the nearest hundred)
e) $987 - 234 \times 5$ (First give the exact answer and then the answer rounded off to the nearest million)

14. Write down the decimal form of $21\frac{1}{4}$.

15. Write the decimal 1,4 as a fraction.

16. How many blocks will 20% of the following be?



17. What is the remainder if 87 is divided by 7?

18. Write down any of these numbers that are equal to $\frac{1}{8}$:

$\frac{-1}{-8}$ 0,25 0,125 $\frac{8}{0}$ 0,8 $\frac{2}{16}$

19. Write these decimal fractions in order of size from **smallest** to **largest**:

–0.2 –3.1 –2.2 –0.02 15.01 15.11 –15.2

20. A shirt costs R108 but the store manager offers you 10% discount. What must you pay for the shirt?

21. How many millilitres are there in 20 litres?

22. You are teaching a class of 32 learners. On a certain day you decide to do group work with them. If you want to have eight groups of learners, how many learners will there be in each group?

23. You write a test out of 25 marks. Indicate the mark of $\frac{16}{25}$ as a percentage.

24. Bongani had R20 to buy milk, bread and eggs. When he got to the shop he found that the prices were those shown below:

Milk	*Eggs*	*Bread*
R9,50	R14,29	R8,44

At which of these times would it make sense to use estimates rather than exact numbers?

a) When Bongani tried to decide whether R40 was enough money.
b) When the cashier entered each amount into the cash register.
c) When Bongani was told how much he was owed in change.
d) When the cashier counted Bongani's change.

25. A newspaper reported that about 18 200 trees had been planted in the park. The number was rounded to the nearest hundred. Which of these could have been the actual number of trees planted?

a) 18 043
b) 18 189
c) 18 289
d) 18 328

26. Tanya has read the first 78 pages in a book that is 130 pages long. Which number sentence could Tanya use to find the number of pages she must read to finish the book?
  a) $130 + 78 = \square$
  b) $\square - 78 = 130$
  c) $130 \div 78 = \square$
  d) $+130 - 78 = \square$

27. Pheladi worked 57 hours in March, 62 hours in April, and 59 hours in May. Which of these is the BEST estimate of the total number of hours she worked for the three months?
  a) $50 + 50 + 50$
  b) $55 + 55 + 55$
  c) $60 + 60 + 60$
  d) $65 + 65 + 65$

28. Jabulani wanted to use his calculator to add 1463 and 319. He entered 1263 + 319 by mistake. Instead of redoing the sum, what could he do to correct his mistake?
  a) Subtract 200
  b) Add 2
  c) Subtract 2
  d) Add 200

29. Marco's garden has 84 rows of cabbages. There are 57 cabbages in each row. Which of these gives the BEST way to estimate how many cabbages there are altogether?
  a) $100 \times 50 = 5000$
  b) $90 \times 60 = 5400$
  c) $80 \times 60 = 4800$
  d) $80 \times 50 = 4000$

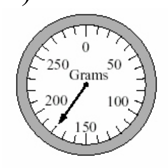
30. A chemist mixes 3,75 millilitres of solution A with 5,625 millilitres of solution B to form a new solution. How many millilitres does this new solution contain?

31. This chart shows the temperature readings made at different times on four days.

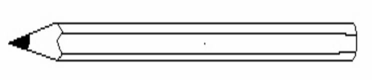
TEMPERATURES					
	6 a.m.	9 a.m.	Noon	3 p.m.	8 p.m.
Monday	15^0	17^0	20^0	21^0	19^0
Tuesday	15^0	15^0	15^0	10^0	9^0
Wednesday	8^0	10^0	14^0	13^0	15^0
Thursday	8^0	11^0	14^0	17^0	20^0

  a) When was the highest temperature recorded?
  b) When was the lowest temperature recorded?
  c) What was the temperature on Wednesday at 3 p.m?
  d) What was the temperature on Monday at 20h00?

32. What is the weight (mass) shown on the scale?
a) 153 g
b) 160 g
c) 165 g
d) 180 g



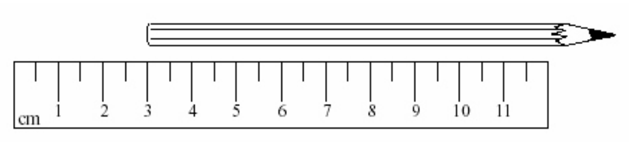
33. About how long is this picture of a pencil?
a) 5 cm
b) 10 cm
c) 20 cm
d) 30 cm



34. Mondi put a box on a shelf that is 96,4 centimetres long. The box is 33,2 centimetres long. What is the longest box he could put on the rest of the shelf (to the nearest centimetre)?

35. A teacher marks 10 of her learners' tests every half hour. It takes her one and a half hours to mark all the tests. How many learners are there in her class?

36. Which of these is closest to the length of the pencil in the figure?
a) 9 cm
b) 10.5 cm
c) 12 cm
d) 13.5 cm



37. Which number is five hundred and four and seven tenths?
a) 54,7
b) 504,7
c) 547
d) 5004,7

38. 0,4 is the same as:
a) four
b) four tenths
c) four hundredths
d) one fourth

39. In which list of fractions are all the fractions equivalent?

a) $\frac{3}{4}, \frac{6}{8}, \frac{12}{14}$

b) $\frac{3}{5}, \frac{5}{7}, \frac{9}{15}$

c) $\frac{3}{8}, \frac{6}{16}, \frac{12}{32}$

d) $\frac{5}{10}, \frac{10}{15}, \frac{1}{2}$

40. Change these fractions into percentages:

a) $\frac{2}{5}$

b) $\frac{9}{20}$

c) $\frac{12}{25}$

d) $\frac{27}{30}$

41. What is the smallest whole number you can make using the digits 4, 3, 9, 6 and 1? Use each digit only once.

42. Sachin buys R4 500 worth of stock for his shop.
a) How much VAT must he pay on that amount?
b) He manages to sell all that stock for R11 000. How much profit does he make?

43. Here is a list of cities and their average maximum temperatures during winter:

Pretoria	-	23°
London	-	4°
Prague	-	–1°
Dubai	-	30°
Oslo	-	–5°

a) Which city has the highest maximum temperature in winter?
b) Which city has the lowest maximum temperature in winter?
c) Which city has a maximum temperature in winter that is closest to zero?

44. The table below reflects the income, expenditure and balance of a company during the first nine months of the year.

Month	**Expenditure**	**Income**	**Balance**
January	2 000	4 500	2 500
February	4 500	3 750	–750
March	1 500	7 500	
April	4 500	1 750	
May	8 500	10 500	
June	6 500	3 500	
July	9 750	2 500	
August	8 000	6 750	
September	2 500	9 850	

a) Complete the balance column on the table.
b) During which months was the balance a positive number?
c) During which months was the expenditure more than the income?
d) During which months was the balance a negative number?

Section B

Time: One hour
Marks: 30

QUESTION 1

1.1 Calculate the following:

1.1.1 $3{,}5(7{,}45 - 2{,}98)$ (1)

1.1.2 $35 + 12 \times 4$ (1)

1.1.3 $\frac{3}{4}$ of 375 (2)

1.1.4 $20\ \%$ of 200 (1)

1.2 A pair of jeans costs R299. How much will you pay for them if you get a $33\frac{1}{3}$ % discount? (2)

1.3 Mr Greenbags has R2000 available in his bank account.
He draws R2500 out at the ATM.
What is the balance on the statement which Mr Greenbags gets from the ATM? (1)

BANKTELLER

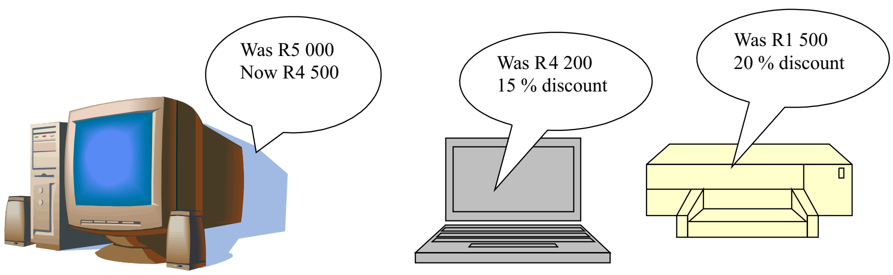
ATM NAME: CENTURION
ATM NO: 08441
DATE: 12/01/98
TIME: 17:55:33
CARD NO: 9876

WITHDRAWAL

FROM ACC: 3987303030
AMOUNT: R2500.00
BALANCE:

QUESTION 2

2.1 Use the following diagram to answer the questions:



2.1.1 By what percentage has the price of the desktop computer been reduced? (1)

2.1.2 Calculate what the price of the laptop computer is now. (1)

2.1.3 Which product has been reduced by the **greatest** percentage and what is the percentage? (1)
2.1.4 What will the reduced price of the printer be? (1)
2.1.5 Which is the best bargain? Why? (2)

QUESTION 3

3.1 Read through the following newspaper article and then answer the questions that follow.

Argentina to Play Bafana Bafana

This Saturday the National soccer squad Bafana Bafana will be playing against Argentina for the first time in ten years.

There is a lot of excitement for the match with people queuing up to buy tickets for the big game. A ticket for the match costs R150.

The kick off time for the match is scheduled for 3:00 pm.

The match is being played at Madiba stadium which can seat 15 000 people.

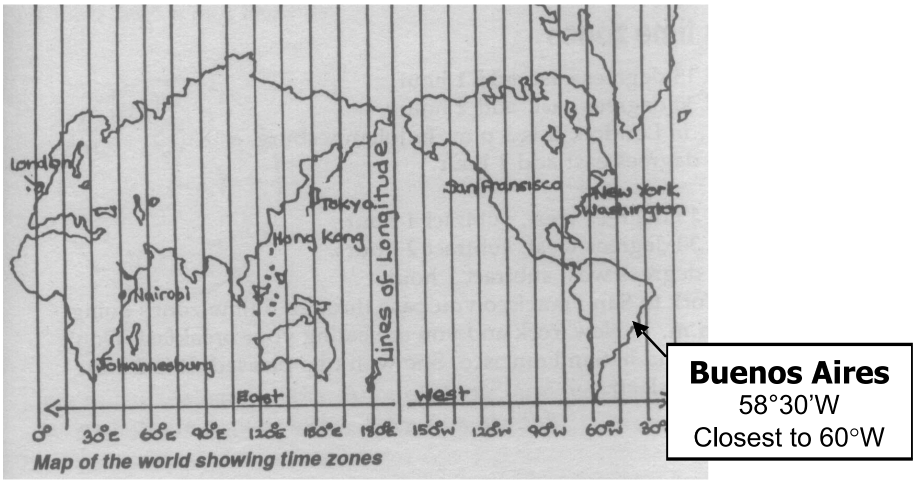
The Bafana Bafana coach is confident that his team will 'wipe the field with the other team'.

3.1.1 Write the starting time of the game using the 24 hour system. (1)
3.1.2 Choose the correct answer. Ten years is a: (1)
a) century
b) decade
c) millennium

3.1.3 A ticket to enter the game costs R150.00 Phil only had $\frac{1}{3}$ of the money. How much money did he still need to buy a ticket? (2)
3.1.4 The soccer stadium below can seat 15 000 people.

3.1.4.1 How many people are there in the stadium if $\frac{3}{5}$ of the stadium is filled? (2)
3.1.4.2 27% of the seats in the stadium are broken. How many broken seats are there? (3)
3.1.4.3 The stadium has 7 entrance gates. If 2 gates are closed, what fraction of the gates are open? (1)
3.1.4.4 Give two equivalent fractions for your answer in 3.1.4.3. (2)
3.1.4.5 The morning of the game, the temperature was 2°C. At the game the temperature rose to 20°C . With how many degrees did the temperature rise? (1)

3.1.5 Use the map below to help you answer the questions:



Map of the world showing time zones

3.1.5.1 The match is being played in South Africa at 3:00 p.m. What will the time then be in Buenos Aires in Argentina? (2)

3.1.5.2 What is the time difference between South Africa and Buenos Aires? (1)

Feedback to Activities

Lesson 1

Activity 1

1.

	Egyptian Symbols	Babylonian symbols	Roman Symbols
1	I	V	I
2	II	VV	II
3	III	VVV	III
4	IIII	VVVV	IV
5	III II	VVVVV	V
6	III III	VVV VVV	VI
7	IIII III	VVVV VVV	VII
8	IIII IIII	VVVV VVVV	VIII
9	IIIII IIII	VVVVV VVVV	IX
10	X	<	X

2. a) 16
   b) 13
   c) 20
   d) 15
   e) 10

Activity 2

1. a) $20 \text{ overs} \times 6 \text{ balls} = 120 \text{ balls}$
   b) $50 \text{ overs} \times 6 \text{ balls} = 300 \text{ balls}$
   c) $72 \div 6 = 12 \text{ overs}$

2. a) 11.2
   b) 30.5

Activity 3

a) b
b) d
c) c
d) a

Activity 4

1.
9 units
0 tens
5 hundreds
7 thousands

2.
8 hundreds
2 tens
0 units

3.
5 units

Activity 5

Round the following amounts off to what you think is the nearest appropriate amount:

1. R3 000 000

2. R306 000

3. R1 060 or R1 100 would be appropriate in this case.

Lesson 2

Activity 1

In this activity there are no right or wrong answers. In fact you probably know things which other people don't know. Surprisingly interesting facts can be learned by talking to different people.

1. Herders may have used stones as symbols. Each stone would represent an animal. As an animal passed through a gate, the herder could put a stone into a bag. In some places stones were easily available, but beads and shells were also used as symbols for numbers.

   The herder would need to count to make sure none of her sheep had got lost. She would need to add, if there were three sheep on one side of the river and twenty on the other side. She may need to subtract to know how many sheep were left after she had given some away.

2. The fishermen would need to measure different lengths of fishing line. They would need to count their fish, add, and subtract. They may need to divide the fish caught among the fishermen. They would need to know the times of going out and coming in. They may need to weigh their fish.

Can you think of other examples where fishermen might use mathematics? Have you ever wondered how the fishermen know how deep to put their fishing lines?

Fishermen know that some fish, like Red Roman, are bottom feeders. They drop their lines to the bottom. Then they pull the line up half a fathom. One fathom is equal to two arms' length. They know snoek are surface feeders. They drop their line one or two fathoms, which is equal to two to four arms' length.

3. The carpenter would need to measure accurately. He would need to add, subtract, multiply and divide. He may have to measure right angles.

Activity 2

1. 20 plus 4 equals 24. Half of 24 is 12. Add 12 to 24. The total is 36.

2. She dropped her line 5 metres to the bottom, then lifted it 2 metres. So she dropped her line at -3 metres or 3 metres below sea level. But her fishing rod was 2 metres above sea level, so her line was about 5 m long.

Activity 3

1. 30 were killed. Take 30 away from 100. That leaves 70. Three more were killed. That leaves 67. 15 visitors each brought her a sheep. 67 + 15 = 82 sheep

2. He would drop his line a bit more than 20 metres.

3. 3 + 4 + 3 + 4 = 14 metres around the edge. Subtract 40 cm for the door. That leaves 13,6 metres.

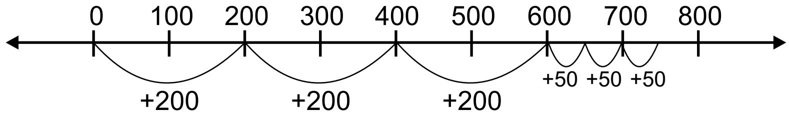
   Draw a picture! Try and find a similar problem in a place you know.

Activity 4

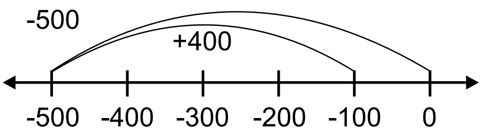
1. There are many ways of approaching this problem. This is one way. 250 ml is 1 cup.

   Step 1: Draw 3 cups.
   Step 2: Each cup has 250 ml. Mark the cup in hundreds.
   Step 3: In each cup there are two hundreds. That makes 200 + 200 + 200 = 600
   Step 4: But there is another 50 in each cup. 50 + 50 + 50 = 150.
   Step 5: 600 + 150 = 750

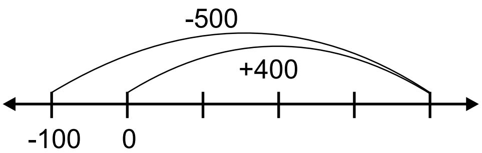
This can be shown on a number line like this:



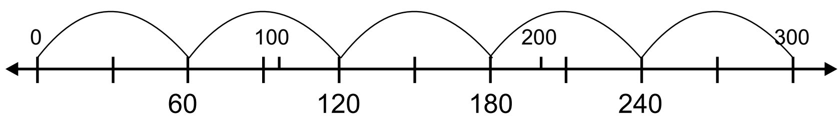
2. R100. This can be shown on a number line like this:



or



3. Here is one way of solving this problem. You may have a different way.
Step 1: Draw 3 metres of wood
Step 2: Mark off 60 cm lengths (1 m = 100 cm)
Step 3: Count the lengths.
This can be shown on a number line like this:

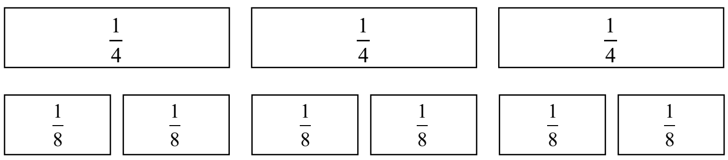


She can make 5 shelves.

Lesson 3

Activity 1

5.



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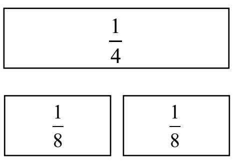
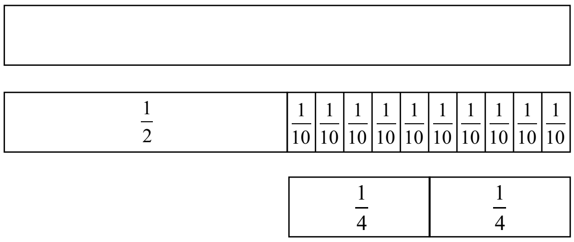


7.



Activity 2

3. 5 pieces

4. 6 pieces

5. $\frac{1}{3}=\frac{2}{6}=\frac{3}{9}=\frac{4}{12}=\frac{5}{15}=\frac{6}{18}=\frac{7}{21}=\frac{8}{24}=\frac{9}{27}=\frac{10}{30}$

A hungry boy may eat all three pieces. He eats $\frac{3}{3}$, which is the same as 1 whole.

Note: $\frac{1}{1} = \frac{2}{2} = \frac{3}{3} = \frac{4}{4} = \frac{5}{5} = \frac{6}{6} = \frac{7}{7}$ etc. = 1 whole

What is the value of $\frac{365}{365}$?

The number below the line (the denominator), is the number of parts the whole has been broken into. The number above the line (the numerator), is the number of parts available, so:

$\frac{365}{365}$ = 1 whole, which is 1.

Activity 3

1. a) $\frac{1}{4}$
   b) $\frac{2}{8} = \frac{1}{4}$
   c) $\frac{3}{12} = \frac{1}{4}$
   d) $\frac{5}{20} = \frac{1}{4}$

2. $\frac{1}{4}=\frac{2}{8}=\frac{3}{12}=\frac{4}{16}=\frac{5}{20}=\frac{6}{24}=\frac{8}{32}=\frac{16}{64}$

3. a) 30 minutes b) 15 minutes

4. 4 hours

5. $\frac{1}{2}$

6. a) chocolate b) sponge c) $\frac{1}{3}$
   d) $\frac{1}{5}$ e) $\frac{1}{10}$

Activity 4

1. $\frac{5}{10} = \frac{6}{12} = \frac{10}{20} = \frac{50}{100}$

2. $\frac{4}{12} = \frac{5}{15}$

3. $\frac{12}{16} = \frac{15}{20} = \frac{24}{32}$

Activity 5

1. a) 5 quarters in $\frac{5}{4}$
   b) $\frac{4}{4} = 1$
   c) $\frac{5}{4} = \frac{4}{4} + \frac{1}{4}$
   $= 1\frac{1}{4}$

2. a) 17 fifths in $\frac{17}{5}$
   b) $\frac{5}{5} = 1$
   c) $\frac{17}{5} = \frac{15}{5} + \frac{2}{5}$
   $= 3 + \frac{2}{5}$
   $= 3\frac{2}{5}$

3. a) $\frac{3}{3} = 1$
   b) $4 = \frac{12}{3}$
   c) $4\frac{1}{3} = \frac{12}{3} + \frac{1}{3}$
   $= \frac{13}{3}$

Activity 6

1. 0,5
2. 0,25
3. 0,75
4. 0,36
5. 0,175
6. 0,6

Activity 7

a) 1,44 b) 3,89 c) 0,80

Activity 8

1. a) 10% b) 60% c) 5% d) 80%

2. a) $66\frac{2}{3}$% b) $12\frac{1}{2}$% c) $16\frac{2}{3}$% d) $37\frac{1}{2}$%

Activity 9

Common Fractions	Decimal Fractions	Percentage
1	1,0	100%
$\frac{1}{5}$	0,2	20%
$\frac{1}{4}$	0,25	25%
$\frac{1}{3}$	0,33	33,3%
$\frac{1}{2}$	0,5	50%
$\frac{3}{4}$	0,75	75%
$\frac{9}{10}$	0,9	90%

Activity 10

a) $\frac{70}{100}$ or $\frac{7}{10}$
b) $\frac{55}{100}$ or $\frac{11}{20}$
c) $\frac{24}{100}$ or $\frac{6}{25}$
d) $\frac{50}{100}$ or $\frac{1}{2}$

Activity 11

1. a) 10 per cent of R75 = R7,50.
Subtract R7,50 from R75. Answer: R67,50.

b) 10 per cent of R150 = R15.
Subtract R15 from R150. Answer R135.

Activity 12

Use your calculator if it makes it easier. Here are the answers. Look back at the table.

For R1 the VAT is 14c

For R2 the VAT must be 2 × 14c = 28c

For R5 the VAT must be 5 × 14c = leave that one for a minute.

For R10 the VAT is R1,40. Now R5 is half of ten, so the VAT on R5 must be half of R1,40, which is 70c.

For R20 the VAT is 2 × R1,40 = R2,80

For R50 the VAT is leave that one for a minute.

For R100 the VAT is R14. Then the VAT for R50 is R7.

Your table should look like this:

Value of article decided by seller	Value Added Tax (sent to government)	Total amount paid by the buyer
loaf of bread R10	R1,40	R11,40
packet of flour R24	R2,80 (for R20) + 56c (for R4)	R27,36
block of cheese R35	R4,20 (for R30) + 70c (R5)	R39,90
pocket of potatoes R50	R7,00	R57,00
2 kilograms of beef R100	R14,00	R114,00
an electric hot plate R300	R42,00	R342,00
Total cost before VAT: = R519	Total VAT: = R72,66	Total cost after VAT: R591,66

The shopkeeper receives R519. The government gets R72,66 and the buyer pays R591,66.

Lesson 4

Activity 1

1. $+\frac{2}{5}, -3, -\frac{1}{6}$
2. positive, positive, neither positive nor negative, positive negative

Activity 2

Hole	*1*	*2*	*3*	*4*	*5*	*6*	*T*
Number of strokes (Tsepo)	1	6	2	3	1	3	+1
Score for Tsepo	+1	+3	–1	0	–2	0	1
Number of strokes (Lerato)	3	6	1	3	2	5	+2
Score for Lerato	0	+3	–2	0	–1	+2	2

So Tsepo had a final score of +1 and Lerato had a final score of +2 making Tsepo the winner of that round only just!

Activity 3

1. 15 or + 15
2. –3

Activity 4

1. (+2) or 2
2. –4
3. –6
4. $(-5) - (-18) = -5 + 18$
   $= 13°C$
5. a) R150 credit
   b) R120 credit

Activity 5

1. 12
2. –35
3. 40
4. a) $2 \times 8 = 16$
   b) $3 \times (-11) = -33$

Activity 6

1. +2
2. +3
3. –3
4. $(-10) + (-8) + 1 + (-5) + 3 + 0 + (-2) + (-15) = -36$
   As there we 8 temperature readings, we divide the total reading by 8 to get the average temperature.
   $-36 \div 8 = -4{,}5$
   The average temperature was –4,5°C that December.

Lesson 5

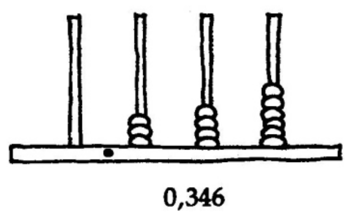
Activity 3

1. a) 4,5 cm or 45 mm
   b) 7,3 cm or 73 mm
   c) 10,3 cm or 103 mm

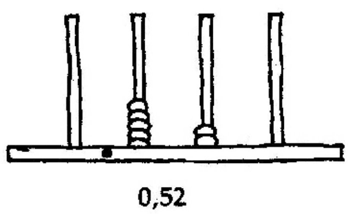
2. a) 50 mm × 30 mm
   b) 50 mm; 40 mm; 30 mm

Activity 4

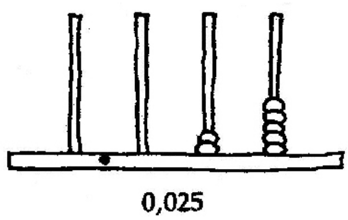
1. a) 0,346 m



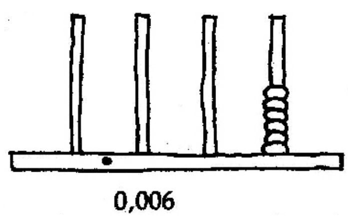
b) 0,52 m



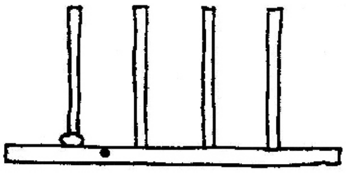
c) 0,025 m

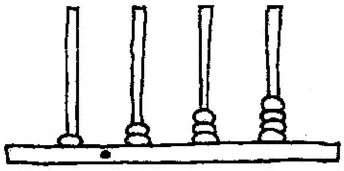


d) 0,006 m



e) 1,00 m f) 1,234 m





Activity 5

2. 1 kg, 750 g, 500 g, 400 g, 250 g

3. 1 litre, 750 ml, 500 ml, 375 ml, 125 ml

4. A litre of milk has a mass of 1 kilogram.
   1 000 ml = 1 000 g
   1 ml = 1 g
   So, a litre and a kilogram of milk have the same mass.
   This is not the case for all substances.
   e.g. 1 litre of sugar has a mass of 800g or 0,8 kg.

Activity 7

1. 36 litres

2. 13,2 litres

3. a) 22,8 l
   b) 22 800 cm^3

4. a) 4,578 l
   b) more than Thandi uses; by 0,422 litres or 422 cm^3

Lesson 6

Activity 2

1 century = 100 years, 1 decade = 10 years, 1 year = 12 months, 1 year = about 365 days, 1 day = 24 hours, 1 hour = 60 minutes, 1 minute = 60 seconds.

Activity 3

1. 5 time zones. West. She subtracts 5 hours. She turns her watch back 5 hours.
2. 10 time zones. East. She adds 10 hours. She turns her watch forward 10 hours.
3. 2 time zones. West. Subtract 2 hours. Turn your watch back 2 hours.
4. Difference of 8 time zones. East. 8 hours later. It is 7 p.m.

Activity 4

1. a) 06h00 b) 08h30 c) 12h00
   d) 17h00 e) 22h55
2. a) 20 minutes
   1 hour and 3 minutes
   b) 1 minute longer
   2 minutes shorter

Lesson 7

Acvitity 1

1. a) Answer on the calculator is 0.5. You write it R0,50 or 50c. Notice the 0 at the end is not shown.
   Depending on the settings of your calculator, your answer may at first be given as $\frac{1}{2}$.
   In order to convert this to a decimal, you need to push the  key (just above the DEL key). This key converts the fraction to a decimal automatically for you.

   b) Answer on the calculator 1.55. You write it R1,55.

   c) Answer 1.55. Note: For 0,50, you need only press .5. If you are not sure whether to put a 0 in or leave it out, put it in.

2. a) Answer: 12
   b) Answer: –6
   c) Answer: 60
   d) Answer: 0.15

Activity 3

1. A is best: $6 \times 40 = 240$ and 240 is closest to 226,044.
2. D is best: $9 – 20 = 70$ and 70 is closest to 65,6.
3. B is best: $120 + 100 = 220$ and 220 is closest to 221,21.
4. C is best: $90 \div 18 = 5$ and 5 is closest to 4,8.
5. B is closest: 3 016 is close to 3 000; and 29 is close to 30
   $3\ 000 \div 30 = 100$ and 100 is closest to 104
6. C is closest: 300 is closest to 298.

Activity 5

1. 4
2. 2
3. 0
4. –2
5. –4
6. 40 000
7. 0
8. –40 000
9. 133 225

When you multiply two negative numbers together, your answer is always positive.

Feedback to Self-check exercises

Lesson 1

1. The Hindu-Arabic number symbols, 1 2 3 ... and the roman numerals, I II III IV V VI ....

2. In seconds, minutes and hours. There are 60 minutes in an hour and 60 seconds in a minute. Also in degrees. There are 360 degrees in one revolution, 60 minutes in one degree and 60 seconds in one minute.

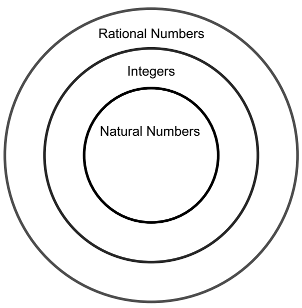
3.

10 units = 1 ten	10
10 tens = 1 hundred	100
10 hundreds = 1 thousand	1 000
10 thousands = 1 ten thousand	10 000
10 ten thousands = 1 hundred thousand	100 000
10 hundred thousands = 1 million	1 000 000

4. Firstly, let's forget about all those 0's. We know we are dealing with millions. We want to compare 3 265 with 19. Let's round off both numbers. 3 000 and 20. We see that when you have R3 000 rand, R20 is not a lot. The answer to the question is that the mistake is very small.

Lesson 2

1. Counting, adding, subtracting
2. Natural numbers
3. Telling the depth of the fish, the length of the line, and the weight of the fish.
4. Integers
5.



6. No, he should have got R20,40.
7. 10 months
8. 80 + 80 + 80 + 80 + 80 = 400 cm. This is 4 metres.

Lesson 3

1. a) $\frac{1}{5}$ b) $\frac{2}{10} = \frac{1}{5}$ c) $\frac{3}{15} = \frac{1}{5}$ d) $\frac{4}{20} = \frac{1}{5}$
2. $\frac{4}{20}$ $\frac{5}{25}$ $\frac{6}{30}$ $\frac{12}{60}$
3. a) 12 hours b) 8 hours
4. $\frac{4}{12}$ $\frac{5}{15}$ $\frac{10}{30}$ $\frac{20}{60}$
5. a) R240 b) R360
6.

Value of article	Value Added Tax	Cost of item for the buyer
watch R400	$\frac{400}{1} \times \frac{14}{100}$ = R56,00	R456,00
soccer boots R280	$\frac{280}{1} \times \frac{14}{100}$ = R39,20	R319,20
dress R360	R50,40	R410,40
basket R175	R24,50	R199,50
jacket R420	R58,80	R478,80
Total cost before VAT: = R1635	Total VAT: = R228,90	Total cost after VAT: R1863,90

7. a)

Common fraction	Decimal fraction	Percentage
$\frac{12}{25}$	$12 \div 25 = 0{,}48$	$\frac{12}{25} \times \frac{100}{1} = 48\%$
$\frac{2}{5}$	0,4	40%
$\frac{18}{40}$	0,45	45%
$\frac{5}{8}$	0,625	62,5%
$\frac{3}{20}$	0,15	15%

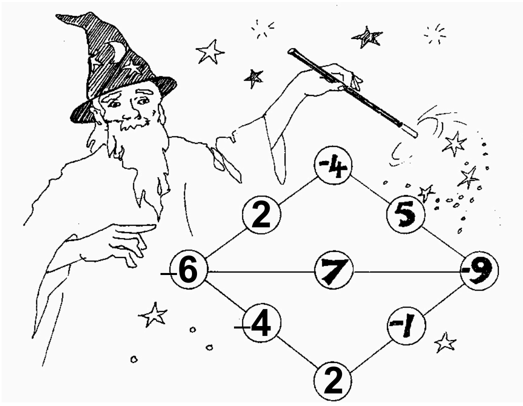
b) From smallest to biggest: $\frac{3}{20}$; $\frac{2}{5}$; $\frac{18}{40}$; $\frac{12}{25}$; $\frac{5}{8}$

8. a)
$\frac{12}{7} = \frac{7}{7} + \frac{5}{7} = 1\frac{5}{7}$
$\frac{8}{5} = 1\frac{3}{5}$
$\frac{23}{2} = 11\frac{1}{2}$
$\frac{19}{5} = 3\frac{4}{5}$
$\frac{13}{3} = 4\frac{1}{3}$

b)
$1\frac{2}{3} = \frac{3}{3} + \frac{2}{3} = \frac{5}{3}$
$2\frac{3}{4} = 2 \times \frac{4}{4} + \frac{3}{4} = \frac{8}{4} + \frac{3}{4} = \frac{11}{4}$
$5\frac{1}{7} = \frac{36}{7}$
$6\frac{1}{2} = \frac{13}{2}$
$3\frac{1}{5} = \frac{16}{5}$

Lesson 4

1. a) −17
   b) −8
   c)



2. a) −8
   b) +56
   c) −72
   d) −40
   e) +20
   f) +2

3. The change in temperature is given by:
   21°C – (–1°C) = 21°C + 1°C
   = 22°C
   So, the temperature rose 22°C.

4. a) Windhoek is the warmest. New York is the coldest.
   b) 31°C – (–3°C)
   = 34°C
   c)

From Paris to Rome:	–2 – 3 = –5°C
From Rome to Lesotho:	25 – (–2) = 27°C
From Lesotho to Namibia:	31 – 25 = 6°C
From Windhoek to New York:	–3 – 31 = –34°C
From New York to Accra:	18 – (–3) = 21°C
From Accra to London:	5 – 18 = –13°C

5. First add all the positive numbers together, that is, credits. Then add all the negative numbers together, that is, debits.

Credits		**Debits**	
01-07	+200,00 brought forward	19-07	−50,00 withdrawal
25-07	+200,00 deposit	21-07	−40,00 withdrawal
Total credits:	+ 400,00		−4,00 charges
		Total debits:	−94,00

∴R400,00 − R94,00 = R306,00
So, the new balance is R306,00.

6. a) whale is at (–15) m, submarine is at (–30) m.
   b) new position of whale: (–15) –10 = –25 m (i.e. 25 m below sea-level)
   new position for submarine: (–30) + 20 = –10 m (i.e. 10 m below sea-level)

Lesson 5

As this was a practical exercise the answers cannot be shown here, But perhaps you had some fun doing the exercise!

Lesson 6

1. 1 century = 100 years
   1 decade = 10 years
   1 year = 12 months
   1 year = about 365 days
   1 day = 24 hours
   1 hour = 60 minutes
   1 minute = 60 seconds

2. 7 time zones west. Subtract 7 hours. Turn your watch back 7 hours.

3. 6 a.m.

4. a) Total time for tasks = 2 hours + $\frac{1}{2}$ an hour + 20 minutes
   = 2 hours + 30 minutes + 20 minutes
   = 2 hours 50 minutes
   b) She left at: 12h30
   Time returning: 12h30 + 2 hrs 50 minutes
   = 14h30 + 50 minutes
   = 15h20 is when Ndu gets back home
   c) How much time at home
   = 17h00 - 15h20
   = 16h60 - 15h20
   = 1 h 40
   = 1 hour 40 minutes

Lesson 7

1-5. I hope you are feeling more comfortable with your calculator after doing these calculations.
1. 1,90 2. 1,6 3. 34,1 4. –32,6 5. $\frac{2}{3}$

6. $\frac{53}{12} = 4\frac{5}{12}$ (both these are in simplest form; one is just written as an improper fraction and the other as a mixed number.

7. 0,25 0,666666 or $0,\dot{6}$

8. $1,\overline{09}$ (which means that the 09 is recurring)

9. 32

10. –32

11. –32

12. a) $\frac{41}{12}$
    b) $\frac{11}{8}$

13. a) $3\frac{1}{8}$
    b) $1\frac{5}{7}$
    c) $1\frac{1}{2}$
    d) $4\frac{1}{5}$

14. a) Best estimate is B. (800×6)
    b) Best estimate is A. (5×13)
    c) Best estimate is C. (17×9)