Mathematical Literacy

Unit 2

Laying foundations for further application

Acknowledgements:

Editors: Allison Kitto, Jennifer Rabinowitz
Writer: Hayley Barnes
Layout: Lidia Kruger

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Unit 2

Laying foundations for further application

A course for adults at secondary level by distance education

1. More calculations

Introduction

This lesson is a follow-up of what you learned in Unit 1 Lesson 7. Look back at Lesson 7 in Unit 1 if you don't remember how to use your calculator.

In this lesson you will:

- learn the order of operations
- use this order when operating a calculator
- learn about squaring, cubing and square rooting
- use the memory key(s)
- use a calculator for everyday activities.

When you are shopping, or buying and selling things, you need to do calculations. A calculator helps you to do your calculations quickly and accurately. Have fun with your CASIO *fx*-82*ES PLUS*.

Example

Mrs Ntene is a hawker. She originally had R2 000 in her purse. She bought 5 hats at R70 each, and 10 dresses at R150 each. She wants to calculate how much money is left.

Mrs Ntene wrote on a piece of paper:
$2000-(5\times 70+10\times 150)=20\ 949\ 000$

She could not understand why she got such a big number. She could see that there was something wrong. She went over the calculations many times and still got this big number. Mrs Ntene did not know how to use brackets, and what the order of arithmetic operations was about.

Now try to do this calculation using your scientific calculator by typing the calculation exactly as it is written. Your key sequence should look like this:

2000 ; 

; 5 ; 

; 70 ; 

; 10 ; 

; 150; 

Answer: 3150

After you have completed the lesson, have another look to see if you can work out how she got the answer 20 949 000.

Order of arithmetic operations

In arithmetic there are certain rules that we have to follow. The arithmetic operations: =,+, –, ÷, ×, ( ), are used in a certain order in calculations.

This order is:
1. ( )
2. ×, ÷
3. +, –

The order of operations is important. If there are brackets in our arithmetic expression, we need to start with whatever operation is in the brackets and calculate this part of the expression first. The second on the list are the × and ÷ operations. We do these next. The last of the operations are +, –. We do these parts of the expression last.

Example:
$(5+4)\div 2$

We first start with (5 + 4) which is 9. We then divide 9 by 2, that is, 9 ÷ 2 = $4\frac{1}{2}$. Note here that although addition is last on the list, in this case it is within brackets, and so we do the addition first.

However, what do we do in this case?
$5+4\div 2$

Division is higher up on the list than addition, so we start with 4 ÷ 2 which is 2. We then add 2 to 5, which is 7.

Do you notice the difference when we have brackets and when we do not have brackets.

Now try the following activity.

First try the calculations without using your calculator and then use your scientific calculator to check your answers. Remember that your scientific calculator has been pre-progammed to carry out the calculations in the correct mathematical order. So when using your scientific calculator you can just type in the calculation as it appears. Normal calculators (that are not scientific) cannot do this.

ACTIVITY 1

1. Without using your calculator do the following exercise:
   a) $4+3\times 2$
   b) $(20+4)\times -9$
   c) $4\times(3+2)$
   d) $4\div 2\times 4$
   e) $4\times 2\div 4$

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When you have two operations of the same level, the rule is start with the operation on the left.

Let us now see how we can use a calculator with this order of operations. When you go shopping, a calculator is very handy.

For example if you buy:

sugar	9,46
salt	10,05
milk	9,45
butter	16,59

then you can key in:
[9.46] [+] [10.05] [+] [9.45] [+] [16.59] [=]   The answer is R45,55

So if you had R50,00 in your pocket you would not have any trouble at the till (cashier). Note that [=] is left for the end. If you are adding a list of numbers you need an equal sign at the end.

ACTIVITY 2

1.

Tsepo falls for this advertisement, and buys a battery.

a) At 2 hours a day, how many days should the battery last?

b) Why is this length of life unlikely?

2. Mrs Jack is at the supermarket. She has in her basket 12 cans of dog food at R7,60 each and 6 cans of cat food at R3,50 a can. What is the total cost of the food in her basket? Use your calculator and indicate the order of the keys you pressed.

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Squaring, cubing and square rooting

Any number or letter multiplied by itself gives the square of that number or letter.

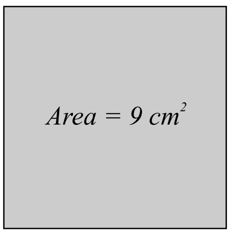
3×3 can be written as 3^2

$3 \times 3 = 9$ and $\sqrt{9} = 3$

We say "three squared".

We say 'the square root of nine is equal to three'.

If we know what the area of a square is we can find out the length of the sides by finding the square root of the area.



The square root of the area of 9 is 3 so the square has sides that each have a length of 3 cm.

You will learn more about area later in this unit.

Working out squares and square roots on your calculator

[√] is the square root sign and the [x^2] the squared sign. Can you find the key for these two operations on your calculator? To find the square root of a number, you must press the [√] key, enter the number, then press [=] . To find the square of a number, enter the number, then press the [x^2] key and then the [=] key.

So to calculate 3^2 on the calculator we will press:

3 ; [x^2] ; [=] ; and you should get 9.

To calculate the square root of 9, we press:

[√] ; 9 [=] and you should get an answer of 3.

ACTIVITY 3

Work out the following square roots on your calculator:

1. $\sqrt{121}$
2. $\sqrt{225}$
3. $\sqrt{1900}$ (round off to the nearest whole number)

Work out the following squares:

4. 17^2
5. 25^2
6. $(3{,}35)^2$ (round off to the nearest whole number)

ANSWERS ON PAGE 99

Cubing numbers

We say 'two cubed'.

Any number multiplied by itself twice gives the cube of that number. 2 x 2 x 2 can be written as 2^3.

Examples:

$3 \times 3 \times 3 = 3^3 = 27$

$5 \times 5 \times 5 = 5^3 = 125$

Working out cubes on your calculator

x^3 is the cubed sign. To find the cube of a number, enter the number, then press the x^3 key and then the = key.

ACTIVITY 4

Work out the following on your calculator:

1. 6^3
2. 4^3
3. $7^3 + 9^3$
4. $(6,7)^3$

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Let's look now at one of the most powerful features of a calculator.

Memory

Your calculator has a store room that you cannot see. This storage is called **memory**.

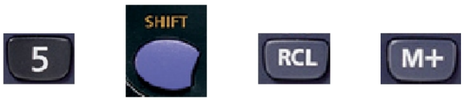
The memory is used to keep numbers that we might need later in our calculations. The calculator you are using has a few **independent memories**. When we say an independent memory we mean the number of values you can store. For the purpose of this course, we will stick to working with only four independent memories, although the calculator can hold up to nine independent memories. Let's see how the memory works. First look at the memory keys.

Try to find the following keys on your calculator:

RCL   M+   (–)   ° ' "   hyp

How the memory works

The memory key stores values (numbers) in memory. This key clears whatever is in memory and then stores the value displayed on the calculator in the independent memory. For example, key in 5 and store it in memory by pressing:



Remember that the "SHIFT" key allows you to access the 2^{nd} functions of certain keys that are written in yellow above that particular key. That is why you needed to press the SHIFT key to access the STORE function. Your screen should indicate that 5 has been stored into memory by:

$5 \longrightarrow M$

Switch off your calculator completely. Now switch it on again, and your display window should look like this:

M D Math
|

The M at the top left corner of your display window shows that there is something in the memory. Your calculator has got a good storage system which does not fail even if the calculator is completely switched off. Switching off the calculator does not erase what is in memory.

How do you get back what you stored?

You can recall the value in the independent memory by pressing the **recall key** 

. Let us now recall the 5 that we stored earlier by pressing:

The value of 5 should now appear on your display screen.

How can you store more than one independent value in a memory?

If you need to store some more values on your calculator for later use, there are also other memory slots to store them in. For example, let us do the following calculation and store the answer for later use:
$4 + 6 - 18 = -8$

If you try to store your –8 answer into memory using the procedure we did above, the calculator will automatically wipe out the 5 we stored and replace it with a –8. But we want to keep the 5 stored. So we use another one of the memory slot keys as follows:

Your display screen should show an answer of: –8

To store the –8, press: 

And your display should show: –8 ⟶ A

Now clear your calculator by pressing the 

. The value of –8 should now be stored in your A memory holder. Remember that 5 has already been stored in your M memory holder.

To retrieve the value of –8 from memory holder A, press the following keys: 

Your calculator should display the value of –8.
Try adding this value of –8 to the value of 5 that you stored previously in memory holder M by pressing (first clear your screen):

Your calculator should display the value of –3.

ACTIVITY 5

Let us try to store more values in the memory holders C and D on your calculator. Do the following calculation and then store the answer in memory holder C (you shouldn't have trouble finding it next to memory holder B key):

10 + 25 – 14 =

Once you have stored it and cleared the answer try to recall that value again from memory holder C. If it worked, you can clear the screen again. If you did not get it or were unable to recall an answer of 21, then try again.

Now do the following calculation and store the answer in memory holder D.
415 – 36 =

Try to recall the answer of 379 from memory holder D. Now clear the screen completely.

Then try to add the two values in memory C and memory D by recalling them.

ANSWERS ON PAGE 100

How do you delete a value from memory?

We have [5] in memory and now we want to store [8] in memory. Remember that your calculator can only store one number at a time. Do we have to delete [5] and then put [8] in memory? We can do that.

delete *- rub or erase, remove, cancel*

But how do we delete 5 from memory? We delete a value from memory by storing 0. To do this, clear the display by pressing the 

key. You will now have:

To store the value of 0, press: 

Notice the M at the top left corner of the display window has disappeared. Now switch off your calculator completely.

Another way of deleting what was previously stored in memory is by storing a new value. The last value you put in memory overwrites anything that is in memory.

Try this on your own, and also try playing around with the other memory holder keys labelled A, B, C, D, etc.

How to add or subtract values from the value in memory

If you want to add a number, for example 2, to the value that is in memory, you use the key [M+]. This key adds the displayed value to the value stored in the independent memory. Store 50 in the M memory and let us now add 2 to it. Key in:

and you should get an answer of 52 on your display screen. If we want to make 52 our new value in the memory, then we simply press the keys:

52 is the number now stored in our memory M holder. You can check this by clearing your display screen with the AC key and then pressing: 

to see 52 on your display screen again.

Now let us try to subtract a value from memory. Try the next activity.

ACTIVITY 6

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Subtract 2 from memory holder A.

ACTIVITY 7

Use the memory keys to solve this problem. Show which keys you pressed. Mr Thebe works in a bank, and receives a monthly cheque of R23 108,40. Mrs Thebe is a part-time teacher, and receives a monthly cheque of R9 026,10. Each month they have the following bills to pay:

House mortgage	R3 182,50	Electricity	R343,75
House insurance	R488,90	Car payment	R1200,00
Life insurance	R561,70	Rates	R626,85

How much is left each month to buy food, clothing, petrol and any extra items?

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Why do you need the memory?

To find out how to keep using your memory keys more effectively, consult the instruction manual of your calculator. You will find quite a number of ways to save time through knowing how to work with your memory keys properly.

How can you use memory in your everyday activities?

Some people have small businesses which sell goods to the public. Because the seller has to give people change, arithmetic computation is involved. Let us look at the following example.

You sell apples for R2,50 each. If someone buys 7 apples and gives you R20,00 you can use your calculator to help you quickly find how much change you should give them.

7 apples at R2, 50 each will cost 7 × R2,50:

The answer is R2,50 change.

ACTIVITY 8

You have R100,00 in your pocket. You want to buy soap for R8,39. Handy Andy for R15,49, Dettol R16,59 and toilet rolls which cost R4,00 each. How many toilet rolls can you buy?

Soap	R8,39
Handy Andy	R15,49
Dettol	R16,59
Toilet Rolls	R4,00 each.

ANSWERS ON PAGE 100

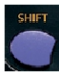
Summary

In this lesson you learned the order of arithmetic operations.

That is,	( ),	1
	×, ÷	2
	+, –	3

When you use these operations by hand you need to do them in the order given in the list. You saw that when you use your scientific calculator to perform arithmetic operations, the calculator sorts the order itself, you do not need to order the operations for it. (Only scientific calculators obey the order of operations; others do not.) You learned how to use your calculator in everyday activities. You saw how powerful the calculator is, how much memory storage it has. This is done through the use of the memory keys.

Your calculator allows you to store a value, recall it, and add or subtract values from the memory. Some of the memory keys are:

For the keys that are written in yellow, you need to press 

before you can use them.

You can use your calculator every time you do a long calculation. You saw how fast the calculator can perform arithmetic operations. You should use the manual (information book) that comes with your calculator. This manual will give you more information about your calculator.

Self-assessment checklist:

Are you able to:

- apply the order of operations
- use this order when operating a calculator
- use the calculator to find the square, cube and square root of any number
- use the memory key
- use a calculator for everyday activities.

SELF-CHECK EXERCISE

1. Without using your calculator do the following:
   a) $27 + 9 \div 3 =$
   b) $(25 - 5) \div (6 - 2) \times 5 =$

2. Repeat exercise 1 using your calculator. For each calculation:
   - write the order in which you pressed the keys
   - give the answer

3. Use your calculator to calculate:
   a) $6^2 + 4^3$
   b) $9^3 - 3{,}3^2 \text{ x } \sqrt{250}$ (round off to one decimal place)

4. Do the following two calculations mentally. Then use the memory keys to do the following calculations. For each calculation write down the keys you pressed, the order in which you pressed them and the answer.
   a) $(95 + 5) \div (30 - 10) =$
   b) $(500 \div 5 - 50 + 3 \times 2) \div [(75 + 6 \div 2) - (59 + 3 \times 3)] =$

5. The Holy Cross Choir held a concert to raise money for choir funds.

   They sold 284 tickets at R47,00 each and 167 programmes at R5,00 each. The costs of the show were R1193,75 for scenery, R570 for costumes, R161,75 for programmes and R250 for producing the tickets. How much money did the choir make? Use your calculator. Indicate which keys you pressed.

ANSWERS ON PAGE 120

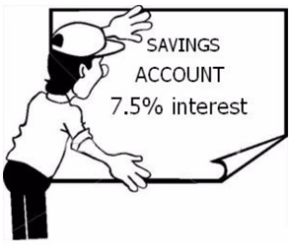
2. Fractions: addition and subtraction

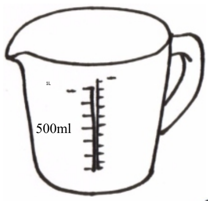
Introduction

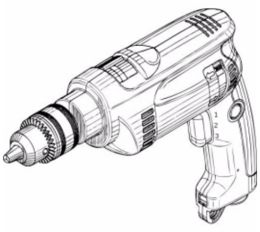
This lesson is divided into two main sections. These sections are addition and subtraction of fractions with the same denominators; and addition and subtraction of fractions with different denominators.

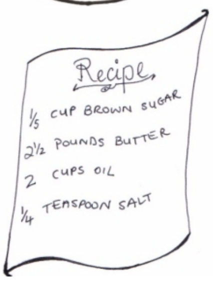
In Unit1lesson 3 you learned about fractions and their different types. If you still have difficulties with fractions, look back at lesson 3 in Unit 1.

We use fractions every day. When we cook we sometimes need half a teaspoon of salt in our food. We sometimes need todivide a piece of breadforourchildren. We give each child a fraction of the bread. We often hear people talk about fractions. We hear them say half-hour, half-day, half-a-dozen, half-moon shaped, quarter-deck and many other words. Let us look at the fractions around us. Each of the following pictures has something to do with fractions.







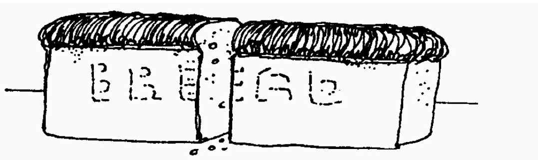


In this lesson you will:

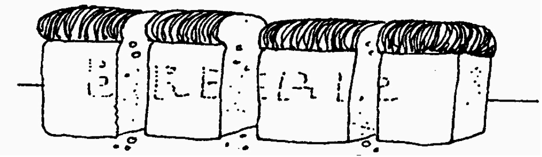
- add and subtract simple fractions, that is, fractions with the same denominator
- understand what a lowest common multiple (LCD) is
- add and subtract fractions with different denominators by finding the lowest common denominator (LCD).

Fractions with the same denominators

Remember lesson 3 of Unit 1 on equivalent fractions?
If you have a loaf of bread and you want to share it equally between your two children, how do you do this? You cut the loaf in half.



You give one child $\frac{1}{2}$ a loaf and the other $\frac{1}{2}$ a loaf.
If each child wants to share his/her portion equally with a friend, the friend would do the same. Each will give a friend half of what he/she has. That means that the loaf of bread will now have been divided up into 4 equal portions. So each child receives $\frac{1}{4}$ of the loaf.



Adding fractions

If we now want to put the pieces back together to make a full loaf, we will add the portions together. In the case where each child got $\frac{1}{2}$ a loaf we will have $\frac{1}{2}$ loaf + $\frac{1}{2}$ loaf = 1 loaf. We can write the above expression without 'loaf', like this:

$\frac{1}{2} + \frac{1}{2} = \frac{2}{2} = 1$

In the second case, the four children, your two children plus their two friends, each got $\frac{1}{4}$ loaf. We can put the portions together by saying,

$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{4}{4} = 1$

Remember that the numerator is the top part of the fraction and the denominator the bottom part. When we have fractions with the same denominator, we can keep the denominator and add the numerators.

Examples

1. $\frac{1}{2} + \frac{1}{2}$

   Keep the denominator of 2

   Then add the numerators so, $\frac{1}{2} + \frac{1}{2} = \frac{2}{2}$

2. Let us try another example:

   $\frac{2}{4} + \frac{1}{4}$

   Keep the denominator of 4

   Add the numerators: 2 + 1 = 3

   so $\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$

ACTIVITY 1

Calculate the following:

1. $\frac{1}{3} + \frac{2}{3} =$
2. $\frac{3}{6} + \frac{2}{6} =$
3. An adult has 32 teeth, 8 of the 32 teeth are incisors, 4 are canines, 8 are premolars and 12 are molars. What fraction of an adult's teeth are incisors and canines?
4. A working week, Monday to Friday, is split into 10 equal shifts. What fraction of the week is worked by someone who is employed for:
   a) 2 days
   b) $2\ \frac{1}{2}$ days
   c) 3 days and 1 half-day?

ANSWERS ON PAGE 101

Subtracting fractions

We follow the same rule with subtraction. When we have the same denominators, we keep the denominators and subtract the numerators as we do with whole numbers. Remember if you are subtracting $\frac{3}{6}$ from $\frac{5}{6}$, ie. $\frac{5}{6} - \frac{3}{6}$ you are subtracting 'sixths' ie. $\frac{5-3}{6} = \frac{2}{6}$.

1. $\frac{5}{5} - \frac{1}{5}$
   1. Keep the denominator of 5
   2. Subtract the numerators: $5 - 1 = 4$

      So $\frac{5}{5} - \frac{1}{5} = \frac{4}{5}$
2. $\frac{7}{8} - \frac{2}{8}$
   1. Keep the denominator of 8
   2. Subtract the numerators: $7 - 2 = 5$

      So $\frac{7}{8} - \frac{2}{8} = \frac{5}{8}$

ACTIVITY 2

Calculate the following

a) $\frac{7}{9} - \frac{4}{9} =$

b) $\frac{4}{8} - \frac{1}{8} =$

c) Mr Ramalane leaves home with a full tank of petrol. When he arrives at work in Polokwane, the petrol indicator shows that his tank is till a quarter full. What fraction of petrol did Mr Ramalane use to reach Polokwane?

(Hint: a tankful = 1, 1 is equivalent to $\frac{1}{1}, \frac{2}{2}, \frac{3}{3}, \frac{4}{4}, ...$)

ANSWERS ON PAGE 101

Fractions with different denominators

How can we add or subtract fractions with different denominators?

Fractions need to have the same denominator before we can add or subtract the fractions. How can we ensure that the fractions have the same denominator? Remember in lesson 4, Unit 1 you multiplied the numerator and denominator by the same number. We are going to use the same rule in this section.

Let us look at the following diagram:

1								1 whole
$\frac{1}{2}$				$\frac{1}{2}$				2 halves
$\frac{1}{4}$		$\frac{1}{4}$		$\frac{1}{4}$		$\frac{1}{4}$		4 quarters
$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	8 eighths

From the diagram, you can see that $\frac{1}{2}$, $\frac{1}{4}+\frac{1}{4}=\frac{2}{4}$, $\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}=\frac{4}{8}$ are equal.

Therefore, we can write $\frac{1}{2}=\frac{2}{4}=\frac{4}{8}$

We can write $\frac{1}{2}$ in different forms. We can change a fraction so that it has a different denominator but still has the same value.

When we change the denominator we change the numerator as well in order to retain the value of the fraction. Let us see how this is done:

$$\frac{1}{3}=\frac{\square}{6}$$

$$\frac{1\times 2}{3\times 2}=\frac{2}{6}$$

We multiply the denominator by 2 to get 6. So, we therefore also need to multiply the numerator by 2 so that we are multiplying the fraction by $\frac{2}{2}$ which equals 1. Remember that multiplying any number by 1 does not change the value of the number. For example:

$9\times 1=9$

$108\times 1=108$

$\frac{1}{2}\times 1=\frac{1}{2}$

$\frac{2\times 3}{5\times 3}=\frac{6}{15}$ (Notice that we multiplied by $\frac{3}{3}=1$ so we changed the numerator and denominator without changing the value of the fraction.)

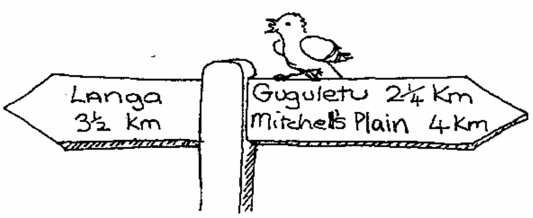
We have multiplied the numerator and denominator by the same number.

ACTIVITY 3

1. Write the following fractions in a different form without changing the value:

   a) $\frac{3}{7}$ with a denominator of 21

   b) $\frac{1}{5}$ with a denominator of 25

2. 

How far is it from a) Langa to Gugulethu?

b) Langa to Mitchell's Plain?

ANSWERS ON PAGE 101

Examples

1. $\frac{1}{2} + \frac{1}{4}$ can be written as $\frac{2}{4} + \frac{1}{4}$

Multiplying the numerator and the denominator by the same number helps us write the fractions in the same form.

2. $\frac{3}{4} - \frac{2}{16}$

We need to find an equivalent fraction to $\frac{3}{4}$ that has the denominator 16.

In this case it is easy because $4 \times 4 = 16$

So: $\frac{3}{4} \times ? = \frac{\square}{16}$

$\frac{3}{4} \times \frac{4}{4} = \frac{12}{16}$

Then: $\frac{12}{16} - \frac{2}{16} = \frac{10}{16} = \frac{5}{8}$

We simplified $\frac{10}{16}$ to $\frac{5}{8}$ by dividing the numerator and denominator each by 2.

You saw that the larger denominator is a **multiple** of the smaller denominator. In our example,

16 is a multiple of 4 because $4 \times 4 = 16$.

If you were counting in 4's, e.g. 4 ; 8 ; 12; 16; 20; etc. these would all be multiples of 4.

ACTIVITY 4

Calculate the following:

1. $\frac{4}{9} + \frac{1}{3}$

2. $\frac{5}{8} - \frac{1}{4}$

3. A DVD lasts 3 hours. A programme lasting $\frac{3}{4}$ hour is recorded.
   (Hint: 3 is equivalent to $\frac{3}{1}$)
   a) How much time is left on the tape?
   b) How much time is left after another $\frac{3}{4}$ hour programme is recorded?
4. A motorcycle's petrol tank holds 9 litres. $3\frac{1}{2}$ litres are used. How much petrol is left?
   (Hint: 9 is equivalent to $\frac{9}{1}$)

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To have the same denominator for both fractions we need to find an equivalent fraction which has the same denominator as the other fraction.

LCD - lowest common denominator: a number that can be the denominator for both the fractions. This number needs to be a multiple of all the denominators of the fractions we start with.

Finding the lowest common denominator (LCD)

We need to have the same denominator before we can add or subtract fractions. You have seen and learned in the previous section that we can change the form (not the value) of a fraction so that it has the same denominator as another fraction.

If we cannot write one fraction in such a way that it has the same denominator as the other fraction, we need to look for a **lowest common denominator (LCD)**. Let us look at the following examples, and see what a LCD is, and how it can be found.

Examples

1. $\frac{3}{4} + \frac{1}{6}$

   We need the same denominator before we can add these two fractions. Is it possible to get an equivalent fraction to $\frac{3}{4}$ which has a denominator 6?

   Let us see.

   $\frac{3}{4} = \frac{\square}{6}$ or $\frac{1}{6} = \frac{\square}{4}$

   What can we multiply 4 by, to get 6? We cannot find any whole number that we can multiply 4 by to get 6. We also can not change the denominators to 4, as there is not a whole number that we can multiply 6 by to get 4.

   So, we need to look for the LCM.
   Let us look for a number that is a multiple of both 4 and 6. This way we will be able to change both denominators. That is, we will find equivalent fractions for both fractions which have the same denominator.

Multiples of 4		**Multiples of 6**	
4	(because $4 \times 1 = 4$)	6	(because $6 \times 1 = 6$)
8	(because $4 \times 2 = 8$)	12	(because $6 \times 2 = 12$)
12	(because $4 \times 3 = 12$)	18	(because $6 \times 3 = 18$)

Have you noticed that the number 12 is a multiple of both 4 and 6? It is common to both 4 and 6. If we continue listing the multiples of the two numbers (4 and 6), we will find more multiples that are common to them both, but we usually take the lowest common multiple (LCM) which in this case is 12 and make that the lowest common denominator (LCD).

We now want to write equivalent fractions for each of our fractions above so that they each have a denominator of 12.

$\frac{3}{4} = \frac{\square}{12}$ and $\frac{1}{6} = \frac{\square}{12}$

$\frac{3}{4} \times \frac{3}{3} = \frac{9}{12}$ and $\frac{1}{6} \times \frac{2}{2} = \frac{2}{12}$

So, $\frac{3}{4} + \frac{1}{6} = \frac{9}{12} + \frac{2}{12} = \frac{11}{12}$

2. $\frac{1}{4} - \frac{3}{7} = ?$

First find the LCM.

Multiples of 4: 4; 8; 12; 16; 20; 24; 28; 32
Multiples of 7: 7; 14; 21; 28
28 is a multiple of both 4 and 7. So the LCM is 28.

Now we need to write equivalent fractions for $\frac{1}{4}$ and $\frac{3}{7}$ where they each have a denominator of 28.

$\frac{1}{4} = \frac{\square}{28}$ and $\frac{3}{7} = \frac{\square}{28}$

$\frac{1}{4} \times \frac{7}{7} = \frac{4}{28}$

$\frac{3}{7} \times \frac{4}{4} = \frac{12}{28}$

So: $\frac{1}{4} - \frac{3}{7} = \frac{4}{28} - \frac{12}{28} = \frac{-8}{28} = -\frac{2}{7}$

When there are two different denominators, where one is not the multiple of the other, we have to look for the lowest common denominator (LCD). We then have the same denominator, so we can proceed as before.

If the two denominators do not have any common factors, then it may take some time to find the lowest common multiple. In that case, we just multiply the denominators with each other to get the lowest common denominator. For example: $\frac{1}{5} + \frac{1}{8}$: the denominators 5 and 8 do not have any common factors (factors of 5 are 1 and 5 and factors of 8 are 1; 2; 4; 8) so we say 5×8 to get the LCD of 5 and 8 which is then 40.

ANSWERS ON PAGE 102

ACTIVITY 5

1. Find the LCD for the following and calculate.

   a) $\frac{1}{8} + \frac{2}{7} =$

   b) $\frac{5}{6} + \frac{7}{8} =$

   c) $\frac{1}{2} - \frac{1}{3} =$

Summary

You saw that when you add or subtract two fractions that have the same denominator, you keep the denominator and add or subtract the numerators as you do with whole numbers.

When you add or subtract two fractions that have different denominators, you need to find equivalent fractions in such a way that they both have the same denominator.

In order to find the common denominator you have to follow these three rules:

Rule 1: Check if one of the denominators is a multiple of the other. If it is, then find an equivalent fraction for the fraction with the smaller denominator, such that it has the denominator of the other fraction.

Rule 2: If the denominators have a common factor, then make a table of multiples and look for the **lowest common multiple** (LCM). The LCM becomes the new denominator for both fractions.

Rule 3: If the denominators do not have any common factors then multiply the denominators to find the LCD. Then convert the fractions so that they have the same denominators.

Self-assessment checklist:

Are you able to:

- add and subtract fractions with the same denominator
- understand what a lowest common denominator (LCD) is
- add and subtract fractions with different denominators by finding the lowest common denominator (LCD).

SELF-CHECK EXERCISE

1. Calculate the following:

   a) $\frac{3}{4} + \frac{1}{2} =$

   b) $\frac{1}{3} + \frac{2}{9} =$

   c) $\frac{5}{8} - \frac{2}{4} =$

   d) $\frac{6}{18} - \frac{2}{9} =$

2. Find the LCD for the following, and calculate:

   a) $\frac{3}{7} + \frac{3}{15}$

   b) $\frac{4}{5} + \frac{1}{20}$

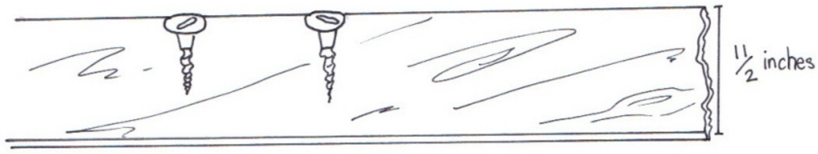
   c) $\frac{14}{35} - \frac{2}{7}$

   d) $\frac{1}{7} + \frac{2}{9}$

   e) $\frac{4}{15} - \frac{2}{25}$

3. Mrs Dlamini comes back from the butcher's with $\frac{3}{4}$ kg mince, $1\frac{1}{2}$ kg steak and $1\frac{1}{4}$ kg chops. Calculate the total weight in kg of meat in her basket.

4. Leba drives nails $\frac{3}{4}$ inch and $\frac{7}{8}$ inch long into a piece of wood $5\frac{1}{2}$ inches thick. How far is the point of each from the other side of the wood?



3. Foundations of finance: Simple interest and discounts

Introduction

This lesson focuses on some of the practical uses of percentages. You learned about decimals and percentages in lesson 3 of Unit 1. Go back to this lesson if you have forgotten about decimals and percentages. You should also revise using a calculator, since you will often need to use a calculator in this lesson.

In this lesson you will learn how interest is calculated. We use money every day. We often borrow or lend money. We can lend money to our friends. We can also lend money to the bank. When we save money in a bank we are in fact lending the bank our money. The bank can use the money we save. When we save our money in the bank we earn interest. For example, if you put R20,00 in the bank, at the end of the month you have more than R20,00 because your money has earned some interest.

Earn: being paid

We can also borrow money from the bank. We pay interest on the money we owe the bank. What is this interest? Read through the lesson and see what interest is.

In this lesson you will:

- learn what simple interest is
- calculate simple interest for given amounts of money
- understand how bank interests are calculated
- understand how discounts are calculated.

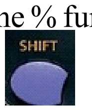
Finding percentages of amounts of money

Did you know:
It is a sin in the Islamic religion to earn interest.

Do you remember the general method of calculating percentages? (If you don't, look at Unit 1 lesson 3 again.) Let us look at how you work out percentages of amounts of money. As there are 100 cents in R1,

1% of R1 $= \frac{1}{100} \times 100$ cents
$= 1$ cent

Your calculator has been programmed to assist you with percentages with the use of a percentage key. It is in the first row of keys above the number keys and looks like this: 

As you will notice the % function of the key is a 2nd function of the key so we need to use the  to access it.

To carry out the calculation correctly, you will also need to remember in mathematics that "of" means multiplication, for example: I have 4 bunches **of** 5 flowers, so I have $4 \times 5 = 20$.

Keeping this in mind, let us try the example above using our calculators:
1% of R1 or 1% of 100: We do it as we say it.

Keys pressed: 

ACTIVITY 1

Use your calculator to work out the following:

1. What is 1% of R2?

ANSWERS ON PAGE 102

2. What is $3\frac{1}{2}$ % of R24?

We will now look at how we use percentages in our everyday lives.

Pensions

During our working lives, we can pay contributions towards pension funds that will be paid out regularly to us after we reach retirement age, to maintain us after we are no longer earning.

Example:
Nomsa works in a factory for a monthly salary of R8 500. The company has a pension scheme. Each month the company deducts 8% of Nomsa's salary to be paid into the pension scheme. As part of her salary package, each month the company pays an amount equal to 15% of Nomsa's salary into the pension scheme for Nomsa's pension.

Nomsa's monthly pension payments $= \frac{8}{100} \times \text{R}8\,500$

$= \text{R}680$

Company's payments for Nomsa $= \frac{15}{100} \times \text{R}8\,500$

$= \text{R}1\,275$

Monthly pension contributions for Nomsa $= \text{R}680 + \text{R}1275$

$= \text{R}1\,955$

After a year the total amount invested for Nomsa's pension would be

$= 12 \times \text{R}1\,955$

$= \text{R}23\,460$

We will now look at how we use percentages in our everyday lives.

Interest

We use money every day. You should know what is happening to your money when you put it in the bank, or when you ask for a loan.

After you deposit R40,00 into the bank and leave it there, say for a month, when you go to the bank to look at your balance, you will find that you have more than R40,00. Remember you only put in R40,00 and nothing more. The extra money you have is the **interest** you have earned. That is, the money the bank has paid you for using your money, in other words, for having your money.

If you deposit money into a bank, interest is the money that the bank pays for using your money. If you borrow money, interest is the money that you pay for using the lender's money.

The **interest rate**, or **rate of interest**, is the rate at which interest is earned (paid). The rate of interest is written as a percentage of the sum of money borrowed or saved. The longer you keep the money, the more interest you will have to pay. The longer you leave your money with a bank, the more interest the bank will pay you on your savings.

Per annum: per year

An interest rate of 10% per annum (written as p.a.) means that R10 has to be paid on every R100 for each year the money is borrowed. That is, R10 is 10% of R100.

Interest you pay

When you go to the bank to ask for a loan of R1 000,00, the bank will lend you the amount under certain conditions. The bank will ask you to pay back the money with some interest. So you might borrow R1 000,00 from the bank and end up paying them back R1 600,00 due to interest that they charge you for lending the money. That is, after paying, a portion of the loan, say R200,00, when you ask for the balance, you find that your balance is more than the R800,00 because it includes the interest that you have to pay the bank for using its money.

Example
If you want to buy a house you will probably borrow money from a bank. The bank will charge you about 16% interest on the money you borrow. Note that building societies and banks change their interest rates from time to time.

Suppose you borrow R3 000 from the bank, and the bank charges you 16% interest. You have agreed with the bank that you will pay back R100 for your loan every month.

So you will pay R100 × 12 = R1 200 each year (× 12 because there are 12 months in a year). You might think that at the end of the first year you will owe the bank R3 000 – R1 200 = 1 800. But this is not so.

The bank will take 16% of the R3 000 you owe at the beginning of the year as interest:

16% of R3 000 = $\frac{16}{100} \times 3\,000$
= R480

So if you pay R1 200 in the first year, part of what you pay will go to pay the interest of R480. R1 200 – R480 = R720, so in the first year R720 will be paid off the loan. At the end of the first year you will owe the bank:
R3 000 – R720 = R2 280

In the second year, again the bank will take 16% of the money you owe at the beginning of the year as interest, that is:

16% of R2 280 = $\frac{16}{100} \times 2\,280$

= R364,80

You will again pay R1 200, so this year R1200 – R364,80 = R835,20 will be paid off the loan.

You will then owe R2 280 – R835,20 = R1 444,80
The following table shows the working for the first two years.

Year	**Calculations**		**Money you owe the bank**
1st year	You pay:	R1 200	
	Interest due on	R3 000:	
	16% of R3 000 =	R480	
	Amount taken off loan = R720		R2 280
2nd year	You pay:	R1 200	
	Interest due on	R2 280:	
	16% of R2 280 =	R364,80	
	Amount taken off loan = R835,20		R1 444,80

ACTIVITY 2

ANSWERS ON PAGE 102

Complete the table for the third year. Do you see that you are paying for using somebody else's money? You are also paid when somebody, the bank in this case, is using your money. In the next section we will look at the interest you earn when you put money in the bank.

Interest you receive

You have R200 in a People's Bank Savings account where the bank will pay you 2,53% interest each year. This is called the **interest rate**. Different banks have different interest rates. Also, the interest rate changes if you have different amounts of money in the bank. For example:

People's Bank Savings account

Amount of money	**Interest rate**
R0 - R199	1%
R200 - R999	2,53%
R1 000 - R4 999	3,56%
R5 000 - R9 999	5,12%

Everybody's Bank Savings account

Amount of money	**Interest rate**
R0 - R199	1%
R200 - R499	1%
R500 - R999	2,5%
R1 000 - R4 999	3,5%
R5 000 - R9 999	5%

Example
How much interest will you receive in one year if you have R200 in your account at the People's Bank, where the interest rate is 2,53%. You can use your calculator.

Answer
Interest = amount × interest rate
= R200 × 2,53%
= R5,06

To do this on your calculator press the following keys:
200 ; 

; 2.53 ; 

The calculator will display: $\frac{253}{50}$ so remember to press the  to convert the fraction to a decimal of 5,06.

The interest will be R5,06 in one year.

Note that the total amount in the bank becomes R205,06 after the one year.

ACTIVITY 3

Use the method given in the above example and work out how much interest you will receive after 1 year if you have the following amounts in your account and the interest rate is 0,5%. You can use your calculator.

a) R100
b) R50
c) R35

ANSWERS ON PAGE 103

Simple interest

When we talk of **simple interest** we talk about the interest on the amount that you deposited or borrowed. The amount you deposited or borrowed is called an **unchanging principal**. The **principal** is the amount of money invested.

The number of years for which the principal is invested is **time**. **Amount** is the principal plus the interest.

These definitions will help you to find a standard method for calculating interest. In the next example you will see how interest is calculated.

Invest: to put money in the bank or to buy shares so that you will receive profit.

Example

Interest on R100 at 1% for 1 year = (1% of R100) × 1 year

$= 1\% \text{ or } (\frac{1}{100}) \times 100 \times 1 \text{ year}$

$= R1 \times 1 = R1$

Using your calculator:

Interest on R100 at 1% for 5 years = (1% of R100) × 5 years

= 1% × 100 × 5 years

= R5

Interest on R100 at 5% for 1 year = (5% of R100) × 1 year
= R5

Interest on R100 at 5% for 5 years = (5% of R100) × 5 years
= R25

Interest on R500 at 5% for 5 years = (5% of R500) × 5 years
= R125

Do not rely only your calculator to be able to do these calculations. You should also do them manually. For example:

5% interest on R500 for 5 years:

$$5\% = \frac{5}{100}$$

$$\text{So } 5\% \text{ of R500} = \frac{5}{\cancel{100}_1} \times \frac{\cancel{500}^{5}}{1} = \frac{5}{1} = 5$$

For 5 years:

$5 \times 5 = 25$

From these examples, we can draw up the following formula:

Interest = principal × rate % × time

Simple interest formula

From the example, we found the formula for interest. The **simple interest formula** is given by

SI = P × r × n

where SI = interest, written in rands
P = principal, money deposited or borrowed, written in rands
r = percentage rate, written as a fraction or decimal (sometimes represented by an i)
n = time, written in years.

A formula is a summary of a method for calculating something. But it is always necessary for you to understand the formula and the mathematical reasoning behind it. That way, if you ever forget the formula, you can still use mathematics to work out your answer.

Example
How much interest will Tsepo earn, if he deposited R300 for 2 years in a savings account which pays 5% simple interest?

Step 1: Identify P, r and n.

$P = 300, R = 5\% = \frac{5}{100}, T = 2$

The interest rate, 5% can be written as a fraction $\frac{5}{100}$ to simplify multiplication in step 2.
If you are using the CASIO calculator (recommended), then you can simply work with 5%.

Step 2: Substitute and multiply. Use your calculators.
$SI = Prn$
$= 300 \times 5\% \times 2$
$= 30$
Answer: R30,00

Keys:300 ; [×] ; 5 ; [SHIFT] ; [%] ; [×] ; 2 ; [=]

Example
At the end of 3 years, what is the total amount that Tsepo owes on a R429 loan he borrowed at 16,5% simple interest per year?

Step 1: Identify P, r and n.
$P = 429, r = 16{,}5\%, n = 3$

It is sometimes easier to multiply with decimals than with fractions, especially when there will be little or no cancellation with fractions. Use your calculator to multiply with decimals.

Step 2: Substitute and multiply to found out the interest he owes.

$SI = Prn$
$= 429 \times 16{,}5\% \times 3$
$= R212{,}36$ (Rounded off to two decimal places)

Keys:429 ; [×] ; 16,5 ; [SHIFT] ; [%] ; [×] ; 3 ; [=] ; [S⇔D]

You can leave this amount on your calculator display screen or put it into one of the memory holders.

Step 3: We know the interest that Tsepo owes the bank but he has to pay them back the original amount plus the interest owed, so we need to calculate that.

Total amount owed = Principal amount borrowed + interest owed
= R429 +R212,36
= R641,36
Answer: R641,36

Pension contributions are invested by pension schemes so that they earn interest.

Example

Nomsa's company invests all pension contributions at 13% interest. If Nomsa works for a year, and the total pension contributions made for Nomsa for that year are R9 300, how much interest would that year's pension contributions earn in one year?

P = 9300; r = 13%; n = 1

Interest = $P \times r \times n$

= $9300 \times 13\% \times 1$

= R1 209

Keys: 9300 ; × ; 13 ; SHIFT ; % (; × ; 1 ; =

ACTIVITY 4

1. The following tables give the interest rates for People's Bank (PB) and Everbody's Bank (EB) savings accounts. Look carefully at the rates and decide which is the best bank to put in your money. If they are both the same please say so.

   Remember the bank that will pay the higher interest is the best.

   People's Bank Savings

R0	- R199	1%
R200	- R999	2,53%
R1 000	- R4 999	3,56%
R5 000	- R9 999	5,12%
R10 000	- R14 999	6,17%

   Everybody's Bank Savings

R0	- R199	1%
R200	- R499	1%
R500	- R999	2,5%
R1 000	- R4 999	3,5%
R5 000	- R9 999	5%
R10 000	- R14 999	6%

   The amount of money refers to the amount of money you put in the bank.

1. a) If you have R500 which bank will you choose? Why will you choose this bank?
   b) If you have R200 which bank will you choose? Why will you choose this bank?
   c) If you have R100 which bank will you choose? Why will you choose this bank?

2. How much interest will you earn on R200 deposited for 3 years in a savings account that pays 5% simple interest?

3. At a 10% interest rate, how much interest would Thabo have to pay on R475 borrowed for 2 years?

4. How much interest would Vuyo pay for a loan of R575 at 12,5% if he repaid the bank at the end of one year? ANSWERS ON PAGE 103

Example
Find the total amount when R480 is invested for 8 months at 7,5% p.a.

Answer
Interest $= P \times r \times n$
The Principal amount invested is R480. The interest rate is 7,5% per annum. But the time presents us with a new challenge. We cannot take the time as 8 because that would indicate 8 years, whereas it is actually representing 8 months. So we convert the 8 months into a fraction of a year by saying:

Interest $= P \times r \times n$
$= 480 \times 7{,}5\% \times \frac{2}{3}$
$= 24$

Keys: 480 ;  ; 7.5 ;  ;  ;  ; 2 ;  ; 3 ; 

Amount = Principal + Interest
= R480 + R24
= R504

NOTE: If you had forgotten to simplify $\frac{8}{12}$ to $\frac{2}{3}$, your answer would still have worked out correctly by using $\frac{8}{12}$ on your calculator.

The important thing is to remember when working with months you need to divide by 12 to express the time as a fraction of a year as interest rates are usually given per year.

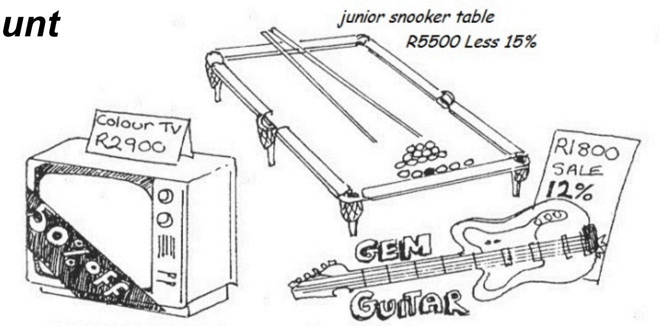
ACTIVITY 5

1. Jane deposited R650 at a $5\frac{1}{2}$% interest rate. What would the total amount in her account after 2 years?

2. What is the total amount owed on a loan if the principal is R1 000, the interest rate is 14%, and the time is 3 years?

3. What is the total amount in a savings account if the amount deposited was R375, the interest rate was $5\frac{1}{4}$%, and the time was 4 years? ANSWERS ON PAGE 104

Discount



Discount: reduction in the usual price.

Deduct: subtract.

We use percentages when we give or receive discounts. When there is a sale, the normal prices of goods are reduced by a certain percentage. For example, if there is a sale at Edgars, the price for children's dresses may be reduced by 20%. The reduction of 20% means that the price is now 20% less than the original price. Can you think of other examples of discounts?

Discount received

Let us say the normal price of the children's dresses is R150 a dress. A 20% reduction would indicate that we need to work out what 20% of the original price (R150) is and then deduct that amount from the original price of R150 to get the new reduced price. Remember that percentage means "out of 100" and the word "of" indicates multiplication. So if we try to do this manually we would say:

$$\frac{20}{100} \times \frac{150}{1}$$
$$= \frac{\cancel{20}^{1}}{\cancel{100}_{5}} \times \frac{150}{1}$$
$$= \frac{150}{5} = 30$$

So the discount is R30. But what is the new reduced price of the dress:
R150 – R30 = R120

You will now only pay R120 for the dress after the 20% discount.

But our calculators can also be useful (and save time) in doing these discount calculations, provided you understand the mathematical process. On the calculator we enter the calculation just as we would write it:

In language: The original price of R150 – 20% of the original price = discounted price.

Mathematical terminology: R150 – 20% of R150

Try it now on your calculator. Did you get R120?

Key sequence: 150 ; 

; 20 ; 

; 

; 

; 150 ; 

Discount given

You sell vegetables for R10 a bag. However, when someone buys 5 bags you give her a discount because they are buying so many. The normal price of 5 bags is R10 × 5 = R50. If you give a person a 2,5% discount on the total price, then you will receive:

R50 – 2,5% of R50
= R48,75 instead of R50

The discount given is R1,25.

Key sequence: 50 ;  ; 2,5 ;  ;  ;  ; 50 ;  ; 

When you give someone a discount you should be careful that you do not lose money. Make sure that the amount you get after giving a discount is not less than the cost price.

Cost: amount paid when buying goods.

ACTIVITY 6

Use your calculator to work out the following:

1. The price of milk is usually R8 per litre. The shopkeeper gives you 2% discount for buying 10 litres of milk.

   a) How much money will you pay if you buy 10 litres of milk?
   b) How much money will you pay if you buy 12 litres of milk?

2. You sell bags of oranges. The price of one bag is R25. You give a 3% discount to someone who buys 5 bags. If Tim buys 5 bags how much money are you going to get from Tim?

3. What percentage discount is this?

ANSWERS ON PAGE 105

Summary

You saw how useful percentages are in our lives. You saw that when you save money, the bank pays you **interest** on your savings. The bank uses the **simple interest formula** to calculate your new balance at the end of each year.

When you borrow money from the bank or building society you have to pay interest for the money you borrow.

We also use percentages when we give or receive **a discount**. A discount is given or received at a certain rate. The rate is usually given as a percentage.

Self-assessment checklist:

Are you able to:
- use the simple interest formula to calculate interest paid or received
- use percentages to calculate discounts

SELF-CHECK EXERCISE

1. John borrowed R124 from his friend at 10% p.a. How much must he repay altogether at the end of 7 months?

2. Which investment will give the greater interest: R420 invested at 8% p.a. for 3 years, or R360 invested at 7% for 4 years?

3. Susan deposits R25 in the Post Office savings bank at $4\frac{1}{2}$% p.a. How much will she have after 2 years?

4. A bank lends R4 900 to a man at 12% p.a. for 1 year 6 months. What amount must he repay?

5. Calculate the simple interest on R480 at $3\frac{1}{4}$% payable monthly.

6. On 1 January Sam had R50 in his Post Office savings account. He deposited a further R20 on 1 July. How much interest would he receive at the end of the year at 5% p.a.?

7. A boy invests R600 at 6% p.a. for 6 months. How much will this investment earn him?

8. A man borrows R5 000 from a bank at 15% p.a. for 2 years. He then lends R6 500 to someone else at 20% p.a. for 2 years. Calculate his profit.

9. If you buy 2 bags of cabbage, you are given a 5% discount on the 2 bags. 1 bag of cabbage costs R17,95. How much money are you going to pay for the 2 bags of cabbage?

10. You are selling a pair of shoes for R250. You give a 2,5% discount if a person pays in cash. If Tom buys this pair of shoes for cash, how much money is he going to pay for the shoes?

ANSWERS ON PAGE 122

4. Foundations of algebra

Introduction

In Unit 1 you studied numbers and their operations. Algebra is an extended study of numerals and letters, and obeys the same rules that numbers obey. You will need to know how to use a calculator in order to do this lesson, and you will need to know how to work with negative numbers (see Lessons 4 and 7 of Unit 1 and Lesson 1 of this unit).
In this lesson you will learn what algebra is, and where it comes from. We will look at why we use algebra in solving problems.

In this lesson you will:

- learn the meaning of the word algebra
- learn who first developed algebra
- discover the meaning of terminology used in algebra such as; variable, constant, term, algebraic expression, coefficient
- learn what a formula is
- be able to change the subject of a formula.

The history of algebra

Algebra means using letters or symbols to represent numbers.

The term "algebra" comes from the Arabic word "al-jabr". The word "algebra" was first used in a book written in 830 by Mohammed ibn Musa al-Khorwarizmi, called *Al-jabr w'al muqabala*. The word *al-jabr* in this context means "restoring" the balance in an equation. *Al'muqabala* meant "simplification" as in $3x + 4x = 7x$.

The algebra of al-Khowarizmi was based on Brahmagupta's work, but was also influenced by the Babylonians and Greeks.

Why do we use algebra?

Algebra helps us to translate from a spoken or written language, such as Sepedi or English, to a mathematical language consisting of mathematical terminology and symbols that are used worldwide. This makes problems easier to solve using mathematical reasoning and approaches.

Example
Try to work out a way in which you can find the number before working through the solution.

If we multiply a certain number by 2, then we subtract 3, the answer is 5. What is the number?

Solution

The number is 4.

Let us call the number we want to find, n. We first multiply the number by 2 and then take away 3. We can write:

$n \times 2 - 3 = 5$

This can be written more conventionally as:

$2n - 3 = 5$

Now it is easier to find the number. What number minus 3 gives an answer of 5?

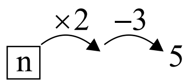
Yes, that number is 8 as $8 - 3 = 5$. So what number multiplied by 2 gives an answer of 8?

Yes, $4 \times 2 = 8$. So $n = 4$.

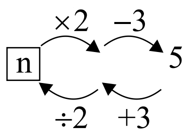
Check: $2 \times \mathbf{4} - 3 = 8 - 3 = 5$

$2n - 3 = 5$ and $3(n + 2) = 12$ is called an **equation**. An equation has two sides, the left-hand side (LHS) and the right-hand side (RHS), and they are connected by an "equal" sign. The name "equation" means that the left-hand side is equal to the right-hand side. The approach we have used above to solve the equation is known as solving by "inspection" (by examining them). In the next section, we will look at another approach to solving the equations that makes even complex equations simple to solve.

Another way to solve the equation $2n - 3 = 5$ is to use a diagram we call a flow chart. It looks like this:



The flow chart helps us to start with the solution given and reverse each operation to find the value of n.



So $5 + 3 = 8$

$8 \div 2 = 4$

Therefore $n = 4$

Try the following activity. However, if we use algebra, it makes long and difficult problems, shorter and easier to solve.

ACTIVITY 1

Write down equations for the following statements and then try to solve those equations to find the unknown number. You may use inspection or a flow chart.

1. Start with a certain number, multiply it by 3, then add 2, and the answer is 8.
2. Start with a certain number, take away 1, then multiply by 2, and the answer is 4.
3. If we subtract 5 from a certain number, then multiply by 2, the answer is 18.
4. Think of a number, multiply it by 5, then take away 4, and the answer is 6.

ANSWERS ON PAGE 106

Equations

An equation is an algebraic statement with two expressions that are equal. For example: $2n-3=5$

The expression on the left hand side (LHS) of this equation is: $2n-3$
The expression on the right hand side (RHS) of this equation is 5.
In solving the equation, we need to find a value for the *n* that will make the LHS=RHS

Up to now we have solved the equations above by inspection or by using a flow chart. But with more complicated equations, that may involve unknown values on both sides, or answers that are fractions, inspection and trial and error approaches may not help us. We are going to look at a standard approach that we can use to solve equations.

Example 1

$2n-3=5$

Solving an equation is about first understanding the concept of "reversing" or "undoing" whatever calculations were done to *n* to start off with. Let us go back to the original statement that led us to formulate this equation:

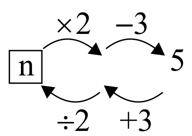
If a certain number is multiplied by 2, then we subtract 3, the answer is 5. What is the number?

We don't yet know what the *certain* number is so we call it *n*.

1. We start with *n*: n
2. It is then multiplied by 2: $n\times 2$ (which we write as $2n$)
3. We then subtract 3: $2n-3$
4. And the answer is 5: $2n-3=5$

Let us now reverse or undo this procedure as we did using the flow chart:

4. The answer is 5: 5
3. The opposite of subtract is add so we add 3: $5+3$
2. The opposite of multiply is divide we divide by 2: $(5+3)\div 2$
1. We should end up with the value of *n*: $(5+3)\div 2 = 8\div 2 = 4$



Let us check:

1. Start with the value of 4 (which we think is the value of *n*): $4\times 2 = 2(4) = 8$
2. Multiply it by 2:
3. Subtract 3: $8-3$
4. What is our answer? 5

$2(4)-3=5$
So $n=4$

Example 2

$3(n+2)=12$

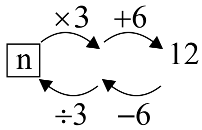
We need to solve the equation. This means we must find the number that n stands for.

We first multiply out the bracket to make the equation easier to solve.

$$3(n+2) = 12$$
$$3n + 6 = 12$$

Now we continue as we did in the previous example.

1. We start with n (the certain number).
2. It is then multiplied by 3: $n \times 3$
3. We then add 6: $3n+6$
4. And the answer is 12: $3n+6=12$



Let us now reverse or undo this procedure.

4. The answer is 12.
3. We subtract 6: $12-6=6$
2. We divide by 3: $6 \div 3 = 2$
1. We should end up with the value of n: $n=2$

Let us check:

1. Start with the value of 2 (which we think is the value of n): 2
2. Add 2: $2+2=4$
3. Multiply by 3: 4×3
4. What is our answer? 12

So: When $n=2, 3(n+2)=3(n+2)=12$

$3(2+2)=12$

So $n=2$

ACTIVITY 2

Solve these four equations from Activity 1 by 'reversing' or 'undoing' the procedure as explained in the examples above:

1. Start with a certain number, multiply it by 3, then add 2, and the answer is 8.
2. Start with a certain number, take away 1, then multiply by 2, and the answer is 4.
3. If we subtract 5 from a certain number, then multiply by 2, the answer is 18.
4. Think of a number, multiply it by 5, then take away 4, and the answer is 6.

ANSWERS ON PAGE 106

It is important that you understand the principle of using a flow chart of 'reversing' the procedure to solve an equation. We will look at another method of solving equations later in Unit 5.

ACTIVITY 3

Solve the following equations by 'reversing' the procedure and sketch a flow chart to indicate what you did.

1. $3n + 2 = 8$

2. $2(n - 1) = 4$

3. $2(n - 5) = 18$

4. $5n - 4 = 6$

ANSWERS ON PAGE 108

Variables and constants

Variable: changing

In the previous activities above we represented the number that we were looking for by the letter *n*. However, we can use any other letter. The letters commonly used by mathematicians are *x* and *y*. The letter *n* in our examples referred to any number that was not known. Since *n* can be anything, its value varies. Hence we call *n* a **variable**.

In the examples we used the numbers 2, 3 , 5 and 12. These numbers are specific. We call these numbers **constants**. A constant has a specific value. It is different from a variable.

Example: $x + 3 = 5$
The variable is *x* while 3 and 5 are constants.

ACTIVITY 4

In each question, say which are variables and which are constants.

1. $y + 4 = 6$
2. $2n + 4 = 3$

In algebra we work with variables and constants. We have represented the questions above using constants and variables. We have expressed these questions using algebra. One of the most important skills in mathematics that you need to master is the ability to translate from a spoken or written language, such as Sepedi or English, to mathematical language consisting of mathematical terminology and symbols that are used worldwide.

If we cannot translate our language problems into mathematical terminology and symbols, we will struggle through the rest of these units. So it is really an important skill to know and master. It is also important to be able to express mathematical notation, terminology and symbols in a language of words.

ANSWERS ON PAGE 108

ACTIVITY 5

Write the letter of the correct expression next to the matching number. The first one has been done as an example for you:

E.g. (1) I

(1)	x increased by 10	A	xy
(2)	x multiplied by y	B	$x \div 2$
(3)	The sum of a certain number and 1	C	$x - 2$
(4)	x divided by 2	D	$\frac{1}{2}x + 3$
(5)	Half of a certain number plus 3	E	$29 + x$
(6)	Thabo is now 29. Thabo's age in x years' time	F	$x \quad 4$
(7)	Two less than x	G	$x + 1$
(8)	A certain number multiplied by itself	H	$2x + 9$
(9)	A certain number decreased by 4	I	$x + 10$
(10)	A certain number multiplied by 2 and increased by 9	J	$x \times x$

ANSWERS ON PAGE 108

An algebraic expression

Decrease: subtract.
Add: increase, sum of.

An **algebraic expression** consists of constants and variables combined by one or more of the operations: addition, subtraction, multiplication or division. For example, $x + 3$ is an algebraic expression which in words means x plus 3. Another example, $\frac{n}{8} + 1$ is an algebraic expression which means n divided by 8 plus 1.

ACTIVITY 6

Write the letter of the correct word expression next to the matching number of the algebraic expression.

Algebraic expression		**Word expression**	
(1)	$3y - 7$	A	x multiplied by a half increased by 5
(2)	$3(y - 7)$	B	x plus 5 multiplied by negative 2
(3)	$\frac{1}{2}x + 5$	C	y decreased by 7 multiplied by 3
(4)	$-2(x + 5)$	D	3 times y minus 7
(5)	$x + 2x$	E	The sum of x and $2x$

ACTIVITY 7

Write down the algebraic expressions for the following, letting y stand for 'a certain number'.

1. A certain number increased by 5
2. A certain number decreased by 5
3. The product of a certain number and 5
4. A certain number divided by 5
5. The sum of twice a number and 4
6. You are y years old this year. What will your age be in 10 years' time?
7. You are y years old this year. What was your age ten years ago?
8. Your friend is x years old. What is your age if it is half your friend's age?
9. How many days are there in x weeks?
10. Write down the number of cents in Rx.

ANSWERS ON PAGE 109

Terms

A combination of numerals and variables involving multiplication or division is known as a term. For example:

$8a$ is a term: $8a = 8 \times a$
y is a term: $y = 1 \times y$
3 is a term: $3 = 3 \times 1$
An expression $x + 3$ has two terms, x and 3.
An expression $2(x + 5)$ is one term, because $2(x + 5) = 2 \times (x + 5)$

Do you see why? When terms are placed in a bracket, the whole bracket is one term. For example, (a + b) + ($2a + 2b$) is an expression consisting of two terms.

Terms are always separated by an addition (+) or subtraction (–) sign.

ACTIVITY 8

How many terms are there in each of the following algebraic expressions?

1. $x + 2y$
2. abc
3. $3(a + 4b)$
4. $5(a + b) + (9a + b)$

ANSWERS ON PAGE 109

The coefficient of a variable

A number that is multiplied by a variable or a product of variables is called the coefficient of that variable. The coefficient is the number or variable that the designated variable is being multiplied by. For example in the expression $2n + 4$, 2 is the coefficient of n.

This sounds like a mouthful, but it is actually quite simple if we look at some examples:

5 is the coefficient of x in the term $5x$
14 is the coefficient of xy in the term $14xy$
1 is the coefficient of x in the term x
4 is the coefficient of a in the term $4a$ and
3 is the coefficient of b in the term $3b$ in the expression $4a + 3b$

As you can see, the coefficient is just the number that the designated variable is being multiplied by.

ACTIVITY 9

Write down the coefficients of the variable x in the following algebraic expressions:

1. $3x + 4$
2. $4y + 5xy$
3. $2y + x$
4. $4xyz$
5. $5 \times m \times 2 \times x$

ANSWERS ON PAGE 109

Formulae

You saw that through algebra we can express ideas about numbers. These ideas, which would otherwise take many words to describe, are known as formulae (plural of **formula**). Remember lesson 4 of this unit? You found a formula for simple interest, $I = P \times R \times T$. We call I the **subject of the formula**.

Example
Go through this example slowly. Make sure you understand it. This example will help you do the next activity.

A recipe for roasting a piece of beef says it take 30 minutes for each kilogram plus 20 minutes extra to cook properly. Find a formula for the time needed to roast a piece of beef. Time is represented by t and mass by m.

Mass in kilograms (m)	**Time in minutes (t)**
1	$30 \times 1 + 20 = 50$
2	$30 \times 2 + 20 = 80 = 1hr20$
3	$30 \times 3 + 20 = 110 = 1hr\ 50$
5	$30 \times 5 + 20 = 170 = 2hrs50$

ANSWERS ON PAGE 109

Therefore, the formula for time is: $\boldsymbol{t = 30m + 20}$
(i.e. time = 30 × mass + 20).

This formula will help you find how long a piece of beef weighing 4 kilograms should stay in the oven. We have to find t if $m = 4$. Use the formula and this gives:

$$\begin{aligned} t &= 30m + 20 \\ &= 30 \times 4 + 20 \\ &= 140 \end{aligned}$$

So, a piece of beef of 4 kilograms should stay in the oven for 140 minutes or 2 hours 20 minutes.

ACTIVITY 10

A friend borrows R800 from you and agrees to repay you at the rate of R20 each month. You kindly choose not to demand any interest (like the bank does). Copy and complete the table below and then make a formula relating the amount which your friend still owes (A) after any number of months (t).

Number of months (t)	**Amount still owing in rands (A)**
1	$800 - (20 \times 1)$
2	$800 - (20 \times 2)$
3	$800 -$
4	
5	
10	

Therefore, the formula is $A =$ ....

Use the formula to find:

1. the amount your friend owes you after 7 months
2. the amount your friend owes you after 15 months
3. how many months it will take for your friend to pay you back the R800.

Changing the subject of the formula

Look back at the example on roast beef. We found the formula:

$t = 30m + 20$.

We say t is the subject of the formula. It is possible to make m the subject of the formula. If, instead of finding the time t when the piece of meat is 4 kilograms, we are given the time, we can find the mass of the meat that will take 140 minutes in an oven.

ANSWERS ON PAGE 109

Do we have to make a table again and find the new formula? No. We already have a formula that links time and mass. What we have to do now is to make mass m the subject of the formula.

Remember the formula, $t = 30m + 20$. Now we want to have m alone on one side of the formula.

Step 1: Subtract 20 from both sides.

$t - 20 = 30m + 20 - 20$

$t - 20 = 30m$

Step 2: Divide both sides by 30 so that we are left with $1m$:

$(t - 20) \div 30 = 1m$, which is the same as:

$(t - 20) \div 30 = m$

$$m = \frac{(t - 20)}{30}$$

This formula will help to find out how many kilograms of meat we will need 140 minutes to roast. We have to find m given $t = 140$. Then using the new formula:

$$m = \frac{(140 - 20)}{30}$$

$$m = \frac{120}{30}$$

$$m = 4$$

Hence we need a piece of meat that weighs 4 kilograms. If you look at the example before Activity 10, the answer there is what we started with here.

ACTIVITY 11

Recall the formula you used in Activity 10. $A = 800 - 20t$. Make t the subject of this formula.

Note that you can also first substitute in the values you know and then make the unknown variable, in this case t, the subject of the formula.

Use the formula to answer these questions:

a) After how many months will your friend owe you R660?

b) After how many months will your friend owe you R500?

c) How many months will it take your friend to pay you back the R800?

Look at your results and the results for activity 10. Do you see any link between the two?

ANSWERS ON PAGE 110

Summary

You saw that it is an important mathematical skill to be able to 'translate' from a language, such as English, into mathematical symbols, terminology and notation used in algebra. It is also important to be able to understand what the algebraic notation and symbols represent so that you can work efficiently with them in solving problems. Algebra enables you to solve problems not only in mathematics but also in other disciplines, for example, science, geography and biology. In order to use algebra, you need to write the problem as an **algebraic expression or equation**. You will then be able to solve the problem.

An algebraic expression consists of **variables** and **constants**. Variables are symbols or letters which stand for any value. Constants have a specific value. Variables can have **coefficients**. A coefficient is the number or letter that is multiplied by the variable.

An algebraic expression is made up of one or more **terms**. A term is a combination of numerals and variables involving multiplication or division. Terms are separated by a plus (+) or a minus (–) sign. Once you have expressed your problem algebraically, you make a **formula** that will help you solve the problem. A formula is an algebraic equation that can be solved to find a particular quantity.

Self-assessment checklist:

Are you able to:

- understand the terms algebra, variable, term, constant and coefficient, formula, equation and expression
- translate a word problem into an algebraic expression or equation
- solve a simple equation
- change the subject of a formula.

SELF-CHECK EXERCISE

1. Write an algebraic expression for each of the following problems:

   a) Thato bought 4 times as many shares on the Johannesburg Stock Exchange as Steve, and Steve bought 3 times as many shares as Peter.

   b) Trucking transportation rates are x rands per metric ton per kilometre. How much does it cost, in rands, to transport one dozen cars which weigh two metric tons each, n kilometres by truck?

2. Solve the following equations:

   a) $x + 7 = 15$
   b) $2y - 5 = 15$

c) $4(a+3)=28$

d) $\frac{n}{3}=4$

e) $\frac{x}{2}+1=6$

3. A student responded to all the 22 questions on a test and received a score of 63,5. The scores were calculated by adding 3,5 points for each correct answer and subtracting 1 point for each incorrect answer. If c represents the number of correct answers;

a) Write an algebraic expression to show the number of incorrect answers (in terms of c).
b) Write an algebraic expression to show the value of the points the student received for:
   i) correct answers
   ii) incorrect answers
c) Write down an algebraic equation that shows that the sum of the value of the correct and incorrect answers is 63,5.
d) Solve this equation.
e) How many questions did the student answer incorrectly?

4. Use the following algebraic expression to answer the questions that follow: $3x+2y-6$

a) How many terms does this algebraic expression contain?
b) What is the constant?
c) Write down the coefficient of x.
d) Write down the coefficient of y.

5. The formula for speed is given by: $s=\frac{d}{t}$

Where: s = speed
d = distance
t = time

a) Make t the subject of this formula.

b) Mr Jones drove from Soweto to Johannesburg at an average speed of 100 km/h. The distance between Soweto and Johannesburg is 10 km. Write down the following:
   i) Mr Jones' average speed (s)
   ii) The distance Mr Jones had to drive (d)

c) Find the time (t) Mr Jones took to get from Soweto to Johannesburg.

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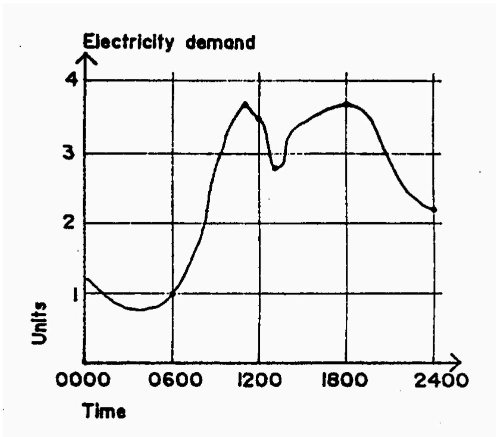
5. Foundations of graphs: Axes and coordinates

Introduction

This lesson is divided into two main sections. The sections are drawing the axes of a graph; and plotting and reading points from a graph.

A graph is a drawing that gives useful information. It is a mathematical diagram which shows how two or more sets of numbers or measurements are related.

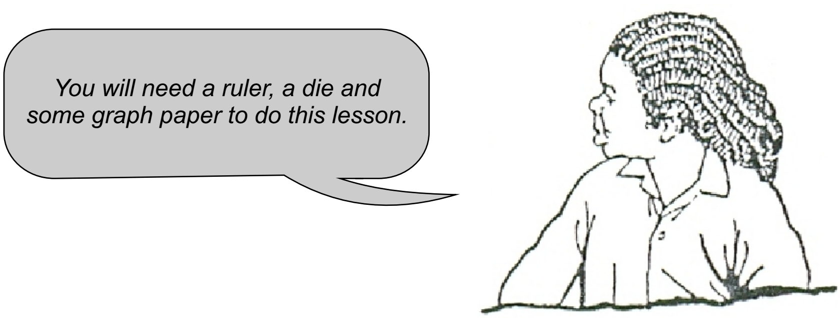
You need to know how to read a position from a graph. This knowledge will allow you to describe exactly where a certain point is on a graph. You need to know how to read graphs because they are often used in newspapers and other media sources.



This graph comes from Eskom's report in a newspaper. The graph shows how much electricity is used during certain times of the day. From the graph we can see how much electricity is used at 12:00 noon or 18:00 pm. From the graph we can see that most people in South Africa need electricity around noon when they prepare lunch, and at 6 o'clock in the evening when they prepare supper. We will be referring you to this graph throughout the lesson.

In this lesson you will:

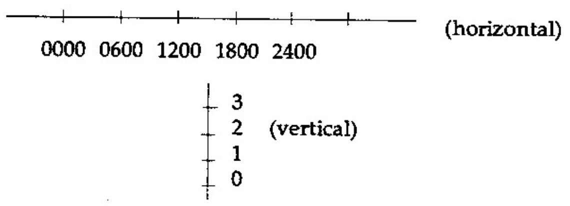
- identify the x - and the y - axis of a graph
- draw x and y - axes
- find coordinates on a graph
- use coordinates to describe a position on a graph
- learn how to plot a point on a graph.



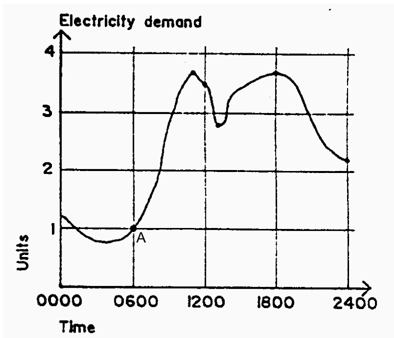
Axes

In order to describe the position of points on a graph, we make use of a set of **rectangular axes**. On the Eskom graph, the axes are labelled "time" and "units".

Remember the horizontal and vertical number lines from Lesson 4 of unit 1?



On the graph the vertical number line is called the *y*-coordinate axis or *y*-axis. The horizontal number line is called the *x*-coordinate or *x*-axis. The two axes cross each other at 0, which we call the **origin** (i.e. the starting point).



When we put the two number lines together, we get a graph with rectangular axes. Let us look at point A on the graph. To find the position of point A, we draw horizontal and vertical lines to link the point to the *x*- and *y*-axes. We end up with a rectangle.

The numbers on each axis are always equal distances apart. These distances represent equal quantities of something. For example, from the Eskom graph, the distances on the *x*-axis represent equal quantities of time. On this axis each square represents 6 hours.

The arrows on the axis show that the numbers can continue. We usually stop at the numbers that we need.

ACTIVITY 1

Draw a graph with rectangular coordinates with:

a) *y*-axis going from –3 to +3 and *x*-axis going from –4 to +4

b) *y*-axis going from 0 to +4 and *x*-axis going from 0 to +4

ANSWERS ON PAGE 110

Points on the graph

Each point on the graph is described by two numbers: the *x*-coordinate and the *y*-coordinate.

The *x*-coordinate tells how far the point is to the right or to the left of the vertical (*y*-coordinate) axis.

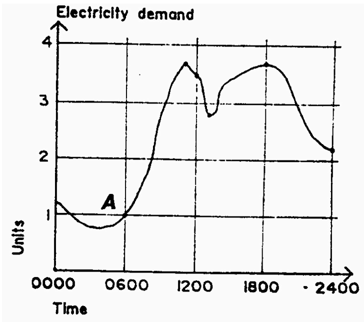
A positive x-coordinate indicates the point is to the right of the y-coordinate axis, and a negative x-coordinate indicates the point is to the left.

For example look at +3 on the horizontal axis, +3 is 3 steps to the right of the vertical axis. In the same manner, –2 is 2 steps to the left of the vertical axis.

The x-coordinate is often called the x-value or just x.

To find the position of point A

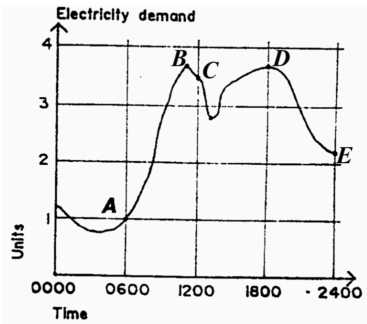
Step 1: To find the x-coordinate:
Start at point A. Move directly up or down from point A to the x-coordinate axis. This point is the x-coordinate.



Step 2: To find the y-coordinate:
Start at point A. Move to the right or left of point A to the y-coordinate axis. This point is the y-coordinate.
The position of point A is described by x-coordinate = 0600 and y-coordinate = 1.

ACTIVITY 2

Find the coordinates of points B, C, D and E on the Eskom graph.



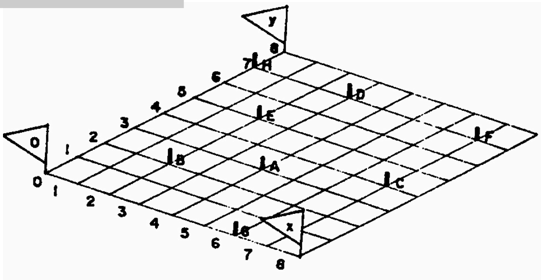
ANSWERS ON PAGE 111

Writing coordinates as an ordered pair

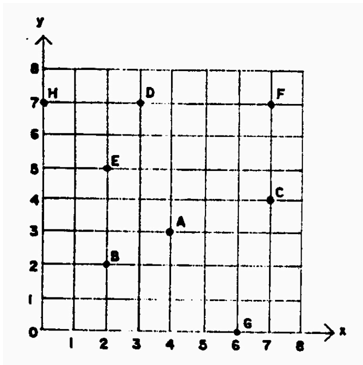
An **ordered pair** is made up of two numbers, the x and y coordinates. These numbers are written in a specific order within brackets. For example, (2400;2,2) is an ordered pair.

In an ordered pair, we write the x-coordinate first within brackets, followed by a semi-colon and then the y-coordinate. That is, (x-coordinate; y-coordinate) or just (x; y).

ACTIVITY 3



A treasure hunt at a competition is laid out using string and pegs. You have to find the point at which the treasure is hidden.
Using x and y axes it looks like this:



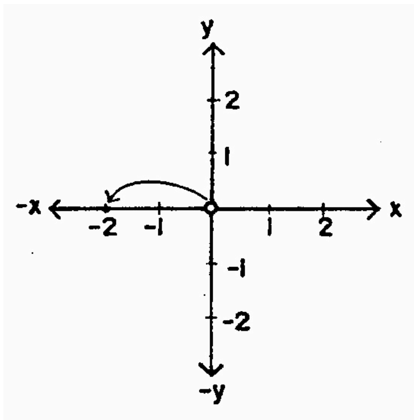
Identify the points at which the treasure could be hidden. For example, A(4;3). (The x-coordinate is always given first).

a) Adam guesses that the treasure is at the point A(4;...)
b) Bill's guess is B(...;2)
c) Caroline's guess is C(7;...)
d) Dave's guess is the point D with coordinates (...;...)
e) The x-coordinate of Eve's guess E is ... and the y-coordinate is ...
f) Fatima's guess is F(...;...)
g) The y-coordinate of Gugu's guess G is ...
h) Harry's guess H has an x-coordinate of ...and a y-coordinate of .....

ANSWERS ON PAGE 111

Plotting points on a graph

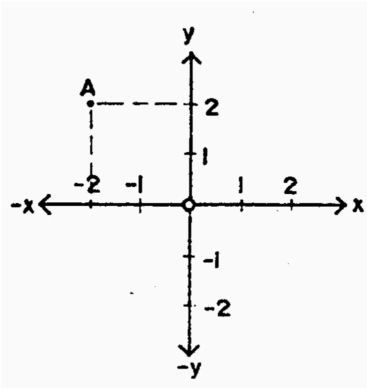
Let us look at how we draw points on a graph. We say we **plot** points on a graph. For example, if we want to plot the point (–2 ; 2), we follow these steps:



Step 1:

Start at $x = 0$.
Find the x-coordinate on the x-axis, i.e. –2. We find the coordinate –2 by moving to the left of $x = 0$. We move to the left because the x-coordinate is negative.

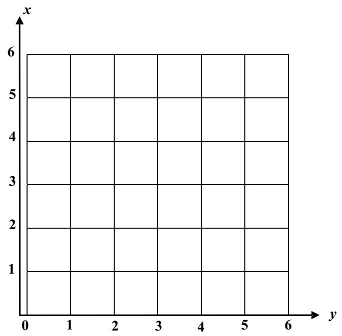
Step 2: From the x-coordinate point on the x-axis, move directly up for a positive y-coordinate. That is, move up to +2 on the y- axis.



Step 3: Use any letter to label this point.

ACTIVITY 4

Prizes are buried at points S(2;3), T(3;0), U(0;6) V(4;1), W(1;4), X(6;6). On squared paper, draw axes *x* and *y* at right angles, and number them from 0 to 6. Plot the positions of the prizes.



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Summary

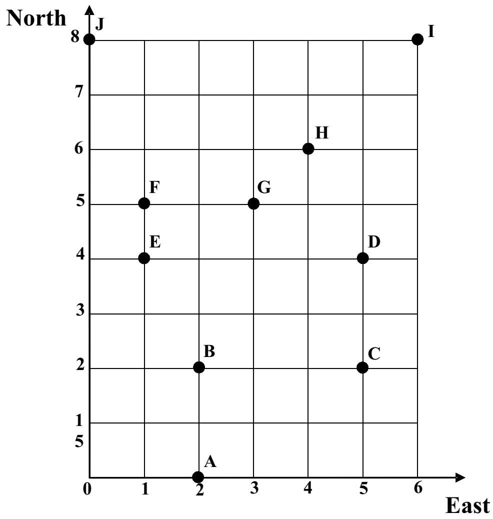
You saw that to draw a graph you need a horizontal and a vertical axis, usually called the *x*- and *y*-axes. You get the *x*- and *y*-axes by putting together a horizontal and a vertical number line. In mathematical terminology we call this a "Cartesian plane". You learned how to plot a point and how to find a point on the graph using the *x* and *y* coordinates. The *x*-coordinate tells how far the point is to the right or to the left of the *y*-axis. The *y*-coordinate tells how far the point is above or below the *x*-axis.

Self-assessment checklist:

Are you able to:
- identify the *x*- and *y*-axes of a graph
- plot a point on a graph
- read the coordinates of a point on a graph.

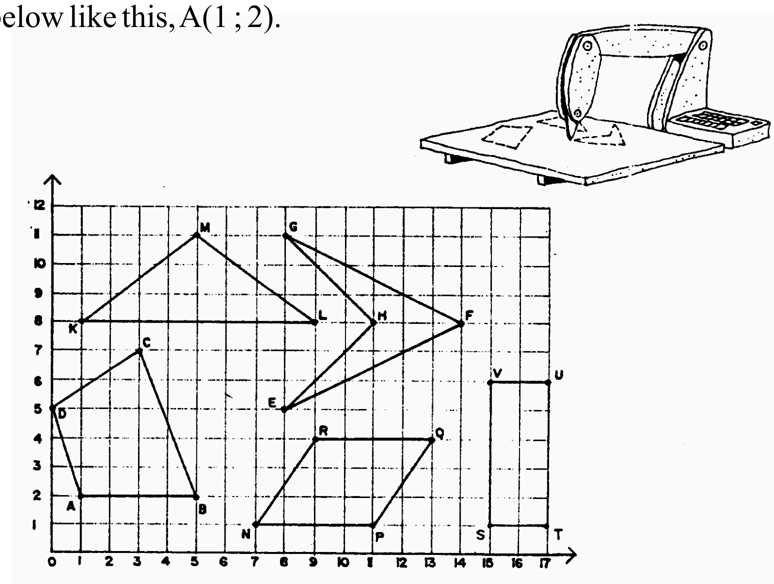
SELF-CHECK EXERCISE

1. Mike was given the basic grid map of the blocks of his neighborhood and told to drop off newspapers at the points marked.
   a) Write down the coordinates of each point.
   b) Write down directions for Mike to go from his first point A to each point (in alphabetical order):



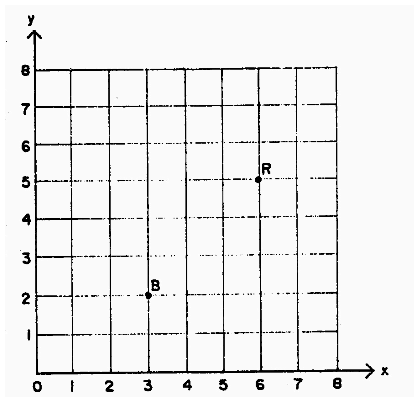
The directions have been started for you:
From A go North 2 blocks to B
From B go 3 blocks East to C

2. In some factories, machines for cutting shapes are controlled by computer. To give the machine its orders, the computer must be told the coordinates of the corners of each shape. Write down the name and coordinates of each corner of each of the shapes shown below like this, A(1 ; 2).



3. Robbers based at R rob a bank at B.
The lines show a network of roads. Copy this onto squared paper.
The police set up road-blocks at (1;2), (2;3), 3;0), (3;4), (4;1), (4;3) and (5;2).

a) Mark the roadblocks.
b) Can the robbers escape from the bank?
c) If you think the robbers can escape, draw their shortest route from B to R.
d) At what other point should the police have set up a roadblock?



ANSWERS ON PAGE 126

6. Foundations of shapes, perimeter and area

Introduction

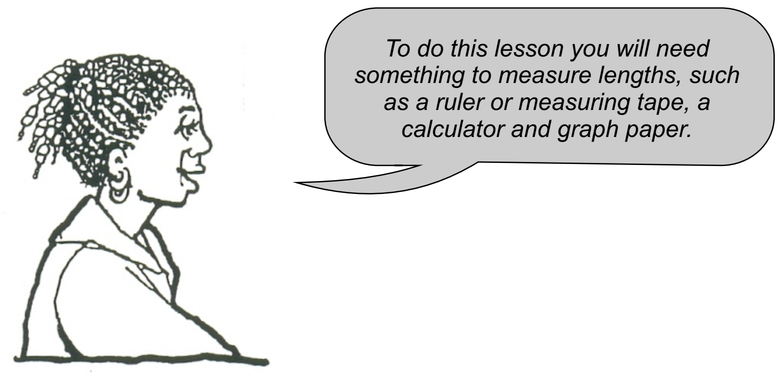
In this lesson you will be studying a few shapes and measuring **lengths**. You will also be working out the **perimeter** and **area** of the shapes. In your daily lives you might measure the length of your plot, work out the quantity of material you would need to make your clothes or work out the number of tiles you would need to tile your kitchen floor.

These examples and many others involve the concept of measurement; that is, measuring length and area. There must be other examples that you experience every day, either at home or at work, that involve measuring objects.

Remember in Unit 1, you learned about society's need for measurement. If you have forgotten how to measure things in cm and mm, please look back at these lessons. You will need to understand these previous lessons in order to develop measurement further in this lesson.

In this lesson you will:

- measure using standard units of measure the way mathematicians do
- know and understand the meaning of perimeter, circumference and area
- calculate the perimeter and areas of shapes
- discover the link between length, perimeter and area
- link your everyday measurements and the standard form presented in mathematics
- reason by using comparisons.



All objects around us have a particular shape. We use shapes to describe objects. For example, a window may be rectangular, or a table may be rectangular. Mathematicians study the properties of shapes. These properties enable them to solve other mathematical problems. Let us look at some of these shapes.

Squares and rectangles

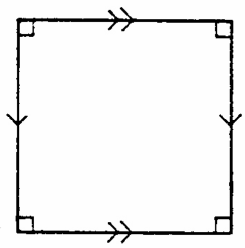
Quadri - four
Lateral - side

A four-sided shape is known as a **quadrilateral**. We are now going to look at two different types of quadrilaterals.

Parallel: when two lines do not meet, and they have constant distance between them

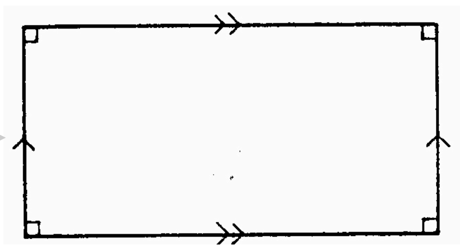
A quadrilateral with two pairs of **parallel** sides, all four sides equal and all of the angles **right-angles** is known as a **square**.

The block indicates that the angle is a right-angle.



A quadrilateral with 2 pairs of parallel sides and opposite sides equal, with all of the angles right-angles is known as a rectangle.

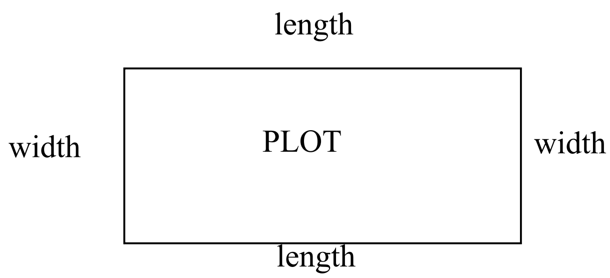
The arrows indicate that sides are parallel.



Both a rectangle and a square are **rectangles**. A rectangle has 2 pairs of parallel sides and all its angles are right angles.

The measurement from one end to the other along the longest side of a rectangle is known as **length**. The measurement along the other side is known as the **width** (or breadth).

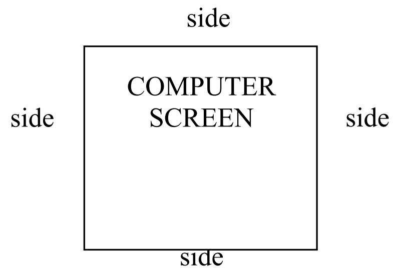
If you want to measure the distance around your rectangular plot, then you will measure the four sides of your plot.



You will measure the two lengths together with the two widths. That is:
length + length + width + width = 2 lengths + 2 widths
= 2 × (length + width)

This measurement is called the perimeter:
Perimeter of rectangle = 2(length + width)

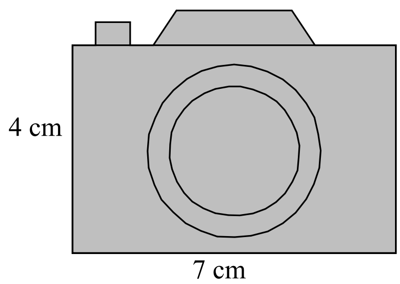
In a square the length = width, so we just talk about the **side**. If we want to measure the distance around a square computer screen, then we need only know the length of one side of square screen.



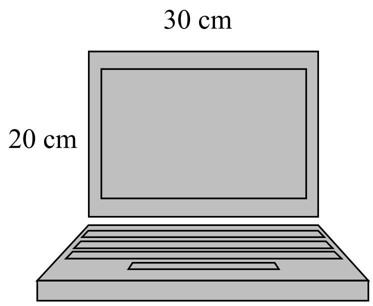
We can then add the lengths of the four equal sides. That is:
side + side + side + side = 4 sides
Perimeter of square = 4 × sides

ACTIVITY 1

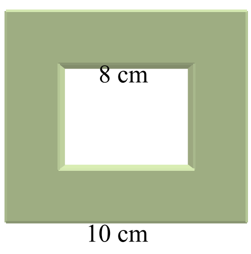
1. Write down the indicated measurements in the pictures below:
   a) Length of rectangular part of camera:
   Breadth/width of rectangular part of camera:



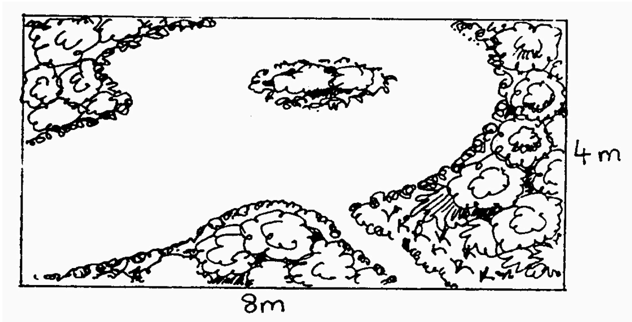
   b) Length of computer screen:
   Width of computer screen:
   Perimeter of computer screen:



   c) Length of outer side of square photo frame:
   Length of inner side of square photo frame
   Perimeter of outer part of photo frame:
   Perimeter of inner part of photo frame:



2. Find the perimeter of the garden in the picture below.



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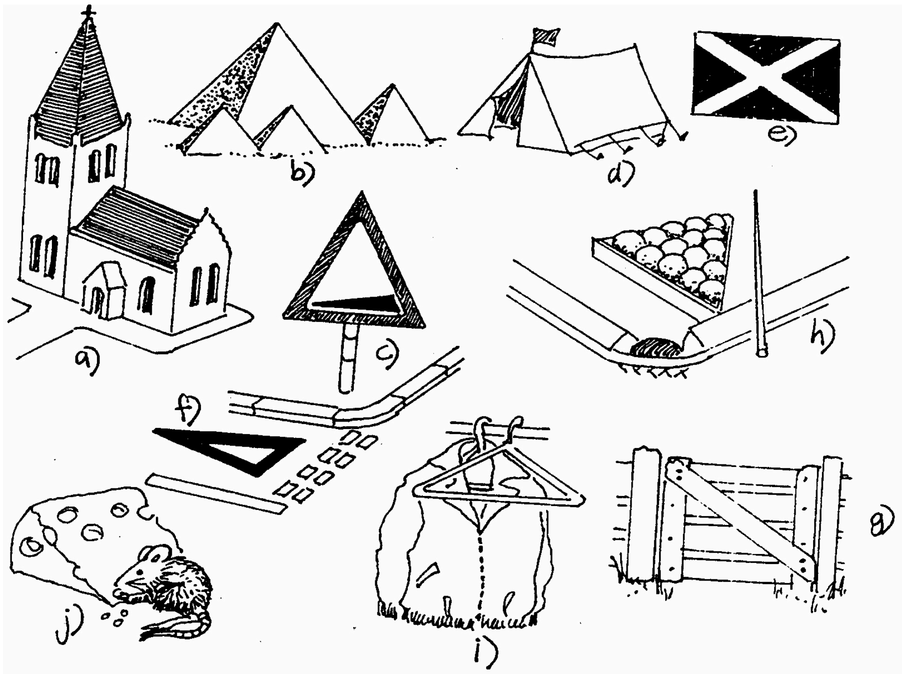
Triangles

ACTIVITY 2

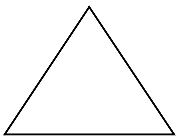
How many triangles can you count?



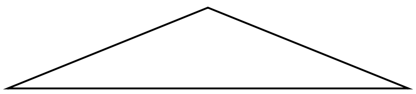
ANSWERS ON PAGE 112

There are different types of triangles. Let us look at the most important ones you need to be able to recognise.

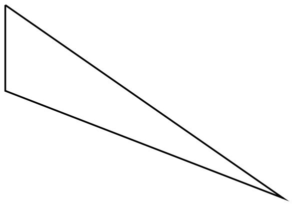
We can have a triangle with all its 3 sides equal. Such a triangle is known as an **equilateral** triangle. The 3 angles of an equilateral triangle are also equal.



We can have a triangle with two sides equal. Such a triangle is known as an **isosceles** triangle. An isosceles triangle also has equal base angles.

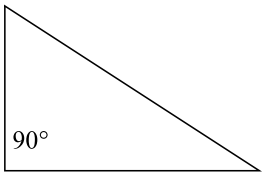


A triangle with none of the 3 sides equal is known as a **scalene** triangle.



We can also have a triangle with a right angle (90°) as one of its angles. This type of triangle is called a **right-angled** triangle.

A right -angle is equal to 90^0.

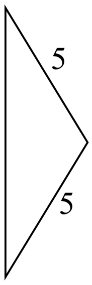


ACTIVITY 3

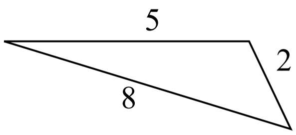
Match the number of the triangle with the correct letter indicating the type of triangle that it is:

A. Right-angled scalene triangle
B. Equilateral triangle
C. Scalene triangle
D. Isosceles triangle
E. Right-angled isosceles triangle

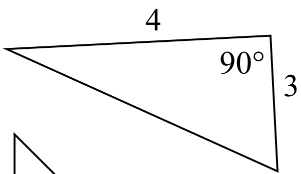
(1)



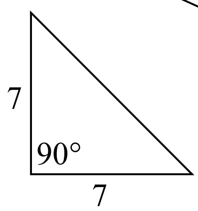
(2)



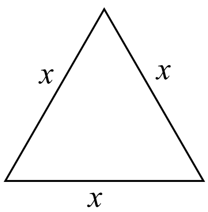
(3)



(4)

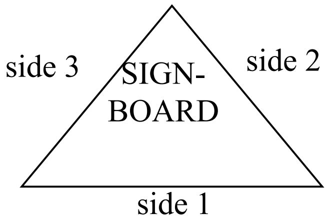


(5)



ANSWERS ON PAGE 113

If you want to measure the distance around a triangular sign-board, you will measure each of the three sides of the board.

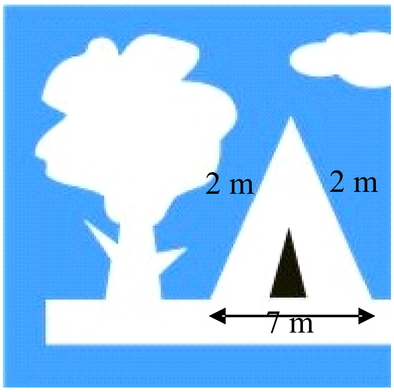


You can then add the lengths of the three sides. That is:
side 1 + side 2 + side 3

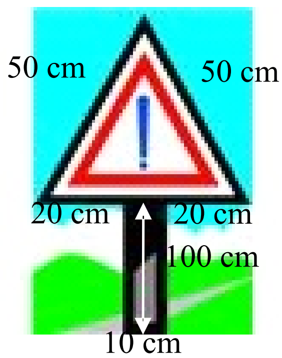
Remember that this measurement is known as the **perimeter:**
Perimeter of any triangle = side 1 + side 2 + side 3

ACTIVITY 4

1. Find the perimeter of the tent in the picture below:



2. Find the perimeter of the traffic sign:

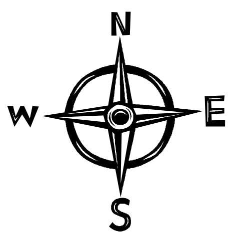


ANSWERS ON PAGE 113

Circles

You will probably agree that circular shapes are as familiar to us as rectangles, squares and triangles.



The earth, our home, is round. We can say that the earth has a spherical shape, and is called a **sphere**. For thousands of years people thought that the world was flat and that they would fall off the edge if they travelled too far.

The astronauts who travelled in space finally proved that the earth is round when they took pictures of the earth from their spaceships. The pictures showed a blue and green sphere floating in the black sky.

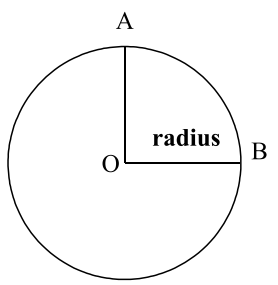
Can you imagine some of the consequences for us if the circular shape did not exist?

Parts of a circle

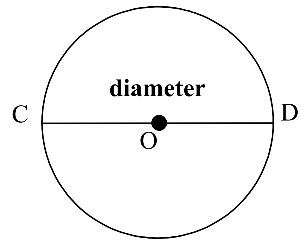
It is difficult to draw a circle free-hand. In earlier times, and even today in some communities, circles were drawn using a length of string and a stick or writing instrument such as a pencil. Some communities still use this method when measuring out the floor for the construction of their houses.

Today mathematicians use a pair of compasses to draw circles accurately.

A pair of compasses



A circle with centre O and **OB** a **radius** and OA another radius.
OA = OB



A circle with centre O and **diameter CD**. OC is a radius and so is OD.
diameter = radius + radius

1 radius
2 radii

The **radius** (*r*) is the line drawn from the centre of the circle to the line around the outside of the circle. The line around the circle is called the circumference. The radius can be drawn anywhere from the centre to the circumference and it will still be the same length. Any straight line drawn from a point on the circumference through the centre to another point on the circumference is called the **diameter** (*d*). So the diameter is made up of two radii or we can say that diameter = radius + radius = 2 × radius.

Circumference of circles and the calculation of π (pi)

Do you remember that the perimeter of a shape is the length of the line around the outside of the shape? When you found the perimeter of a rectangle you measured the length of the line round the outside of the rectangle. The line around the outside of the circle is called the **circumference**. So the length of the circumference is the perimeter of the circle.

The circumference can be found by using a piece of string to measure around the circle, or by rolling the shape along a ruler. You can try to measure the circumference of the circles on page 60 if you have a piece of string. This approach can be awkward and not very accurate if we have to work with very small or very large circles. Mathematicians use a formula for calculating the circumference of a circle.

ACTIVITY 5

Complete and study the table below that presents the radius and circumferences of a number of circles:

	radius	**circumference**	**diameter (2 × radius)**	$\frac{\textbf{circumference}}{\textbf{diameter}}$
1	1 cm	6,283 cm	2 cm	$\frac{6,283}{2} = 3,1415$
2	2 cm	12,566 cm	4 cm	$\frac{12,566}{4} = 3,1415$
3	3 cm	18,850 cm		
4	4 cm	25,133 cm		
5	5 cm	31,416 cm		
6	10 cm	62,832 cm		
7	20 cm	125,664 cm		
8	50 cm	314,159 cm		

You should notice that the answers in the final column of the table are all approximately equal to 3,142 if we round off to 3 decimal places.

We do not know the exact value of this number as it is an irrational number in that it has no recurring decimal pattern and does not terminate. Because of the importance of this number in finding the circumference and area of a circle, it is given a symbol π (pi) which is a letter of the Greek alphabet.

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You will see there is a π key on your calculator.

This is a 2[nd] function key which will give you a long decimal value for π. But for this course and in the examinations, you are expected to work with the approximate value of 3,142 for π in all your calculations unless otherwise indicated.

≈ indicates 'approximately'. So π is approximately equal to 3,142.

So we now know the following:

$$\therefore \pi = \frac{\text{circumference}}{\text{diameter}}$$

We can use this to help us find a formula for the circumference of any circle by changing the subject of the formula (we covered this in Lesson 5 of this unit). Let *c* represent circumference and *d* represent diameter. Then we have:

$$\pi = \frac{c}{d}$$

$$\pi \times d = \frac{c}{d} \times d$$ (Multiply both sides of the equation by *d* to get *c* on its own)

$$\pi d = c$$

$$c = \pi d$$

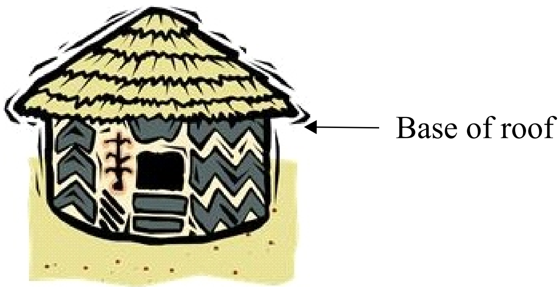
To calculate the circumference (perimeter) of a circle we use the formula:
Circumference of a circle $= \pi \times \text{diameter}$

We know that the lengths of 2 radii make up the length of a diameter. So we can say the circumference (or perimeter) is also equal to:

Circumference of a circle $= \pi \times \text{diameter}$
$= \pi \times (2 \times \text{radius})$

ACTIVITY 6

1.



a) The floor of a circular hut has a radius of 7m. What is the circumference of the floor of the hut? Round off your answer to the nearest metre.

b) The diameter of the base of the roof of the hut is 20 m. What is the circumference of the base of the roof? Round off your answer to the nearest metre.

2. Copy and complete the following table (remember you may need to change the subject of the formula):

	radius	diameter	circumference
a)	9 cm		
b)		30 cm	
c)			34,562 cm
d)			72,8944 cm
e)		90 cm	

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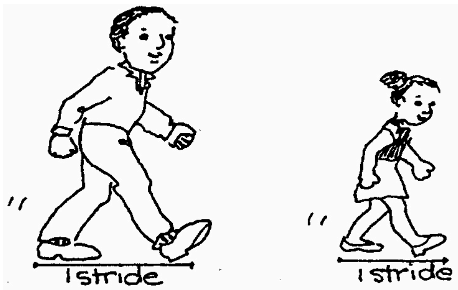
Different methods of measuring

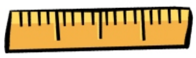
In early societies, people used different parts of their bodies to measure lengths. This way of measuring proved to be one of the most effective ways of measuring. However, this method of measuring does not give correct (or accurate) measurements. Different people give different measurements for one length. For example, the length of your stride will be longer than the length of the stride of a 4 year old girl.



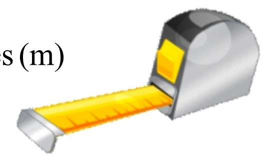
Why is accuracy important? Imagine you buy a window frame that is smaller than the one you need. Instead of measuring 1,5 metres accurately, you estimated 1 metre. How would you feel when something like this happens? Think of other situations where an accurate measurement is needed.

With the introduction of technology, more accurate instruments for measuring were introduced by scientists. These instruments include:

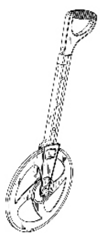
- Rulers in millimetres (mm), centimetres (cm) or in metres (m)



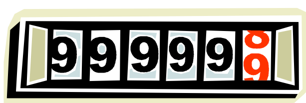
- Measuring tapes in millimetres (mm) or metres (m)



- Trundle wheels in metres (m) These make a click each time the wheel rolls a metre.



- Odometers. These measure speed and distance in vehicles in kilometres per hour (km/h).



- Scales in milligrams (mg), grams (g) or kilograms (kg)



You need to make sure that you can recognise and read all these measuring instruments.

Units of length

Remember in Unit 1 lesson 5 you learned about where standard units come from. You also learned how to convert from one unit to another. The standard units commonly used to measure length are millimetres (mm), centimetres (cm) and metres (m).

10mm 10cm

In different countries different units of measurement are used. In South Africa, we use mm, cm and m to measure lengths. Some countries use inches and yards. You have to convert these units to the units used by everybody so that you do not encounter problems when you have to use units at work or at any other place.

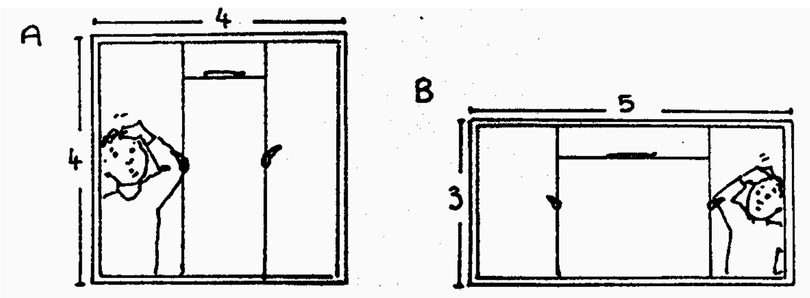
10 mm = 1 cm
100 cm = 1 m
1000 m = 1 km

Units of area

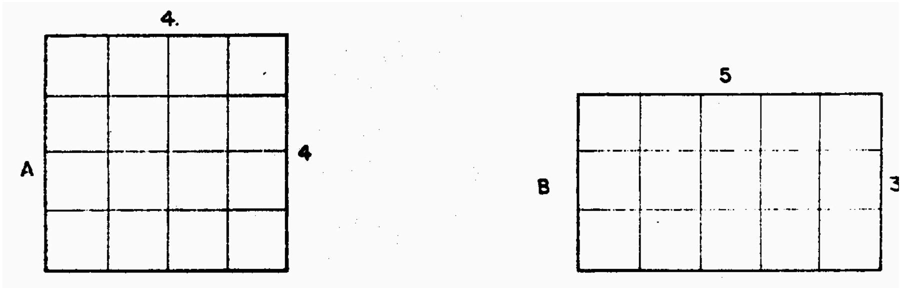
Area is a measurement of a region of a flat surface. We use area to measure the size of our plots, to calculate the total material we would need to make a curtain, to measure the size of the space on the floor to fit tiles. There are many other examples where we use area.

ACTIVITY 7

Thabo is adding a room to his house. He goes to a hardware shop to find out the sizes and shapes of window frames.
He sees two that he likes, and wonders which would let in more light.



The actual unit of length does not matter at present.
Here are two rectangles on squared paper to represent the windows.



a) Which window uses more glass?
b) Which window lets in more light?

ANSWERS ON PAGE 114

Measurement of area

When we want to *compare* the areas of two figures, we choose the most convenient unit of area. When we choose the actual shape of a `tile' to use, we make sure that the final result can be written in units which everybody will understand.

We often need to measure the area of a figure. To do this, we compare it with some fixed standard unit of area which is familiar to everyone. The shape of the standard unit of area can still be chosen for our own convenience. Two common units of area are the square centimetre (cm^2) and the square metre (m^2).

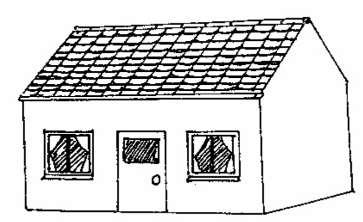
Areas of rectangles and squares

ACTIVITY 8

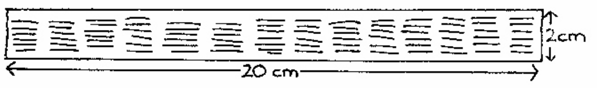
1. To find the areas of windows A and B, you counted the number of squares.

   Counting squares is all right when the numbers are small, but what about Mr. Mosa's problem? The roof of his house needs to be retiled. It is not easy to count the tiles one by one, so he looks for a better way.

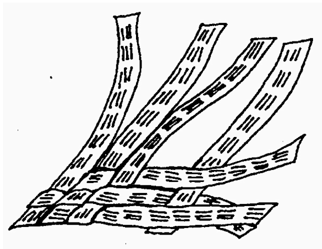
   a) How many tiles are there?
   b) How many tiles are there in each row?



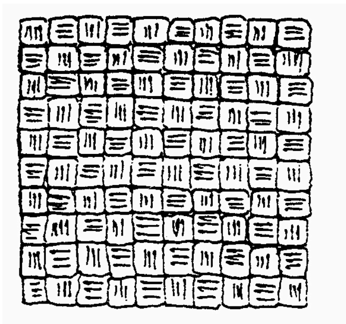
2. Nima is making place-mats for the dinner table. She takes strips of material like this:



and weaves them like this:



into a square-shaped mat like this:



a) What is the area of one strip of the material?
b) What is the area of the top of the finished mat?

ANSWERS ON PAGE 114

Have you noticed that the number of rows (the width) × the number of tiles in each row (the length) = total number of tiles on one side of the roof (area)? If not, in question 1 of the activity above, multiply the answer in a) by the answer in b), and see if it does not give you the answer in c). This finding leads to the general formula of area used by mathematicians for rectangles.

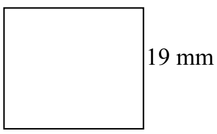
This is:
Area of rectangle = length × width

As we mentioned at the beginning of this lesson, the four sides of a square are equal so we can just refer to the length of one side as length = width. So the area for a square can be written as:
Area of square = side × side

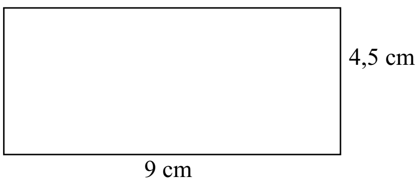
ACTIVITY 9

1. Calculate the areas of a) the square and b) the rectangle:

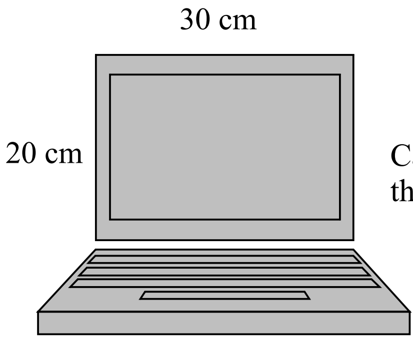
a)



b)



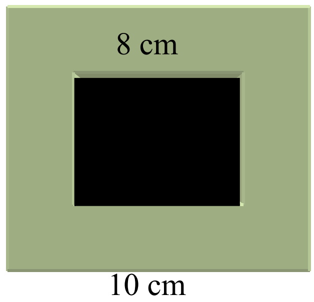
2.



Calculate the area of the screen of the laptop.

3. a) Calculate the area of the glass surface (the black part) of this square photo frame.

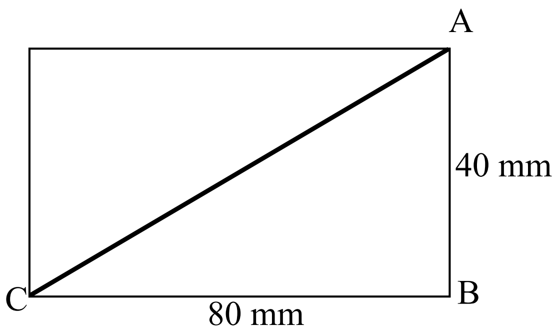
b) If glass costs R0,49 per cm^2, how much will it cost you to replace the glass in this frame?



ANSWERS ON PAGE 115

Areas of triangles

Remember that the area of a rectangle is: length × width. Look at the following rectangle:



The area of this rectangle is $80 \times 40 = 3\,200$ mm^2.

Can you guess the area of triangle ABC?

Yes, it is 1 600 mm^2. The diagonal AC cuts the area of the rectangle in half.

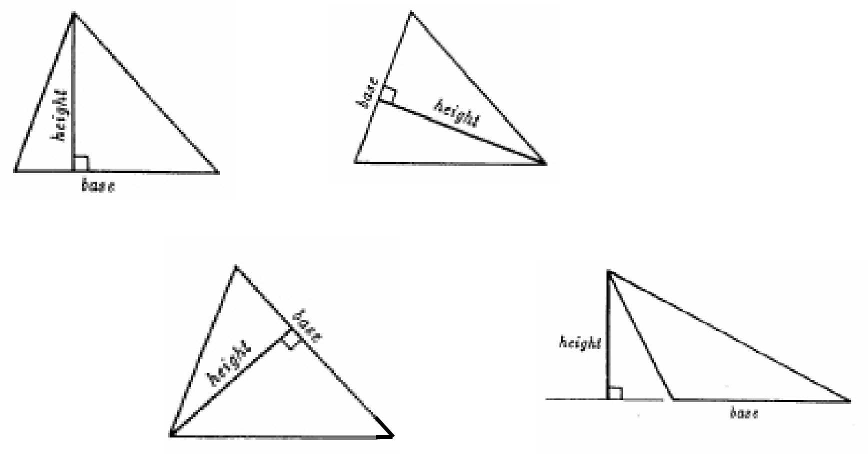
Based on this we can formulate a formula for the area of a triangle:

Area of triangle = $\frac{1}{2} \times \text{base} \times \text{height}$

It is important that the base of the triangle and the height are **perpendicular** to each other.

Perpendicular (⊥): two sides meet at an angle of 90º.

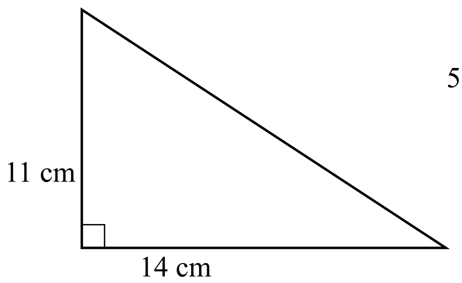
The diagrams illustrate that two of the sides meet at an angle of 90^0.



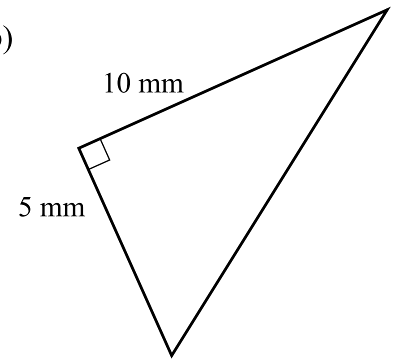
ACTIVITY 10

1. Calculate the area of the following right-angled triangles:

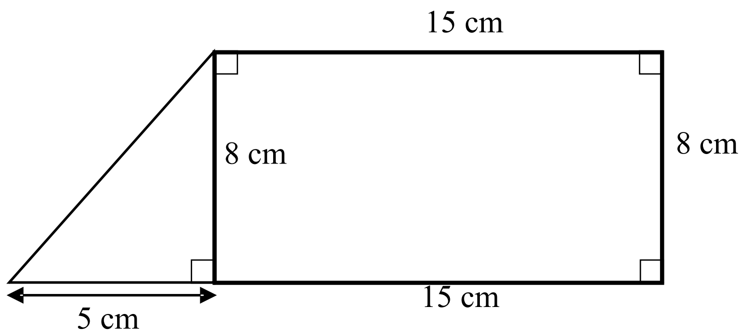
a)



b)



2. Calculate the area of the following figure:



ANSWERS ON PAGE 115

Areas of circles

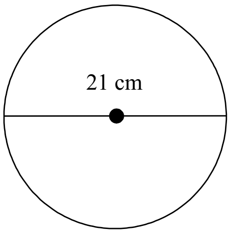
You have learnt how to work out the area of squares, rectangles and triangles. We can also calculate the area of a circle if we know what the radius is. We do this by squaring the radius ($r \times r = r^2$) and multiplying this number by Pi (π) = 3,142. So the formula for the area of a circle is calculated using the following formula:

$$\text{Area of circle} = \pi \times \text{radius} \times \text{radius} = \pi \times (\text{radius})^2$$

ACTIVITY 11

1. Calculate the areas of the circles:

   a) with radius = 3 cm b)



2. Calculate the amount of material, in cm^2, needed to cover the flat front face of the dart board:

3. This clock has a grey metal outer lining and a glass cover. The length of the radius (the longer black arrow) from the centre of the clock to the outer part of the clock is 21 cm. And the radius of the shorter black arrow to the inside part of the metal lining is 18 cm. Calculate the area of the metal lining in cm^2.

ANSWERS ON PAGE 115

Self-assessment checklist:

Are you able to:

-
-
-

Summary

In this lesson you learned about the shapes of rectangles, squares, triangles and circles and how to calculate the perimeter of each:

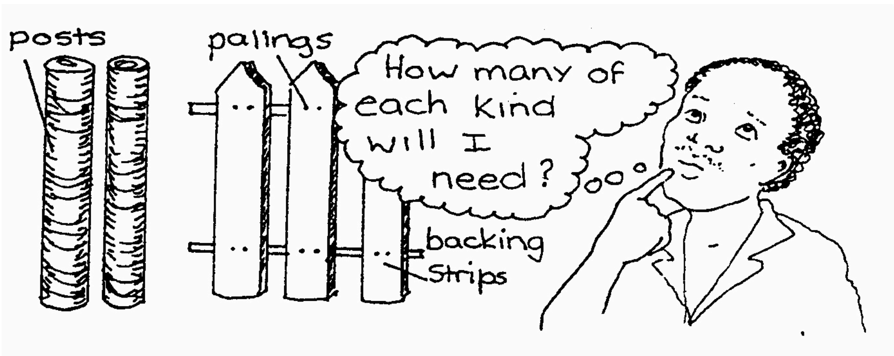
Perimeter of a rectangle	=	2 × (length + width)
Perimeter of a square	=	4 × side
Perimeter of a triangle	=	side 1 + side 2 + side 3
Circumference of a circle	=	2 × π × radius or π × diameter

You also learned about the relationship of area in these shapes:

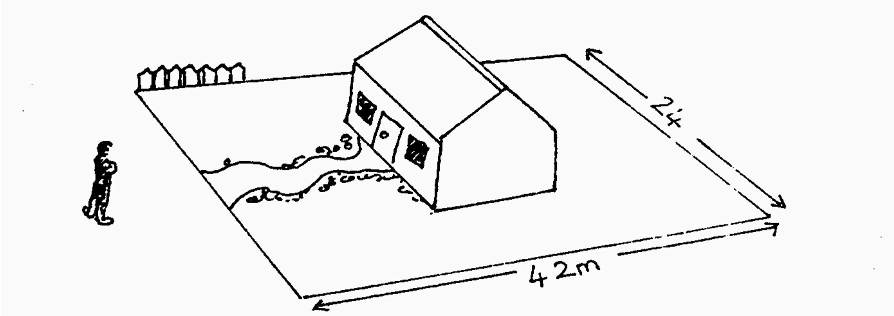
Area of a rectangle	=	length × width
Area of a square	=	side × side
Area of triangle	=	$\frac{1}{2}$ × base × height
Area of circle	=	π × radius × radius

SELF-CHECK EXERCISE

1. Seca wants to build a fence round his garden. He needs palings, backing strips and posts.

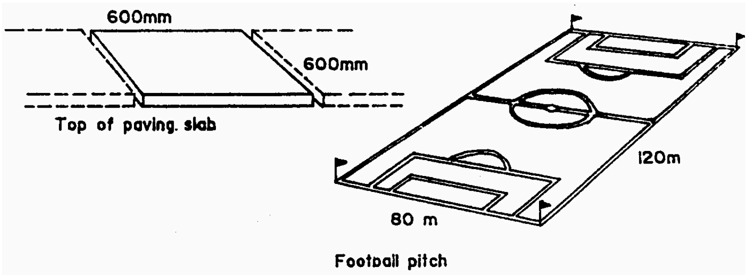


This is a drawing of his rectangular garden. Calculate the total length of fencing needed. (He will put in a gate later.)



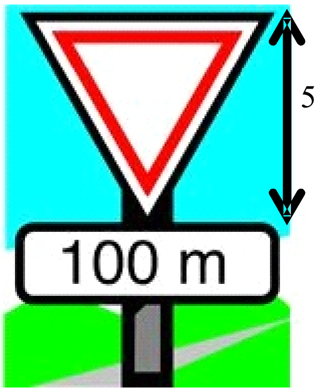
2. Find the area of:

a) the paving slab
b) the football pitch
c) the circle in the middle of the football pitch if the diameter is 10 m.



3. Calculate the area of the triangular sign board if the perpendicular height is 55 cm.

50 cm



55 cm

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7. Foundations of statistics

Introduction

Statistics is a branch of mathematics that deals with the collection, presentation, analysis and interpretation of numerical data. Numerical data are facts about society that can be expressed as numbers. For example, information about how many people are without jobs, how many people own their houses, and how many children go to school are all numerical data. Statistics is part of our mathematics course just like algebra and geometry.

To do the calculations in this lesson, you will need to remember what you learned about **percentages** in Lesson 3 of Unit 1.

For this lesson you need to have mathematical instruments -- in particular, a pair of compasses and a protractor. You will also need your calculator and some sheets of graph paper.

In this lesson, you will:

- understand what statistics are and why people collect statistics
- construct a frequency table for a set of statistical data and interpret the information given in a frequency table
- represent (show) statistical data on pie charts
- interpret information represented on pie charts.

What are statistics?

In one way or the other, each one of us has already been gathering some statistics, for example, you are gathering statistical information even if you counted the number of television programmes you watched daily for the past week. Statistics are number facts or numerical data about the way people live, or the way animals behave, or the way the weather changes, or about anything that happens in the world. The word 'data' means 'unorganised pieces of information'.

Later in this unit we will look at how to organise and analyse statistical data. But for now we will concentrate on just the importance or usefulness of statistics and on how to present and interpret statistics. Here are two more examples of statistics that you may already have gathered.

If you recorded the amount of money spent by your family in a particular month on rent, food, travel, entertainment and savings, then you gathered statistics. If you recorded the age, weight and height of each person in your family, that is also statistics. In general, we say that statistics is the science of collecting, classifying, organising and analysing information shown in numbers.

ACTIVITY 1

ANSWERS ON PAGE 116

Can you think of any examples of statistics? You may use examples from newspapers or magazines.

Why are statistics useful?

Statistics are used to summarise data or information. A few number facts given in a table quite often contain a lot of information. If we had to write the information in words we would need many pages to explain everything.
Look at this example of a statistical table.

Example
Castle League Log

Team	**P**	**W**	**D**	**L**	**F**	**A**	**Pts**
Sundowns	36	22	7	7	63	31	51
Swallows	37	20	10	7	40	24	50
AmaZulu	38	17	14	7	43	34	48
Umtata Bucks	37	15	12	10	49	39	42
Kaizer Chiefs	36	15	12	9	41	31	42
Orlando Pirates	35	13	13	9	48	39	39
Pretoria City	37	11	17	9	38	38	39
Hellenic	37	13	13	11	55	49	49
Rangers	37	9	20	8	43	37	38
Celtic	37	13	11	13	36	42	37
Vaal Pros	37	10	15	12	48	43	35
Spurs	36	12	11	13	40	35	35
Witbank Aces	35	12	10	13	46	48	34
Wits University	37	13	8	16	36	42	34
Dynamos	36	8	17	11	41	38	33
Jomo Cosmos	35	10	12	14	37	41	32
D'Alberton Callies	36	12	8	16	40	48	32
Qwaqwa Stars	34	8	15	11	36	35	31
Santos	37	7	15	15	29	43	29
Welkom Eagles	37	1	6	30	22	93	8

It would take you many pages to write out and explain the information in this league table.

People usually collect information to learn more about society, and so gathering statistics is often part of a process of studying or changing society.

Examples

- Statistics is used by the government to take decisions. For example, before the government builds a school in a particular area, it must find out the number of children of schoolgoing age who live there, and also the number of schools in the surrounding areas.

- Statistics is used to investigate problems, such as why people who eat certain kinds of foods die younger than people who don't eat those foods.

Scientists will gather statistical information from a small group of people in a society. This is called a **sample** in statistics. The scientists will use the result they get from this sample to help them understand what is happening to the whole population.

- Students may keep records of their performance in their different subjects. The records will show them which subjects are strongest and which subjects are weakest. This will help them to make a decision about their future career.

- Before an election, many organisations collect statistics by asking people who they will vote for. They can then see which political parties have the most or the least support. Statistics like these help people to predict what will happen, after the election.

ACTIVITY 2

There are many other uses of statistics. You might like to find some of them yourself by reading newspapers and magazines, watching television and looking at what happens around you in your daily life. Write down three different ways in which people use statistics, and give an example of the statistics used in each of these cases.

ANSWERS ON PAGE 117

Data presentation

Data are facts or information used in deciding or discussing something. One kind of data is statistics. People who do research collect a lot of data. When they have done this, they must organise their data in a way that makes easy reading and understanding.

One way to do this is to put the information in a frequency table. The word 'frequency' comes from 'frequent', which means 'often'. A frequency table is used to show how often something happens.

Frequency table

Suppose in 10 different tests a student obtains the following marks where each mark is out of a total of 5:

2, 2, 2, 3, 3, 3, 3, 3, 5, 5.

We can put information in a frequency table. This will help us to say quickly how many times the student obtains each of the marks. We say the frequency of 2 marks is 3, the frequency of 3 marks is 5 and the frequency of 5 marks is 2. From this can you tell what a frequency is?

Frequency is the number of times a value or a symbol occurs in a set of data.

In other words, if the student gets 2 marks out of 5 for 3 tests, then 2 occurs 3 times so its frequency is 3.

From the data given, we get this frequency table:

Frequency table of marks scored by a student

Marks	Frequency
2	3
3	5
5	2
Total	10

This list of marks for the student's test is called a set of data. The student's marks are all arranged in an ordered form, with all the 2s first, then the 3s, and so on. This made it easy for you to find the frequency of each mark. Sometimes you may get a set of data which is not in ordered form (arranged according to size of the values), or which contains many values. In such a case we use a *tally procedure* to get the frequency tally.

Tally procedure

Tallying is a way to mark off the number of times a certain piece of information occurs, if the information is not easy to group.
We take the pieces of information one at a time, and each time the same value occurs, we make a mark, (|) called a tally, next to that value, symbol or group. We put tally marks in bundles of 5 to make easy and quick counting. So
| = 1 || = 2 ||| = 3 |||| = 4   𝍸 = 5   𝍸 | = 6 and so on.

ACTIVITY 3

A statistics student at a small college in the Western Cape decided to count and group the people living in her street as adults, teenagers and children. She obtained the following table:

Age group	**Tally**
Adults	𝍸 ||||
Teenagers	||||
Children	𝍸 ||

ANSWERS ON PAGE 117

How many of the people in her street are:
a) adults   b) teenagers   c) children

Example
The school guidance teacher wants to find out about the home life of her pupils. She needs to know what size their families are. She therefore asks her pupils to give the number of children in their families. She gets these figures:
2, 3, 1, 2, 4, 3, 3, 6, 2, 2, 2, 4, 1, 3, 5, 3, 2, 3, 1, 2.

Draw a frequency table for the data.

Solution

We need to make three columns with headings: Number of children in a family, Tally and Frequency. We put the numbers 1, 2, 3, 4, 5 and 6 under the heading 'Number of children in a family'. These are the possible number of children in a family from our data. We then cross out the values as we make our tallies.

2, 3, 1, 2, 4, 3, 3, 6, 2, 2, 2, 4, 1, 3, 5, 3, 2, 3, 1, 2

Number of children in a family	Tally	Frequency
1	|||	3
2	~~||||~~ ||	7
3	~~||||~~ |	6
4	||	2
5	|	1
6	|	1
Total		20

Always check to see whether the total frequency is equal to the number of observations. If it is not equal then you might have made a mistake somewhere so you need to go over your work.

Sometimes instead of having number values, you may be given symbols and you will be required to construct a frequency table for them.

ACTIVITY 4

1. 25 students from a high school in Langa wrote Mathematical Literacy in last year's Matric examinations. They obtained the following grades:

   B, D, B, C, F, G, A, B, G, F, B, C, C, D, A, G, E, E, B, D, E, C, G, F, D.

   Draw a frequency table to find out how many students obtained each grade.

2. The number of bankrupt businesses recorded yearly over a period of 20 years for a small town in the Eastern Cape is as follows:
   1 0 3 2 5 6 3 5 3 0
   4 1 3 2 3 3 5 2 1 3
   Represent this on a frequency table.

ANSWERS ON PAGE 117

Grouped frequency table

Sometimes you need to divide data into groups before you can construct the frequency table. Here is an example to illustrate this.

Example
The following marks were obtained by a class of 30 students when they wrote their first test in Mathematical Literacy.

54	56	50	44	58	55	49	64	44	48
50	48	59	47	58	54	51	56	51	37
54	40	51	44	53	43	38	51	60	54

Construct a frequency table to find out how many students got marks in each of the groups: 36-40, 41-45, 46-50, 51-55, 56-60, 61-65.

Solution
We use the same method here as we used earlier. As you cross out a value, you mark the tally next to the group to which the value belongs. Then you add up the tallies to get the frequency for each group.

54	56	50	44	58	55	49	64	44	48
50	48	59	47	58	54	51	56	51	37
54	40	51	44	53	43	38	51	60	54

Marks	**Tally**	**Frequency**
36-40	|||	3
41-45	||||	4
46-50	~~||||~~ |	6
51-55	~~||||~~ ~~||||~~	10
56-60	~~||||~~ |	6
61-65	|	1
Total		30

ACTIVITY 5

24 students obtained the following marks out of 50 for a test:

49	38	31	27	20	48	37	31
23	41	33	10	15	34	22	35
21	39	31	27	20	19	35	26

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Construct a frequency table for these marks using the groups 10-17, 18-25, 26-33, 34-41 and 42-49.

Interpretation of data from a frequency table

There are many questions that can easily be answered once the data is put in a frequency table.

Example

Let us take our example of the number of children in 20 different families. We had the following frequency table:

Number of children in a family	Frequency
1	3
2	7
3	6
4	2
5	1
6	1
Total	20

From this we can answer questions such as:

1. How many families had 4 children?
2. What fraction of the families had 2 children?
3. How many families had fewer than 3 children?
4. How many families had more than 3 children?

Solution

1. The frequency for 4 children is 2; we say 2 of the families had 4 children.
2. Out of the 20 families, 7 (check the frequency table) had 2 children in their families. Therefore the fraction is $\frac{7}{20}$.
3. Families with fewer than 3 children are families with either 1 or
2 children. There are 3 families with 1 child each and 7 families with 2 children each so we get 3 + 7 = 10 families.
4. Families with more than 3 children are families with either 4, 5 or 6 children in them. Therefore we have 2 + 1 + 1 = 4 families.

ACTIVITY 6

A farmer wants to study the way the birds in a certain area breed. He counts the eggs found in 20 different nests of birds at the start of the breeding season and obtains the following data:

0 1 0 3 1 0 0 0 2 2
1 2 0 3 2 0 1 2 3 1

Construct a frequency table for the data and use it to find:

a) the number of nests without eggs
b) the number of nests that contain fewer than 2 eggs
c) the fraction of nests that contain more than 1 egg
d) the percentage of nests that contain 3 eggs.

ANSWERS ON PAGE 118

Pie chart

Data can also be shown on a pie chart. A pie chart is a circle cut into sectors (sections). Each sector shows the frequency of one group compared with the total of all frequencies. In other words, a pie chart shows proportions by the different sectors of a circle. The whole circle represents the total number involved.

Components: parts

A pie chart is a useful way to represent statistics because it shows how much of the whole is being used for each item. It is also useful for comparisons. For example, if the government's national budget is represented on a pie chart, we can compare the different components of the budget, to see what the government spends on different things. You will look at pie charts again in later units.

Drawing a pie chart

The total angle of a circle is 360°. Therefore, to draw a pie chart:

- we equate the total value of the data to 360°
- we then work out the different angles for the various parts of the data.

Example

The transport officer of a small firm of 36 workers did a survey to find out how the workers travelled to work. His survey showed that 13 workers walked to the firm, 8 cycled, 4 came by bus, 9 by train and 2 by car. Represent this information on a pie chart.

Solution

Make a simple table of the information before you construct your pie-chart. So we have:

Walk	Cycle	Bus	Train	Car
13	8	4	9	2

We then work out the degrees for each number:

Walk $= \frac{13}{36} \times 360° = 130°$

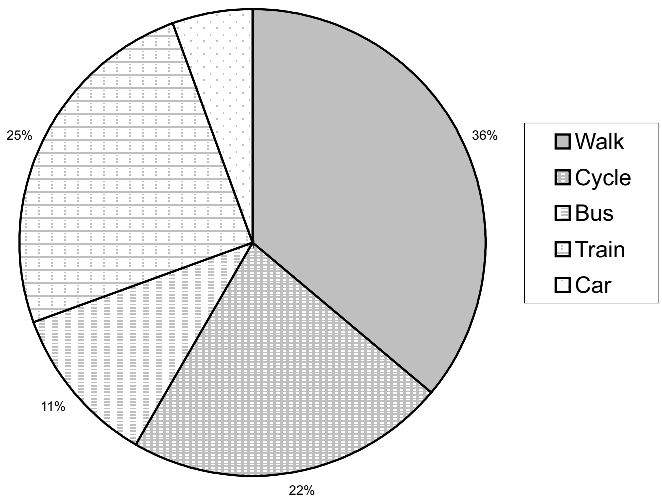
Cycle $= \frac{8}{36} \times 360° = 80°$

Bus $= \frac{4}{36} \times 360° = 40°$

Train $= \frac{9}{36} \times 360° = 90°$

Car $= \frac{2}{36} \times 360° = 20°$

Once your data is in degrees, use your pair of compasses to draw a circle. Now use your protractor to divide the circle into sections. Each section should have as many degrees as one of your calculations. Your pie chart should look like this:



Note: The different parts of the pie chart are shaded in different ways. Each pattern refers to one way of travelling. The explanation of which pattern refers to which kind of transport is given next to the pie chart. This explanation is called the "legend" or the "key".

ACTIVITY 7

Draw a pie chart to represent the number of learners in each grade of a high school of 720 pupils.

Grade 8	208
Grade 9	180
Grade 10	158
Grade 11	94
Grade 12	80

ANSWERS ON PAGE 118

Percentage pie chart

At times the data will be given in percentages and you will have to represent it on a pie chart. We use the same method to do this as we used in the last example, except that our total in this case is 100% instead of the total frequency. So we equate 100% to 360°.

Example
A family spends the following percentage of their annual income on various items they need:

Item	Percentage
Food	30
Rent	20
Clothing	20
Travel	10
Entertainment	5
Savings	15

Represent the information on a pie chart.

Solution
Change the percentages to degrees:

Food $= \frac{30}{100} \times 360° = 108°$

Rent $= \frac{20}{100} \times 360° = 72°$

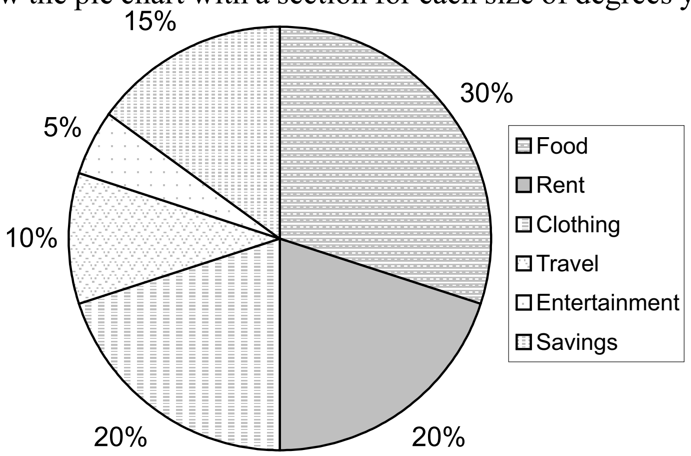
Clothing $= \frac{20}{100} \times 360° = 72°$

Travel $= \frac{10}{100} \times 360° = 36°$

Entertainment $= \frac{5}{100} \times 360° = 18°$

Savings $= \frac{15}{100} \times 360° = 54°$

Now draw the pie chart with a section for each size of degrees you have found:



ACTIVITY 8

Planners from the Education Department conducted a survey to find out whether a high school should be built in a township in Gauteng Province. When they asked people in the township if they wanted another school, 75% of the people interviewed said 'yes', 10% said 'no' and the rest said they 'don't care'. Represent this information on a pie chart.

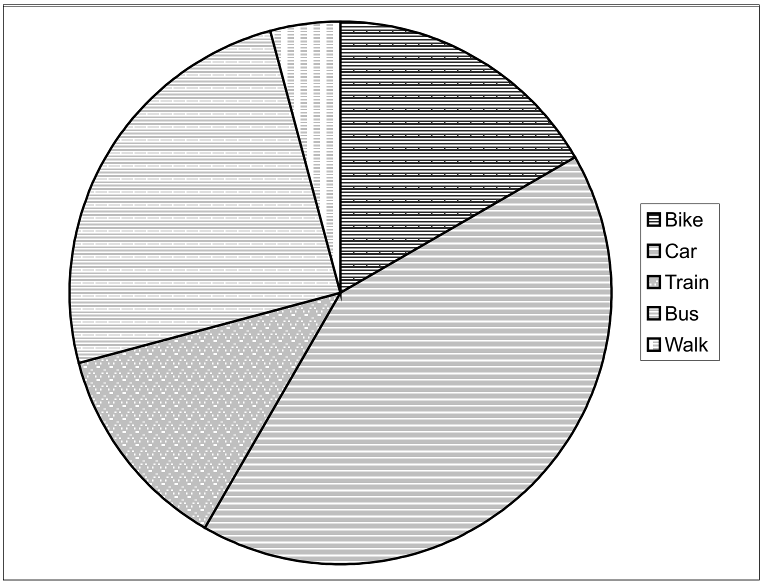
ANSWERS ON PAGE 118

Interpretation of information from a pie chart

We shall use an example to show how much information we can get from a pie chart.

Example

This pie chart shows how 120 people travel to work.



a) How many people travel by bike?
b) What fraction of people travel by train?
c) What is the size of the unlabelled angle?
d) How many people travel by car?
e) By what means do most people travel?
f) How is this information useful to people who plan the transport system of the city?

Solution

a) The angle for bike is 60°
So the number of people who travel by bike =

$$\frac{60}{360} \times 120 = 20$$

Here we have to change from the value of the angle (60°) to the number of people. Can you see how this method is different from the method we used in the last example?

b) Angle for train $= 45°$
Therefore the fraction of people who travel by train is $\frac{45}{360} = \frac{1}{8}$

c) Let x be the unlabelled angle, then since the total angle is $360°$,
we have: $x° + 60° + 45° + 15° + 90° = 360°$
which gives: $x = 360° \ (60° + 45° + 15° + 90°)$
$x = 150°$

d) Angle for car: $x = 150°$
Therefore number of people who travel by car

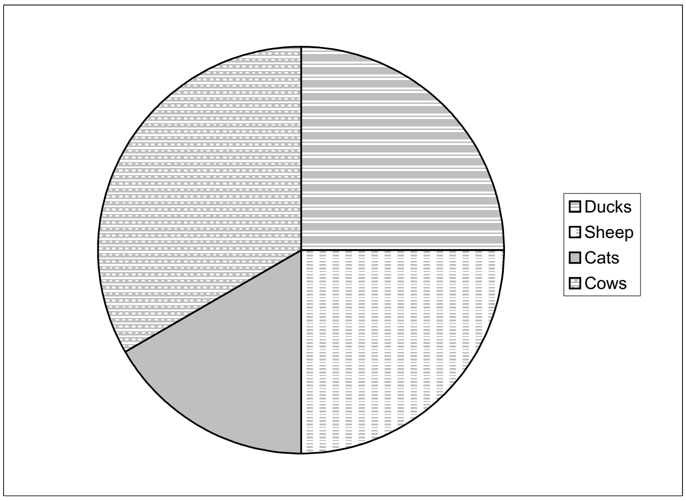
$$\frac{150}{360} \times 120 = 50$$

e) Comparing the angles we see that the angle for car is the largest. So most people travel by car.

f) The information shows that more roads are needed for the cars, and also more parking space near the places where people work.

ACTIVITY 9

The following pie chart shows the number of animals on a farm.



If the farm has 100 cows,

a) How many cats does it have?
b) How many sheep does it have?
c) How many animals are there on the farm?
d) What percentage of the animals are sheep?

ANSWERS ON PAGE 119

Summary

In this lesson we dealt with the concept of statistics and how statistics are used. We looked specifically at how to construct and interpret pie charts using angles, fractions and percentages.

Self-assessment checklist:

Are you able to:

- represent data on a frequency table with the help of tallies
- answer simple questions based on a frequency table
- draw a pie chart
- answer simple questions from a pie chart

SELF-CHECK EXERCISE

1. As part of its audience research, a TV company interviewed 85 people who had watched one or more programmes in an adult education series. Each person was asked how many programmes he or she had watched. The replies were
5, 8, 6, 7, 8, 3, 2, 1, 1, 4, 1, 4, 2, 3, 3, 8, 6, 7, 7, 3, 3, 3,
2, 1, 1, 2, 1, 4, 5, 5, 6, 3, 7, 8, 2, 2, 4, 3, 7, 8, 8, 2, 3, 5,
4, 6, 6, 7, 7, 3, 3, 4, 2, 6, 6, 8, 3, 1, 1, 2, 4, 5, 3, 7, 8, 8,
8, 5, 2, 1, 3, 1, 5, 5, 4, 2, 8, 7, 4, 8, 8, 7, 1, 2, 6

   a) Draw a frequency table to illustrate this.
   b) How many of the people interviewed watched 5 programmes?

2. A statistics student was asked to do a project of his choice. He decided to find the height of 20 people standing near to him. He obtained the following heights in cm (to the nearest cm).

151	162	174	168	185	156	172
167	144	162	148	174	176	166
171	160	168	181	157	174	

   Complete this grouped frequency table for the heights.

Height (cm)	**Tally**	**Frequency**
140-149		
150-159		
160-169		
170-179		
180-189		

3. Sandile calculates the percentage of time he spends on each subject. His results are below:

   English: 20%
   Mathematical Literacy: 40%
   Social Studies: 15%
   Sciences: 25%

   a) Draw a pie chart of his results.
   b) Sandile has divided his time for studies into 25 equal periods in a week. How many periods does he use for English?
   c) If each period is an hour long, how much time does he spend on Mathematical Literacy each week?

ANSWERS ON PAGE 127

8. Revision and consolidation

Introduction

The objective of this final lesson is to revise and consolidate the skills you learnt in this unit and to build your confidence in preparing you for the exam. This is a 'test-yourself' lesson and the answers are not contained in this unit but in a separate booklet entitled "Revision and consolidation answer booklet". Section A is revision of the whole unit combined to see that you are able to understand and integrate the various topics dealt with in the unit. Section B is in an examination form and the questions are taken from previous Mathematical Literacy tests and exams.

Summary of unit

In this unit we covered the following knowledge and skills:

- More concepts related to numbers and calculations with numbers including:
  - Further calculating skills
  - Addition and subtraction of fractions
- Finance
  - Simple interest and discount
- Patterns, relationships and representations including:
  - An introduction to algebra
  - An introduction to graphs
- Measurement
  - Shapes, perimeter and area
- Data handling
  - An introduction to statistics

Section A

1. Calculate:
   a) $24^2 + 6^3$
   b) $2 - 3^2 \times (114 + 25)$
   c) $\sqrt{729}$
   d) $\sqrt{144 + 25}$
   e) $-3126 + 879 - 321 + 18\,901$

2. Calculate without your calculator:
   a) $3 + (5 - 8) - 2$
   b) $4 + 3 \times 5$
   c) $(4 + 3) \times (5 - 3)$
   d) $35 + 12 \times 4$
   e) $\frac{3}{4} + 1\frac{1}{2}$

3. Calculate first without your calculator and then use your calculator to check your answers:

a) $\frac{1}{3}+\frac{2}{9}$

b) $\frac{2}{7}+\frac{1}{3}$

c) $2\frac{1}{5}-\frac{2}{9}$

d) $2\frac{5}{6}-\frac{3}{8}$

e) $4-\frac{3}{4}\times\frac{5}{7}$

4. Use your calculator to calculate. Try to use the memory keys when doing the calculations.

a) $\left(\frac{9}{10}-\frac{8}{9}\right)\div\frac{3}{5}$

b) $2\times\left(\frac{4}{5}\right)^2-\left(\frac{19}{25}\right)$

c) $\sqrt{\frac{9}{4}-\frac{4}{16}}$

5. a) How many minutes in an hour?
b) How many minutes in half an hour?
c) How many minutes in a quarter of an hour?
d) How many half hours do you need to make up an hour?
e) How many quarter hours do you need to make up an hour?

6. There are 48 men and 30 women in a church who need to board a bus. If $\frac{3}{4}$ of the men and $\frac{2}{3}$ of the women board the first bus, how many people boarded the first bus?
A. 36
B. 32
C. 56
D. 50

7. Some students from the community are selling raffle tickets. The table shows how many tickets they have sold so far.

Student's name	**Number of tickets sold**
Maria	4
Jakob	7
Shahewa	3
Bayani	7
Martin	6
Paulina	9

a) Who has sold the most tickets?
b) Who has sold the least number of tickets?
c) The students need to sell 75 tickets altogether. How many more tickets must they sell?

8. Complete the following equivalent fractions:

Equivalent: equal

a)	$\frac{1}{2} = \frac{\_\_}{60}$	b)	$\frac{3}{7} = \frac{\_\_}{21}$
c)	$\frac{2}{3} = \frac{6}{\_\_}$	d)	$\frac{10}{10} = \frac{50}{\_\_}$
e)	$\frac{3}{4} = \frac{24}{\_\_}$	f)	$\frac{1}{6} = \frac{\_\_}{36}$
g)	$\frac{5}{6} = \frac{\_\_}{30}$	h)	$\frac{5}{\_\_} = \frac{10}{15}$
i)	$\frac{6}{9} = \frac{\_}{3}$	j)	$\frac{21}{35} = \frac{\_}{5}$

9. In which pair of numbers is the second number 100 less than the first number?
   A. 209 and 199
   B. 4 246 and 4 236
   C. 9 735 and 9 635
   D. 52 863 and 51 863

10. □ stands for a number. □ × 4 will always give the same answer as:
    A. □ ÷ 4
    B. □ + □ + □ + □
    C. □ − 4
    D. □ + 4

11. Solve the following equations:
    a) $x + x + x + x = 12$
    b) $3x = 27$
    c) $2x - 8 = 4$
    d) $4x + 7 = 9$
    e) $2(x + 6) = 20$

12. Simplify the following fractions to their simplest form:

Note: Simplest form is when the numerator and the denominator cannot be divided any more by the same number.

a)	$\frac{30}{60}$	b)	$\frac{20}{30}$
b)	$\frac{2}{6}$	d)	$\frac{3}{15}$
e)	$\frac{2}{5}$	f)	$\frac{7}{28}$
g)	$\frac{25}{35}$	h)	$\frac{14}{49}$
i)	$\frac{12}{72}$	j)	$\frac{10}{28}$

13. Derika takes 20 minutes to paint a picture onto a mug. How many mugs can she paint in 3 hours?

14. If Johan can lay 150 bricks in 30 minutes, how many bricks can he lay in 8 hours?

15. Sophie takes $\frac{1}{6}$ of an hour to bake muffins. How many minutes is this?

16. Thabi can run one km in $\frac{1}{10}$ of an hour. How many minutes does it take Thabi to run 3 km's?

17. Akash cycles 2 km's in $\frac{1}{12}$ of an hour. How long will it take Akash to cycle 26 km's?

18. Write down the answers to the following questions:
    a) How many metres are there in one kilometre?
    b) How many metres are there in five kilometres?
    c) How many metres are there in x kilometres?

19. Write down an algebraic expression for the following:
    a) The area of a rectangle with a length of x and a breadth of y.
    b) The perimeter of an equilateral triangle with one side equal to k.
    c) The number of days in x weeks.
    d) A certain number doubled and then increased by 10.
    e) The product of 5 and a certain number decreased by 4.

20. Use the following algebraic expression to answer the questions that follow:
    a) How many terms does this algebraic expression contain?
    b) What is the constant?
    c) Write down the coefficient of x.
    d) Write down the coefficient of y.

21. Use the following list of numbers to answer the questions that follow:

    $-12\,;0{,}4\,;\,;-100\,;\ (-1)^3;\ 1$

    a) Write down the number with the smallest value.
    b) Write down the number with the greatest value.
    c) Rewrite the decimal as a fraction in its simplest form.

22. R5 000 is deposited into a savings account at a bank for 5 years. The interest paid is 15% per annum. Calculate the value of the investment at the end of each year for the 5 year period if interest is calculated as simple interest.

23. You decide to buy a lounge suit from 'Lazy Lounges' store and to pay it off over 12 months. The conditions of sale are that you pay 10 % of the total price of the lounge suite immediately as a deposit and then pay the rest off over 12 months.

a) How much money will you have to pay as a deposit if the lounge suite costs R3500?

b) How much money will you have to pay as a deposit if the lounge suite costs R3000?

c) If your deposit is R50, how much do you think the lounge suite costs?

d) If the lounge suite costs R4000:

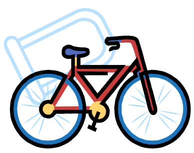
  i) what will the deposit be?

  ii) what will the payments be each month for the next 12 months?

24. R20 000 is invested for 15 years at 12% p.a. Calculate the value of the investment if the interest is calculated as simple interest.

25. Mpho buys unit trusts for R 25 000. After 8 years he sells them for R63 000. Calculate a simple interest rate that would provide the same growth.

26. **Summer sale**



10 % discount

15 % discount

20 % discount

At the summer sale, how much, in rands, is the discount you will get on:

a) a bicycle that costs R500?

b) a cellphone costing R900?

c) a laptop costing R6 600?

At the summer sale, how much will you pay for:

d) a bicycle that cost R990?

e) a cellphone that costs R1 200?

f) a laptop that costs R4 000?

g) At the summer sale, which would be the cheapest item to buy?

  A. a bicycle that was advertised for R4 000?

  B. a cellphone that was advertised for R3 500?

  C. a laptop that was advertised for R4 500?

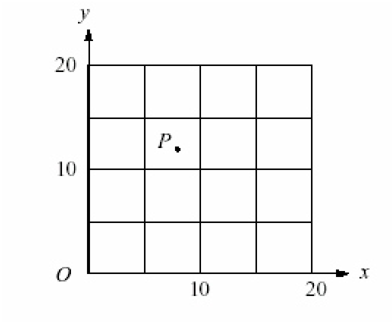
27. Which of the following are most likely to be the coordinates of point *P*?

A. (8 ;12)

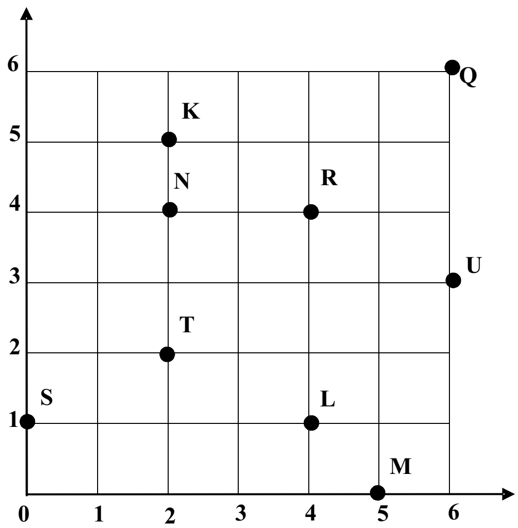
B. (8 ; 8)

C. (12 ; 8)

D. (12 ; 12)



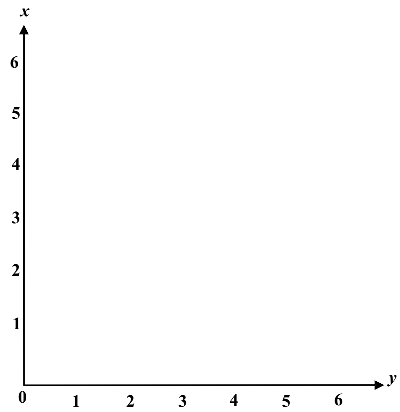
28.



Name the following points (by the letters indicated):
a) Point located at (2; 2)
b) Point 2 units North of (2; 2)
c) Point located at (5; 0)
d) Point 3 units East of (1; 1)
e) Point located at (6; 6)
f) Point 3 units South of (6; 6)
g) Point where $x = 2$ and $y = 5$
h) Point 2 units West of (6; 4)
i) Point 1 unit North of (0; 0)
j) Point 2 units South of (4; 3)

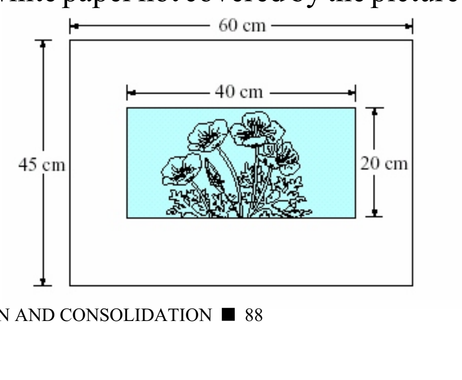
29. Redraw the following axes, plot the stated points and label them as indicated by the letters:

A(1 ; 2)
B(5 ; 2)
C(5 ; 5)
D(1 ; 5)



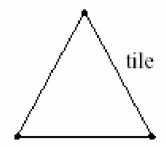
a) What shape does ABCD form?
b) Calculate: i) the perimeter of the shape
ii) the area of the shape

30. You decide to frame a picture for your room. The rectangular picture is pasted onto a white sheet of paper as shown. What is the area of the white paper not covered by the picture?

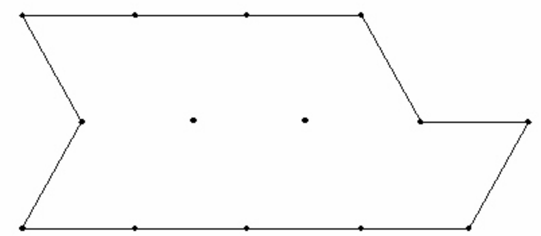


31. Your rectangular room needs to be retiled. If the length of the room floor is 3 m and its perimeter is 10 m, what is the area of your classroom floor in square metres?

32. The triangle represents one tile in the shape of a triangle.



How many tiles will it take to cover the figure below?

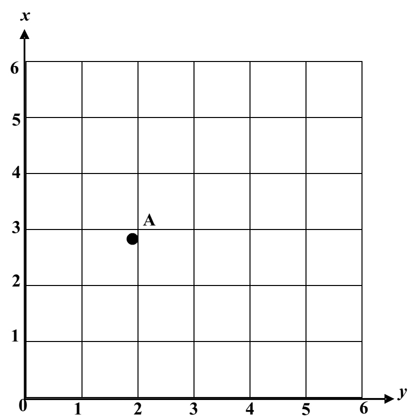


33. If there are 300 calories in 100 grams of a certain food, how many calories are there in a 30 g portion of that food?
A. 90
B. 100
C. 900
D. 1000
E. 9000

34. The weight of a standard pen is 9.2 g. Which of these is the best estimate of the total weight of 1000 standard pens?
A. 900 g
B. 9 000 g
C. 90 000 g
D. 900 000 g

Weight: mass

35. Start at point A. Follow the directions indicated and write down the coordinates of the point where you end.
a) From A, move 2 units to the right
b) Now move 1 unit up.
c) Add 2 units to the x-coordinate
d) Subtract 1 unit from the y-coordinate
e) Move 1 unit to the left.



36. In South Africa, $\frac{1}{3}$ of the year is Winter and $\frac{1}{2}$ of the year is Summer. The rest of the time is shared between autumn and spring.
a) Which is the longest season in South Africa?
b) For how many months of the year is it Winter?
c) How many months of the year are left to be shared between Autumn and Spring?

Note: one day has 24 hours

37. The following table explains how Martha spends her day. Complete the table by filling in the fraction of the day she spends doing each activity. Write the simplified fraction in the last column.

Activity	Number of hours	Fraction of day	Simplified fraction
sleeping	6		
cooking and cleaning	4		
working	8		
watching TV	2		
eating	1		
other	3		

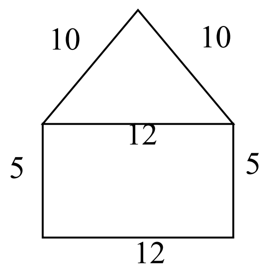
a) What activity takes up most of Martha's day?
b) Calculate the angles and draw up a pie chart to indicate this data.

38. Use the following expression to answer the questions that follow:

$-7 + x + (4y + 1)$

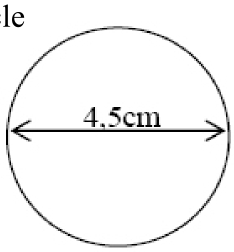
a) How many terms does the expression contain?
b) Write down the constant term of the expression.
c) Write down the coefficient of y.
d) Write down the coefficient of x.
e) Are there any like terms in this expression?

39. Use the following figure to calculate the:



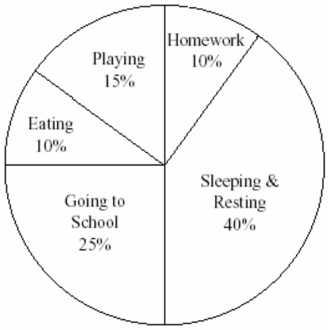
a) perimeter of the rectangle.
b) area of the triangle.
c) area of the rectangle.
d) perimeter of the triangle.
e) perimeter of the figure.
f) area of the figure.

40. Calculate: a) the circumference of the circle
b) the area of the circle



41. The area of a square is given as 36 cm^2. What is the length of one side of the square?

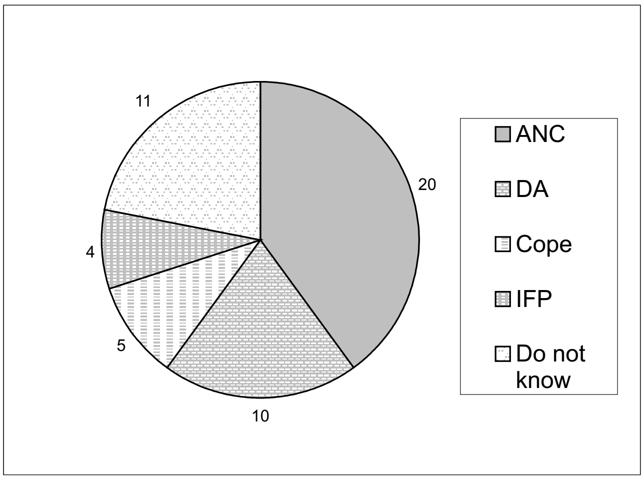
42. The figure shows how Matsepho spent her time one day.



What percent of time altogether did she spend playing and doing homework?

A. 10 %
B. 15 %
C. 20 %
D. 25 %

43. Zaheeda asked 50 people in her neighbourhood which political party they planned to vote for in the next elections. The results are represented on the pie chart.

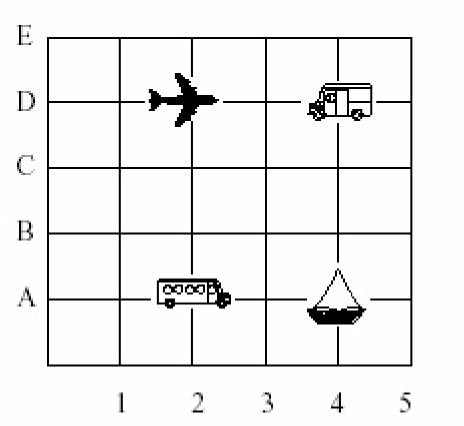


a) What political party has the most support according to Zaheeda's survey?
b) What political party has the least support according to Zaheeda's survey?
c) What percentage of people in the survey support the DA?
d) What percentage of people in the survey support Cope?
e) Calculate the angle on the pie chart that represents the support for the DA.

44. The formula for simple interest can be written as: **I = P × R × T** where:
I = interest, written in rands
P = principal, money deposited or borrowed, written in rands
R = percentage rate, written as a fraction or decimal
T = time, written in years.

a) Make T the subject of the formula.
b) Make P the subject of the formula.
c) Loabe invested R300 at his bank in 2001 and after 3 years he had earned R60 interest. Calculate the percentage rate of his interest.

45. This is a game board.
Which object is located at (2, D)?



A. the plane
B. the truck
C. the bus
D. the boat

46. Jabu wanted to work out how many hotdogs he sold in a working week at his 'Snack box' stand. He decided to keep a tally which he recorded:

Monday: 𝍸 𝍸 ||

Tuesday: 𝍸 𝍸 ||||

Wednesday: ||||

Thursday: 𝍸 𝍸 𝍸 ||

Friday: 𝍸 𝍸

Draw a frequency table for the data.

Section B

Time: 2 hours
Marks: 80

QUESTION 1 **[14]**

1.1 Notice that this assessment is two hours long and out of a total of 80 marks.

1.1.1 How many minutes do you have to complete the assessment? (1)

1.1.2 How many marks out of 80 do you need to get 50 % for this assessment? (1)

1.1.3 How many marks out of 80 do you need to get 80 % for this assessment? (2)

1.1.4 If you get $\frac{45}{80}$ for the assessment, what is your percentage? (2)

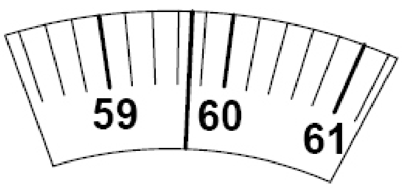
1.3. Your favorite shop is offering a discount of 20 % on an item of clothing which costs R180. How much does it cost now? (2)

1.4 Roelof spends $\frac{1}{3}$ of the year working in Cape Town and $\frac{1}{2}$ of the year working in Pretoria. The rest of the year he spends on holiday.

1.4.1 What fraction of the year does Roelof spend working? (3)

1.4.2 What fraction of the year does Roelof spend on holiday? (1)

1.5 What is the mass (weight) shown on the scale below:



(2)

QUESTION 2 **[14]**

2.1 Kuketso works at a car-wash. He earns R60 per day plus R5 for every car he washes.

Calculate how he earned in a day if he:

2.1.1 washed 5 cars. (2)
2.1.2 washed 7 cars. (2)
2.1.3 washed *n* cars. (2)

2.2 Renuka gets a 5 % increase in salary and Fatima gets an increase in salary of R292,50 more per month. Renuka earns R14 575 per month and Fatima earns R16 500 per month.

2.2.1 Determine Renuka's new salary per month. (2)

2.2.2 Who received the greater increase in terms of actual money? (1)

2.2.3 Who received the greater percentage increase? Show your working. (2)

2.3 A person invests R1 000 at an annual interest rate of 12 % for 6 years.

2.3.1 Calculate the simple interest earned over this time. (2)

2.3.2 What amount will the person have after the 6 years? (1)

QUESTION 3 **[12]**

3.1 Choose the correct answer for $7x-2$.

A. A certain number multiplied by 7 and subtract 2 from the product.

B. A certain number minus 2 multiplied by 7.

C. 7 plus a certain number minus 2. (1)

3.2. Write down the algebraic equation for the following:
The sum of 12 and a number divided by 2. (2)

3.3 How many terms are there in the following algebraic expressions?

3.3.1 $4(x+3)$ (1)

3.3.2 $5x+3y+2$ (1)

3.4 Write down the coefficient of x in each of the following expressions.

3.4.1 $3x+4$ (1)

3.4.2 $-3-9x$ (1)

3.5 Change the subject of the formula for each of the following:

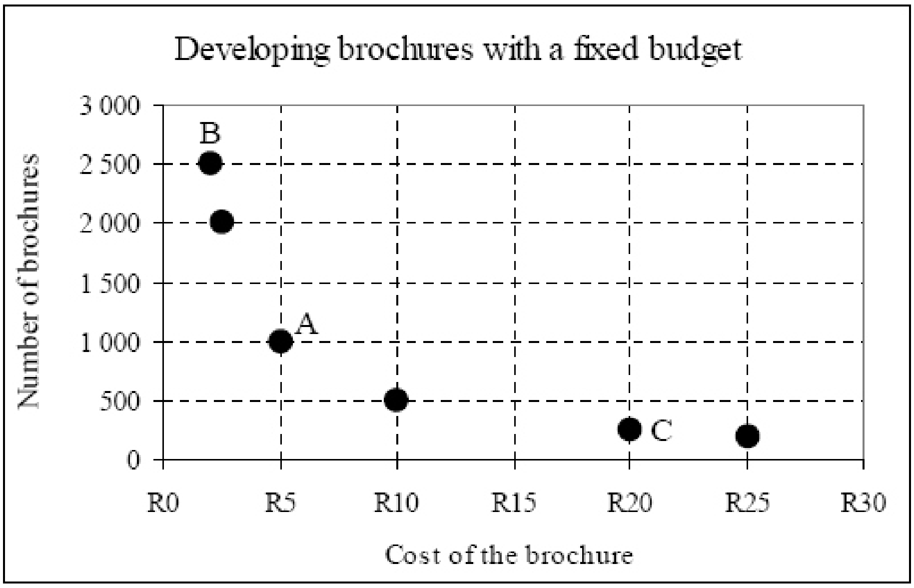
3.5.1 Make *t* the subject of the formula.
$A = 800 - 20t$ (3)

3.5.2 Make *n* the subject of the formula and work out its value
$3(n + 2) = 12$ (2)

QUESTION 4 **[7]**

The graph below depicts the summary of Shaheda's research into the number of advertising brochures that can be printed with a fixed budget. Answer the questions that follow.

Depicts: shows



4.1 Write down the coordinates of points A, B and C. (3)

4.2 What is the number of brochures represented at point A? (1)

4.3 If each brochure costs R5 to print, how many brochures can Shaheda print? (1)

4.4 What is the **cost** of each brochure represented by point B? (1)

4.5 What is the **number** of brochures represented by point B? (1)

QUESTION 5 **[13]**

5.1 The timetable for the movies at the local cinema is shown below. Use it to answer the questions that follow.

Date	Cinema	Available show times					
		1	2	3	4	5	6
Wed 17 Oct	18	09:45	12:15	15:00	17:45	20:15	22:45
Thu 18 Oct	18	15:00	17:45	20:15	22:45		
Fri 19 Oct	17	09:15	11:45	14:45	17:00	19:30	22:15
Sat 20 Oct	17	09:15	11:45	14:45	17:00	19:30	22:15
Sun 21 Oct	17	09:15	11:45	14:45	17:00	19:30	
Mon 22 Oct	17	09:15	11:45	14:45	17:00	19:30	
Tue 23 Oct	17	09:15	11:45	14:45	17:00	19:30	22:15

5.1.1 In which cinema will the movie show on Friday? (1)

5.1.2 On which day of the week is it possible to watch a movie at 3:00pm? (2)

5.1.3 On Saturday, how long after the start of show 2, does show 3 start? (3)

5.1.4 On Saturday, how long after the start of show 3, does show 4 start? (3)

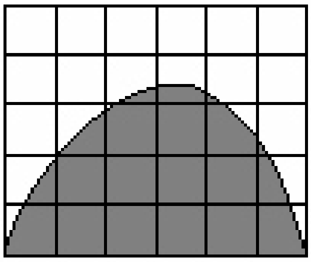
Justify: explain

5.1.5 From the above, estimate how late the last movie will end on Saturday. Justify your answer. (4)

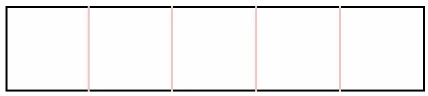
QUESTION 6 **[10]**

6.1 Each of the small squares in the figure is 1 square unit. Which is the best estimate of the area of the shaded region?

A. 10 square units
B. 12 square units (1)
C. 14 square units
D. 16 square units



6.2 The figure consists of 5 squares of equal size. The area of the whole figure is 405 cm^2.



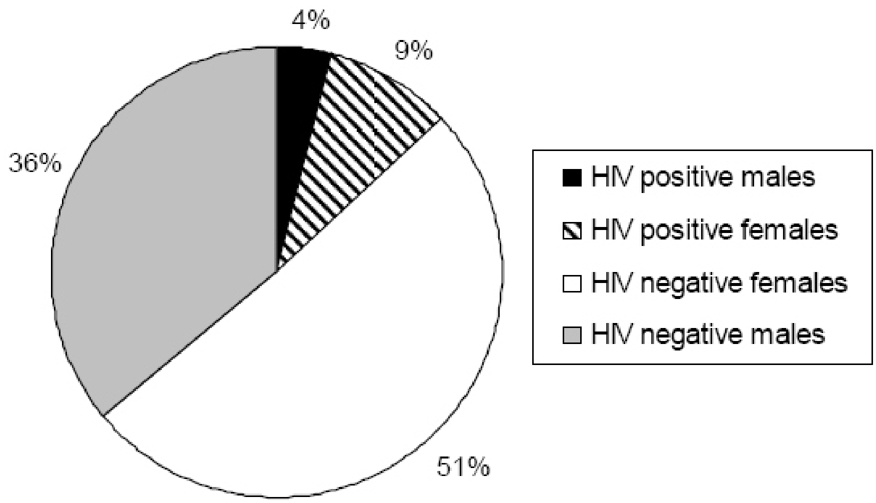
6.2.1 Calculate the area of one square. (1)
6.2.2 Calculate the length of one side of one square. (1)
6.2.3 What is the length of the whole figure? (1)
6.2.4 What is the width of the figure? (1)
6.2.5 Calculate the perimeter of the figure. (2)

6.3 The formula for the circumference of a circle is:
circumference = $\pi \times$ diameter or $c = \pi d$

The circumference of the circle is 37,704. Calculate:
(a) the diameter (2)
(b) the radius (1)

QUESTION 7 **[10]**

In a survey of 2 435 people in 2005, researchers tested participants' HIV status. The graph below shows the results.



7.1 What percentage of the total number of participants were male and HIV positive? (1)

7.2 What percentage of the total number of participants were male and HIV negative? (1)

7.3 Calculate the number of males that took part in the survey. (3)

7.4 What percentage of men who participated in the survey were HIV positive? (4)

7.5 How many females took part in the survey? (1)

Feedback to Activities

1. More calculations

Activity 1

		Manual calculation
1.	a) $4 + 3 \times 2$	10 first do $3 \times 2 = 6$ then $4 + 6 = 10$
	b) $(20 + 4) \times -9$	-216 first do $20 + 4 = 24$ then do $\times -9 = -216$
	c) $4 \times (3 + 2)$	20 first do $(3 + 2) = 5$ then $4 \times 5 = 20$
	d) $4 \div 2 \times 4$	8 first do $4 \div 2 = 2$ then $2 \times 4 = 8$
	e) $4 \times 2 \div 4$	2 first do $4 \times 2 = 8$ then $8 \div 4 = 2$

In d) and e) both ÷ and × are at the same level in the list. When this happens you should start with the operation on the left. You will get the same answer if you start from the right, but to avoid confusion, let us stick to the rule that we start with the operation on the left, i.e. work from left to right.

Activity 2

1. a) $10\,000 \div 2 = 5\,000$ days
   b) There are 365 days in 1 year. So 5 000 days is equivalent to 13 years.
   Therefore, the advertisers are saying the battery will last for 13 years, which is unlikely.

2. $12 \times 7{,}6 + 6 \times 3{,}5 =$ total cost
   [12] [×] [7.6] + [6] [×] [3.5] [=]
   The total cost is R112,20

Activity 3

1. 11
2. 15
3. 44
   Keys pressed:  ; 1900 ;  
   (answer displayed is 43.58898…)

4. 289
5. 625
6. 11
   Keys pressed:  3,35 ;  ;  ;
   (answer displayed is 11.2225)

Activity 4

1. 216
2. 64
3. 1072
4. 300,763

Activity 5

Your keys pressed should go something like this:

10 + 25 – 14 = 

: Now clear your screen

415 – 36 = 

: Now clear your screen

To add the values stored in memory holders C and D:

You should get an answer of 400.

Activity 6

From the previous work, you should still have -8 stored in our memory holder A. Try subtracting 2 from that amount using the memory key. Did you get –10 as your answer? And your order of working with the keys should have looked something like this:

and you should get an answer of -10 displayed on your screen.

Activity 7

There are many ways of calculating this problem. This way is just one of many ways.
Add the income and store it in memory holder A:
(R32134,50)
Add the expenses and store the amount in memory holder B:
(R6403,70)
Subtract the expenses from the income to find the money left.

32135.50 ; 

6403,70 ; 

and your display should show: $\frac{128654}{5}$

In order to convert this fraction into a decimal value, press the 

key and you should get your answer of R25 730,80.

Activity 8

Try to use your memory keys on the calculator to help you with this calculation. First add up the cost of all the items, except the toilet rolls, and store this amount in a memory (R40,47). Then subtract this amount from the R50 you have in your pocket (R9,53) and divide this amount by 4 to see how many toilet rolls you can buy. You should be able to buy 2 toilet rolls @ R4,00 each. Remember to try and practice your memory keys in this activity.

2. Fractions: addition and subtraction

Activity 1

1. $\frac{1}{3}+\frac{2}{3}=\frac{1+2}{3}=\frac{3}{3}=1$

2. $\frac{3}{6}+\frac{2}{6}=\frac{3+2}{6}=\frac{5}{6}$

3. $\frac{8}{32}+\frac{4}{32}=\frac{12}{32}=\frac{3}{8}$

4. a) $\frac{2}{10}+\frac{2}{10}=\frac{4}{10}=\frac{1}{5}$ b) $\frac{2}{10}+\frac{2}{10}+\frac{1}{10}=\frac{5}{10}=\frac{1}{2}$

   c) $\frac{2}{10}+\frac{2}{10}+\frac{2}{10}+\frac{1}{10}=\frac{7}{10}$

Activity 2

a) $\frac{7}{9}-\frac{4}{9}=\frac{7-4}{9}=\frac{3}{9}=\frac{1}{3}$

b) $\frac{4}{8}-\frac{1}{8}=\frac{4-1}{8}=\frac{3}{8}$

c) $1-\frac{1}{4}=\frac{4}{4}-\frac{1}{4}=\frac{3}{4}$ so $\frac{3}{4}$ has been used.

Activity 3

1. a) $\frac{3}{7}=\frac{\square}{21}$

   $\frac{3\times 3}{7\times 3}=\frac{9}{21}$

   b) $\frac{1}{5}=\frac{\square}{25}$

   $\frac{1\times 5}{5\times 5}=\frac{5}{25}$

2. a) $2\frac{1}{4}+3\frac{1}{2}=2+\frac{1}{4}+3+\frac{1}{2}$

   $=2+3+\frac{1}{4}+\frac{1}{2}$

   $=5+\frac{1}{4}+\frac{2}{4}$

   $=5\frac{3}{4}$ km

> ***Remember:*** *Before we can add or subtract fractions we have to change the fractions so they have the same denominators.*

b) $4+3\frac{1}{2}=4+3+\frac{1}{2}$

$=7+\frac{1}{2}$

$=7\frac{1}{2}$ km

Activity 4

1. $\frac{4}{9}+\frac{1}{3}=\frac{4+3}{9}=\frac{7}{9}$

2. $\frac{5}{8}-\frac{1}{4}=\frac{5-2}{8}=\frac{3}{8}$

3. a) $\frac{3}{1}-\frac{3}{4}=\frac{12-3}{4}=\frac{9}{4}=2\frac{1}{2}$ hrs

b) $\frac{3}{1}-\left(\frac{3}{4}+\frac{3}{4}\right)=\frac{3}{1}-\frac{6}{4}=\frac{12-6}{4}=\frac{6}{4}=1\frac{2}{4}$ hrs = $1\frac{1}{2}$ hrs

4. $\frac{9}{1}-3\frac{1}{2}=\frac{9}{1}-\frac{7}{2}=\frac{18-7}{2}=\frac{11}{2}=5\frac{1}{2}$ litres left

Activity 5

a) 8 and 7 do not have any common factors.

LCD: $8\times 7=56$

$\frac{1}{8}+\frac{2}{7}=\frac{7}{56}+\frac{16}{56}$

$=\frac{23}{56}$

b) 6 and 8 have a common factor. Hence we need to list the multiples:

Multiples of 6: 6; 12;18; 24
Multiples of 8: 8; 16; 24
LCD: 24

$\frac{5}{10}+\frac{7}{8}=\frac{20+21}{24}=\frac{41}{24}$

You can also write $\frac{41}{24}$ as a mixed number if you like as $1\frac{17}{24}$

c) 2 and 3 do not have any common factors. LCD: $2\times 3=6$

$\frac{1}{2}-\frac{1}{3}=\frac{3}{6}-\frac{2}{6}=\frac{1}{6}$

3. Foundations of finance: Simple interest and discounts

Activity 1

1. $\frac{1}{100}\times 200$ cents $=2$ cents
2. $\frac{3,5}{100}\times 2\,400$ cents $=84$ cents

Activity 2

In the third year the bank will take 16% of the money you owe at the beginning of the year as interest, that is,

16% of R1 444,80 = $\frac{16}{100}\times 1\,444,80$

= R231,168 = R231,17

You will again pay R1 200, so this year R1 200 – R231,17 = R968,83 will be paid off the loan.

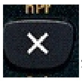
You will then owe R1 444,80 R968,83 = R475,97

So the third year on the table will now look like this:

Year	Calculations	Money you owe the bank
3rd year	You pay R1 200	
	Interest due: R1 444,80	
	16% of R1 444,80 = R231,17	
	Amount taken off loan = R968,83	R475,97

Activity 3

a) Interest = amount of money × interest rate
= R100 × 0,5%
= R0,50

Keys pressed: 100 ; 

; 0.5 ; 

; 

; 

; 

b) Interest = amount of money × interest rate
= R50 × 0,5%
= R0,25
The interest will be 25 cents in one year.

c) Interest = amount of money × interest rate
= R35 × 0,5%
= 0,175
= R0,18
The interest will be 18 cents in one year.

Activity 4

1. a) Choose People's Bank because the interest rate for R500 in this bank is 2,53% while the interest rate for R500 at Everybody's Bank is 2,5%.

Let us check by calculating the interests for both banks.
PB: 2,53% of R500 = R12,65
EB: 2,5% of R500 = R12,50

You see that the interest at PB is higher than the interest at EB. Therefore, the best bank at which to save R500 is PB.

b) Choose People's Bank because the interest rate for R200 in this bank is 2,53% while the interest rate for R200 at Everybody's Bank is 1%. Let us check by calculating the interests for both banks.

PB: 2,53% of R200 = R5,06
EB: 1% of R200 = R2,00

You see that the interest at PB is higher than the interest at EB. Therefore, the best bank at which to save R200 is again PB.

c) Both banks have the same interest rate of 1% for saving R100. You can therefore choose either bank because you will earn the same interest. Let us check by calculating the interest for both banks.

PB: 1% of R100 = R1,00
EB: 1% of R100 = R1,00

You see that the interest at the People's Bank is the same as the interest at Everybody's Bank. Therefore, you can choose either bank to save R100.

2. $SI = P \times r \times n$
$= 200 \times 5\% \times 3$
$= R30$
The interest is R30. You will earn R30 interest.

3. $SI = P \times r \times n$
$= 475 \times 10\% \times 2$
$= R95$
The interest is R95. Thabo will have to pay R95 interest.

4. $SI = P \times r \times n$
$= 575 \times 12{,}5\% \times 1$
$= 71{,}875$
$= R71{,}88$
The interest is R71,88. Vuyo would pay R71,88 interest.

Activity 5

1. $P = 650, r = 5\tfrac{1}{2}\%, n = 2$
Interest $= P \times r \times n$
$= 650 \times 5.5\% \times 2$
$= R71{,}50$

Amount = Principal + Interest
$= R650 + R71{,}50$
$= R721{,}50$

The total amount in Jane's account will be R721,50 after 2 years.

2. $P = 1\,000, r = 14\%, n = 3$
Interest $= P \times r \times n$
$= 1\,000 \times 14\% \times 3$
$= R420$

Amount owing = Principal + Interest
$= R1\,000 + R420$
$= R\,1\,420$

The total amount owed on a loan will be R1 420.

3. $P = 375, r = 5\frac{1}{4}\ \%, n = 4$

Remember that if you want to convert the rate of $5\frac{1}{4}\%$ into a decimal and you cannot remember how to do it mentally, then use your calculator by entering the following sequence:

5 ; 

;  ; 1 ;  ; 4 ; 

; 

(press the down arrow)

Your display should show: 5,25

Now we can continue:

Interest $= P \times r \times n$

$= 375 \times 5{,}25\% \times 4$

$= R78{,}75$

Amount = Principal + Interest

$= R375 + R78{,}75$

$= R453{,}75$

The total amount in a savings account will be R453,75.

Activity 6

1. a) If 1 litre is R8, 10 litres will be $10 \times R8 = R80$

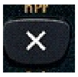
R80 – 2% of R80

=R78,40

Keys: 80 ; 

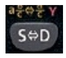
; 2 ; 

; 

; 

; 80 ; 

; 

Note: if you are using the CASIO calculator your answer is given as a fraction and you use the 

to convert your answer into decimal form.

b) If I buy 12 litres then I will get a discount for 10 litres and pay the normal price for 2 litres.

The price for 10 litres is R78,40. The price for 2 litres is $2 \times R8 = R16$.

So the total is R78,40 + R16,00 = R94,40

2. If 1 bag is R25, then 5 bags should sell at $5 \times R25 = R125$. You give Tim a 3% discount for buying 5 bags.

Tim will pay you:

R125 – 3% of R125

R121,25

Tim will have to pay you R121,25 for 5 bags of oranges.

3. To get the percentage find a fraction with a denominator of 100. Remember from lesson 4 Unit 1, per cent means per hundred. The price of two tapes is now half-price. So, we need to find an equivalent fraction to $\frac{1}{2}$. That is, $\frac{1}{2}$ is equivalent to $\frac{50}{100}$. The discount is 50%.

$$\frac{1}{2} \times \frac{\square}{\square} = \frac{?}{100}$$

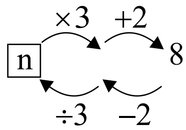
$$\frac{1}{2} \times \frac{50}{50} = \frac{50}{100}$$

So the discount is 50%.

4. Foundations of algebra

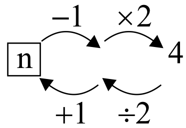
Activity 1

1. By inspection:
   $3n + 2 = 8$
   What number added to 2 is equal to 8? 6
   So $3n = 6$. What number multiplied by 3 is equal to 6? 2
   So $n = 2$
   Check: $3 \times \mathbf{2} + 2 = 6 + 2 = 8$
   Solving using a flow chart:



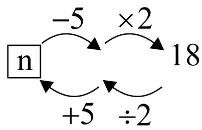
$n = 2$

2. $2(n-1) = 4$
   What number multiplied by 2 is equal to 4? 2
   So $n-1 = 2$. What number will give an answer of 2 if we subtract 1 from it? 3
   So $n = 3$
   Check: $2(\mathbf{3} - 1) = 2(2) = 4$
   Solving using a flow chart:



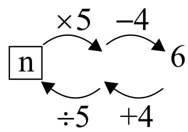
$n = 3$

3. $2(n-5) = 18$
   What number multiplied by 2 is equal to 18? 9
   So $n-5 = 9$. What number will give an answer of 9 if we subtract 5 from it? 14
   So $n = 14$
   Check: $2(\mathbf{14} - 5) = 2(9) = 18$
   Solving using a flow chart:



$n = 14$

4. $5n - 4 = 6$
   What number can we subtract 4 from that will give an answer of 6? 10
   So $5n = 10$. What number multiplied by 5 is equal to 10? 2
   So $n = 2$
   Check: $5 \times \mathbf{2} - 4 = 10 - 4 = 6$
   Solving using a flow chart:



$n = 2$

Activity 2

1. Start with a certain number, multiply it by 3, then add 2, and the answer is 8.
   We don't yet know what the *certain* number is so we called it *n*.
   1. We start with *n*: n
   2. It is then multiplied by 3: $n \times 3$ (which we write as $3n$)
   3. We then add 2: $3n + 2$
   4. And the answer is 8: $3n + 2 = 8$

   Let us now reverse or undo this procedure:
   4. The answer is 8: 8
   3. The opposite of add is subtract so we subtract 2: $8 - 2$
   2. The opposite of multiply is divide we divide by 3: $(8-2) \div 3$
   1. We should end up with the value of *n*: $(8-2) \div 3 = 6 \div 3 = 2$

   Check: $3(2) + 2 = 8$
   So $n = 2$

2. Start with a certain number, take away 1, then multiply by 2, and the answer is 4.

We don't yet know what the *certain* number is so we called it n.

1. We start with n: n
2. We then subtract 1: $n-1$
3. We then multiply by 2: $(n-1)\times 2$ which we write as $2(n-1)$
4. And the answer is 12: $2(n-1)=4$

Let us now reverse or undo this procedure:

4. The answer is 4: 4
3. The opposite of multiplication is division so we divide by 2: $4\div 2$ $(4\div 2)+1$
2. The opposite of subtracting 1 is adding 1: $(4\div 2)+1= 2+1=3$
1. We should end up with the value of n:

Check:

So $n=3$

3. If we subtract 5 from a certain number, then multiply by 2, the answer is 18.

We don't yet know what the *certain* number is so we called it n.

1. We start with n: n
2. We then subtract 5: $n-5$
3. We then multiply by 2: $(n-5)\times 2$ which we write as $2(n-5)$
4. And the answer is 18: $2(n-5)=18$

Let us now reverse or undo this procedure:

4. The answer is 18: 18
3. The opposite of multiplication is division so we divide by 2: $18\div 2$
2. The opposite of subtracting 5 is adding 5: $(18\div 2)+5$
1. We should end up with the value of n: $(18\div 2)+5=9+5=14$

Check: $2(14-5)=18$

So $n=14$

4. Think of a number, multiply it by 5, then take away 4, and the answer is 6.

We don't yet know what the *certain* number is so we called it n.

1. We start with n: n
2. It is then multiplied by 5: $n\times 5$ (which we write as $5n$)
3. We then subtract 4: $5n-4$
4. And the answer is 6: $5n-4=6$

Let us now reverse or undo this procedure:

4. The answer is 6: 6
3. The opposite of subtract is add so we add 4: $6+4$
2. The opposite of multiply is divide; we divide by 5: $(6+4)\div 5$
1. We should end up with the value of n: $(6+4)\div 5=10\div 5=2$

Check: $5(2)-4=6$

So $n=2$

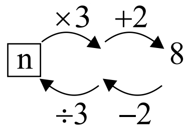
Activity 3

1. $3n + 2 = 8$

$8 - 2 = 6$

$6 \div 3 = 2$

$\therefore\ n = 2$

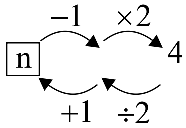


2. $2(n - 1) = 4$

$4 \div 2 = 2$

$2 + 1 = 3$

$\therefore\ n = 3$

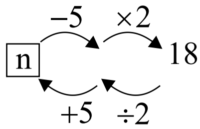


3. $2(n - 5) = 18$

$18 \div 2 = 9$

$9 + 5 = 14$

$\therefore\ n = 14$

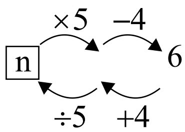


4. $5n - 4 = 6$

$6 + 4 = 10$

$10 \div 5 = 2$

$\therefore\ n = 2$



Activity 4

a) *y* is a variable. 4 and 6 are constants.
b) *n* is a variable. 4 and 3 are constants.

Activity 5

(1) I
(2) A
(3) G
(4) B
(5) D
(6) E
(7) C
(8) J
(9) F
(10) H

Activity 6

(1) D
(2) C
(3) A
(4) B
(5) E

Activity 7

1. $y+5$
2. $y-5$
3. $y \times 5$
4. $\frac{y}{5}$
5. $2y+4$
6. $y+10$ years old
7. $y-10$ years old
8. $\frac{1}{2}x$
9. $7x$ (we multiply by 7 because there are 7 days in a week)
10. $100x$ (we multiply by 100 because there are 100 cents in R1,00)

Activity 8

1. 2 terms: x and $2y$
2. 1 term: abc because $abc = a \times$ product of b and c
3. 1 term: $3(a+4b)$ because $3(a+4b) = 3 \times (a+4b)$
4. 2 terms: $(5a+b)$ and $(9a+b)$

Activity 9

1. 3 is the coefficient of x.
2. $5y$ is the coefficient of x.
3. 1 is the coefficient of x. (We usually do not write the 1 but it is not wrong to do so)
4. $4yz$ is the coefficient of x.
5. $10m$ is the coefficient of x.

Activity 10

Number of months (***t***)	**Amount still owing in rands** (***A***)
1	$800-(20 \times 1)$
2	$800-(20 \times 2)$
3	$800-(20 \times 3)$
4	$800-(20 \times 4)$
5	$800-(20 \times 5)$
10	$800-(20 \times 10)$

Therefore, the formula is $A = 800 - 20t$

1. When $t = 7$
   $A = 800-(20 \times 7)$
   $=$ R660
   After 7 months your friend owes you R660.

2. When $t = 15$ months
   $A = 800-(20 \times 15)$
   $=$ R500
   After 15 months your friend owes you R500.

3. A little different this one: we know A is the amount still owing and want to find t (number of months to pay off the R800). So we still use our same formula:

$$A = 800 - 20t$$

We substitute in the values we know:

$$0 = 800 - 20t$$

(A = 0 because we are trying to find out how long it will take for your friend to pay you the full amount.)

So now it is the value of t (number of months to pay back the R800) that we are looking for.

$$0 = 800 - 20t$$
$$20t = 800$$
$$t = \frac{800}{20}$$
$$t = 40$$

It will take 40 months for your friend to pay you back.

Activity 11

$$A = 800 - 20t$$

Step 1: Subtract 800 from both sides (as we are trying to isolate the t)

$A - 800 = 800 - 20t - 800$ (which is the same as the following)

$A - 800 = 20t$

Step 2: Divide both sides by 20, to get t on one side.

$$\therefore t = \frac{A - 800}{-20}$$

a) $A = \text{R}660$:
So it will take 7 months for your friend to pay you R660.

b) $A = \text{R}500$:
So it will take 15 months to pay back R500.

c) $A = 0$ (everything has been paid back)
So it will take 40 months (or 3 years and 4 months) for your friend to pay the R800 back.

We have reversed the answers of Activity 10. Here we are given the amount owed and we have to find the number of months. In Activity 10 we were given the number of months and had to find the amount owed.

5. Foundations of graphs: Axes and coordinates

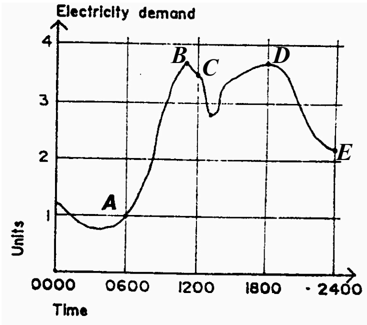
Activity 1

a) b)

INSERT A/W L6-4

Make sure you have equal distances between points on each axis.

Activity 2



Point B: x-coordinate = 1100 (estimate the points that are between marked points on the axis)
y-coordinate = 3,7

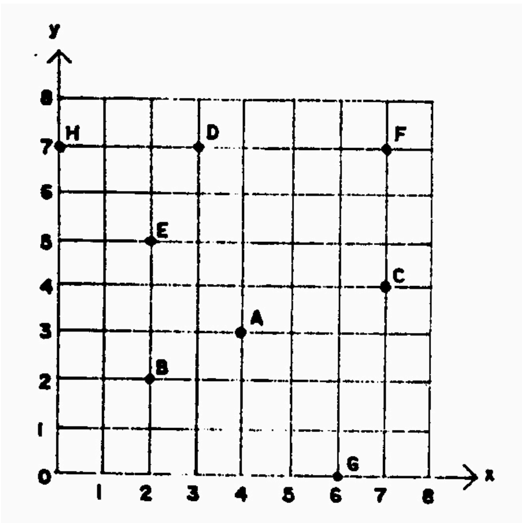
Point C: x-coordinate = 1200
y-coordinate = 3,5

Point D: $x = 1800$
$y = 3,7$

Point E: $x = 2400$
$y = 2,2$

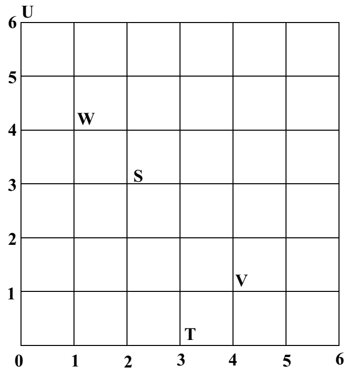
Read the x-coordinates from the x-axis, and the y-coordinates from the y-axis.

Activity 3



a) (4;3)
b) (2;2)
c) (7;4)
d) (3;7)
e) x-coordinate is 2, y-coordinate is 5
f) (7;7)
g) y-coordinate is 0
h) x-coordinate is 0 and y-coordinate is 7

Activity 4



S(2;3), T(3;0), U(0;6), V(4;1), W(1;4), X(6;6)

6. Foundations of shapes, perimeter and area

Activity 1

1. a) Length: 7 cm
   Width: 4 cm

   b) Length: 30 cm
   Width: 20 cm
   Perimeter: = 2 (length + width)
   = 2 (30 + 20)
   = 2 × 50
   = 100 cm

   c) Outer length: 10 cm
   Outer width: 8 cm
   Outer perimeter: = 4 × side
   = 4 × 10
   = 40 cm
   Inner perimeter: = 4 × side
   = 4 × 8
   = 32 cm

2. Perimeter = 2 (length + width)
   = 2 (8 + 4)
   = 2 × 12
   = 24 m

Activity 2

a)	2	b)	8
c)	3	d)	1
e)	4	f)	2
g)	2	h)	1
i)	1	j)	1

Activity 3

(1) D
(2) C
(3) A
(4) E
(5) B

Activity 4

1. Perimeter $= \text{side } 1 + \text{side } 2 + \text{side } 3$
$= 2 + 2 + 7$
$= 11 \text{ m}$

2. Perimeter $= \text{sum of all outer sides}$
$= 50 + 20 + 100 + 10 + 100 + 20 + 50$
$= 350 \text{ cm}$

Activity 5

	radius	**circumference**	**2 × radius**	$\frac{\textbf{circumference}}{\textbf{2} \times \textbf{radius}}$
1	1 cm	6,283 cm	2 cm	$\frac{6{,}283}{2} = 3{,}1415$
2	2 cm	12,566 cm	4 cm	$\frac{12{,}566}{4} = 3{,}1415$
3	3 cm	18,850 cm	6 cm	$\frac{18{,}850}{6} = 3{,}1416$
4	4 cm	25,133 cm	8 cm	$\frac{25{,}133}{8} = 3{,}141625$
5	5 cm	31,416 cm	10 cm	$\frac{31{,}146}{10} = 3{,}1416$
6	10 cm	62,832 cm	20 cm	$\frac{62{,}832}{20} = 3{,}1416$
7	20 cm	125,664 cm	40 cm	$\frac{125{,}664}{40} = 3{,}1416$
8	50 cm	314,159 cm	100 cm	$\frac{314{,}159}{100} = 3{,}14159$

Activity 6

1. a) Circumference of a circle $= \pi \times (2 \times \text{radius})$
$= 3{,}142 \times 2 \times 7$
$= 43{,}988$

Circumference of floor to the nearest metre is 44m

Keys pressed: 3,142; 

; 7; 

; 2; =; 

b) Circumference of a circle $= \pi \times \text{diameter}$

$=3{,}142 \times 20$

$=62{,}84$

Circumference of base of roof to the nearest metre is 63 m

Keys pressed: 3,142; ; 20; =; 

2.

	radius	diameter	circumference
a)	**9 cm**	18 cm	56,556 cm
b)	15 cm	**30 cm**	94,26 cm
c)	5,5 cm	11	**34,562 cm**
d)	11,6 cm	23,2 cm	**72,8944 cm**
e)	45 cm	**90 cm**	282,78 cm

Activity 7

The number of squares gives a measure of the area of each window, or the amount of surface of each window. The area of window A is 16 square units, the area of window B is 15 square units.

a) window A uses more glass.
b) window A lets in more light.

Activity 8

1. a) 14 rows
   b) 9 tiles
   c) $14 \times 9 = 126$ tiles

2. a) $20 \times 2 = 40\text{cm}^2$
   b) $20 \times 20 = 400\text{cm}^2$

Activity 9

1. a) Area of square = side × side = $19 \times 19 = 361\ \text{mm}^2$
   b) Area of rectangle = length × width = $9 \times 4{,}5 = 40{,}5\ \text{cm}^2$

2. Area of rectanglular laptop screen
   = length × width = $30 \times 20 = 600\ \text{cm}^2$

3. a) Area of glass surface of square frame
   = side × side = $8 \times 8 = 64\ \text{cm}^2$
   b) $64\ \text{cm}^2 \times$ R0,49 per cm^2 = R31,36

Activity 10

1. a) Area of triangle $= \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 11 \times 14$

$= 77 \text{ cm}^2$

b) Area of triangle $= \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 10 \times 5$

$= 25 \text{ mm}^2$

2. a) Area of triangle $= \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 5 \times 8$

$= 20 \text{ cm}^2$

b) Area of rectangle $= \text{length} \times \text{width}$

$= 15 \times 8$

$= 120 \text{ cm}^2$

Total area $=$ area of triangle $+$ area of rectangle

$= 20 \text{ cm}^2 + 120 \text{ cm}^2$

$= 140 \text{ cm}^2$

Activity 11

1. Radius $= 3$ cm

a) A $= \pi \times (\text{radius})^2$

$= \pi \times (3)^2$

$= 3{,}142 \times 9$

$= 28{,}278 \text{ cm}^2$

Keys pressed: 3,142 

; 3 ; 

; 

; 

b) Diameter is 21 cm, so radius $= 10{,}5$ cm

A $= \pi \times (\text{radius})^2$

$= \pi \times (10{,}5)^2$

$= 3{,}142 \times (10{,}5)^2$

$= 346{,}4055 \text{ cm}^2$

Keys pressed: 3,142 

; 10,5 ; 

; 

Material is measured in metres making this the more appropriate unit of measurement.

2. Radius = 22 cm

   a) A $= \pi \times (\text{radius})^2$

   $= \pi \times (22)^2$

   $= 3{,}142 \times (22)^2$

   $= 1520{,}728 \text{ cm}^2$

   $= 1521 \text{ cm}^2$ (correct to nearest cm^2)

   Keys pressed: 3,142 [nPr ×] ; 22 ; [x^2] ; [=]

3. Radius of outer circle of metal lining = 21 cm

   A $= \pi \times (\text{radius})^2$

   $= \pi \times (21)^2$

   $= 3{,}142 \times (22)^2$

   $= 1385{,}622 \text{ cm}^2$

   Radius of inner circle of metal lining = 17 cm

   A $= \pi \times (\text{radius})^2$

   $= \pi \times (17)^2$

   $= 3{,}142 \times (17)^2$

   $= 908{,}038 \text{ cm}^2$

   Area of metal lining is the difference between the two areas above:
   $1358{,}622 - 908{,}038 = 477{,}584 \text{ cm}^2$

   This is 478 cm^2 to the nearest cm^2.

7. Foundations of statistics

Activity 1

You could give many different examples of statistics. If your example is not on this list, it does not mean it is wrong. A list of some examples of statistics:

- the temperatures of the main cities of South Africa on a particular day
- the increase of the price of petrol over a period of years
- the record of population growth of a country over the years
- the number of accidents recorded on a major road over the past 5 years
- the performance of the various teams of the National Soccer League
- the interest rates of a bank on savings accounts for various periods
- the number of different types of cars bought in a year
- the record of the student population of a university

Activity 2

We will not give feedback as we cannot give all the uses of statistics.
Your tutor will help you to decide if your examples are suitable.

Activity 3

a) 9 b) 4 c) 7

Activity 4

1. B D B C F G A B G F B C C D A G E E B D E C G F D

Symbols	**Tally**	**Frequency**
A	||	2
B	𝍸	5
C	||||	4
D	||||	4
E	|||	3
F	|||	3
G	||||	4
Total		25

2. 1 0 3 2 5 6 3 5 3 0
4 1 3 2 3 3 5 2 1 3

Number of bankrupt businesses	**Tally**	**Frequency**
0	||	2
1	|||	3
2	|||	3
3	𝍸 ||	7
4	|	1
5	|||	3
6	|	1
Total		20

Activity 5

Marks	**Tally**	**Frequency**
10-17	||	2
18-25	𝍸 |	6
26-33	𝍸 ||	7
34-41	𝍸 ||	7
42-49	||	2
Total		24

Activity 6

Number of eggs	Tally	Frequency
0	𝍸 ||	7
1	𝍸	5
2	𝍸	5
3	|||	3
Total		20

a) 7, that is the frequency for 0.
b) $7+5=12$ (frequency of 0 + frequency of 1).
c) More than 1 egg means 2 or 3 eggs. Sum of the two frequencies is $5+3=8$.
Total frequency is 20. Therefore the proportion is $\frac{8}{20}=\frac{2}{5}$.
d) The frequency of nests with 3 eggs is 3. So 3 out of the 20 nests contain 3 eggs.

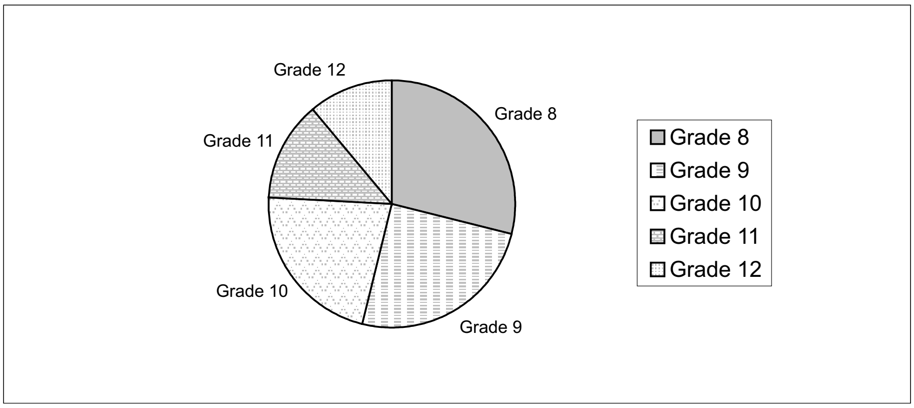
$\frac{3}{20}\times 100\% = 15\%$. Therefore 15% of the nests contain 3 eggs.

Activity 7

	Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
Number	208	180	158	94	80
Degrees	104	90	79	47	40

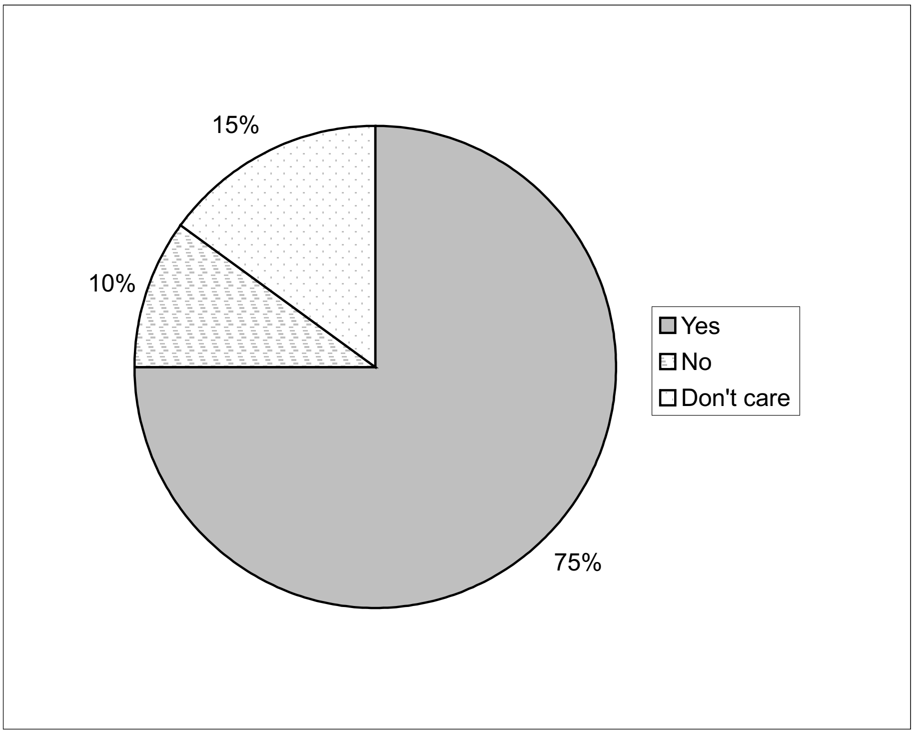
We got the degrees by dividing each of the numbers by 720 (the total number of learners) and then multiplying by 360.

That is, for Grade 8 we have: $\frac{208}{720}\times 360=104°$



Activity 8

	Yes	No	Don't care
Percentage	75	10	$100\ (75+10)=15$
Degree	$\frac{75}{100}\times 360=270°$	$\frac{10}{100}\times 360=36°$	$\frac{15}{100}\times 360=54°$



Activity 9

a) If 120° represents 100 cows, then:

$60° = \frac{60}{120} \times 100 = 50$ (the cats are represented by 60° on the pie chart)
Therefore there are 50 cats on the farm.

b) Number of sheep: $\frac{90}{120} \times 100 = 75$

c) Total number of animals on the farm: $\frac{360}{120} \times 100 = 300$

d) Percentage of animals that are sheep: $\frac{75}{120} \times 100 = 25\%$

Feedback to Self-check exercises

1. More calculations

1. a) $27 + 9 \div 3 =$
   $9 \div 3 = 3$, then $27 + 3 = 30$

   b) $(25 - 5) \div (6 - 2) \times 5$
   $= 20 \div 4 = 5$ (Brackets first)
   $= 5 \times 5$ (Working left to right)
   $= 25$

2. a) $27 + 9 \div 3 =$ [27] [+] [9] [÷] [3] [=]
   Answer = 30

   b) [( ] [25] [–] [5] [)] [÷] [(] [6] [–] [2] [)] [×] [5] [=]
   Answer = 25

3. a) 556,8 = 100

   b) (rounded off to one decimal place)

4. a) [30] [–] [10] [=] [SHIFT] [STO] [M+] Clear [95] [+] [5] [=] [÷] [RCL] [M+] [=]
   Answer = 5

   b) [75] [+] [6] [÷] [2] [=] [SHIFT] [STO] [A] Clear [59] [+] [3] [×] [3] [=] [SHIFT] [STO] [B] Clear [500] [÷] [5] [–] [50] [+] [3] [×] [2] [=] [÷] [(] [RCL] [A] [-] [RCL] [B] [ )] [=] [S⇔D]
   Answer = 5,6

5. Calculate the amount of money raised by selling tickets and programmes and store it.

   Now calculate the money spent for the preparation of the concert. Then subtract the money spent from the money raised to see how much money was earned.

   So the money raised was R12 007,50

2. Fractions: addition and subtraction

1. a) $\frac{3}{4} + \frac{1}{2}$

   $\frac{1}{2} = \frac{2}{4}$

   $\therefore \frac{3}{4} + \frac{1}{2} = \frac{3}{4} + \frac{2}{4}$

   $= \frac{5}{4}$

   b) $\frac{1}{3} + \frac{2}{9}$

   $\frac{1}{3} = \frac{3}{9}$

   $\therefore \frac{1}{3} + \frac{2}{9} = \frac{3}{9} + \frac{2}{9}$

   $= \frac{5}{9}$

c) $\frac{5}{8}-\frac{2}{4}$

$=\frac{5}{8}-\frac{4}{8}$

$=\frac{1}{8}$

d) $\frac{6}{18}-\frac{2}{9}$

$=\frac{6}{18}-\frac{4}{18}$

$=\frac{2}{18}$

$=\frac{1}{9}$

2. a) 7 and 15 do not have a common factor. Therefore:
LCD $= 15 \times 7 = 105$

$$\frac{3}{7} + \frac{3}{15} = \frac{45}{105} + \frac{21}{105} = \frac{66}{105}$$

b) 20 is a multiple of 5. Therefore: $\frac{4}{5}$ can be written as:

$\frac{4}{5}\times ? = \frac{}{20}$

$\frac{4}{5}\times\frac{4}{4}=\frac{16}{20}$

$\therefore \frac{4}{5}+\frac{1}{20}$

$=\frac{16+1}{20}$

$=\frac{17}{20}$

c) 35 is a multiple of 7. So the LCD $= 35$.

$\frac{14}{35}-\frac{2}{7}$

$=\frac{14-10}{35}$

$=\frac{4}{35}$

d) 7 and 9 do not have a common factor. So: LCD $= 7 \times 9 = 63$.

$\frac{1}{7}+\frac{2}{9}$

$=\frac{9}{63}+\frac{14}{63}$

$=\frac{9+14}{63}$

$=\frac{23}{63}$

e) 15 and 25 have a common factor. So we make a table of multiples.

Multiples of 15	**Multiples of 25**
15 (since $15 \times 1 = 15$)	25 (since $25 \times 1 = 25$)
30 (since $15 \times 2 = 30$)	50 (since $25 \times 2 = 50$)
45 (since $15 \times 3 = 45$)	75 (since $25 \times 3 = 75$)
60 (since $15 \times 4 = 60$)	
75 (since $15 \times 5 = 75$)	

STOP!!

So, the LCD = 75.

$$\frac{4}{15} - \frac{2}{25}$$
$$= \frac{20}{75} - \frac{6}{75}$$
$$= \frac{20 - 6}{75}$$
$$= \frac{14}{75}$$

3. $\frac{3}{4} + \frac{3}{2} + \frac{5}{4} = \frac{3}{4} + \frac{6}{4} + \frac{5}{4}$

$= \frac{14}{4}$

$= \frac{32}{4}$ kg or $3\frac{1}{2}$ kg

4. For a $\frac{3}{4}$ inch nail: $5\frac{1}{2} - \frac{3}{4}$ $= \frac{32}{4} - \frac{3}{4}$

$= \frac{22}{4} - \frac{3}{4}$

$= \frac{19}{4} = 4\frac{3}{4}$ inches

For a $\frac{7}{8}$ inch nail: $5\frac{1}{2} - \frac{7}{8}$ $= \frac{11}{2} - \frac{7}{8}$

$= \frac{44}{8} - \frac{7}{8}$

$= \frac{37}{8} = 4\frac{5}{8}$ inches

3. Foundations of finance: Simple interest and discounts

1. I $= P \times R \times T$

$= 124 \times \frac{10}{100} \times \frac{7}{12}$ year (12 since 1 year = 12 months)

$=$ R7,23

Amount: $= P + I$

$= 124 + 7{,}23$

$=$ R131,23

To do this using your calculator, press the following key sequence:

[10]; SHIFT; (; ×; [124]; ×; [7] ÷ [12]; =; S⇔D; +; [124]; =; S⇔D

You will notice that the calculator keys pressed are not in the same order as the calculations that are written out above. This is because logically we are telling the calculator to work out what 10% is of 124 and multiply that by $\frac{7}{12}$ which is the interest period.

This gives us the amount of R7,23 that we then keep in the calculator and just add to the R124 to get the final answer.

Play around with your calculator though. You can try to do the calculations in the order they appear in the calculation, without then using the % key.

2. a) $I = P \times R \times T$ b) $I = P \times R \times T$

$= 420 \times \frac{8}{100} \times 3$ $= 360 \times \frac{7}{100} \times 4$

$= R100{,}80$ $= R100{,}80$

Both investments yield the same interest.

3. $I = P \times R \times T$

$= 25 \times 4{,}5\% \times 2$

$= R2{,}25$

Amount $= R25 + R2{,}25$

$= R27{,}25$

Keys pressed:

[4,5]; [SHIFT]; [% (]; [×]; [25]; [×]; [2]; [=]; [S⇔D]; [+]; [25]; [=]; [S⇔D]

4. $I = 4\,900 \times \frac{12}{100} \times 1{,}5$ years (1,5 year = 1 year 6 months)

$= R882{,}00$

Amount owing $= 4900 + 882$

$= R5\,782$

5. $I = P \times R \times T$

$= 480 \times 3{,}25\% \times \frac{1}{12}$ year (since 1 year = 12 months)

$= R1{,}30$ ($3\frac{1}{4}\% = 3{,}25\% = 3\frac{25}{100}$)

The interest payable monthly is R1,30.

6. Interest earned on R50 for whole year is:

$I = 50 \times \frac{5}{100} \times 1$

$= R2{,}50$

Interest earned on R20 for the half a year from 1 July to the end of the year is:

$I = R20 \times \frac{5}{100} \times \frac{6}{12}$

$= R0{,}50$

Total interest earned is $R2{,}50 + R0{,}50 = R3{,}00$

7. $I = P \times R \times T$

$= 600 \times \frac{6}{100} \times \frac{6}{12} = R18{,}00$

8. a) Bank: $I = 5000 \times \frac{15}{100} \times 2$

$= 1\,500$

Amount owing $= 5000 + 1\,500$

$= R6\,500$

b) To borrower: $I = 6\,500 \times \frac{20}{100} \times 2 = R2\,000$

Amount $= 5\,000 + 2\,000 = R7\,000$

His profit is: $R7\,000 - R6\,500$

$= R500$

9. You will pay:
$R17{,}95 \times 2\ \ 5\%$ of $(7{,}95 \times 2)$
$= R35{,}90 - 1{,}80$
$= R34{,}10$

10. Tom will pay:
$R250 - 2{,}5\%$ of 250
$= R250 - 6{,}25$
$= R243{,}75$
Keys pressed: [250] ; [–] ; [2,5] ; [SHIFT] ; [(] (%) ; [×] ; 250 ; [=] ; [S⇔D]

4. Foundations of algebra

1. a) Let T = Thato's shares, S = Steve's shares, P = Peter's shares. Then
$T = 4S$
$S = 3P$
$T = 4(3P) = 12T$

b) The total weight of the 12 (dozen) cars to be transported is 12(2) = 24 metric tons. Since the cost per kilometre of 1 metric ton is x rands, the cost per kilometre for 24 metric tons is $24x$ rands. If the cost of transport the entire shipment is $24x$ rands per kilometre, the cost is $24xn$ rands for a distance of n kilometres.
The answer is $24xn$.

2. a)
$x + 7 = 15$
$x + 7 - 7 = 15 - 7$
$x = 8$

b)
$2y - 5 = 15$
$2y = 20$
$y = 10$

c)
$4(a + 3) = 28$
$a + 3 = 7$
$a = 4$

d)
$\frac{n}{3} = 4$
$\frac{n}{3} \times 3 = 4 \times 3$
$n = 12$

e)
$\frac{x}{2} + 1 = 6$
$\frac{x}{2} = 5$
$x = 10$

3. a) $22 - c$

b) i) $3{,}5 \times c = 3{,}5c$
ii) $(22 - c) \times (-1) = -1(22 - c) = -22 + c$

c) $3,5c + (-22 + c) = 63,5$

d) $3,5c + (-22 + c) = 63,5$

$3,5c - 22 + c = 63,5$

$3,5c + c - 22 = 63,5$

$4,5c - 22 = 63,5$

$4,5c = 63,5 + 22$

$4,5c = 85,5$

$c = 19$

e) Incorrect answers: $22 - c = 22 - 19 = 3$
3 incorrect answers

4. a) 3

b) −6

c) 3

d) 2

5. The formula for speed is given by: $s = \frac{d}{t}$

Where: s = speed
d = distance
t = time

a) $s = \frac{d}{t}$

$s \times t = \frac{d}{t} \times t$

$s \times t = d$

$\frac{st}{s} = \frac{d}{s}$

$t = \frac{d}{s}$

b) i) $s = 100$
ii) $d = 10$

c) $t = \frac{d}{s}$

$t = \frac{10}{100}$

$t = \frac{1}{10}$ (which is $\frac{1}{10}$ of an hour)

$t = 6$ min (1 hour = 60 minutes and $\frac{1}{10}$ of 60 = 6)

5. Foundations of graphs: Axes and coordinates

1. a)

A(2; 0)	F(1; 5)
B(2; 2)	G(3; 5)
C(5; 2)	H(4; 6)
D(5; 4)	I(6; 8)
E(1; 4)	J(0; 8)

   b) From A go North 2 blocks to B
   From B go 3 blocks East to C
   From C go 2 blocks North to D
   From D go 4 blocks West to E
   From E go 1 block North to F
   From F go 2 blocks East to G
   From G go 1 block East and then 1 block North to H or
   From G go 1 block North and then 1 block East to H
   From H go 2 blocks East and then 2 blocks North to I or
   From H go 2 blocks North and then 2 blocks East to I
   From I go 6 blocks West to J

2.

A(1;2)	B(5;2)	C(3;7)	D(0;5)
E(8;5)	F(14;8)	G(8;11)	H(11;8)
K(1;8)	L(9;8)	M(5;11)	N(7;1)
P(11;1)	Q(13;4)	R(9;4)	S(15;1)
T(17;1)	U(17;6)	V(15;6)	

3. a) INSERT A/W A-2

   b) Yes.

   c) INSERT A/W A-3

   d) At (2;1) or at any other point along the escape route.

6. Foundations of shapes, perimeter and area

1. Perimeter $= 2(24 + 42)$
$= 2(66)$
$= 132$ m
He needs 132 m of fence.

2. a) Area $= 600 \times 600$ mm
$= 360\,000 \text{ mm}^2$
b) Area $= 80 \times 120$ m
$= 9\,600 \text{ m}^2$
c) Area $= \pi \times 5 \times 5$
$= 78{,}55 \text{ m}^2$

3. Area of triangle $= \frac{1}{2} \times \text{base} \times \text{height}$
$= \frac{1}{2} \times 50 \times 55$
$= 1375 \text{ cm}^2$

7. Foundations of statistics

1. a)

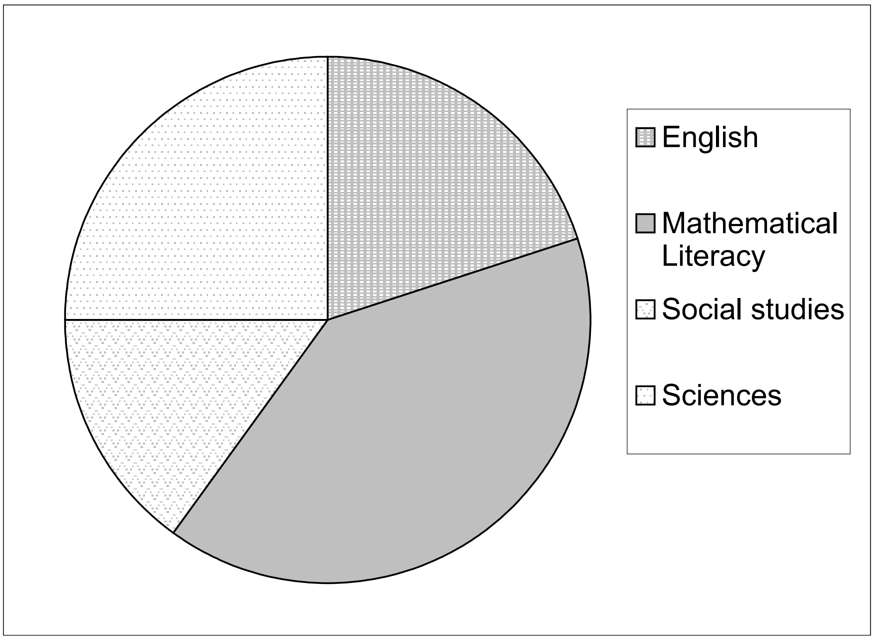
Number of programmes watched	Tally	Frequency
1	𝍸 𝍸 |	11
2	𝍸 𝍸 ||	12
3	𝍸 𝍸 ||||	14
4	𝍸 ||||	9
5	𝍸 |||	8
6	𝍸 |||	8
7	𝍸 𝍸	10
8	𝍸 𝍸 |||	13
Total		85

b) 8 people watched 5 programmes

c)

Height (cm)	Tally	Frequency
140-149	||	2
150-159	|||	3
160-169	𝍸 ||	7
170-179	𝍸 |	6
180-189	||	2
Total		20

3. a)



b) 5 periods

c) 10 hours