Mathematical Literacy

Unit 3

Building on the foundations

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Unit 3

Building on the foundations

A course for adults at secondary level by distance education

1. Ratio and direct proportion: Comparing and combining different amounts

Introduction

Think about the title of this lesson. Do you know what it means? When we **compare** the amounts of different things we look at how much of each thing we've got, and we see if the amount of each thing is the same or different. **Combining** means mixing these amounts together.

Often we might need to mix or combine things together in the right **proportion**. The things that are mixed together are often called the **ingredients**.

Do you know any people who mix ingredients in this way? Think of the baker who needs the right amounts of ingredients such as milk, flour and sugar to make good bread. And consider the builder, who must mix or combine the right amounts of ingredients to make cement. The chemist or herbalist must mix the right amounts of ingredients to make the appropriate medicine.

Appropriate: most suitable.

We call the different amounts of things that must be mixed **proportional parts** and we compare them mathematically by writing down a **ratio**. The ratio shows us what fraction of the total amount each part is.

In this lesson you will meet Seca who, like you, is a student. One day he decided to show his friends Thandi and Thabo that they were using maths in the things they do every day at home. He thought that was one way he could share his studies with his friends.

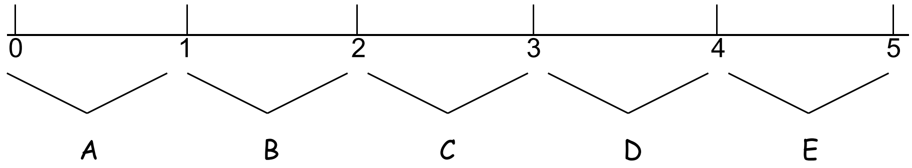
Seca showed Thabo and Thandi how to use proportion in their baking and building. In this lesson Seca also helps Thandi to draw plans for the house she is building. He shows Thandi how to make a drawing of the house on paper using a scale.

In this lesson you will:

- understand the meaning of ratio, direct proportion, exchange rates and scale drawings
- write down the amounts of ingredients as a ratio
- convert (change) ratios into simplified fractions
- convert (change) fractions into simplified ratios
- work with exchange rates
- calculate distances using the scale of a map.

Fractions and ratio

What do you know about fractions? If you can't follow this section on fractions look back at Unit 1, lesson 3. Have you got a ruler? Find the number line on the ruler. Look at the number line below.



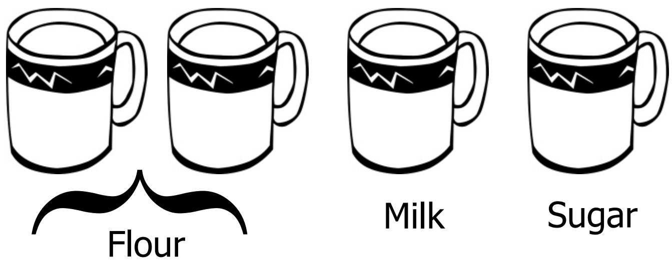
Remember that we can divide the number line into equal parts. We can write these equal parts as fractions. We can say that A is one of the five equal parts of the number line, so A is one fifth ($\frac{1}{5}$) of the length of the line. B, C, D and E are each $\frac{1}{5}$ of the length of the line. If we compare A with B we can see that they are the same length. A and B are each $\frac{1}{5}$ of the line. B, C, D and E together make up $\frac{4}{5}$ of the line. Check that this is true by counting how many parts B, C, D and E are of the five-part line.

We can see that all the parts are the same length.
When we combine the five parts we get the whole line:
$A + B + C + D + E = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{5}{5} = 1$ line

Writing ingredients as a ratio

When Seca went to visit Thabo and Thandi, Thabo was taking amounts of different ingredients and combining them to make a cake for a few friends. Thabo had to get the right amount of each ingredient to mix. He didn't want to make a cake that no one would eat! Thandi told him to combine 2 cups of flour with 1 cup of milk and 1 cup of sugar. Seca told Thabo that this was the **proportion** in which to combine the ingredients. Thabo asked Seca to explain what he meant by proportion.

Look at the picture Seca drew to show Thabo.



Seca showed that for two cups of flour you need one cup of sugar and one cup of milk.

There is a mathematical way of writing down the proportions in which amounts are mixed. This is called *ratio*.

Look again at Seca's drawing. We compare the amounts of each ingredient by writing:
2 cups of flour:1 cup of milk:1 cup sugar
When we read this we say 2 cups of flour to 1 cup of milk to 1 cup of sugar.

When we write this mathematically as a ratio, we write:
2:1:1 and when we read this we say ‘2 to 1 to 1’.

Writing ratios as fractions

We can use this ratio to find the total number of cups of ingredients we added together.

We get 2 cups + 1 cup + 1 cup = 4 cups. Each cup is a fraction of the total number of cups of ingredients. The total number of ingredients equals 4 cups. Let's work out what fraction each ingredient is of the whole mixture.

ACTIVITY 1

1. Work out what fraction the cups of flour are of the total mixture.
2. What fraction is the milk?
3. How many times bigger was the amount of flour compared to the amount of milk?

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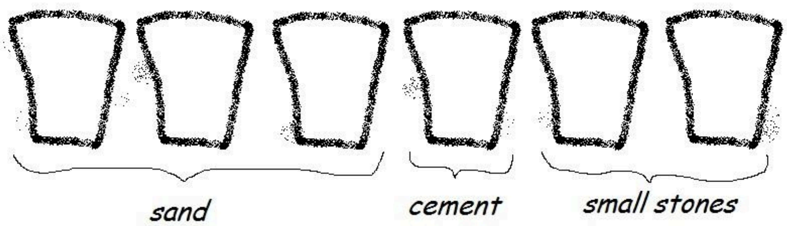
Thabo then gave Thandi advice about making the concrete blocks she needed. He told her to mix 3 buckets of sand with 1 bucket of cement and 2 buckets of small stones to make one block. Seca picked up a stick and drew the proportions in the sand.



Note: we can get the total number of parts in the mixture from the ratio.
Total number of parts (buckets)
= 3 + 1 + 2 = 6 buckets

ACTIVITY 2

1. Write down the ratio in which Thandi added the ingredients. Start with sand, then cement, then stones. Next, write the ratio in words.
2. Use the ratio to work out what fraction of each ingredient was added. Try and simplify the fractions.

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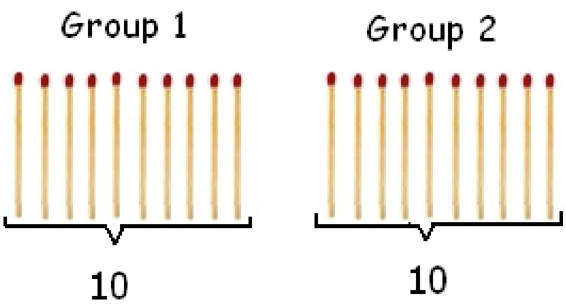
Try the next activity. All you will need is a box of at least 20 matches, or you can use 20 little sticks.

ACTIVITY 3

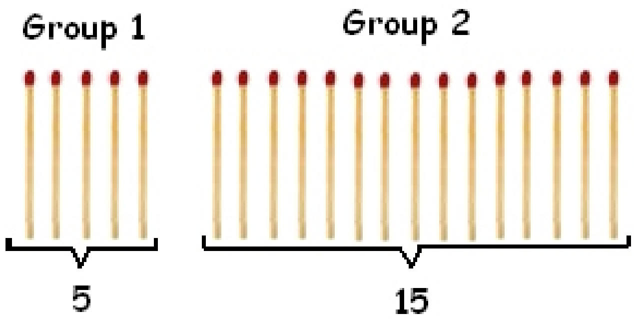
1. Group the 20 matches in the following way:



The ratio 1:1, called the simplified ratio of 10:10, tells us that we have the same amount in each group. But this ratio does not tell us how many matches there are in each group like the ratio 10:10 did.

You have divided the matches in groups so that for 10 matches in group 1 there are 10 matches in group 2. The matches have been divided in the ratio of 10:10.
What is the smallest number in the ratio 10:10?
10 of course, because both groups have the number 10. 10 divided by 10 is 1. So if we divide both numbers by 10 our ratio becomes 1:1.

2. Now divide the 20 matches as follows:



You have divided the matches in the ratio 5:15.
Simplify this ratio.

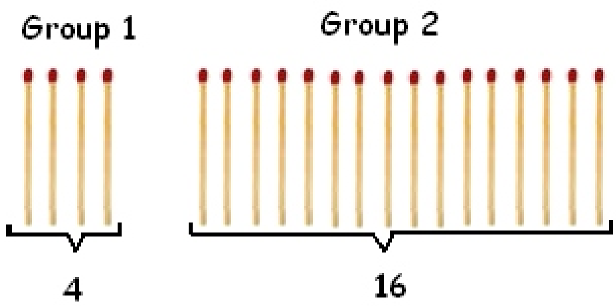
What is the smallest number in the ratio? The smallest number is 5. So we must divide 5 into 5 and 15. We get the ratio 1:3.

This simple ratio helps us to compare the two groups. From the ratio 1:3 we can see that group 2 has three times as many matches as group 1.

Is this true?

Check your thinking. How many times bigger is 15 than 5? 3 times of course. Did you follow this? $5 \times 3 = 15$.

3. Now divide the matches into 2 groups, one of 4 and the other of 16, like the matches in the next picture and answer the questions that follow.



Work out the ratio in which you have divided the matches.

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What fraction of the total is in each group?

Writing fractions as ratios

You already know how to write ratios as fractions. Remember, the ratio tells us how many parts there are of the whole. In the ratio 4:16 there are 20 parts (matches) altogether. There are 4 parts for group 1 and 16 parts for group 2 which makes 20 parts (the total of all the matches). Group 1 had 4 of those 20 so the fraction is $\frac{4}{20}$.

Remember, with ratios we compare two amounts. We can compare two fractions by writing them as a ratio. To do this we have to make the denominators of the fractions the same.

Do you remember the equivalent fractions $\frac{1}{3}$ and $\frac{2}{6}$ from Unit 1, lesson 3?

Equivalent: having the same value.

We can prove that they are equivalent by making their denominators the same. If we turn the fraction with denominator 3 into a fraction with denominator 6, we can show that these fractions are equivalent. To get a denominator of 6 we have to multiply denominator 3 by 2. We must also multiply the numerator by the same number. By multiplying the numerator and the denominator by the same number, we are multiplying the fraction by the number 1, which ensures that we are not changing the value of the fraction but keeping it equivalent.

So: $\frac{1}{3} \times \frac{2}{2} = \frac{2}{6}$

The fraction $\frac{1}{3}$ has the same number of parts out of 6, 2 parts, as the fraction $\frac{2}{6}$ has.

They are therefore equivalent fractions.

If you want to compare fractions you can do that by changing the fractions to have the same denominator. We have already shown this when we compared equivalent fractions. We can also use this method to find out if one fraction is bigger or smaller than another fraction. This will give us the ratio of the fractions.

To write the ratio of the fractions, we need to get the denominators of the fractions the same, and then write down the value of the numerator of each fraction.

Example

Let's write down the ratio of the two fractions, $\frac{2}{5}$ and $\frac{4}{20}$. To get the same denominator we must multiply $\frac{2}{5}$ by $\frac{4}{4}$.

We get: $\frac{2}{5} \times \frac{4}{4} = \frac{8}{20}$

So we can write the ratio of $\frac{2}{5} : \frac{4}{20}$ as the ratio of $\frac{8}{20} : \frac{4}{20}$.

We simplify this to get the ratio 8:4 or 2:1.

ACTIVITY 4

Write down the ratio of the following fractions:

$\frac{3}{12}$, $\frac{1}{4}$ and $\frac{2}{6}$.

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Direct proportion

Seca saw Thabo again the day after he had his friends over for tea and cake. Thabo's cake was such a success with his visitors that he decided to try and bake an even bigger cake for his family of 8 people.

ACTIVITY 5

Remember the recipe for a cake for 4 people was:
2 cups flour
1 cup milk
1 cup sugar
What amount of ingredients does Thabo need to make a cake for 8 people?

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Before we discuss direct proportion, for example the increase in the amount of cake needed as the number of people increase, first try and see what you remember about ratios and fractions.

ACTIVITY 6

1. Write down the ratio of the ingredients Thabo would need for 8 people without simplifying the ratio.

2. Now simplify the ratio.

3. Write each ingredient in the ratio 4:2:2 as a fraction of the total number of cups used in the mixture.

4. Simplify these fractions. Then simplify the ratio in which they are mixed.

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The increase in the number of people taught Thabo a lesson in direct proportion. Thabo learnt that the amount of cake he needed to make, increased as the number of people increased. He found out that if the number of people doubled then the amount of cake he needed must also double.

ACTIVITY 7

1. Write down the ingredients needed to make enough of Thabo's cake for:
   a) 12 people
   b) 16 people
   c) 20 people

2. Write down the ratios (in their simplest form) of the ingredients needed for:
   a) 12 people
   b) 16 people
   c) 20 people

3. Write down the **total number of cups** of ingredients needed to make a cake for:
   a) 4 people
   b) 8 people
   c) 12 people
   d) 16 people
   e) 20 people

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In question 2, the simplified ratio remains the same. This is known as direct proportion. The number of people and the amount of ingredients required for the cake increase *in proportion* to each other. We multiply the people and the cups of ingredients by the same number. This number is known as a *constant ratio.*

Look at the following patterns in adapting the recipe:

People	Cups of flour	Cups of milk	Cups of sugar	Ratio
4	2	1	1	2: 1: 1
8	4	2	2	4: 2: 2
12	6	3	3	6: 3: 3
16	8	4	4	8: 4: 4
20	10	5	5	10: 5: 5

When the number of people (4×2=8) are doubled, each cup of ingredient (2: 1: 1) is also doubled:
(2×2=**4**; 1×2=**2**; 1×2=**2**) = (4: 2: 2) = (2: 1: 1) in its simplest form.

If you have 3 times the number of people, each cup of ingredient (2: 1: 1) is also multiplied by 3:
(2×3=**6**; 1×3=**3**; 1×3=**3**) = (6: 3: 3) = (2: 1: 1) in its simplest form.

If you have 5 times the number of people, each cup of ingredient (2: 1: 1) is also multiplied by 5:
(2×5=**10**; 1×5=**5**; 1×5=**5**) = (10: 5: 5) = (2: 1: 1) in its simplest form.

Do you remember how to draw axes and to plot co-ordinates from Unit 2, lesson 5? In the next activity you will draw a graph of direct proportion. We can use graphs to show how one amount changes as the other amount changes.

ACTIVITY 8

In the table below the amounts of ingredients needed for a cake to feed
(a) 4 visitors
(b) 8 visitors are shown.

No. of people	Total cups of ingredients needed
(a) 4	4 cups
(b) 8	8 cups

Because the number of people doubled, the amount of cake needed also doubled.

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Draw a pair of axes. Let the number of people be on the horizontal axis and the number of ingredients be on the vertical axis. Mark off points 0 to 8 along both axes and plot the co-ordinates from the table. Draw a line by joining up the points of the plotted co-ordinates.

How do we work out the co-ordinates? Since the number of people is along the horizontal or x-axis and number of cups of ingredients along the vertical or y-axis and we write the co-ordinates as (x,y), we put down the number of people first and then the number of cups of ingredients needed for that number of people. We get the co-ordinates: (4;4) and (8;8). Now draw the graph.

The co-ordinates tell us the number of people and the total number of cups of ingredients needed at the same time. Have a look at the graph again.

When the amounts are in proportion, then the line joining the marks will pass through the point where the vertical and horizontal number lines (called axes) join. This point is known as the **origin** of the graph. But on this particular graph we cannot join the dots as you cannot have 0,5 people!

ACTIVITY 9

1. A re-hydrate mixture used to treat diarrhea recommends that the patient be given 1 sachet of powder mixed with 250 ml of water. The number of times per day that the patient needs to drink the re-hydrate mixture is calculated using the weight of the person. For every 10 kg the person weighs, they should have 1 re-hydrate measure of the mixture. How many re-hydrate mixtures should a patient have who weighs:
   a) 20 kg
   b) 50 kg

2. A baby buffalo needs to be fed 20 % of its body weight each day. How much should the baby buffalo be fed per day when she weighs:
   a) 40 kg
   b) 60 kg
   c) 100 kg

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ACTIVITY 10

1. a) Use question 1 from Activity 9 to complete the following table:

Weight of patient	Number of re-hydrate mixtures
10 kg	
20 kg	
30 kg	
40 kg	

b) Use the table in a) to help you plot the points and draw a graph. Let the weight of the patient be on the x-axis and the number of re-hydrate mixtures be represented on the y-axis.

2. a) Use question 2 from Activity 9 to complete the following table:

Weight of baby buffalo	Kilograms of food per day
10 kg	
20 kg	
30 kg	
40 kg	

b) Use the table in a) to help you plot the points and draw a graph. Let the weight of the buffalo be on the x-axis and the amount of food (in kg) be represented on the y-axis.

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Exchange Rates

In ratios we compare quantities which are expressed in the same units. For example in the previous activities in this lesson, we compared ***cups*** *of flour to* ***cups*** *of milk or water, i.e. the* ***cups*** *being the units. We usually talk of rates when we compare two quantities which are of different types, .e.g. distance and time, or rands and dollars.*

The money that is in use in a country is called the *currency* of that country. The currency of South Africa is the Rand (R). In Japan the currency is the Yen (¥). In the United States of America, the currency is the Dollar ($), and in Britain, the Pound (£).

When we visit another country, we have to buy money that can be used in that country. We say that we 'exchange currency'. The price that we pay is called the 'exchange rate'. The exchange rate is a type of ratio. Because the quantities are each in a different currency though, we can also call it a rate.

Example

Tumi wanted to visit her cousin in Zambia. She found out that the exchange rate was 701,73 Zambian Kwacha (ZK) to the Rand (R). Tumi had saved R1 000 to spend while she was in Zambia.

- How many Zambian Kwacha could Tumi buy with her R1 000?
- Tumi would need to keep ZK300 000 to pay the taxi back to South Africa, and ZK40 000 for her food and refreshments on the trip back home. How many Zambian Kwachas remained for Tumi to spend on her holiday?

Solution

The exchange rate of Rands to Zambian Kwachas is 1:701,73
This is the ratio in unit form. A ratio in unit form is one where the first amount in the ratio is a unit of 1.

Each R1 buys ZK701,73. The ratio/rate is written as 1:701,73
So for R1 000: $1 \times 1\,000 : 701{,}73 \times 1\,000$
$= 1\,000 : 701\,730$
Tumi would buy ZK701 730 for her R1 000.
Tumi's spending money must pay the taxi: $701\,730 - 300\,000 = 401\,730$
Allow for refreshments on the way home: $401\,730 - 40\,000 = 361\,730$
ZK361 730 remains for Tumi's spending money on her visit to Zambia.

ACTIVITY 11

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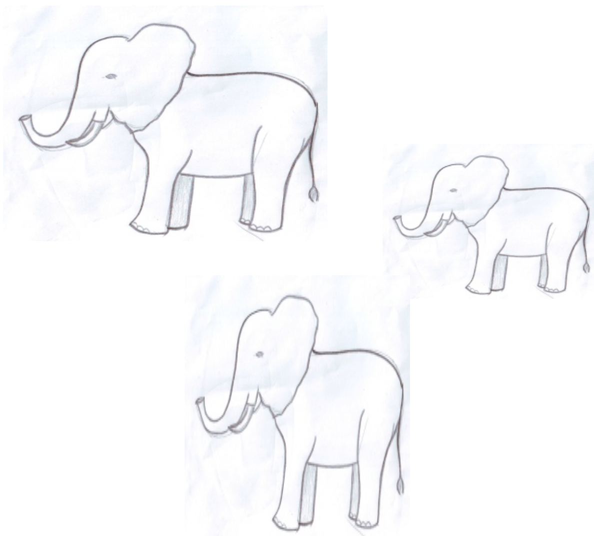
When Tumi's cousin Barbara had visited South Africa 6 months earlier, the exchange rate was 655,37 Zambian Kwacha to the Rand. How many Rands did Barbara buy for ZK1 000 000?

Proportion and drawing maps

Most maps are so big that you can't draw them to their real size on paper. People who make maps have to draw big countries on paper. When they draw these maps they make all the parts of the maps the same number of times smaller than the real size of the parts. This is called *drawing maps to scale*.

To draw big things like South Africa to scale on paper you have to make the country a lot smaller. If you need to compare the sizes of, say the different regions, then you have to draw them the same number of times smaller. In other words, you must draw them in the right *proportion*. The sizes of mountain ranges and the lengths of rivers are all drawn to scale in the right proportion so we can compare their sizes and lengths. When we compare all the sizes of the parts on the map or scale drawing, they must compare in the same way they do in real life.

Look at the following pictures. Try to see if you can identify the original picture, the picture that has been enlarged in proportion to the original picture and the picture that has been enlarged out of proportion to the original picture.



ACTIVITY 12

Itinerary: travel plan.

Here is part of a map of the Kruger National Park. The scale shows that 1 cm measurement on the map drawing represents each 5 kilometres along the ground. Chimy works as a tour guide in the Kruger National Park. He must take a tour group in their bus from Phalaborwa Gate. The bus is only able to travel on the main roads. His itinerary is as follows:

Day 1: Phalaborwa gate - Masorini Archaeological Site picnic area (lunch)
Masorini Archaeological Site - Letaba camp (sleep)

Day 2: Letaba camp - Mooiplaas picnic site (lunch)
Mooiplaas picnic site Mopani camp (sleep)

Day 3: Mopani camp back to exit at Phalaborwa gate

a) Use a pencil to trace the route along the main road that the group travelled.

b) On day 1 Chimy wanted to know the approximate distance from the Masorini Archaeological Site to the Letaba camp. He measured as closely as possible the distance on the map and found it to be approximately 8 cm. Using the scale ratio of 1 cm : 5 km, determine approximately how many kilometres they would have to travel?

c) Use a ruler or another measuring instrument you have to measure the distance on the map from Letaba camp to Mopani camp to check the distance the tour group will travel on their second day. Calculate how far the distance is in kilometres using the 1cm : 5 km scale.

d) If the total distance of the tour from Phalaborwa gate to Letaba camp, then Mopani camp and then back to Phalaborwa gate is approximately 38 cm, calculate the actual distance in kilometres of the bus tour.

c) Use a ruler or another measuring instrument you have to measure the distance on the map from Letaba camp to Mopani camp to check the distance the tour group will travel on their second day. Calculate how far the distance is in kilometres using the 1cm : 5 km scale.

d) If the total distance of the tour from Phalaborwa gate to Letaba camp, then Mopani camp and then back to Phalaborwa gate is approximately 38 cm, calculate the actual distance in kilometres of the bus tour.

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ACTIVITY 13

1. If the distance on the map from Phalaborwa gate to the Masorini Archaeological Site is 2,5 cm, what is the actual distance in kilometres if 1 cm represents 5 km?
2. If the approximate actual distance from the Phalaborwa gate to Letaba is 50 km, calculate how many centimetres this would measure on the map above.

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Summary

In this lesson you learned how different amounts have to be compared before they are combined or mixed by people who make things, like cooks, herbalists and builders. This is to make sure that the ingredients are in the right proportion so that they make the right mixture.
You saw that you can write the parts that are mixed together mathematically as fractions or ratios. The parts have to get bigger if we want to make more of something. If the mixture we want to make is doubled then we must double the amounts of the parts. This is called direct proportion.

You also learned how to work with exchange rates. These involve changing currencies of money from different countries into simple ratios so that you are able to make comparisons. Exchange rates are also examples of direct proportion.

Finally you learned how to use ratio and proportion in scale drawings of maps. People who do scale drawings and map-making draw big things on paper by making every part the same number of times smaller. In this way all of the parts remain in proportion and can be compared. The scale is written to show the reader of the map or drawing how many times bigger the real object is.

Self-assessment checklist:

Are you able to:

- understand the meaning of ratio, direct proportion, exchange rates and scale drawings
- write down the amounts of ingredients as a ratio
- convert (change) ratios into simplified fractions
- convert (change) fractions into simplified ratios
- work with exchange rates
- calculate distances using the scale of a map.

SELF-CHECK EXERCISE

Do this exercise to check that you have understood everything in this lesson. If you are unsure how to answer a question, re-read the part of the lesson that explains it, before you try to do the problem.

Part A

1. Thandi must make 4 more blocks to finish the bottom of a building. The cement mix for one block is:
   3 buckets of sand
   1 bucket of cement
   2 buckets of small stones
   How many buckets of each ingredient will she need for 4 blocks? Remember that the amount she needs for 4 blocks is four times the amount she needs for 1 block.
2. Write the amounts of the ingredients as a ratio.
3. Use the ratio to work out the fractions of each ingredient.
4. Simplify the ratio.
5. Use the simplified ratio to work out the simplified fractions of each ingredient.
6. Now simplify the fractions you got in question 3 and see if you get the same answer as in question 5.

Part B

Oscar is working in Nelspruit in South Africa and decides to visit his family in Mozambique for a few days. The Mozambique currency is called Metical (MT) and the exchange rate at the time of Oscar's visit is:
R1, 00 = MT 4,6

Oscar budgets the following for his trip:

Taxi from Nelspruit to Komatipoort border:	R30,00
Bus trip from Komatipoort border to Maputo:	MT 150,00
Food:	MT 300,00
Bus trip from Maputo back to Komati Poort:	MT 150,00
Taxi from Komatipoort border to Nelspruit:	R30,00

1. How much does the bus trip from Komatipoort border to Maputo cost in Rands (to the nearest Rand)?
2. How many Rands has Oscar budgeted for food in Mozambique?
3. Which is more expensive:
   a) The cost of the taxi from Nelspruit to the border or
   b) The cost of the bus from the border to Maputo?
4. Calculate the cost of Oscar's budget to Mozambique in:
   a) Meticals
   b) Rands

Part C

On the next page there is a map of the Eastern Cape from the website of safariNow.com. Look at the bottom right side of the map. You will notice that there is a small scale. The scale indicates a ratio of:
5 mm : 20 km

1. Write down in your own words what this scale means.

2. If the distance from Graaff Reinet to Middelburg is 1,5 cm, calculate the actual distance between these two towns.

3. Regina is currently working in Port St. Johns and decides to visit her family in Queenstown to show them her new car. A possible route for her journey is written below as an example. Can you find another possible route? Identify the route Regina should take by naming the towns she will pass through on her route to Queenstown.

   Route 1: Port St. Johns - Umtata - Rhodes- Aliwal North - Queenstown (300 km)
   Route 2: Port St. Johns -

4. If the approximate actual distance of route 1 from Port St. Johns to Queenstown is 300 km, what is the measurement on the map?

5. If the approximate measurement of route 2 on the map is 7 cm, calculate the approximate actual distance of this route.

6. Which of the routes is a shorter distance to travel? Route 1 or route 2?

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2. More on ratio and proportion

Introduction

In the previous lesson, you were introduced to the concept of what a ratio is and how that relates to direct proportion. You also learned how to complete a table using ratio and direct proportion and to draw a graph by plotting the points on the table. You learned that the graph of direct proportion is represented by a straight line. One of the applications of direct proportion is the concept of exchange rates. You should also be able to convert to foreign currencies and vice-versa.

In this lesson, we will continue to learn more about ratio and another type of proportion, known as inverse proportion. We will also deal with average speed and how this relates to time and distance.

In this lesson you will:

- understand the meaning of ratio, direct and inverse proportion andaverage speed
- learn how to tell the difference between direct and inverse proportion
- work with direct and inverse proportions in tables and real-life applications
- understand how the graphs of direct and inverse proportion differ
- work with the concept of average speed (in terms of total distance and time) and how to apply this in everyday situations.

Inverse proportion

In the last few activities we have dealt with direct proportion where the ratio remains the same or stays a **constant ratio**. Look at the table below:

Table A

Number of cakes	1	2	3	5
Cups of flour	2	4	6	10

The ratio of each column works out to be $\frac{1}{2}$. As the one quantity, the number of cakes, increases, the other quantity, the cups of flour, also increases in direct proportion to the number of cakes. The ratio of cakes : cups of flour should remain 1 : 2 in its simplified form.

ACTIVITY 1

Use the data from Table A to write down the coordinates of four points that can help you draw a graph. Let the x-coordinate represent the number of cakes and the y-coordinate represent the cups of flour.

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ACTIVITY 2

Use the points to draw a graph.
Did your graph turn out to be a straight line, passing through the origin of (0; 0)? If not, check your graph again before looking at the feedback to this activity.

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ACTIVITY 3

1. Fill in the missing value in Table B.

Table B

US Dollars ($)	1	2	3	4
Rands (R)	6,5	13	19,5	?

2. Is this an example of direct proportion? If so, what is the constant ratio?

3. Use the information from Table B to draw a graph. Let x represent the US dollars and y represent the Rands.

4. On your graph, indicate, using the letter P, the point where you can read off how many Rands you will get for $ 5.

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Note that this graph also forms a straight line that passes through the origin which is the point with co-ordinates (0 ; 0). Now study the following table which shows the number of days it took men painting the inside and outside of a house.

Table C

Number of men	1	2	4	5	10
Number of days	20	10	5	4	2

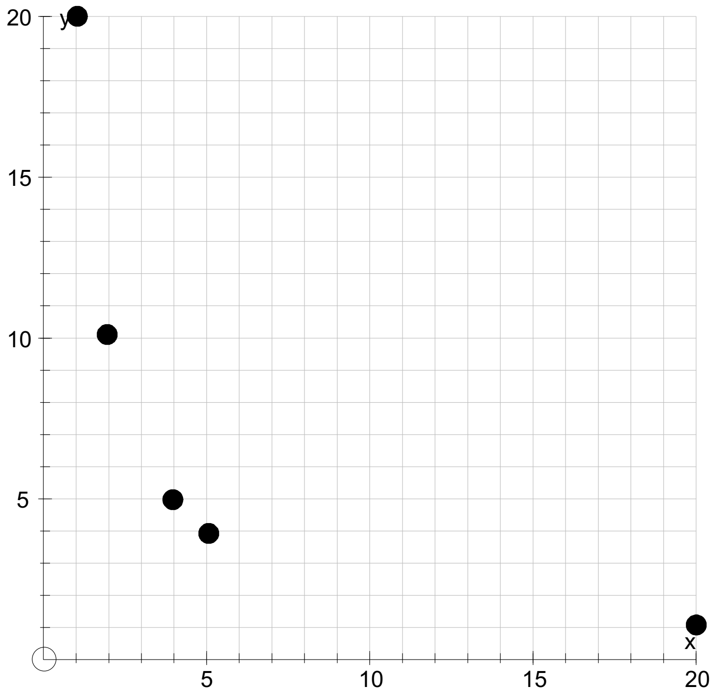
What do you notice about the number of days as the number of men increases? Yes, the number of **days** it takes to paint the house **decreases** as the number of **men increases**. Look at the ratios. You will also notice that:

$$\frac{1}{20} \neq \frac{2}{10} \neq \frac{4}{5} \neq \frac{5}{4} \neq \frac{10}{2}$$

But $\dfrac{1}{\frac{1}{20}} = \dfrac{2}{\frac{1}{10}} = \dfrac{4}{\frac{1}{5}} = \dfrac{5}{\frac{1}{4}} = \dfrac{10}{\frac{1}{2}}$

Can you see the pattern? Look at the denominators again. $\frac{1}{20}$ is called the multiplicative **inverse** of 20 so we call this type of proportion **inverse proportion**. In inverse proportion there is a **constant product**. Product means multiplication, so if you multiply the number of men in Table C with the number of days, you will get a constant product, in each column, of 20.

If we take the information from Table C and draw a graph to illustrate this we get:



We cannot join up the points. Why? Can we have a fraction of a man?

Notice that this graph is not a straight line. Rather it is curved. As the number of men, indicated on the *x*-axis, increases, the number of days it took the men to paint the house decreases in proportion. Inverse proportion is always defined by a curve rather than a straight line. Direct proportion is always defined by a straight line as you saw in the previous lesson and the graphs of Tables A and B above.

ACTIVITY 4

Find the values for the variables in the following tables that are all examples of inverse proportion.

1.

5	10	20	50	100
20	10	x	y	z

2.

1	2	3	4	12
12	6	x	y	z

3.

30	10	50	1	1500
50	150	x	y	z

ANSWERS ON PAGE 96

When you are looking at a table and you are not sure if the table is in direct or inverse proportion, then you need to see if you can identify whether there is a constant ratio or a constant product. Remember that direct proportion has a constant ratio and inverse proportion has a constant product.

ACTIVITY 5

Decide whether the data in the following tables indicates direct proportion or inverse proportion and determine the constant ratio or the constant product.

1.

1	2	3	5	10
3	6	9	15	30

2.

1	2	3	5	10
60	30	20	12	6

3.

2	3	4	5	6
20	30	40	50	60

4.

2	1	200	50	10
200	400	2	40	40

ANSWERS ON PAGE 96

We also need to be able to look at a contextual problem and decide whether it involves direct or inverse proportion so that we can solve the problem.

Example
I can cut 9 pieces each 240 mm long from a piece of wood. How many pieces each of length 180 mm can I cut from the same length of wood? Is this an example of direct or inverse proportion?

If I can cut 9 pieces each 240 mm long, it means the length of the wood is:
$9 \times 240 = 2160$ mm long

I now divide that same length of wood (2160 mm) into pieces that are 180 mm long.
$2160 \div 180 = 12$
I can cut 12 pieces of 180 mm.

This is inverse proportion as we made use of the constant product of 2160 to calculate our answer.

ACTIVITY 6

Solve the following problems and state whether they are examples of direct or inverse proportion.

1. A ferry can carry 8 cars each of mass 1 400 kg. How many cars of mass 1 120 kg each can the same ferry carry?

2. If your 50 cc motorbike goes 280 km on 7 litres of petrol, approximately how far do you expect it to go on a full tank of 20 litres?

3. 6 men can lay 2 000 bricks in a day. How many bricks can 9 men lay in a day?

ANSWERS ON PAGE 96

Average speed

What does this sign mean?

Yes, it means that the speed limit is 60 kilometres per hour. The unit of speed depends on the unit of distance and the unit of time. The distance here is in kilometres and the time is in hours, so the average speed is kilometres per hour.

The main route from Johannesburg to Durban is approximately 600 km. Driving at an average speed of 60 km/h would take 10 hours to drive the route. Driving at an average speed of 100 km/h would take 6 hours to drive the route. Clearly we do not drive at the same speed all the time. Sometimes we can drive fast, on the highway, and sometimes we need to drive slowly through the towns. Sometimes we have to stop at intersections and robots. That is why we talk about average speed; the time it takes is exactly as if we were driving along a straight road at the speed.

As a formula this is written as: $s = \frac{d}{t}$

ACTIVITY 7

1. Make distance (d) the subject of the formula in the average speed formula given above.

2. Make time (t) the subject of the formula in the average speed formula.

ANSWERS ON PAGE 97

Example

1. A car travels a total distance of 200 km in 4 hours. What is its average speed?

$$\text{Average speed} = \frac{\text{Distance travelled}}{\text{Time taken}}$$
$$= \frac{200}{4}$$
$$= 50 \text{ km/h}$$

2. A truck travels for 3 hours at an average speed of 70 km/h. How far does it travel in this time?

$$\text{Distance} = \text{Average speed} \times \text{time taken}$$
$$= 70 \times 3$$
$$= 210 \text{ km}$$

3. A train travels 120 km at 40 km/h. How long does the journey take?

$$\text{Time} = \frac{\text{Distance travelled}}{\text{Average speed}}$$
$$= \frac{120}{40}$$
$$= 3 \text{ hours}$$

ACTIVITY 8

1. How long will a car travelling at an average speed of 120 km/h take to drive the 600 km from Johannesburg to Durban?

2. Oscar's bus trip from Komatipoort border to Maputo takes 2 hours. The distance was 80 km. What was the average speed that the bus was travelling at?

3. Besa walks from Chiawa village in Zambia to the next village called Mgoromeno. He walks at an average speed of 5 km/h and takes an hour and a half to reach Mgoromeno. What is the distance he covered during this time?

ANSWERS ON PAGE 97

Remember that average speed is defined as total distance travelled divided by total time taken.

Example

A car travels at an average speed of 30 km/h on a dirt road for 60 km and then on the main road for 120 km at an average speed of 60 km/h. Work out the average speed of the car for the entire journey.

Time taken to travel 60 km at 30 km/h: $\frac{60}{30} = 2$

Time taken to travel 120 km at 60 km/h: $\frac{120}{60} = 2$

Total time taken: $2+2=4$ hours
Total distance travelled: $60\text{ km}+120\text{ km}=180\text{ km}$

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$
$$= \frac{180}{4}$$
$$= 45 \text{ km/h}$$

ACTIVITY 9

1. A bus travels 160 km the first stage of its journey at an average speed of 40 km/h and 120 km the second stage of its journey at an average speed of 60 km/h. Calculate the average speed of the whole journey.

2. A car travels 200 km at an average speed of 50 km/h. On the return journey the speed is increased to 80 km/h. Calculate the average speed for the journey.

ANSWERS ON PAGE 98

Example
A train travels for 4 hours at an average speed of 64 km/h. For the first two hours the average speed is 50 km/h. What is its average speed for the last two hours?

Total distance travelled in 4 hours
= Average speed × total time
$= 64 \times 4$
= 256 km

Distance travelled in first 2 hours
= Average speed for those 2 hours × time (2 hrs)
$= 50 \times 2$
= 100 km

Distance travelled in last 2 hours
= 256 km − 100 km
= 156 km

Average speed for last 2 hours
= Distance travelled ÷ time taken
= 156 ÷ 2
= 78 km/h

ACTIVITY 10

1. For the first 3 hours of a journey of 182 km the average speed was 30 km/h. If the average speed for the remainder of the journey was 23 km/h, calculate the average speed of the whole journey.

2. John walks at 3km at a speed of 6 km/h. He then cycles 6 km at a speed of 12 km/h. What was the average speed for John's entire journey?

3. A motorist travelling an average speed of 80 km/h covers a section of road in 25 minutes. A few months later, the speed limit is reduced on that section of road and the motorist finds that when she travels at an average speed of the new speed limit, she takes 10 minutes longer to cover the same section of road. What is the new speed limit on that road?

ANSWERS ON PAGE 98

In the activities you have completed, you would have noticed that as the average speed increases, the time taken to complete the distance decreases. For example, at an average speed of 60 km/h it will take a bus 10 hours to complete 600 km. At an average speed of 100 km/h it will take a bus 6 hours to complete the 600 km. Increase in speed results in a decrease in the time taken to complete the distance. This is also another example of inverse proportion. Can you see that the constant product in the table below is 600? That is because the formula for calculating the total distance is:

Average speed × total time.

Average speed for 600 km	20 km/h	60 km/h	100 km/h	120 km/h
Total time for 600 km	30 hours	10 hours	6 hours	5 hours

ACTIVITY 11

Use the table above to draw a graph of the information. Let the average speed be on the x-axis and the total time be represented on the y-axis. Plot the four points. Do you think you can join the points?

ANSWERS ON PAGE 100

Independent and dependent variables

Remember the example of the cake recipe in the previous lesson? The increase in the number of people to eat the cake resulted in an increase in the number of cups of ingredients needed to bake the cake. We say that the number of cups of ingredients is **dependent** on the number of people who will be eating the cake.

Look at the table and the graph in the previous activity. The total time taken to cover the distance of 600 km is **dependent** on the average speed of the bus. We refer to the number of people and the average speed as the **independent variables** and the number of cups of ingredients and the total time as the **dependent variables**. It is standard practice in mathematics to use the independent variable as the x coordinates in tables and graphs and the dependent variable as the y coordinates.

Example
In Activity 3 of this lesson we looked at the number of days it took men to paint a house.

Number of men	1	2	4	5	10
Number of days	20	10	5	4	2

The increase in the number of men resulted in a decrease in the number of days it took the men to complete painting the house. The number of men determines the number of days. So the number of men is the independent variable (x) and the number of days is the dependent variable (y).

This helps us to decide which values in the table, and graph, will be the x-values and which ones will be the y-values.

ACTIVITY 12

1. Mbeki's mother kept records of his height and his mass during the last 5 years.

Age	Height	Mass
11	1,2 m	42 kg
12	1,28 m	43 kg
13	1,35 m	45 kg
14	1,42 m	50 kg
15	1,53 m	53 kg

   a) Use ordered pairs to show how age and height are related.
   b) Use ordered pairs to represent the age-mass relationship.
   c) Mbeki's height depends on his age. Identify the dependent and independent variables.
   d) Does Mbeki's height vary inversely or directly according to his age?

2. The area of a square depends on the length of the side of the square. Identify the dependent and independent variables.

ANSWERS ON PAGE 100

Summary

In this lesson you studied the concept of ratio and proportion in more detail. We introduced the concept of inverse proportion and looked at average speed. You covered examples and activities on the difference between direct and inverse proportion and the tables and graphs that represent them. Direct proportion always has a constant ratio and inverse proportion always has a constant product. Finally we discussed the difference between independent and dependent variables that assist us in drawing graphs.

Self-assessment checklist:

Are you able to:

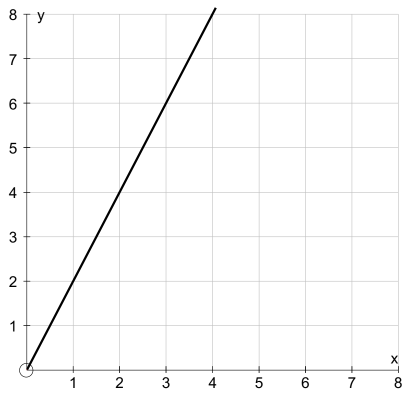
- understand the meaning of ratio, direct and inverse proportion and average speed
- tell the difference between direct and inverse proportion
- complete tables with values of either direct or inverse proportions
- apply direct and inverse proportion in real-life applications
- work with the concept of average speed (in terms of total distance and time) and how to apply this in everyday situations.

SELF-CHECK EXERCISE

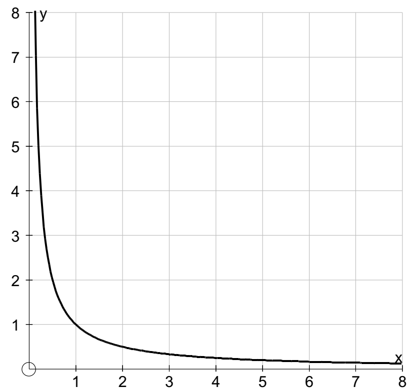
1. Explain in your own words the difference between direct and inverse proportion and how you can tell the difference between them.

2. Study the following graphs. State whether they represent direct or inverse proportion.

a)



b)



3. Study the following tables and decide whether they are in direct or inverse proportion. Then find the missing values.

a)

1	2	3	4	*b*
3	6	*a*	12	15

b)

2	4	1	28	*b*
14	7	*a*	1	2

4. A Jumbo jet flies from OR Tambo airport to Heathrow London in 12 hours at an average speed of 1 000 km/h. If the time is reduced to 10 hours, what must the average speed of the plane be?

5. Brad kept a record of the temperature one winter's day in Pretoria:
   a) Identify the dependent and the independent variables.
   b) Use ordered pairs to show how the temperature depends on the time of day.
   c) Does the temperature vary directly or inversely according to the time?

Time	Temp
00:00	1°
04:00	-1°
08:00	3°
12:00	14°
16:00	12°
20:00	4°
24:00	2°

ANSWERS ON PAGE 85

3. Further finance: Credit and compound interest

Introduction

In lesson 3 of Unit 2 you learnt how to calculate simple interest when you were studying investments and pensions. In this lesson, we discuss the concept of credit in finance and other related terms such as **hire-purchase** and **loans**. We also introduce another type of interest that is used when you use your credit card or take out large loans such as home loans; **compound interest**. In order to understand compound interest, you first need to have fully understood simple interest. So if you have forgotten how to do that, we suggest you revise that lesson first.

In this lesson you will:

- learn about financial credit (borrowing money) and associated terms such as hire-purchase and loans
- calculate compound interest
- recognise the difference between simple and compound interest.

Buying on credit

When money is borrowed from an institution such as a bank, or when you buy an item from a shop and pay it off over a number of months, this is called **credit**. The credit you get from a bank is called a **loan** and when you pay an item off over a period of time at a shop, this is called buying on **hire-purchase**.

When you use a credit system, from a bank or a shop, you usually have to pay back the money with interest. This interest can either be calculated using a flat rate, such as simple interest or interest that is compounded monthly, known as compound interest. When simple interest is used, the total amount of interest is calculated as a percentage of the initial amount borrowed and multiplied by the term of the loan.

*The **term of the loan** is the length of time which the loan is agreed to be paid over.*

Remember that the formula we used for calculating simple interest in lesson 3 of Unit 2 was:

Interest = Principal × rate % × time (number of years) or
SI = P × r × n

where I = interest, written in rands
P = principal, money deposited or borrowed, written in rands
r = percentage rate, written as a fraction or decimal (sometimes this is represented by an *i*)
n = time, written in years.

P.a. - per annum

$$7,5\% = \frac{7,5}{100} = 0,075$$

Example

Calculate the simple interest to be paid on a loan of R20 000 at an interest rate of 8 % p.a. if the loan is to be repaid over 5 years.

$$\begin{aligned} SI &= Prn \\ &= 20000 \times 7,5\% \times 5 \\ &= 20000 \times 0,075 \times 5 \\ &= 7500 \end{aligned}$$

The simple interest on this loan over five years will be R7 500.
That means that the person borrows R20 000 from the bank but pays back an amount of R27 500 to the bank over five years at an interest rate of 7,5% p.a.

ACTIVITY 1

1. Calculate the simple interest to be paid on a loan of R50 000 at an interest rate of 12% p.a. if the loan is to be repaid over 5 years.

2. Zandele borrows R75 000 from a bank to buy a car at a simple interest rate of 14% p.a. Zandele is to repay the loan plus interest, over a period of 4 years, or 48 months. Calculate the total amount of Zandele's loan for her car.

ANSWERS ON PAGE 100

An instalment is the amount that you pay back to the bank each month during the time that you have the loan.

Most loans are repaid on a monthly basis. Once the total amount of the loan has been calculated, this can be divided into monthly instalments.

Example

Mischak borrows R100 000 from the bank to start a small business. The loan needs to be paid off over 5 years and the interest rate is 12,5% p.a.

a) What is the total amount of Mischak's loan?
b) What will Mischak's monthly instalments be for the 5 years?

a)

$$\begin{aligned} SI &= Prn \\ &= 100000 \times 12,5\% \times 5 \\ &= 100000 \times 0,125 \times 5 \\ &= 62500 \end{aligned}$$

$$100000 + 62500 = 162500$$

b)

Total of loan:	R162 500
Total term of loan:	5 years
	= 60 months
Monthly instalment:	R162 500 ÷ 60
	= R2 708,33 per month

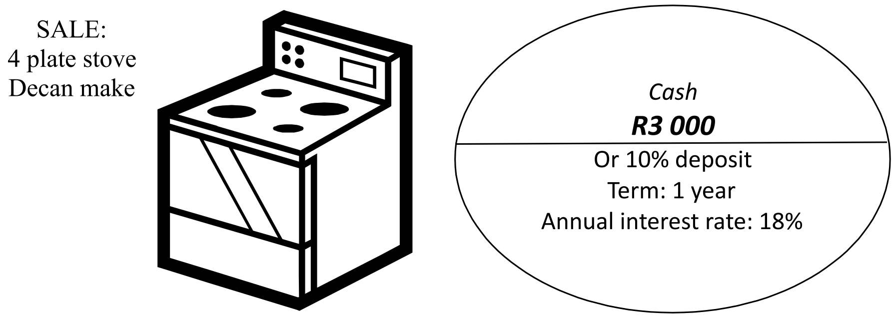
ACTIVITY 2

1. For a loan of R5 000 at an interest rate of 20% over 2 years, calculate:
   a) The simple interest on the loan.
   b) The total amount of the loan.
   c) The monthly instalments of the loan.

2. Masondo borrows R12 000 from the bank at a simple interest rate of 15% over 3 years. Calculate his monthly instalments to pay back the loan over the 3 years.

Look at the advert below. Some shops allow people to buy on credit if the customer is not able to pay the full amount for the product immediately. This is also a type of credit loan, known as hire-purchase. When you use this type of credit, you also usually have to pay monthly instalments until your purchase is paid off. Sometimes you also have to put down a deposit when you buy the product and then pay off the balance over the term of the hire-purchase as indicated by the shop.

ANSWERS ON PAGE 101



Example

Use the advert above to answer the following questions:

a) What is the cash price of the stove?
b) If Sibongile buys the stove on hire-purchase, what deposit will she have to pay?
c) What will the remaining amount be that she will loan on hire-purchase?
d) What will be the simple interest on her hire-purchase loan?
e) What is the total she will have to pay to the shop over the 1 year?
f) What will her monthly instalments be over the 12 months?
g) How much will the stove cost her in total on hire-purchase?

Solution

a) R3 000
b) R300
c) R2 700
d) $SI = Prn$

$$= 2700 \times 20\% \times 1$$
$$= 2700 \times 0{,}2 \times 1$$
$$= 540$$

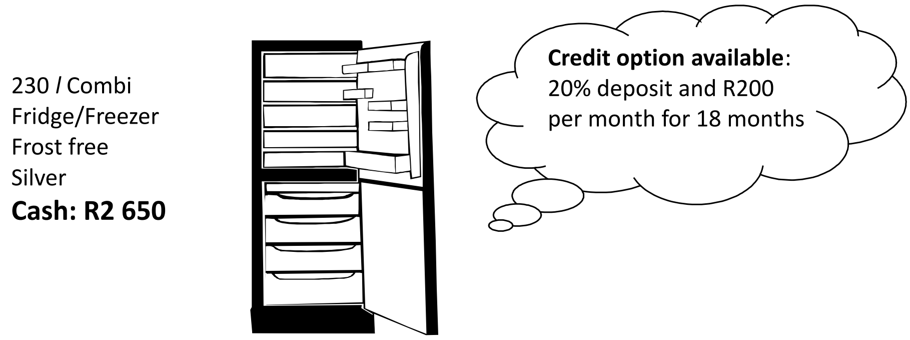
e) $R2\,700 + R540 = R3\,240$
f) $R3\,240 \div 12 = R270$ per month
g) $R300 + R2\,700 + R540 = R3\,540$

ACTIVITY 3

1. Vusi buys a computer on hire-purchase. The cash price of the computer is R4 800. Vusi must pay a 10% deposit and then pay the remaining balance off over 24 months at a simple interest rate of 21%.

   a) Calculate the deposit.
   b) Calculate the balance owing.
   c) Calculate the interest on the loan.
   d) Calculate the total amount to be repaid.
   e) Calculate the monthly instalments.
   f) Calculate the difference between the cash price and the hire-purchase price of the computer.

2. Look at the advert below:



   a) How much will the fridge cost to buy on hire-purchase?
   b) Calculate the difference between the cash price and the hire-purchase price.

ANSWERS ON PAGE 101

You will notice that in the advert in question 2 of Activity 3, the monthly instalments were given instead of the interest rate being charged. If you know the total of the loan and the term of the loan, you can use the simple interest formula to find the interest rate. This is done by changing the subject of the formula (see Lesson 4 of Unit 2).

Example

What is the interest rate being charged on the fridge being advertised in question 2 of Activity 3?

Remember that the formula for calculating simple interest is: $SI = Prn$

In this example we are asked to find the interest rate (r) so we need to know what the other values are so that they can be substituted in and then we can change the subject of the formula to find r.

Let us see what information we have in order to substitute in. We know that the term of the loan (n) is 18 months which is 1,5 years. We can find the principal of the loan amount using the cash price and the deposit:

The cash price of the fridge is R2 650. The 20% deposit is therefore R530. So the remaining loan amount is R 2650 – R530 = R2120. This is the principal amount of the loan (P).

To find the simple interest (*SI*) paid, we need to calculate the difference between the principal amount (*P*) and the total amount of the loan. The monthly instalments and the term of the loan can be used to calculate the total amount of the loan.

Monthly instalments of R200 for 18 months = R200 × 18 = R3 600.

Difference between principal amount (R2 120) and the total amount of the loan (R3 600) = R3 600 – R2 120 = R1 480 (*SI*).

So our known values are:

SI = R1 480
P = R2 120
n = 1,5 years

Let us substitute our known values into the formula:

$$SI = Prn$$
$$1480 = 2120 \times r \times 1{,}5$$
$$1480 = 3180r$$
$$\frac{1480}{3180} = r$$
$$r = 0{,}47$$
$$r = 47\%$$

The interest rate when buying the fridge on loan was almost 50%. This means that you end up paying almost one and a half times the cash price for the fridge if you buy it using hire-purchase.

Shops often use this type of credit system to attract people to buy items they cannot afford to purchase with cash. When the item is paid off, you as the customer may end up paying a lot more for the item than you would have if were able to purchase it with cash.

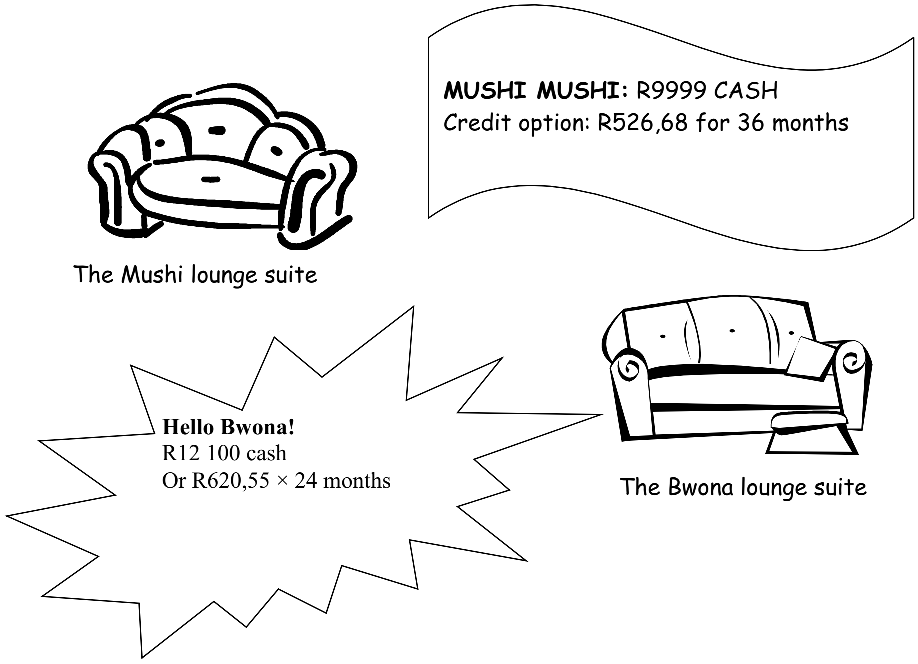
So you need be careful of credit systems and loans and always calculate what the loan or hire-purchase will cost you in interest. If possible, it is better to save money until you have enough to buy the item with cash.

ACTIVITY 4

1. Dr Banda borrows R230 000 to buy a car. This is to be repaid over 36 months with monthly instalments of R8 000 per month.

   a) What does the car end up costing Dr Banda to buy?
   b) Calculate the simple interest Dr Banda was charged.
   c) Calculate the interest rate of Dr Banda's loan.

2. Use the following advert:



a) What is the interest rate on the Mushi lounge suite if you buy it on hire-purchase?

b) What is the interest rate on the Bwona lounge suite if you buy it on hire-purchase?

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c) Which lounge suite do you think is the better deal? Why?

Compound interest

The biggest loan most people ever take out is a home loan to buy a house. These loans are often for large amounts of money (usually more than R200 000) and are paid back over a long period of time, such as 10, 15, 20 or even up to 30 years.

Home loans are not charged at a simple interest rate. Remember that on a simple interest rate, interest is only charged on the principal amount borrowed, or invested. The other type of interest rate is called compound interest. In this case the interest is calculated on the principal amount borrowed as well as the interest that is added to the interest rate at regular intervals.

It is called compound interest because the interest is compounded (added onto the principal amount) daily, weekly, monthly or annually. This can be a good type of interest for investing money. Let us look at an example that illustrates the difference between simple and compound interest:

Example
Let us start off with a simple example where the interest is calculated annually. If an investment of R4 000 is placed at a bank over 4 years at an interest rate of 20%, the table on the next page shows the difference between calculating the simple and compound interest on such an investment.

	Simple Interest	Compound Interest
End of 1st year	$4000 + 0{,}2 \times 4000 =$ R4 800	$4000 + 0{,}2 \times 4000 =$ R4 800
End of 2nd year	$4\,800 + 0{,}2 \times 4000 =$ R5600	$4800 + 0{,}2 \times 4800 =$ R5760
End of 3rd year	$5600 + 0{,}2 \times 4000 =$ R6400	$5760 + 0{,}2 \times 5760 =$ R6912
End of 4th year	$6400 + 0{,}2 \times 4000 =$ R7200	$6912 + 0{,}2 \times 6912 =$ R8294,40

Notice that the principal amount (R4 000) for the simple interest remains the same each year that the interest is calculated. The interest each year is R800 and just the total amount increases. But with the compound interest, the interest from the previous year is added to the principal amount of that year to make up the new principal amount the following year. The amount of interest changes each year and so does the principal amount.

So with an investment of R4 000 at an interest rate of 20% over four years, you end up earning R7 200 if a simple interest rate is applied and R8 294,40 if a compound interest rate is applied.

ACTIVITY 5

1. Complete the following table that shows the difference between simple and compound interest rate on an investment of R20 000 over 5 years at a rate of 12% p.a.

	Using simple Interest	Using compound Interest
End of 1st year	$20000 + 0{,}12 \times 20000 =$ R22 400	$20000 + 0{,}12 \times 20000 =$ R22 400
End of 2nd year	$22400 + 0{,}12 \times 20000 =$	$22400 + 0{,}12 \times 22400 =$
End of 3rd year		
End of 4th year		
End of 5th year		

2. Calculate the difference between the total investment amount that will be earned back over the 5 years if simple interest or compound interest are applied.

3. Which is the better investment option here: simple interest or compound interest?

ANSWERS ON PAGE 103

Consider an investment of 20 years. To draw up a table of interest calculations could be quite time consuming. Let us look at a way of calculating the total amount of the investment using:

a) simple interest and
b) compound interest

Formula for simple interest: $SI = Prn$

So the total amount (A) can be calculated by:

$$A = P + SI$$
$$A = P + P \times r \times n$$

The total loan amount is equal to the principal amount plus the simple interest.

Example

Alex invests R12 500 in the bank to further her studies later. Calculate the total amount (A) of her investment if the investment is taken over 2 years at a simple interest rate of 11,5% p.a.

$P = 12\,500$

$r = 11{,}5\% = 0{,}115$

$n = 2$

$A = P + P \times r \times n$

$A = 12500 + 12500 \times 0{,}115 \times 2$

$A = 15375$

The total amount of the loan is R15 375.

ACTIVITY 6

1. A television with a cash price of R5 400 is advertised on hire-purchase to be paid over 18 months at a simple interest rate of 21% p.a. Calculate the cost of the television on hire-purchase.

2. Calculate the total amount of an investment of R50 000 over 5 years at a simple interest rate of 15% p.a.

ANSWERS ON PAGE 103

There is also a formula that helps us to calculate the total amount of an investment taken using compound interest. The formula for this is:

$A = P(1 + r)^n$

Where:
- A = Total Amount of investment
- P = Principal amount of investment
- r = rate of interest (written as a decimal)
- n = term of investment

Example

If Alex invests R12 500 in the bank, calculate the total amount (A) of her investment if the investment is taken over 2 years at a compound interest rate of 11,5%.

$P = 12\,500$

$r = 11{,}5\% = 0{,}115$

$n = 2$

$A = P(1 + r)^n$

$A = 12500(1 + 0{,}115)^2$

$A = 15540{,}31$

The total amount of the investment is R15 540,31.

ACTIVITY 7

1. A television with a cash price of R5 400 is advertised on hire-purchase to be paid over 18 months at a *compound interest rate* of 21% p.a. Calculate the cost of the television on hire-purchase.

2. Calculate the total amount of an investment of R50 000 over 5 years at a *compound interest rate* of 15% p.a.

ANSWERS ON PAGE 103

So the formulae for calculating the **total amount** of an investment (or a loan) are:

Simple interest: $A = P + P \times r \times n$ or $A = P(1 + rn)$
Compound interest: $A = P(1 + r)^n$

ACTIVITY 8

1. Use the formulae to determine which of the following investments would provide a better option:

   a) R9 000 at 14% p.a. compound interest over 4 years.
   b) R9 000 at 18% p.a. simple interest over 4 years.

2. Mr Tau takes out a loan of R180 000 from his father to start a business. The loan is repaid in one amount at the end of four years. Calculate how much money Mr Tau would need to repay the loan if the interest is calculated as:

   a) 15% p.a. simple interest
   b) 12% p.a. compound interest

3. Calculate the compound interest on a home loan of R550 000 over 20 years at a rate of 11% p.a.

ANSWERS ON PAGE 103

Summary

In this lesson you learned about credit in the form of loans and hire-purchase. You also learned more about simple interest and were introduced to another type of interest called compound interest. The difference between simple and compound interest was discussed and formulae to calculate the total amount of a loan was given for each of these types of interest. You worked through some examples of calculating and using monthly instalments but most of this lesson focussed on interest rates that are charged per annum. In Unit 4 you will learn more about compound interest that is compounded monthly, weekly or daily rather than just annually.

Self-assessment checklist:

Are you able to:

- understand the concept of financial credit (borrowing money) and associated terms such as hire-purchase and loans
- use the necessary formulae to calculate simple interest and compound interest and the total amount of an investment or a loan using either simple or compound interest
- recognise and calculate the difference between simple and compound interest.

SELF-CHECK EXERCISE

Bayani decides to buy himself a car for R89 000 from his brother. He has the following options to consider when paying off the car:

a) R89 000 at a simple interest rate of 16% p.a. over 3 years.
b) R89 000 at a compound interest rate of 12% p.a. over 30 months.
c) To pay a 10% cash deposit and then pay off the balance at a compound interest rate of 13% p.a. over 2 years.

1. Calculate the amount of interest that Bayani will be charged in each of the options above.

2. Calculate the total loan amount that Bayani will have to repay his brother in each of the loan options.

3. If you were Bayani which of the loan options would you choose and why?

ANSWERS ON PAGE 85

4. Graphs of linear equations

Introduction

In the foundations of the algebra lesson in Unit 2 lesson 4, you learned that x is called a variable because it can have different values. You also learned how to solve equations, and show solutions of equations on a set of x and y axes.

This set of axes is made up of the horizontal (x) number line and the vertical (y) number line. This is known in mathematics as a **Cartesian plane**, named after the famous mathematician Descartes.

On these axes we can draw the solutions of equations. The line that draws the solution of the equation is called a graph. You learned about the foundations of graphs in lesson 5 of Unit 2 so you may want to revise that if you have forgotten the basics of axes and coordinates. In this lesson you will learn more about graphs and representing equations as graphs.

In this lesson you will:

- draw the graph of an equation with one variable. We will draw the graph for the x variable first and then we will draw one for the y variable.
- draw a graph of an equation with two variables, where x and y are changing at the same time.
- use substitution to find the co-ordinates that make the equation true, and then to find the graph of the equation by plotting these co-ordinates on a set of x and y axes.
- plot graphs of $y = x$ and $y = x$ and find out from the graphs what happens to x when y changes and what happens to y when x changes.
- translate real life situations into linear equations with two variables, x and y, and draw the solution as a graph.
- read information from a graph to help solve a problem.
- find the solution that satisfies two separate equations (makes both equations true) by drawing graphs for both equations and finding the point where the two graphs cut each other.

Drawing the graphs of equations with one variable

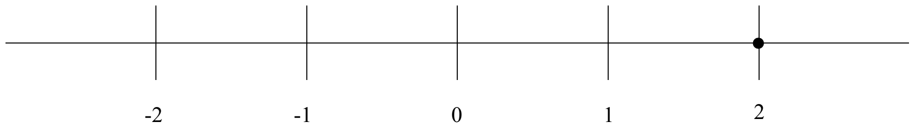
Equations with variable x

Let us represent the following equation on a graph:
$x = 2$

Example 1
$x = 2$

If we show the solution to this on a **number line** we place a solid dot at number 2.

This indicates that there is only one position where $x = 2$. A number line is a single axis.



Remember from the Introduction that we can also work with two axes; an x-axis which is horizontal, like the number line, and a y-axis which is vertical. This is called a Cartesian plane and has more than one axis.

If we show the solution to this as a graph on a *Cartesian plane* we draw a solid line through number 2 on the x-axis, parallel to the y-axis as in the drawing below. This indicates all the positions, and different y-values, where $x = 2$.

This graph shows that no matter what the value for y, the x-value for this equation will always be the same, equal to 2. This is because y is not in the equation and will not affect x.

ACTIVITY 1

Place your finger on any y-value on the y-axis of the graph of the Cartesian plane in Example 1. Then move your finger straight across to the $x = 2$ line and down or up to the value of x on the x-axis. You will notice that you keep ending on the x-value of 2.

Now represent the solutions to the equations: $x = 1$ and $x = -4$.

a) On a number line
b) On a Cartesian plane

You can represent both solutions on one number line and then on one Cartesian plane.

ANSWERS ON PAGE 104

Equations with variable y

ACTIVITY 2

Look at the equation below. See if you can draw the solution of this on a *Cartesian plane graph* before you look at the feedback.

$y = -3$

On this graph the value for y does not change no matter what the value is for x. Take any point along the x-axis and place your finger on this. Then move your finger straight up or down to the line of $y = -3$ and across to the point parallel to the y-axis. Check the corresponding value for y. Try a couple of values for x to convince yourself that y still has the same value. Try the next activity to see if you have learnt how to graph the solutions of equations on a Cartesian plane of x and y axes.

ANSWERS ON PAGE 105

ACTIVITY 3

Draw the following graphs on the same Cartesian plane:

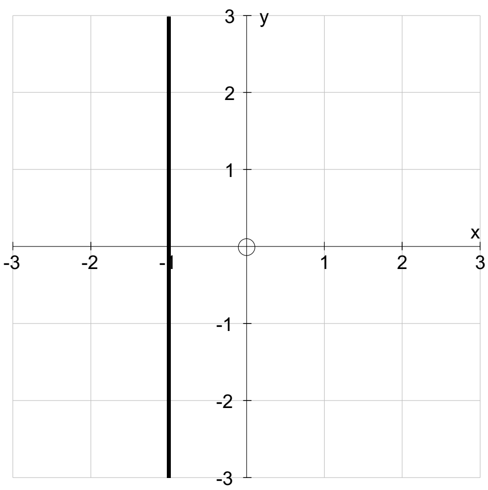
1. $x = -3$
2. $y = 4$

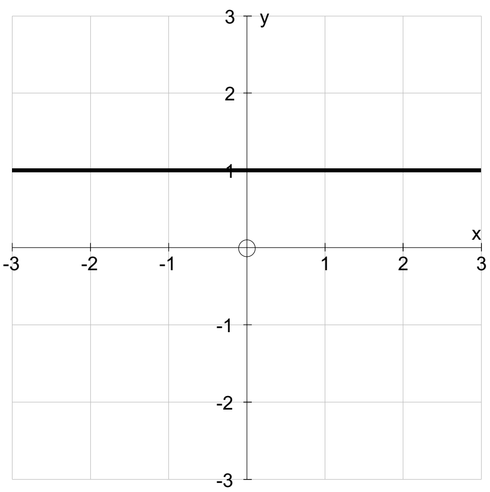
ANSWERS ON PAGE 105

ACTIVITY 4

Write out the equations of the following graphs:

1.





ANSWERS ON PAGE 105

Equations with two variables

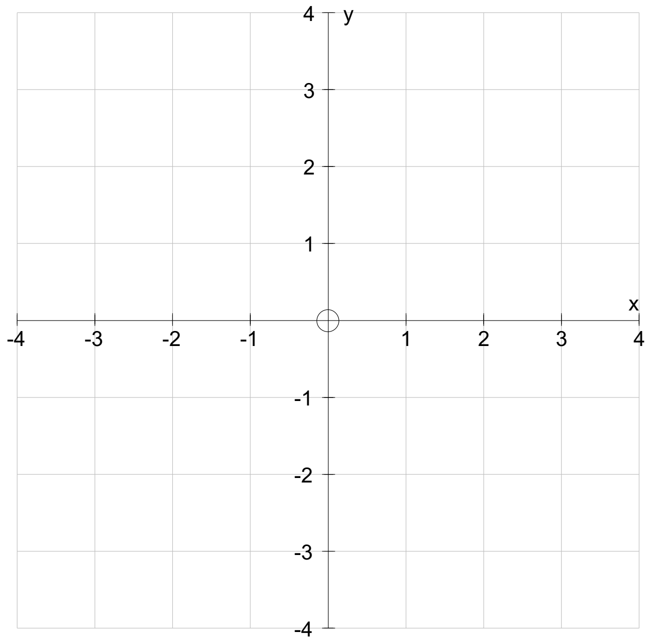
Finding the line $y = x$ by plotting points on the graph

For any point, the co-ordinates for x and y are the same when $y = x$
This is because they must be equal to satisfy the $y = x$ equation (make the $y = x$ equation true by making sure that the LHS = RHS).

Here are some of the co-ordinates. Can you think of more?
$(x ; y) = (0;0), (2;2), (1;1), (3;3)$

ACTIVITY 5

Take a piece of graph paper and draw in x- and y-axes as in the drawing below. Plot the points of the co-ordinates above and join them up.



ANSWERS ON PAGE 106

Increasing graphs and the linear equation $y = x$

Do you remember ratio and proportion from Lesson 1 of this unit? We drew a graph of direct proportion to show that as one amount increases, the other amount also increases in proportion.

Have a look at the x- and y-values in the equation $y = x$:
As the x-value increases so does the y-value.

ACTIVITY 6

Let's look at changes in the y co-ordinate as x changes.

What happens to y when x gets smaller? Choose a negative value on the x-axis and see what the corresponding y co-ordinate is. Now find a larger number for x and see what the y co-ordinate is. How has the y co-ordinate changed? Has it got smaller (decreased) or bigger (increased) as the x-value has increased?

ANSWERS ON PAGE 106

Finding the line $y = -x$ by plotting points on the graph

ACTIVITY 7

Here are some co-ordinates for the $y = x$ graph:
$(x;y) = (0;0), (1;-1), (-1;1), (-3;3)$

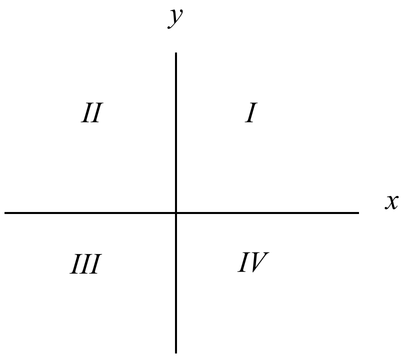
Write down another three sets of co-ordinates for $y = -x$.
Plot these points on a Cartesian plane and join them up to draw the line graph.

ANSWERS ON PAGE 106

Quadrants

Have a look at the set of axes drawn below. The sections of the graph that lie between the axes are labelled I to IV.

The four quadrants of a Cartesian plane are conventionally labeled as follows:



These sections are called the *quadrants* of the graph. In the first quadrant both the x and y-values are positive. In the second quadrant the y-values are positive and the x-values are negative. In the third quadrant both x and y-values are negative and in the fourth quadrant the x-values are positive and the y-values are negative.

In the graph $y = x$, both x and y must have the same sign. That is why this graph lies in quadrant I (x and y both positive) and quadrant III (x and y both negative). For the graph $y = -x$, x and y have opposite signs and this graph lies in quadrants II and IV.

Decreasing graphs and the linear equation $y = -x$

When you studied the graph of the linear equation $y = x$ in Activity 6, you noticed that the y–values of the graph increased as the x–values of the graph increased (moving from left to right on the x-axis). In the following activity we are going to investigate if this is also the case for the $y = -x$ linear equation.

ACTIVITY 8

Substitute the values –3, 0 and 2 for x into the linear equation $y = -x$ and solve for y. Then check your answer by looking at the graph you drew in Activity 7and finding out the y co-ordinates for these values of x.

Find out what happens to y when x increases.

Have a look at the $y = -x$ graph in the previous activity. Find –3 on the x-axis. Now move your finger up to the linear graph and across right to the corresponding value on the y-axis. You should get $y = 3$. Now use the graph to find the values for y where $x = 0$ and then $x = 2$. Do you notice that as the x–value increases (–3 ; 0 ; 2), the corresponding y–values decrease (3 ; 0 ; –2)? We call this a *decreasing graph* or function as the y-values decrease as the x-values increase.

When you check to see if a graph is an increasing or a decreasing graph, you need to always move from left to right on the x-axis; that is you need to increase the x-values. The x- values increase and the corresponding y–values (using the graph or substitution) also increase, then the graph is an increasing graph. If the corresponding y–values decrease, then the graph is a decreasing graph.

So: if x-values increase and y-values increase - increasing graph
if x-values increase and y-value decrease - decreasing graph

ANSWERS ON PAGE 107

Drawing graphs of word sums and equations in two variables

During the last flood, Sipho and Simphiwe both lost their houses. They had no blankets, or food to eat. Sipho and Simphiwe went to a flood relief station to get food. Sipho was part of a family of 5 and Simphiwe lived on his own. Sipho had to get five times as many rations as Simphiwe because of his larger family.

Sipho got cross with the storeperson who was handing out the rations because she took a long time to work out how much Sipho was meant to get.

Sipho was doing the Mathematical Literacy course and he thought that the store-person had a very simple multiplication sum to do. This waste of time frustrated Sipho. He decided to draw a graph to show how many more rations he must get compared to Simphiwe, and to show the store-person how to read the amount that he must get from the graph. This is how he worked out the graph:

Let the amount of rations that Simphiwe gets be x. Sipho's family has 5 members and Simphiwe has only 1 member, himself. Sipho has five times the number of people in his family that Simphiwe has so if Simphiwe must get x rations then Sipho gets five times that amount which is $5x$.

Let Sipho's amount equal y, so we get the equation $y = 5x$.
This equation compares the amount of food needed by Sipho and Simphiwe.

Let's plot this graph.
Once again we must use substitution to work out what x- and y-values satisfy the equation. When y is the subject of the equation like this, it is easier to substitute values for x and then find the corresponding y-values.

Because a linear equation has a straight line graph, we only need a minimum of two sets of co-ordinates. When we join them up we draw a straight line.

Let's take two values for x. Any two will do. How about 0 and 3? When Simphiwe gets no rations then Sipho will also get no rations;
If $x = 0$ then $y = 5(0) = 0$
If $x = 3$ then $y = 5(3) = 15$

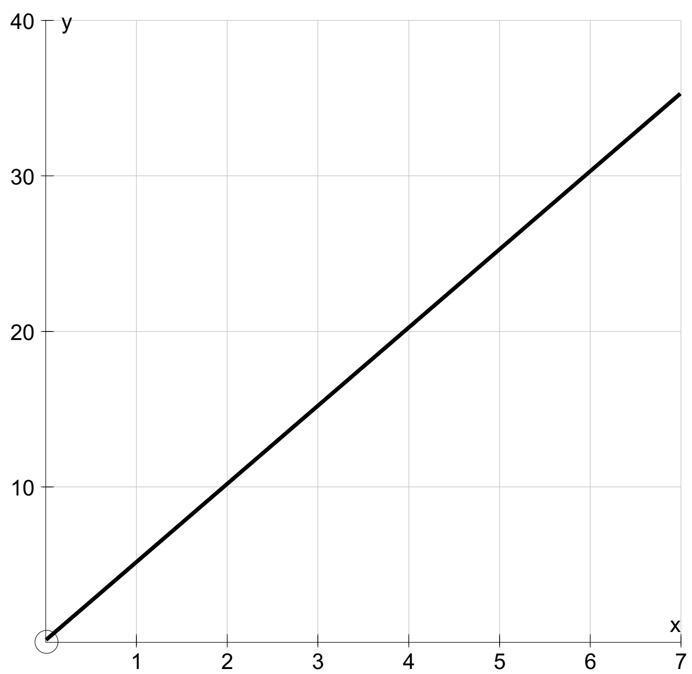
So our co-ordinates are (0;0), (3;15).

ACTIVITY 9

Plot the two points given above on a graph and draw the line that compares the amount of rations that Sipho and Simphiwe are to receive.

Feedback

Did your graph look like this?



The last time Sipho and Simphiwe went to collect rations, Simphiwe was given 7 tins. Sipho showed the storeperson how to work out how many tins he should get by reading it from the graph. Read from your graph how many tins Sipho should get.

Now check that you read this correctly by using your equation to solve this problem.

If $x = 7$ then $y = 5(7) = 35$ tins.

Did you find the point where x was 7 and then go up to the line directly above that point? Looking across from that point onto the y-axis did you find that y was equal to thirty five?

ACTIVITY 10

Draw these two linear equations on the same graph by finding two points to use to plot each graph. Take any values for x and work out the corresponding y co-ordinate so that you can plot two points to join up and create each straight line graph.

$y = 2x + 1$

$y = 3x + 1$

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What are the coordinates of the point where the two graphs intersect each other?

Intersect: cross. The point where the two graphs cross each other.

Finding a value that will satisfy two separate equations (make both equations true)

If we draw a graph for each linear equation and find the point where they intersect, we can find the value that satisfies both equations.

Example

Two years ago a woman was six times older than her son and two years from now she will be four times as old as her son.

If we make linear equations from these two statements and sketch the graphs of these equations we can find out from the graphs what the present ages of the woman and her son are.

Let the present age of the mother be y and the age of the son be x.

Equation 1:

2 years before the present age the woman was six times older than her son.

If her present age is y, then two years before her present age was $y - 2$. At that time the mother was six times older than her son.

How old was her son two years before? If her son is now x years old, then 2 years before he must have been $x - 2$ years old.

In words: The mother's age two years ago $(y - 2)$ is equal to six times $(6 \times)$ her son's age two years ago $(x - 2)$.

Mathematical notation: $y - 2 = 6 \times (x - 2)$.

Let's solve this equation:

$y - 2 = 6x - 12$ (6 times $x = 6x$ and 6 times $-2 = -12$)

Making y the subject: $y = 6x - 10$ (add 2 on both sides)

This is one part of the problem. To go further, we must organise the rest of the information into another equation.

Equation 2:

2 years from now (the present age) the woman will be $y + 2$ years old and her son will be $x + 2$ years old.

The woman will be four times the age of her son so: $y + 2 = 4(x + 2)$

Let's solve this equation by making y the subject.

$y = 4x + 8 - 2$ (subtract 2 from both sides to make y the subject.)
$y = 4x + 6$

We now have two separate linear equations with the variables x (the son's age) and y (the woman's age).

ACTIVITY 11

Find two points to help you to draw a graph of each of these two equations from the example above:

$y = 6x - 10$

$y = 4x + 6$

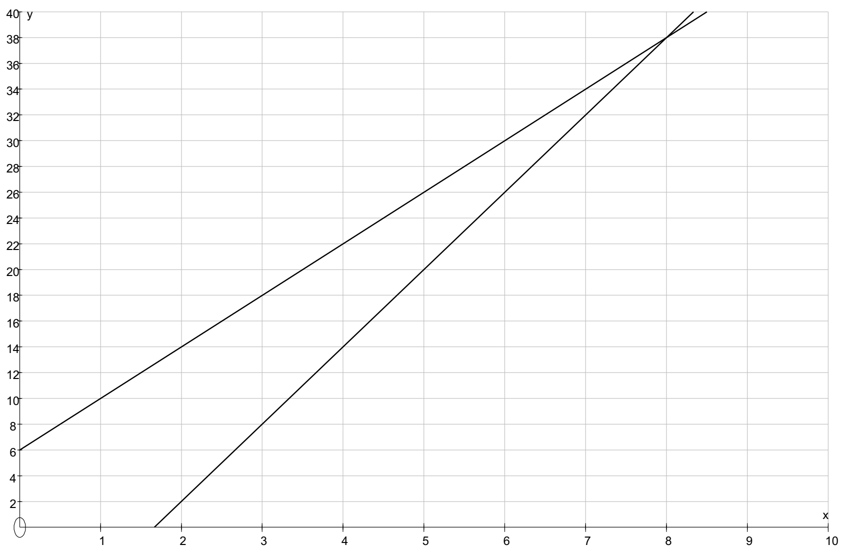
a) Choose two values for x and find the corresponding y-values for each equation by substituting in the chosen x-values.
b) Draw these two linear equations on the same Cartesian plane using the two points that satisfy each equation.
c) What are the co-ordinates of the point where the two graphs intersect each other?

Note that in this example you should not choose any negative values for x as x represents age and this is always a positive number (for example, you cannot be –5 years old!)

Feedback

This is the point that satisfies both equations; the one point (set of co-ordinates) where the same x and y-values make both equations true at the same time (simultaneously).

Did your graphs look like the ones below?



We chose x-values of 2 and 5:

Substituting these values into equation 1, $y = 6x - 10$, we get:

If: $x = 2$
$y = 6(2) - 10$
$\therefore$ $y = 12 - 10$
$\therefore$ $y = 2$

If: $x = 5$
$y = 6(5) - 10 = 20$

So when $x = 2, y = 2$
This gives us the point with co-ordinates (2 ; 2)

And when $x = 5, y = 20$
This gives us the point with co-ordinates (5 ; 20)

We substitute the same values into equation 2, $y = 4x + 6$, and repeat the process:

$x = 2: y = 4(2) + 6 = 14$: (2 ; 14)
and $x = 5: y = 4(5) + 6 = 26$: (5 ; 26)

Can you see these points we plotted on the graph?
What points did you use?

Did you get (8 ; 38) as your point of intersection? These are the present ages of the son and his mother that make both equations true.

We can check this algebraically by making the two equations equal to each other:

$$6x - 10 = 4x + 6$$
$$2x = 16$$
$$x = 8$$
$$y = 6(8) - 10$$
$$y = 38$$

So we know that our point of intersection is definitely (8 ; 38)

ACTIVITY 12

In this activity, we're going to test values in the equation from the activity above.

1. Substitute the values you got for the intersection of the graphs, (8;38), into both equations and see if these values for x and y make both equations true.

2. Now try and substitute a different point into both equations, for example (2 ; 6).

ANSWERS ON PAGE 107

Did you notice that the point (8 ; 38) in question 1 satisfied both equations? We can therefore say that this point is the simultaneous solution for the two graphs (i.e. the point lies on both graphs).

The point (2 ; 6) in question 2 did not satisfy either of the equations so this particular point does not lie on either of the two graphs.

We say that these equations provide the conditions or *limits* for x and y. This means that the equations allow only certain values for x and y. Only the point of their intersection has x- and y-values that make both equations true.

We have come to the end of this lesson. Read through the summary before you answer the questions in the exercise that follows.

Summary

In this lesson you found out that linear equations are equations that have straight lines as solutions when they are plotted on a graph (set of x- and y-axes which is also known as a Cartesian plane).

You learned to show the solutions of equations by drawing a graph for the (horizontal) variable x and another graph for the other (vertical) variable y.

You also learned to draw the solution of equations that have two variables, x and y by substituting values for x and finding the corresponding y-value.

You plotted two points given by these co-ordinates, and you used them to draw the straight line of the linear equation through these points.

You learned that the values for y change when the values for x change on a graph. Looking at how they change helps us to know whether the graph is an increasing or a decreasing graph. If we increase the x-values and the corresponding y-values also increase, then the graph is an increasing graph. But if we increase the x-values and the y-values decrease, then the graph is called a decreasing graph.

You also learned that if you draw the graphs of two separate equations, then the place where the two graphs cut has an x- and y-value that satisfies both equations.

Finally, you learned that examples from daily life can be turned into linear equations and straight line graphs. We can use these equations and graphs to get answers to questions.

Self-assessment checklist:

Are you able to:

- draw the graph of an equation with one variable.
- draw a graph of an equation with two variables, where x and y are changing at the same time.
- use substitution to find the co-ordinates that make the equation true, and then to find the graph of the equation by plotting these co-ordinates on a set of x and y axes.
- plot graphs of $y = x$ and $y = x$ and identify whether a graph is increasing or decreasing.
- translate real life situations into linear equations with two variables, x and y, and draw the solution as a graph.
- read information from a graph to help solve a problem.
- find the solution that satisfies two separate equations (makes both equations true) by drawing graphs for both equations and finding the point where the two graphs cut each other.

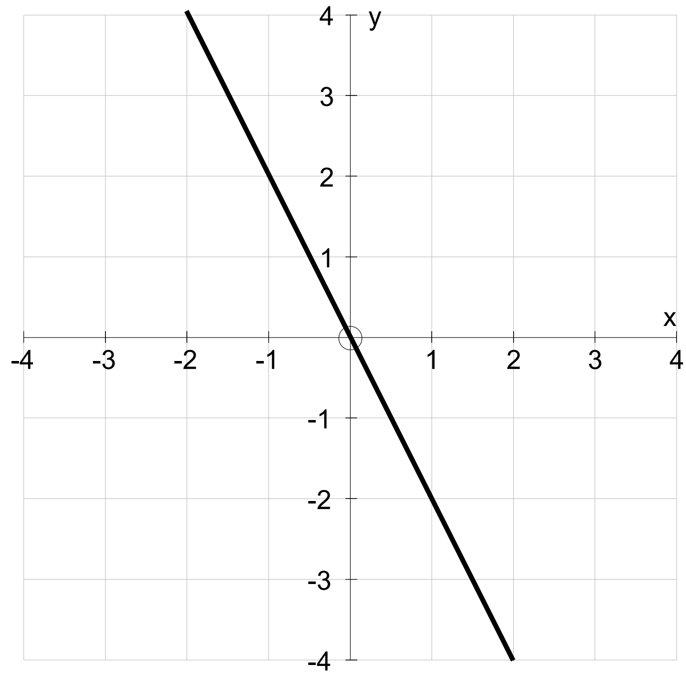
SELF-CHECK EXERCISE

1. Draw the following graphs on the same set of axes:
   a) $x = 2$
   b) $y = 2$

2. Make sketches of the following graphs (on the same set of axes) and say whether each graph is an increasing or a decreasing graph.
   a) $y = 3x$
   b) $y = -3x$

3. From the graph below, write down the corresponding y co-ordinate when:
   a) $x = -1$
   b) $x = 0$
   c) $x = 1$

   Is this graph increasing or decreasing? Explain why.



4. Two years ago a man was six times the age of his son. In four years' time the man will be three times the age of his son.

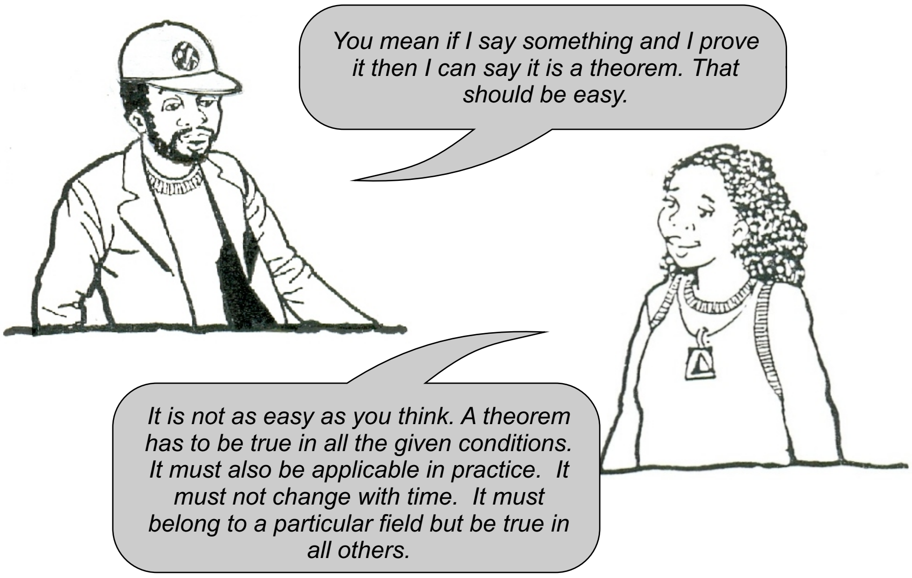
   a) Write two linear equations that describe these two statements.
   b) Draw the equations on to a Cartesian plane by finding and plotting points that satisfy the equations.
   c) Find the point where the two graphs intersect and label it with a P. This point will give you the present ages of the man and his son.
   d) Calculate the point of intersection algebraically to find the actual co-ordinates for point P.

ANSWERS ON PAGE 86

5. The theorem of Pythagoras

Introduction

What is a theorem? This is a good question. A theorem is a statement that has to be proved through methods of reasoning.



Theorems are useful in mathematics, because if mathematicians have proved beyond doubt that a theorem works, this theorem can be applied anywhere in any problem, without further argument or proof.

There is not much background necessary for this lesson. You only need to remember what a right-angled triangle is. Unit 2, lesson 6 introduced triangles. A right-angled triangle was one of them.

There are many theorems in mathematics. The theorem of Pythagoras is just one example.

In this lesson you will:

- put the theorem into your own words
- learn briefly about the history of the theorem of Pythagoras
- use the theorem in practical ways.

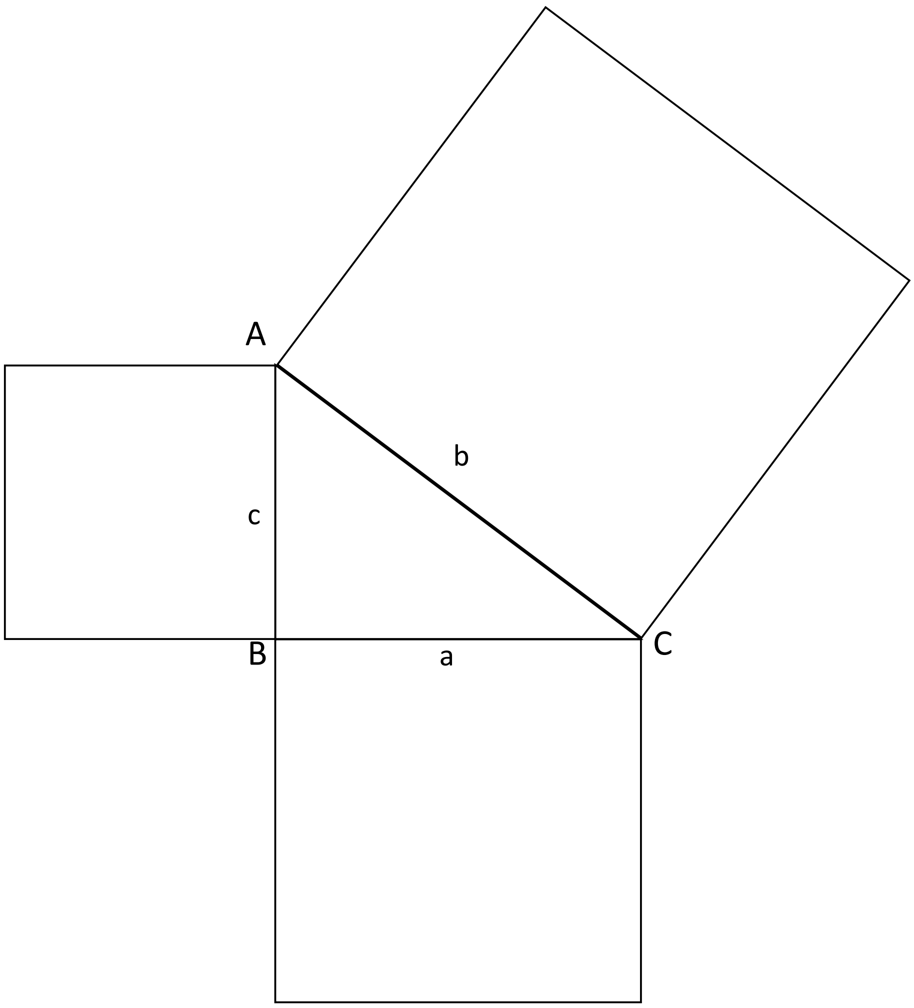
It is useful to remember this theorem in your own words. As you will find out, this will help you to understand the reasoning behind the theorem. This will also show you one of the uses of this theorem. You can discover this theorem yourself, through the following activity.

ACTIVITY 1

Draw a right-angled triangle ABC. Make AB = 3 cm and BC = 4 cm. Make the right angle at B. From each side of this triangle, draw a square.

This means that you will have three squares, where each side of the triangle is one of the sides of a square.

Remember, a square has four sides that are equal in length. This drawing will give you an idea of what we are talking about.



The hypotenuse is the name given to the side of a right-angled triangle that is opposite the right angle.

Note that usually we name the side of a triangle by the angle opposite it. For example, the side opposite angle A is named side *a*, and we use small letters. Can you see what we called the sides opposite angles B and C? Do you recognise the hypotenuse?

It is side AC, called b, the longest side of a right-angled triangle. Now calculate the area of the squares. Note that the area of the smallest square is c^2. What is $c^2 + a^2$ equal to? Write out what you have found in words.

Feedback

Did you find that $a^2 + c^2 = b^2$, where b is the hypotenuse? In words we could write this result like this:

In a right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides. This is known as the *theorem of Pythagoras*. Try to remember it. It is useful in many ways.

ACTIVITY 2

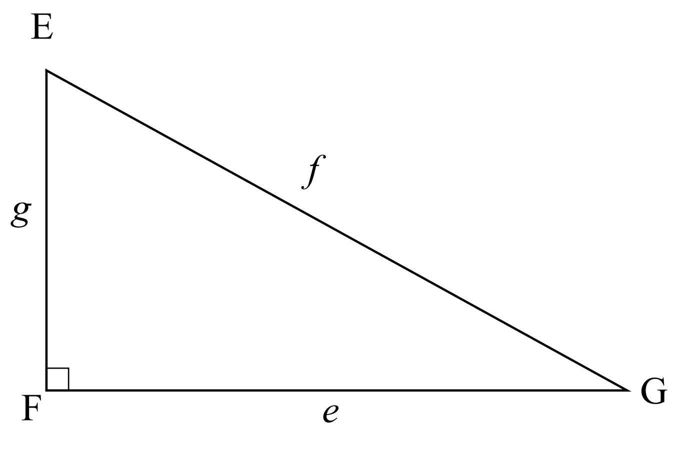
In a right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

Let's check what we have just said in practice. Take the right-angled triangle in Activity 1 and your mathematical instruments. Measure the lengths of its sides. Check that if you square all the sides, the square of the longest side, the hypotenuse, is equal to the sum of the squares of the other two sides.

ACTIVITY 3

In triangle EFG below:

a) State which angle is the right angle.
b) State which side is the hypotenuse.
c) Write down the theorem of Pythagoras in terms of the sides of this triangle.
d) Calculate the length of the hypotenuse if EF = 6 cm and FG = 8 cm



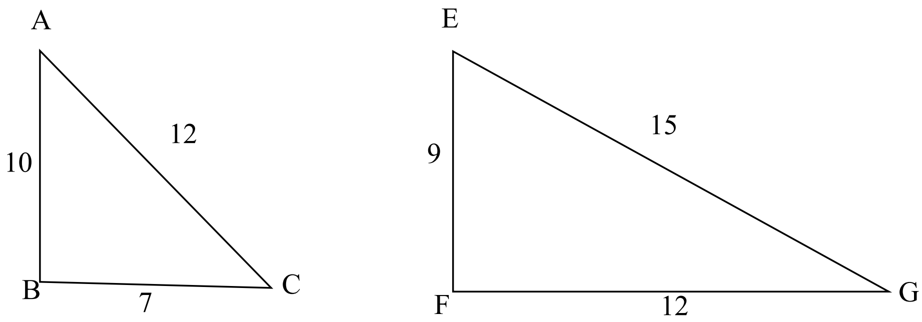
ANSWERS ON PAGE 108

Uses of the theorem of Pythagoras

The theorem can be used to determine whether a triangle is a right-angled triangle or not. Try the following activity.

ACTIVITY 4

The dimensions of triangles are given as shown in the figure below. Both triangles appear to look like right-angled triangles but one of them actually has an angle that is just less then 90°. Find out which of the two is a right-angled triangle.



ANSWERS ON PAGE 108

There are problems in our daily lives which we solve with the use of the theorem of Pythagoras. The following activity illustrates this.

ACTIVITY 5

A neighbour came to my house to borrow a ladder. She told me she wanted to get to the roof of her house. She told me that the wall of the house was 2 metres high, so I gave her a ladder 2 metres long. Later she came back to tell me that the ladder was too short. She told me that she did not want the ladder to destroy her flowers. The flower garden on the side of the house is 1,5 metres wide. We then decided to use the theorem of Pythagoras to find out how long the ladder should be.

1. Draw a diagram to represent this information.

2. Find the shortest length that the ladder can be so that my neighbour can reach her roof without destroying her flower garden.

ANSWERS ON PAGE 108

ACTIVITY 6

An aeroplane flies due East from an airport for 30 km (from A to B). The pilot then does a 90 degree turn and travels North for 40 km (to point C).

a) Draw and label a sketch representing this problem.
b) What distance would the pilot fly if he flew directly from A to C? Indicate this on your sketch.
c) How many kilometres shorter is the route from A to C than the route from A to B to C?

ANSWERS ON PAGE 109

Summary

In this lesson we studied the theorem of Pythagoras and how to apply it practically.

Self-assessment checklist:

Are you able to:

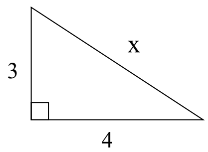
- say the theorem of Pythagoras in your own words
- calculate the length of sides of right-angled triangles
- use the theorem of Pythagoras to check whether or not a triangle is a right-angled triangle
- apply the theorem in solving practical problems.

SELF-CHECK EXERCISE

Question 1 to 8

In numbers 1 - 7 solve for x. In number 8 find the perimeter of the rhombus.

1)



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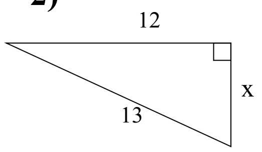

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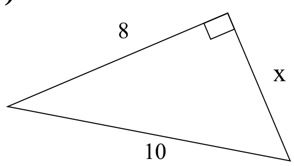

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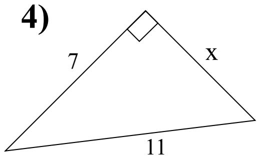
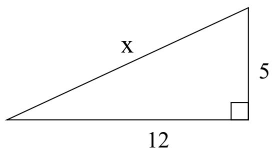
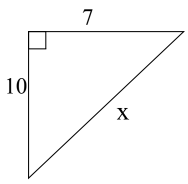


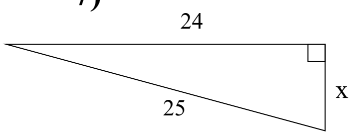
5)



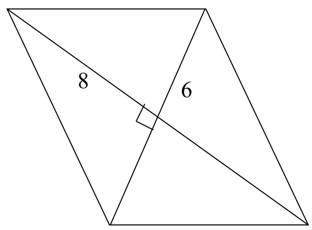
6)



7)



8)



Question 9

ANSWERS ON PAGE 88

When building the foundation of a rectangular house, the supervisor tells the workers that the length of the house should be 16 metres, and the width, 12 metres. To check whether the measurements are correct, the supervisor only measures the distance between two opposite corners. What length is the supervisor looking for? Why?

6. Presenting and interpreting statistics

Introduction

When we did proportion in Lesson 1 of this unit, we compared amounts of ingredients used by builders and bakers. We showed the comparison of these amounts mathematically as fractions or as ratios. Each ratio or fraction is a proportional part of the total amount.

There are other ways that we can compare different amounts. Statistics is one way of doing this. You were introduced to statistics in lesson 7 of Unit 2. There you learnt to draw up frequency tables and work with pie charts.

In statistics we compare different amounts by making tables, and by drawing pie charts, line graphs and bar graphs. Sometimes we show the different amounts as percentages and at other times as numbers. In this lesson you will learn about all these different methods that compare amounts. You will learn to read information affecting our daily lives from these charts and graphs.

You will find out that the people who prepare statistics often choose certain information so as to influence the reader. They present this information in different ways in order to lead people to think in particular ways.

For example, when factory owners or shopkeepers want to sell the things they make, they provide statistical information that makes the readers believe they need these products. A company selling burglar alarms may use a table, chart or graph in their advertising, to show an increase in housebreaking and theft. The advertisers can also draw the graphs and charts in such a way that the increase in housebreaking and theft can be made to look as great as possible. In this way they can convince the readers that they need burglar alarms for protection.

In this lesson you will:

- discover where statistical information is found, for example, in newspapers and company reports.
- compare some of the ways in which statistical information is represented in newspapers and company reports.
- study a pie chart, a bar graph and a broken line graph, and draw the bar and broken line graphs.
- take information given statistically on a pie chart and draw it on a bar graph.
- calculate proportional parts, ratio, fractions and percentage from a pie chart.
- read information from these different charts and graphs and study the information and the way the information is presented.
- compare the different pieces of information you read from the chart or graph.
- look at information critically. This means that you should be able to find the motive or reasons why people present certain information in a certain statistical way.

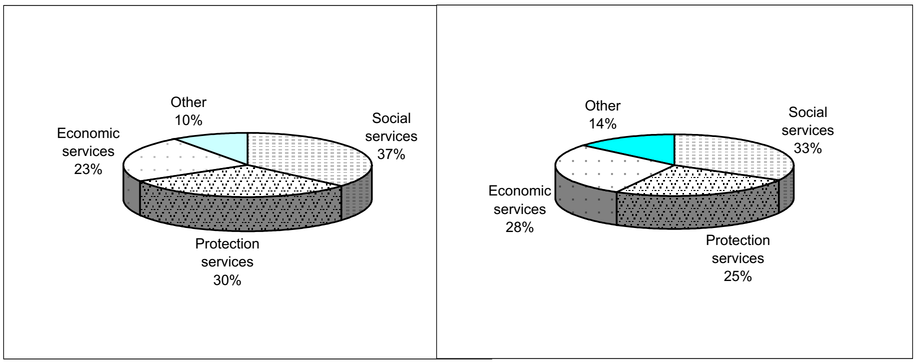
Reading statistical information critically

Reading and comparing statistical information in one chart

Have a look at the picture below of a pie chart taken from a newspaper. Pie charts are charts that show the proportional parts that make up a whole. The pie chart below shows the proportional parts that make up the total government budget for the years 2005/2006 and 2008/2009. These proportional parts of the 'pie' are given as percentages. When you add the percentages for each proportional part of the budget together, they make up a 100% of the budget.

How Government Spends

2005/2006 2008/2009



What information can we read from the pie charts? What type of comparisons can we make from the charts?

Look at the pie chart on the left showing government spending during 2005/2006 and work through the example below.

Example

Work out a simplified ratio that compares the different proportional amounts spent during 2005/2006.

The proportional amounts are given as 'pieces' of the pie chart with these percentages:

10% : 37% : 30% : 23%

To simplify, we divide all of them by the lowest percentage - 10%. Do this on paper or with your calculator.

Simplified ratio = 1 : 3,7 : 3 : 2,3

This simplified ratio means that the amount the government spent on 'social services' was 3,7 times greater than the amount spent on 'other'. The amount spent on protection services was 3 times greater than the amount spent on other. And the amount spent on economic services was 2,3 times greater than the amount spent on other.

ACTIVITY 1

Work out a simplified ratio that compares the different proportional amounts spent during 2008/2009.

ANSWERS ON PAGE 109

ACTIVITY 2

Have a good look at the pie chart for 2005/2006. See if there is some information that is difficult to recognise or if there is some information that is missing. If you think that certain information is difficult to read or has been left out, try and think why this information may have been left out or is difficult to understand.

If you were a person who did not understand percentages and you had to get information from the chart, what information would you find difficult to get? What information on the chart would confuse you? Once you have thought about these things, look at the feedback below.

Feedback

This chart can be very misleading for someone who doesn't understand percentages.

In this pie chart it is difficult to see where the shading for 'other' ends and where the shading for protection services begins. Someone who is literate, but not numerate can easily be confused about the amount of proportional parts given to social services and protection services.

Literate: able to read with understanding.
Numerate: able to work with numbers.

Someone who is numerate can use the percentages on the chart to realise that protection services and social services are each approximately one third of the total pie chart of the 2005/2006 spending. Can you see that?

The people, in presenting this pie chart, may have had the purpose of covering up the amount of money spent on protection services.

The words 'protection services' are themselves misleading. They refer to the army, the police force, the traffic police, and money spent on the prisons and justice system.

The chart does not give information about 'other'. What is this 'other' the government is spending money on? How can we keep a check on how the government spends our taxes if we don't even know what they spend the money on?

Reading and comparing statistical information from two charts

If we compare the proportional parts in the 2005/2006 pie chart, we can see that more money was spent on protection services than on economic services.

During the 2008/2009 years the amount of money spent on economic services increased and the amount spent on protection services decreased. Look for this change on the pie charts and then do the activity below.

ACTIVITY 3

Answer the following questions by reading the information from the pie charts.

1. By what percentage did the amounts given to economic services and 'other' increase from 2005/2006 to 2008/2009?

2. By what percentages did protection services and social services decrease from 2005/2006 to 2008/2009?

3. Why do you think that there have been changes in the percentages of money spent on the services mentioned in questions 1 and 2? Do you think that these changes are reasonable?

ANSWERS ON PAGE 109

Changing the way we show statistical information

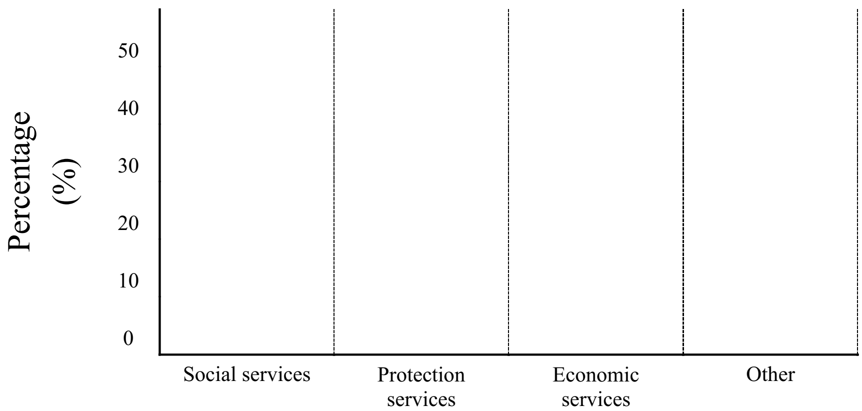
We can draw the information from pie charts to draw other graphs such as bar graphs. We can also take statistical information from other graphs and give the information in the form of a pie chart.

Drawing a bar graph from information on a pie chart

A bar graph is another way of comparing different amounts of proportional parts. We can compare the different amounts by looking at the heights of the bars.

ACTIVITY 4

When we draw a bar graph we write the different names of things being compared on the x-axis. We write the amounts of each thing on the y-axis. We can give these amounts as numbers or as percentages. Have a look at the diagram below and draw one like this on graph paper.



ANSWERS ON PAGE 110

Now fill in the percentages for each of the things compared in the 2005/2006 pie chart.

ACTIVITY 5

ANSWERS ON PAGE 110

Use the pie chart from 2008/2009 to draw a bar graph for 2008/2009 data.

ACTIVITY 6

We can compare the spending for the different years 2005/2006 and 2008/2009 by drawing them together on the same bar graph. Do this by drawing the amounts of both 2005/2006 and 2008/2009 in the same 'bar' above the name.

You just need to divide the space of the 'bar' into two so you can show the percentages of both years. Now there will be two bars for each name (8 bars all together). Don't forget to put your percentages on the *y*-axis and make sure your bar for each is the right height.

ANSWERS ON PAGE 110

ACTIVITY 7

1. This was the information given to show the average temperatures in Port Elizabeth during January, February, March and April of 2009.

   January = 23°C
   February = 21°C
   March = 18°C
   April = 17°C

   Draw a bar graph to show how the temperatures changed over the months. Make the *y*-axis equal to degrees in centigrade °C.

2. In 2010 the average temperatures were:

   January = 25°C
   February = 24°C
   March = 19°C
   April = 17°C.

   a) Draw a bar graph to show these temperatures for 2010.
   b) Now draw a graph of two bars for each month to show how the temperature compared over the years 2009 and 2010.

3. After you have drawn your graph, look at it quickly and try to guess which month showed the greatest change in temperature and which month showed the least change.

   Now work out the change in temperature for each month and see if you guessed right.

4. In your opinion, how did the temperature generally change from 2009 to 2010?

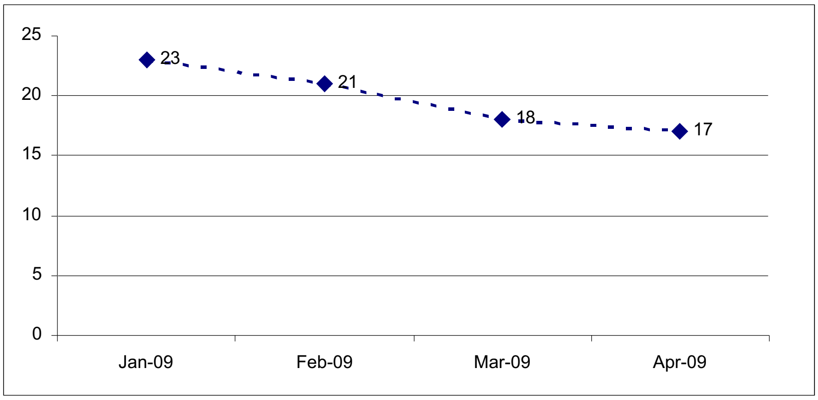
ANSWERS ON PAGE 111

In the next section we will be looking at presenting statistical information on a broken line graph. You will draw the information on your bar graph as a broken line graph.

From bar graph to broken line graph

Instead of having a whole bar to show the amount of something, we can show it with a point. We can do this in the same way as when we plotted points in lesson 4 of this unit. When you join up the points you get a broken line.

Example
We will make the bar graph you drew in Activity 7, question 1, into a broken line graph. Use graph paper for this activity. Start at the origin and label the month as you did for the bar graph. Label the *y*-axis and plot the point that lies above each line that shows the temperatures for each of the four months in 2009.
Now join up the dots.
Your broken line graph should look like this?



Remember, we said that a linear equation is called *linear* because it has a straight line graph. Notice that the graph above is not a straight line. We are also only looking at the averages for each month and not at values between these points. This is why it is called a broken line graph.

ACTIVITY 8

1. Draw a broken line graph of the average temperatures in 2010 as indicated in question 2 of Activity 7.

2. Draw a broken line graph to compare the average temperatures over those four months for both 2009 and 2010.

ANSWERS ON PAGE 112

Sometimes it is useful to put two different things on the same graph so that you can compare them. You can do this by making a *y*-axis on each side of the graph. You put the units (the quantities being measured) for each different thing on either side of the graph. You can make the two dotted lines different colours, so that you can tell the difference between the two different graphs.

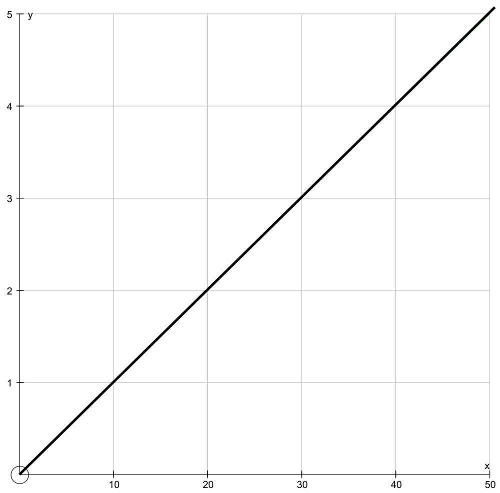
Fog: water vapour

Example
Have a look at the double broken line graph on the next page. It compares deaths and pollution during the days of December. The pollution is called smog. Smog is a thick mist that forms from fog and the smoke of pollution.

The thick mist makes it difficult for people to see and more car accidents can happen. People with lung sicknesses also find it harder to breathe when there is smog.

The graph is drawn to see whether there are more deaths when the amount of smog pollution increases. See if you can tell from the graph if death rates increase as pollution levels increase.

This graph was taken from the *London Times,* 19 April, 1953. You can see on the y-axis there are two different amounts being measured. On the left side the y-axis shows the amount of pollution. On the right side the y-axis shows the number of deaths during the same days. Note that the amount of pollution is shown by the solid line graph and the number of deaths is shown with a dotted line. Answer the questions in Activity 8 below the graph.



ACTIVITY 9

1. On what day in December was the pollution the highest?
2. On what day in December did most deaths occur?
3. How many deaths occurred on this day?
4. During what period of the month (which days) was the pollution level highest?
5. During what period of the month (which days) was the number of deaths highest?
6. Do you think that pollution can cause an increase in the number of deaths?

ANSWERS ON PAGE 112

Even though most deaths did not occur on days that pollution was highest, the graph shows a period of time when both deaths and pollution were high. Study this time when both are high and also see that at other times both are low. This means that when pollution is high, deaths are high and when pollution is low, deaths are low.

In statistics we say that there is a strong *correlation* between the number of deaths and the level of pollution. Statistics like this can show us patterns in certain kinds of events. These patterns can give us insight into cause and effect. This means that statistics help us find the causes of the events that take place.

Correlation: strong link.

For example, scientists have discovered that aerosol sprays, like deodorants or insect sprays, cause the ozone layer of the earth to break up. Ozone is a gas that protects living creatures from the sun's harmful rays. A thin layer of ozone gas surrounds the earth. Statisticians can show that the cause of this loss of ozone is aerosol spray. By plotting the amount of ozone with the amount of aerosol spray on a double broken line graph, they can show how the ozone is affected by aerosol spray.

In the next part of this lesson we will show how we can draw double broken line graphs in different ways to influence the reader.

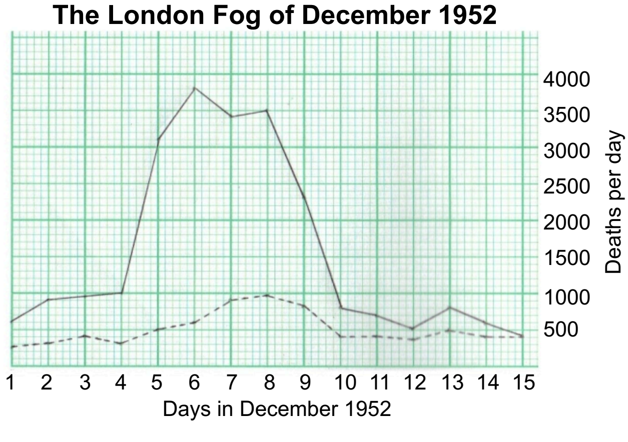
Drawing graphs to influence the reader

For an industrialist with a big factory that pollutes the area, the graph in the *London Times* should be taken seriously. This is because the industrialist may be forced to cut down on production to lower pollution levels. Cutting down on production means losing money. The industrialist therefore asked the company statistician to redraw the graph so that it was not so easy to see the close link between pollution levels and deaths. He asked the statistician to put the graph in a company report so that most workers would not be so aware of the strong correlation between pollution and deaths.

Here is a graph with the same information as in the *London Times* graph. But this graph looks different. Study it carefully to see how the same information can be drawn in a different way, to give the reader a different idea of what is happening.

Answer the questions in Activity 10 that follows.

The London Fog of December 1952



ACTIVITY 10

ANSWERS ON PAGE 113

Study the two graphs of the London Fog. Compare the graph that appeared in the *London Times* with the one that was written as a company report. See if you can find the difference between the two graphs, then look at the feedback.

More on pie charts, percentages, degrees and fractions

Re-look at the pie chart in the section 'Reading statistical information critically' just before Activity 1.

In Activity 1 we worked out the ratios of spending using percentages. In this part of the lesson we will work out the degrees for each shaded section of the pie chart and use this to work out fractions and percentages.

ACTIVITY 11

Look carefully at the 2005/2006 pie chart just before Activity 1. The area shaded for each section is a part of the total circle.

Do you remember when you studied pie charts in Unit 2, lesson 7?

You learned that all the angles at the centre of a circle make up a total of 360°. In the pie chart, each proportional part of the circle makes up a certain amount of the total degrees in the circle.

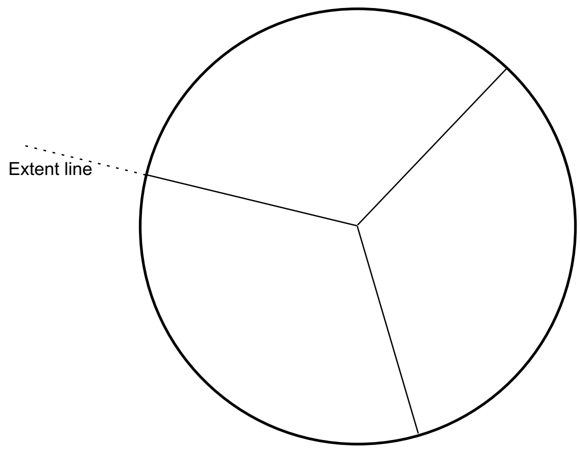
1. Use your protractor to work out the sizes of the angles in degrees for each proportional part of the circle.

2. Write these proportional parts as fractions and decimals.

3. Show the proportional parts as percentages.

   Hint: Extend the line that divides each proportional part so that you can measure the angle accurately with your protractor. Have a look at the figure below to see how to measure the angles accurately.

ANSWERS ON PAGE 113



We have come to the end of this lesson, and of the unit. Before you read the summary on the next page look back to the beginning of this lesson to see if you can do the things we said you should be able to do. Then try to do the Self-check exercise that follows.

Summary

In this lesson you learned some of the ways in which people can show statistical information, some of the places where you could find statistical information and some of the reasons why and how statistical information is presented.

We saw statistical information that came from newspapers and we spoke about the use of statistical information in company reports.

You learned about three methods of representing statistical information. We took information from a pie chart and changed this to a bar chart. We also took information from a bar chart and changed this to a broken line graph.

You learned to look at the information critically and to watch out for being misled. You learned to read information from graphs and charts and to compare separate pieces of information.

You learned about the connection between the work we did in earlier lessons on proportional parts, ratios, fractions and percentages and the information in pie charts.

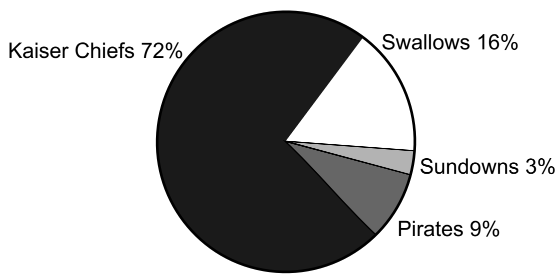
Self-assessment checklist:

Are you able to:

- read information from a pie chart, and draw it onto a bar chart or a broken line graph
- critically analyse information given statistically
- calculate the proportional parts, ratios, fractions and percentages from a pie chart.

SELF-CHECK EXERCISE

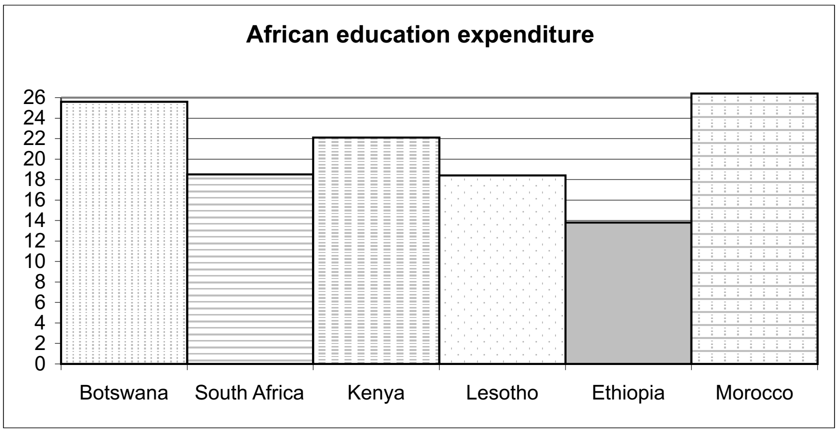
1. Have a look at the pie chart below. It shows the percentage of points won by different soccer teams in a tournament in 1980. Answer the questions that follow.



   a) Use a protractor to work out the fraction of the circle for each soccer team.
   b) Change these fractions to percentages and see how close you are to the percentages given on the pie chart.

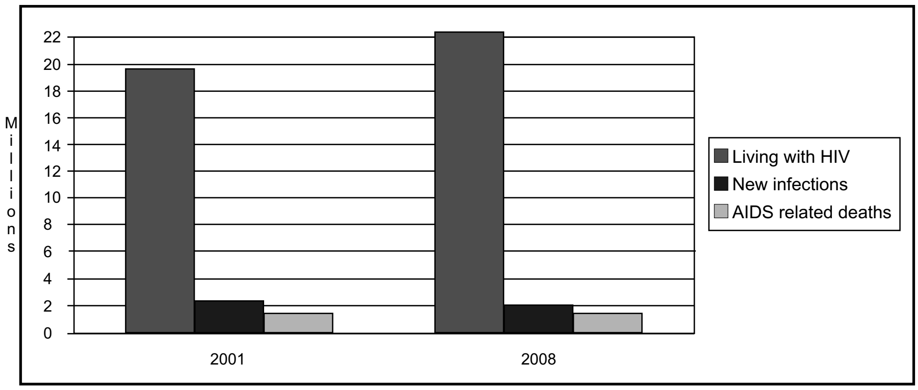
c) If the total number of points gained in the tournament was 200, work out the number of points each team gained out of the total number of points. Work this out by using the percentages given on the pie chart.

2. Look at the bar graph below. It compares the average percentage of the total government expenditure different African countries spent on education in 2003-2008. Answer the questions that follow. The values on the vertical axis are in percentages.



a) Which of the African countries above spent the highest percentage of their total government expenditure on education?
b) Which of the African countries above spent the lowest percentage of their total government expenditure on education? Give a reason why you think this is the case.
c) Write down the order of the African countries above from the country that spent the highest percentage of their government expenditure to the country that spends the lowest.
d) Where does South Africa rank in comparison to these other African countries?

3. The bar graph below was taken from the United Nations AIDS website. It compares HIV/AIDS statistics in Sub-Saharan African in 2001 and 2008. Note that the vertical column indicates the number of people in millions.



a) How many more millions of people were living with HIV in 2008 compared to 2001?

b) Estimate how many millions of people you think will be living with HIV in 2015. Give a reason for your answer.

c) Did the number of people with new infections increase or decrease between 2001 and 2008? Why do you think this happened?

d) Did the number of AIDS related deaths increase or decrease between 2001 and 2008? Do you think this will still be the case in 2015? Give a reason for your answer.

ANSWERS ON PAGE 89

7. Revision and consolidation

Introduction

The objective of this final lesson is to revise and consolidate the skills you learnt in this unit and to build your confidence in preparing you for the exam. This is a 'test-yourself' lesson and the answers are not contained in this unit but in a separate booklet entitled 'Revision and consolidation answer booklet'. Section A is revision of the whole unit to see whether you are able to understand and integrate the various topics dealt with in the unit. Section B is in an examination form and the questions are taken from previous Mathematical Literacy tests and exams. Try to stick to the time given in Section B to ensure that you are working fast enough. If you find that you are not able to finish, continue working through problems in the units so that your calculation and interpretation speed will improve. This will also help you improve your confidence, accuracy and timing in the examinations.

Summary of unit

In this unit we covered the following knowledge and skills:

- More concepts related to numbers and calculations with numbers including:
  - ratio, direct proportion and exchange rates
  - inverse proportion and average speed

- Finance
  - credit and compound interest
  - the difference between simple and compound interest

- Patterns, relationships and representations including:
  - drawing linear graphs
  - increasing and decreasing graphs
  - using graphs to solve contextual problems
  - independent and dependent variables

- Measurement
  - the theorem of Pythagoras

- Data handling
  - presenting and interpreting statistics
  - bar graphs
  - broken line graphs
  - pie charts
  - how statistics are used to influence readers

Section A

1. Match the correct answers from column B with the questions from column A. In your books, write down only the correct answer next to the corresponding letter.

Column A		**Column B**
a)	The number of matches Brendo will get if he receives $\frac{3}{5}$ of a total of 30 matches.	8
b)	The cost of 5 apples if 10 apples cost R28,00	18
c)	The distance climbed in 7 minutes by an aeroplane with a climb rate of 300 ft/minute.	6
d)	The price of one fruit-juice if 8 bottles cost R46,00.	14,00
e)	The number of days it takes 3 people to paint a house if 6 people take 4 days.	12
f)	The cost of hiring a car for 2 days at a cost of R500 per day.	2100
g)	The number of minutes a watch will lose in 24 hours if it loses 5 minutes in 20 hours.	5,75
h)	The distance a car travelling at a constant speed of 80km/h will go in $2\frac{1}{4}$ hours.	216
i)	The amount I will pay for a meal in rands if I pay $30 for the meal in America and the rate of exchange is $7,20 to the rand at the time.	1000
j)	The number of days it will take 4 identical pumps to empty a dam if 3 pumps take 16 days to empty the dam.	180

2. Use the questions above and give one example of:
   a) direct proportion and
   b) inverse proportion

3. If one night at the Hotel Shake-up costs R350:
   a) copy and complete the following table:

No. of nights	3	4	5
Total cost	1050		

   b) State whether this is an example of direct or inverse proportion
   c) Decide whether the total cost is dependent on the number of nights or if the number of nights are dependent on the total cost. Then write down which is the dependent and which is the independent variable.
   d) Draw a graph representing this information and show on the graph with the letter P where we could read off what the total cost of 6 nights at Hotel Shake-up will cost.
   e) Can you join up the points on the graph? Explain.

4. An eight-man life-raft is equipped with food to last 8 men for 30 days.

   a) copy and complete the following table:

Number of survivors	4		6	
Number of days food will last		120 days		24 days

b) Is there a constant ratio or constant product? State what it is.
c) State whether this is an example of direct or inverse proportion.
d) Identify the dependent and independent variables.
e) Draw a graph to represent this information.
f) Can you join up the points on this graph? Explain.

5. a) Complete the table, without changing the area of the rectangle below, using only whole numbers.

Length of rectangle (in cm)	6	4	1	12
Breadth of rectangle (in cm)	2			

b) Complete:
i) $12 = 2 \times 6$
ii) $12 =$ ____ $\times$ ____
iii) $12 =$ ____ $\times$ ____
iv) $12 =$ ____ $\times$ ____

c) Use the table or part b) to write down four ordered pairs that can help you to plot the graph representing the information provided in the table. Let the length of the rectangle be represented by the x-coordinate and the breadth by the y-coordinate.
d) Plot these points on a Cartesian plane with your x and y-values going from 0 - 12 on each axis.
e) The numbers 1, 2, 3, 4, 6, 12 are called ______ of 12.
f) Is this an example of direct or inverse proportion? Explain your answer and give the constant ratio or product.

6. Solve the following problems:

a) If a bag of toffees is shared among 8 children they get 15 toffees each. How many toffees will each child get if 4 more children join the group?
b) 6 packets of fudge cost R10,74. What will 10 packets cost?
c) At the supermarket there were two different brands of rice crispies, 500 g for R5,89 and 300 g for R3,85. Which is the better buy?
d) On my weekly budget for petrol I could buy 30 *l* of petrol when it cost 913 c per litre. Now that it has increased to R10,20 per litre, how many litres can I get for the same amount?
e) A man has three daughters aged 15 years, 12 years and 9 years. He divides R720 among them in the ratio of their ages. How much does each get?

7. The plan in this activity is to show how the ratio of a mother's age to her daughter's age depends on how old the daughter is. By graphing the equation we will see how kids catch up in age to their parents.

Suppose Ms Beverly Cornel gives birth to her daughter Heidi when she is 24 years old. When Heidi has her first birthday, Beverly will be 25. The ratio of Beverly's age to Heidi's age will be 25 : 1 which can also be written as $\frac{25}{1} = 25$. When Heidi turns 3, Beverly will be 27. The ratio of their ages will then be 27 : 3 or $\frac{27}{3} = 9$.

Heidi is catching up! What will the ratio of Beverly's age to Heidi's age be when Heidi turns 6?

If x = Heidi's age, then Beverly's age = $x + 24$. Let y = the ratio of their ages.

Therefore: $y = \frac{x+24}{x}$

a) Copy and complete the following table to keep track of Heidi's age (x), Beverly's age ($x + 24$), and the ratio of their ages (y). Only use values of x between 1 and 24 that make y come out to be an integer. Also calculate values for $x = 15$ and $x = 20$.

Heidi's age (x)	**Beverly's age (x + 24)**	**Ratio of their ages (y)**
1	25	25 ÷ 1 = 25
2	26	26 ÷ 2 = 13
3	27	27 ÷ 3 = 9
4	28	
	30	
	32	
	36	36 ÷ 12 = 3
15	39	
20	44	

b) Graph your data using the x and y columns. Use the same scale for both axes, and make sure that both scales go from 0 to 36. Draw a smooth curve.

c) Now answer the following questions based on the graph.
  i) Is this an example of direct or inverse proportion?
  ii) Describe what is getting bigger and what is getting smaller in this set of data?
  iii) Use your calculator to find *y*-values for $x = 50$, 75 and 100.

8. The table below shows the time taken by Mr Malome to drive a certain number of kilometres in his car, if he were able to drive at a constant speed. Use the table to answer the questions that follow:

Distance (km)	60	120	180	210	240
Time taken (hours)	1	2	3	3½	4

a) How far did Mr Malome drive in one hour?
b) How far did Mr Malome drive in 4 hours?
c) How far would Mr Malome be able to drive in 6 hours?
d) If Mr Malome drove 90 km, how long would it take him?
e) At what constant speed is Mr Malome driving?

9. A car travels the first stage of its journey on a sand road (100 km) at an average speed of 40 km/h and the second stage of its journey (300 km) at an average speed of 80 km/h. Calculate the average speed of the whole journey.

10. A plane travels 1 500 km from Johannesburg to Cape Town at an average speed of 800 km/h. On the return journey there is a tailwind so the speed is increased to 980 km/h. Calculate the average speed of the plane for the journey.

11. In the following formula for simple interest:
a) Explain what each of the variables represents:

SI represents:
P represents:
r represents:
n represents:

b) Make the rate the subject of the formula.
c) Make the term of the loan the subject of the formula.
d) Calculate the term of a loan if the principal amount is R4 000 at a rate of 12,5% interest p.a. and the simple interest worked out to be R2 500.

12. Find the annual rate of interest if the simple interest on R1 050 for 36 months is R514,50.

13. Calculate the total amount of the following:
a) A car loan of R58 000 at a simple interest rate 11% p.a. over 4 years.
b) An investment of R200 000 at a compound interest rate of 9% p.a. over 15 years.

14. Simphiwe wants to buy a new car costing R220 000. Her father will lend her all the money at a simple interest rate of 12% p.a. for 5 years. Her brother offers her a loan of all the money at a compound interest rate of 16% p.a. for 3 years.

    Calculate the total loan amount of each offer and state which would be the better deal for Simphiwe.

15. Fulu moves into her new flat and buys R16 500 worth of new furniture on hire-purchase.

    The company charges her a simple interest rate of 22,4 % p.a. for 3 years.
    a) Calculate the interest over 3 years.
    b) Determine her monthly instalments if she is to pay it all off in 3 years.
    c) What would she would have had to pay back if the finance charges had been 18% compound interest p.a. for 3 years.

16. Draw the following graphs on one Cartesian plane and label them with the letters indicated:
    a) $x = 3$ (A)
    b) $y = -1$ (B)

17. Follow the steps to draw the graphs of $y = 2x - 1$ and $y = x + 3$
    a) Complete the following table of ordered pairs for each graph:

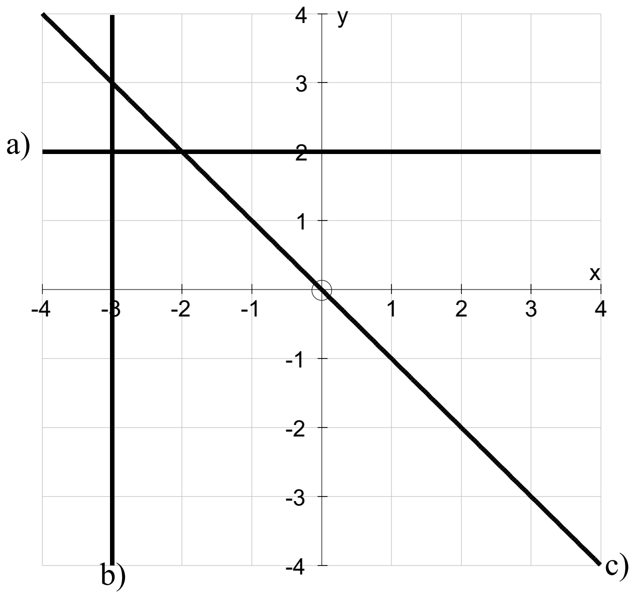
	***x*-values**	***y*-values**	**Ordered pairs**
$y = 2x - 1$	-1 ; 0 ; 1		
$y = x + 3$	-1 ; 0 ; 1		

    b) Now use the ordered pairs to draw both graphs on one Cartesian plane.
    c) Use the letter P to show the point where the graphs intersect each other.
    d) Calculate the coordinates of point P algebraically (i.e. by making the two equations equal to each other).
    e) Are both of these graphs increasing or decreasing graphs? Explain your answer.

18. 1. Write down the equations of the graphs a), b) and c) on the next page.

    2. Write down the co-ordinates of the point of intersection of:

       i) graphs a) and c)
       ii) graphs a) and b)
       iii) graphs b) and c)



19. a) Copy and complete the following table showing the cost of a call on a cellphone if calls are charged at a rate of R2, 00 per minute.

Number of minutes	1	2	3	4	5
Total cost of call	R2,00			R8,00	

b) Decide if the number of minutes determines the cost of the call or the cost of the call determines the minutes and state which is the dependent and independent variable.

c) Write down at least 3 ordered pairs from the table above to assist you in drawing a graph.

d) Draw a graph representing the information.

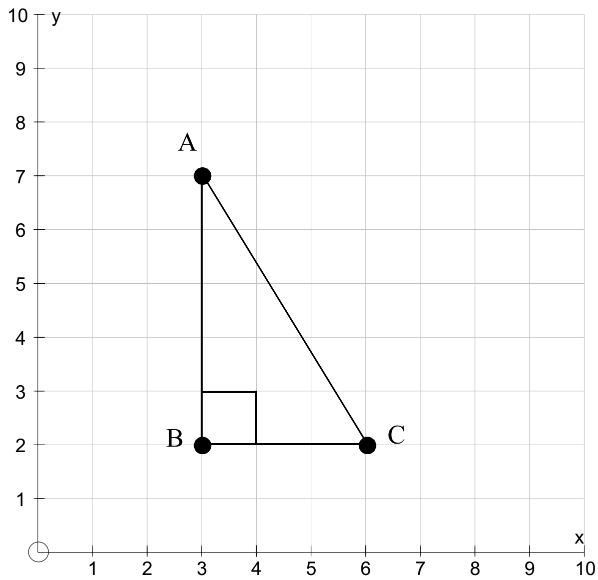
e) Is this an example of direct or inverse proportion? Explain your answer.

f) Write down the constant ratio or constant product.

g) What would the total cost of a call be if the person spoke for 25 minutes?

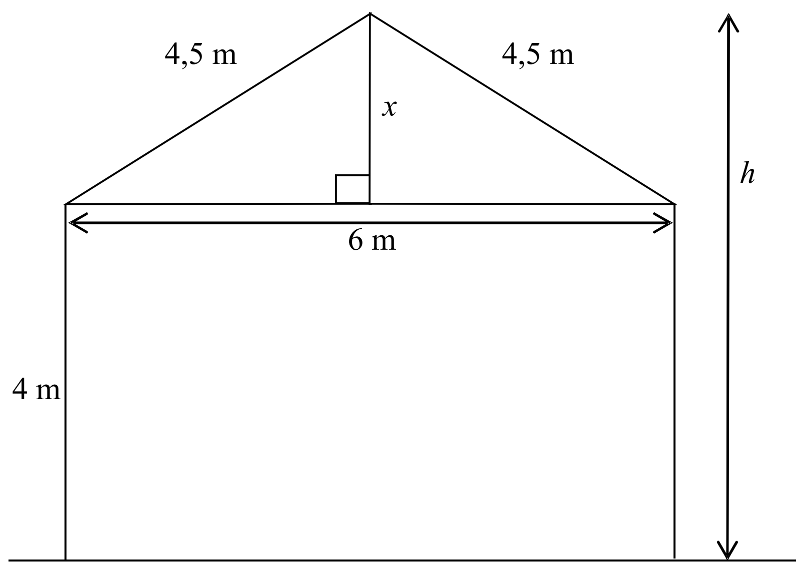
h) If the total cost of a call is R100, how long was the call?

20.



a) Write down the co-ordinates of each of the points A, B and C.
b) What type of triangle is triangle ABC?
(Hint: look at angle B)
c) Calculate the lengths of:
i) AB
ii) BC
ii) AC
d) Calculate:
i) The perimeter of triangle ABC
ii) The area of triangle ABC
e) Write down an equation for the line BC.
f) Write down an equation for the line AB.

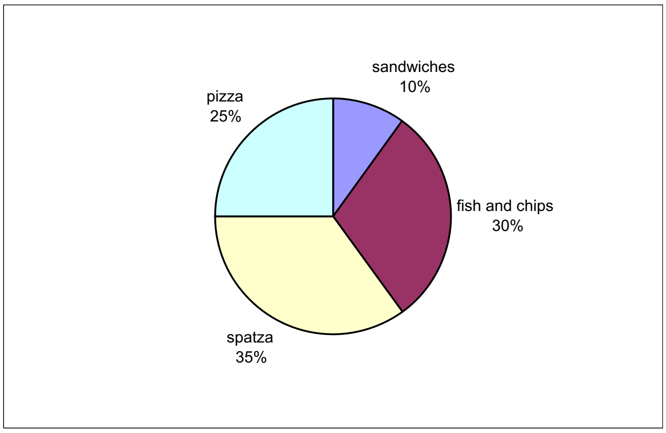
21. This is a side-view of Thabi's house showing one wall and the roof.



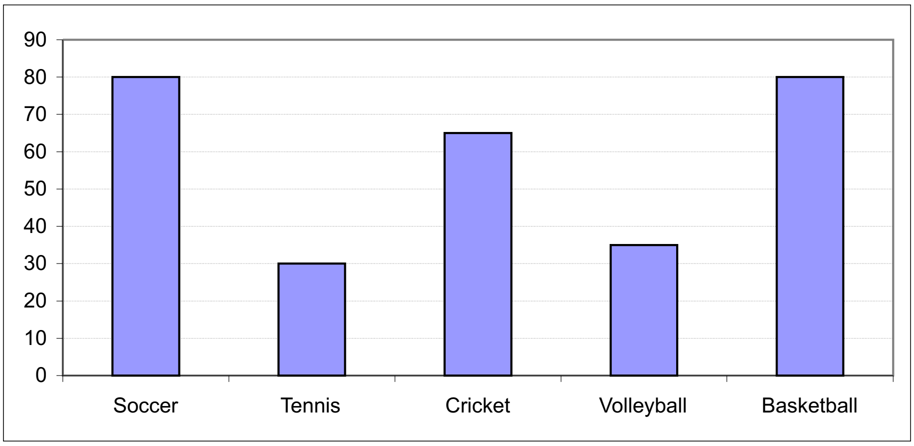
a) Calculate the height of the roof (x).
b) Calculate the height of the house (h).
c) Calculate the perimeter of the part of the roof visible from this side-view.
d) If Thabi wants to make a scale drawing of this side-view and wants the scale to be: 500 mm: 1 m
How long must she draw:
i) the length of the wall and the roof (indicated as 6 m on this diagram above)?
ii) each side of the roof?
iii) the height of the house from the floor to the highest point of the roof (h)

22. Modjadji sells food at a local high school during break. She decided to conduct a survey on the food the learners liked to most to inform her how much stock to buy each week. Modjadji represented the results on a pie chart. Use the chart on the next page to answer the questions.
a) Which food was the most popular?
b) What percentage of learners prefer fish and chips?

c) What food was the least popular?
d) Write each percentage of the foods given as a fraction in its simplest form.
e) Use the data from the pie chart to draw a bar graph representing the same information.



23. The following graph shows the number of students taking part in sport at a university on a particular day. Study it and answer the questions that follow:

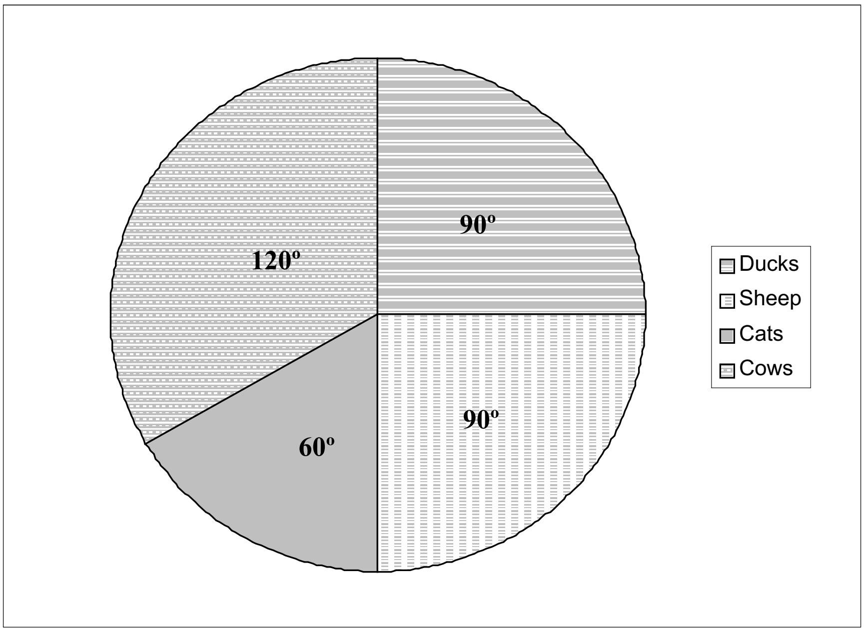


a) Which was the most popular sport(s) on that particular day?
b) How many more students were playing cricket than volleyball?
c) How many more students were playing basketball than tennis?
d) How many students in total were counted in this survey on this particular day?
e) What fraction of the total number of students counted, were playing soccer?
f) What percentage is this of the total number of students?

24. The following data tally was collected on the first examination of 30 Mathematical Literacy students.
a) Complete the frequency column of the table.
b) Draw a bar graph using the data.

Marks	Tally	Frequency
36-40	|||	3
41-45	||||	
46-50	~~||||~~ |	
51-55	~~||||~~ ~~||||~~	
56-60	~~||||~~ |	
61-65	|	
Total		30

25. This graph shows the percentages of different types of animals on a farm:



a) What type of graph is this?
b) What percentage of the animals were cows?
c) Ducks represented one quarter of the total number of animals. There were 60 ducks. How many sheep were there?
d) How many animals in total were there on the farm?
e) Draw a bar graph showing the number of each of these animals on the farm.

26. The scale on a map is 1: 50 000. If the distance between two towns on the map is 5,7 cm, determine the distance between the towns in kilometres.

Section B

Time: $2\frac{1}{2}$ hours

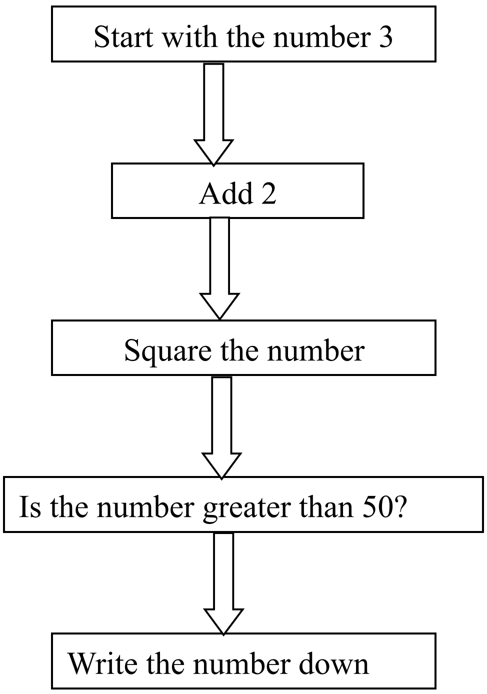
Marks: 135

QUESTION 1 **[59]**

1.1 Calculate:

1.1.1	3^2	(1)
1.1.2	1^3+9^2	(2)
1.1.3	$5\times8\times9\times0$	(1)
1.1.4	$\frac{3\times4+1\times8}{2}$	(2)
1.1.5	$\sqrt{64+36}$	(2)

1.2 Follow the instructions in the flow diagram:



1.3 Complete the table, filling in values where there are variables a - j: (10)

Number	No. of tens in that number	Remaining Units	No. of sixes in that number	Remaining units
21	2	1	3	3
34	3	*a*	5	4
20	*b*	0	*c*	2
5	0	*d*	*e*	5
18	*f*	*g*	*h*	*i*
54	*j*	4	9	0

1.4 Vusi works at a car-wash. He earns R75 per day plus R5 for each car he washes.

Calculate how much Vusi earns in a day if he washes:

1.4.1 5 cars (1)

1.4.2 7 cars (1)

1.4.3 x amount of cars (2)

1.5 The dilution instructions on an energy sports drink concentrate are:

Mix concentrate and water in the ratio of 1: 4.

1.5.1 Explain what is meant by this instruction. (2)

1.5.2 How many *ml* of concentrate and how many *ml* of water do you need to make up 1 litre energy drink? (2)

1.5.3 If your friend throws 500 *ml* concentrate into a bottle, how many litres of water must you add? (2)

1.6 The following exchange rates were valid in May 2011.

Complete the table and then answer the questions that follow, using these rates.

1 British Pound = R9,78
1 Euro = R11,14
1 USA dollar = R6,84
1 Israeli new Shekel = R1,99

Name of country/citry	Price of a standard MacDonald's burger	Price of burger converted into Rands
England	£3.50	
France	5 € (Euros)	
Israel	6 nis	
New York	\$5,50	

1.6.1 Which is the most expensive country to buy a MacDonald's burger? (5)

1.6.2 How much would you spend in Rands if you ate one MacDonald's burger in each of these countries? (2)

1.7 The scale on a map is 1: 25 000. If the distance between two train stations on the map is 5,5 cm, determine the actual distance (in kilometres) between the two stations. (4)

1.8 Tom is 12 while his sister Emma is 4. Tom is also three times as tall as Emma.
If Tom's height is 5,1 feet, how tall is his sister? (2)

1.9 In a bus, the ratio of women to men to children is 5 : 3 : 1. If there are 36 people on the bus, how many of them are women? (3)

1.10 In a **right-angled triangle**, the ratio of the sizes of the two other angles is 4 : 5. Find the size of each of the three angles. (3)

1.11 In each of the following, write down:

i) whether the values in the following tables have a constant ratio, a constant product or neither. (5)

ii) whether they are examples of direct or inverse proportion or not in proportion at all. (5)

a)

X	3	5	7	9	11
Y	6	10	14	18	22

b)

X	15	45	135	210	270
Y	5	15	45	70	90

c)

X	2	4	6	8	48
Y	24	12	8	6	1

d)

X	100	80	60	20	40
Y	24	30	40	120	60

e)

X	12	13	14	15	16
Y	6	7	8	9	10

QUESTION 2 [21]

The table on the next page is an extract from a letter from Sanlam to Mr Moloke. It shows the amounts that are available on instant loan from Sanlam and the monthly repayments involved.

Dear Mr Moloke,
As a valued Sanlam customer, we are pleased to be able to offer you a personal loan at the following rates.

Loan Amount	**24 months**	**36 months**	**48 months**	**60 months**
R4 000	R229	R174	R147	R131
R8 000	R448	R338	R285	R253
R16 000	R864	R643	R534	R470
R25 000	R1 344	R1 000	R830	R730

FIXED REPAYMENTS!!!!!!

These loan repayments conveniently **include** a monthly premium of R3,95 per R1000 of the loan and a monthly administration fee of R9,50 for your optional personal protection plan.

2.1 If Mr Moloke chooses to borrow R16 000 from the bank, calculate how much he will repay if he takes the loan over:

2.1.1 24 months (2)

2.1.2 60 months (2)

2.1.3 In general would you advise him to borrow money for a longer or shorter time? Give a reason for your answer. (2)

2.2 If he chooses the 60 month option, calculate the interest that he will pay over the period. (3)

2.3 The loan repayments include insurance premium and administration fees.

Mr Moloke borrows R16 000, how much of each month's payment is the premium and how much is the administration fee? (4)

2.4 Mr Moloke has two other options for borrowing the money.

2.4.1 An aunt has offered him the R16 000 for five years at 18% p.a., simple interest. What would the cost of this option be at the end of 5 years? (4)

2.4.2 The People's bank will lend him R16 000 for 5 years at an interest rate of 16% p.a. compound interest. Determine the total cost of this option. (4)

2.5 If you were the personal banker of Mr Moloke, which option would you recommend to him?

QUESTION 3 **[13]**

3.1 A person invests R1 000 at an annual interest rate of 12% for a period of 6 years. Interest is compounded annually. The table below is a statement of the investment account.

	Interest credited	**Balance**
Opening balance		R1 000,00
End year 1	R120,00	R1 120,00
End year 2	R134,00	R1 254,40
End year 3	R150,53	R1 404,93
End year 4	R168,59	R1 753,52
End year 5	R188,82	(a)
End year 6	(b)	(c)

3.1.1 Why is the interest earned by the person at the end of year 2 not the same as the interest earned at the end of year 1? (2)

3.1.2 Calculate the values of (a), (b) and (c). (6)

3.1.3 Express the total amount of interest earned over the 6 years as a percentage of the amount invested. (2)

3.1.4 How much interest would the person have earned over the 6 year period if the interest had not been compounded but rather earned at a simple interest rate of 12% per year for the period? (3)

QUESTION 4 **[11]**

The monthly income and costs of a company which produces soccer balls can be calculated using the formulae:

income $= 4 \times x$
costs $= x + 120$
income: $y = 4 \times x$
and costs: $y = x + 120$

where x is the number of soccer balls sold.

4.1 On the same set of axes, plot graphs showing the company's monthly income and costs for the values of x from 0 - 500. You may first draw a table of values if this helps you. Remember you only need three points (from ordered pairs) in order to sketch the graph. (5)

4.2 Determine how many soccer balls the company needs to sell if it is to break even (i.e. make no profit but also no loss). (3)

4.3 Indicate your answer to 4.2 on the graph you have drawn in 4.1. (1)

4.4. If the company produces 905 soccer balls in January, what is the profit at the end of January? (2)

QUESTION 5 **[31]**

A school counsellor conducted a survey among a group of high school students using the following survey slip.

Survey (please tick the correct boxes)			
Sex: Male ☐		Female ☐	
Age: 13 – 14 ☐		15 – 16 ☐	17 - 18 ☐
How much pressure do you feel to achieve at school?			
None: ☐	A little: ☐	A lot: ☐	An unbearable amount: ☐

5.1 Show, by completing the survey slip on the answer sheet, how Samantha, a 14-year old girl who feels a lot of pressure to achieve at school would complete the survey form. (3)

5.2 The counsellor has summarised data from all the completed survey forms in the table below. Use this summary to answer the questions that follow:

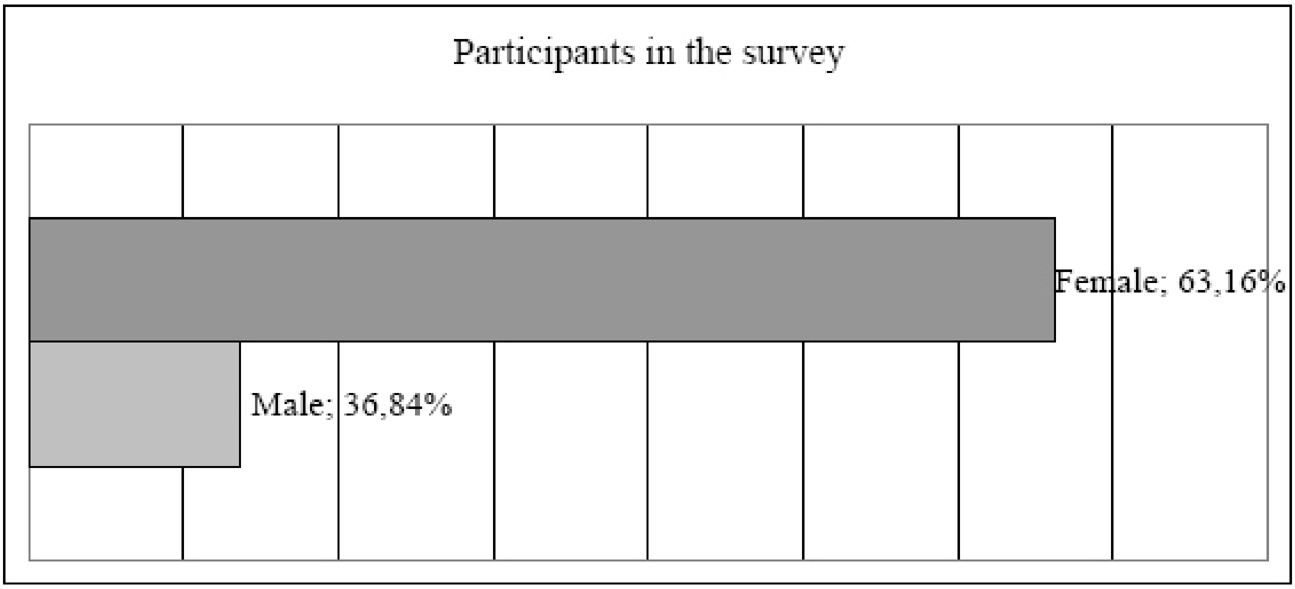
	Male			Female		
	13 – 14	15 – 16	17 – 18	13 – 14	15 – 16	17 – 18
None	4	1	-	5	4	4
A little	9	4	3	7	4	6
A lot	1	3	1	3	6	8
An unbearable amount	3	4	2	2	4	7

5.2.1 How many males and how many females participated in the survey? (2)

5.2.2 The counsellor wrote is his report *"more than two out of every five teenagers feel either a lot or an unbearable amount of pressure to achieve at school."* Show how the counsellor could have come to this conclusion. (4)

5.2.3 Do boys and girls experience this pressure equally or differently? Substantiate your answer using the information in the table. (5)

5.2.4 The counsellor illustrated his report with the following graph:



(a) What impression does the graph create about the number of male and female participants? (2)

(b) Is this impression correct? Substantiate your answer? (3)

(c) What has the counsellor done in developing the graph to create that impression? (2)

5.2.5 The counsellor has summarised the data in a different way in the table below:

	Male			Female		
	13 – 14	15 – 16	17 – 18	13 – 14	15 – 16	17 – 18
None A little	65%	42%	a	71%	44%	40%
A lot An unbearable amount	35%	58%	b	29%	56%	60%

(a) By referring to the earlier table show that the values of *a* and *b* are both 50%. (3)

(b) By comparing the responses for the females according to age, describe the trend in the data by rewriting the sentence, making the best choices from the words in brackets:

"(Older/younger) girls are more likely to experience more pressure than (older/younger) girls." (4)

(c) What graph would you choose to illustrate the observation described in (b)?
Why would this graph illustrate the point most effectively? (3)

Feedback to self-check exercises

Lesson 1

Part A

1. For 1 block: 3 buckets sand : 1 bucket cement : 2 buckets stone
   For 4 blocks: 4 × 3 buckets sand : 4 × 1 bucket cement : 4 × 2 buckets stone
   Thandi needs 12 buckets sand, 4 buckets cement, 8 buckets stone.

2. Sand : cement : stone ⇒ 12 : 4 : 8

3. Sand is 12 buckets out of 24 buckets: $= \frac{12}{24}$
   Cement is 4 buckets out of 24 buckets: $= \frac{4}{24}$
   Stone is 8 buckets out of 24 buckets: $= \frac{8}{24}$

4. Ratio sand : cement : stone = 12 : 4 : 8
   4 can divide into 12, 4 and 8.
   So sand : cement : stone = 12 ÷ 4 : 4 ÷ 4 : 8 ÷ 4
   = 3 : 1 : 2

5. Sand is 3 buckets out of 6 buckets: $= \frac{3}{6} = \frac{1}{2}$
   Cement is 1 bucket out of 6 buckets: $= \frac{1}{6}$
   Stone is 2 buckets out of 6 buckets: $= \frac{2}{6} = \frac{1}{3}$

6. From number 3:
   Sand $= \frac{12}{24} = \frac{1}{2}$ same as in number 5.
   Cement $= \frac{4}{24} = \frac{1}{6}$ same as in number 5.
   Stone $= \frac{8}{24} = \frac{1}{3}$ same as in number 5.

Part B

Oscar is working in Nelspruit in South Africa and decides to visit his family in Mozambique for a few days. The Mozambique currency is called Metical (MT) and the exchange rate at the time of Oscar's visit was: R1, 00 = MT 4,6

Oscar budgets the following for his trip:

Taxi from Nelspruit to Komatipoort border:	R30,00
Bus trip from Komatipoort border to Maputo:	MT 150,00
Food:	MT 300,00
Bus trip from Maputo back to Komati Poort:	MT 150,00
Taxi from Komatipoort border to Nelspruit:	R30,00

1. R33,00
2. R65,00
3. B
4. a) MT 876,00
   b) R190,00 or R191,00

Part C

1. It means that every 5 mm on the map represent 20 km in actual distance.
2. 1,5 cm = 15 mm = 5 × 3. Therefore the actual distance is 20 × 3 = 60 km.
3. *Route 2:* Port St. Johns - Umtata - East London - Queenstown
4. 300 km ÷ 20 = 15
   (Finding out how many 20's there are in 300 km shows us how many 5mm's there are)
   15 × 5 mm = 75 mm or 7,5 cm
5. 7 cm = 70 mm ; 70 mm ÷ 5 = 14
   Actual distance: 14 × 20 = 280 km
6. Route 2 is 20 km shorter.

Lesson 2

1. In direct proportion the one value changes in direct proportion to the other value. For example, as the x-values increase or decrease, the y-values also increase or decrease by a constant ratio. With inverse proportion, the one value decreases as the other value increases to keep a constant product. For example, as the x-value increases, the y-value decreases.

2. a) Direct proportion b) Inverse proportion

3. a) Direct proportion
   $a = 9$ and $b = 5$
   b) Inverse proportion
   $a = 28$ and $b = 14$

4. Average speed of 1 000 km/h for 12 hours gives a distance of 12 000 km. $s = \frac{d}{t} = \frac{12000}{10} = 1200\text{km/h}$

5. a) The temperature appears to be dependent on the time of day. So time is the independent variable and temperature is the dependent variable.
   b) (0; 1), (4; -1), (8; 3), (12; 14), (16; 12), (20; 4), (24; 2)
   c) It varies directly.

Lesson 3

1. a)
$$\begin{aligned} SI &= Prn \\ &= 89000 \times 0{,}16 \times 3 \\ &= 42720 \end{aligned}$$

   b)
$$A = P(1+r)^n$$
$$A = 89000(1+0{,}12)^{2{,}5}$$
$$A = 118150{,}36$$

   So compound interest = Total amount - principal loan
   = 118150,36 − 89000
   = 29150,36

c) 10% of 89 000 = 8 900

89 000 – 8 900 = 80 100

$A = P(1) + r^{n}$

$A = 80100(1 + 0,13)^{2}$

$A = 102279,69$

So compound interest = Total amount – principal loan

= 102279,69 – 80100

= 22179,69

2. a) $A = P + SI$

$A = 89000 + 42720$

$A = 131720$

b) $A = P(1 + r)^{n}$

$A = 89000(1 + 0,12)^{2,5}$

$A = 118150,36$

c) $A = P(1 + r)^{n}$

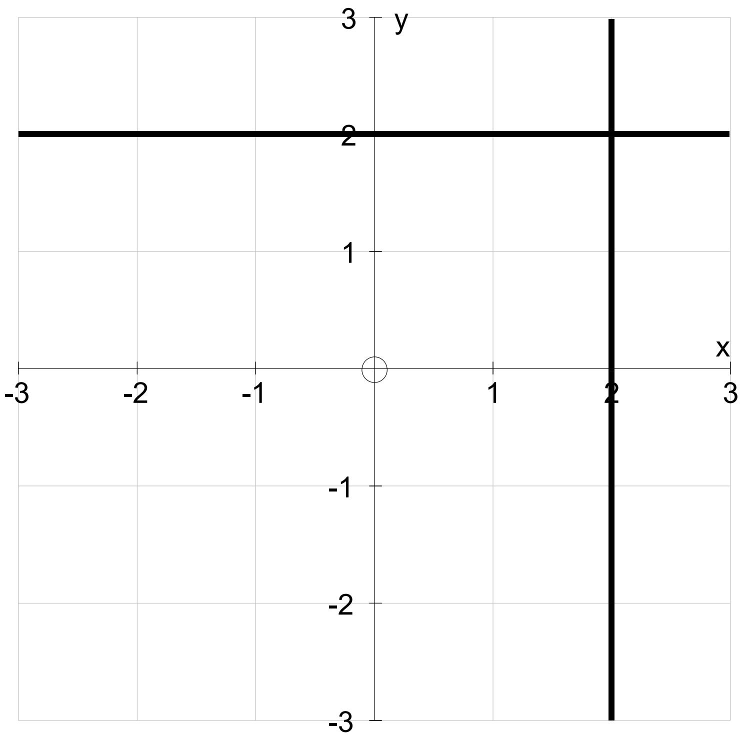
$A = 80100(1 + 0,13)^{2}$

$A = 102279,69$

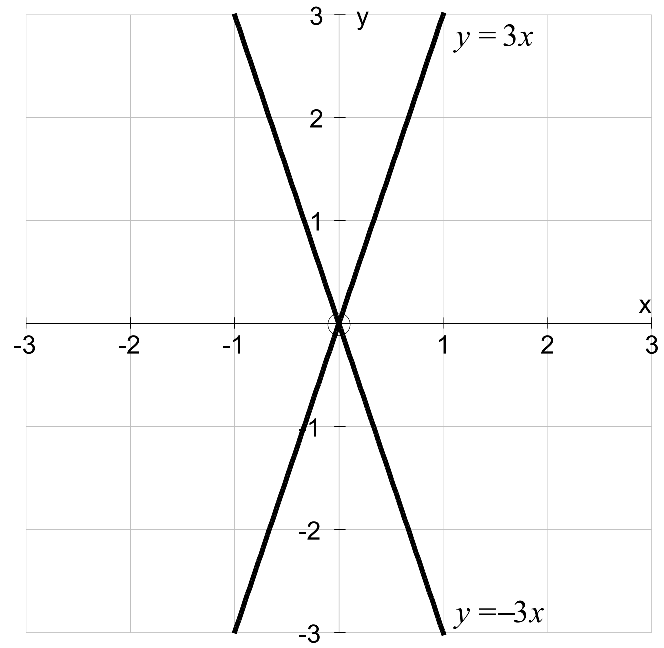
3. Option C works out to be the cheapest option in that you end up paying the least for the car. But it depends on whether Bayani has the 10% cash deposit available. Otherwise his next best option is b).

Lesson 4

1.



2.



a) Increasing graph.
b) Decreasing graph.

3. a) $x = -1$; $y = 2$
b) $x = 0$; $y = 0$
c) $x = 1$; $y = -2$

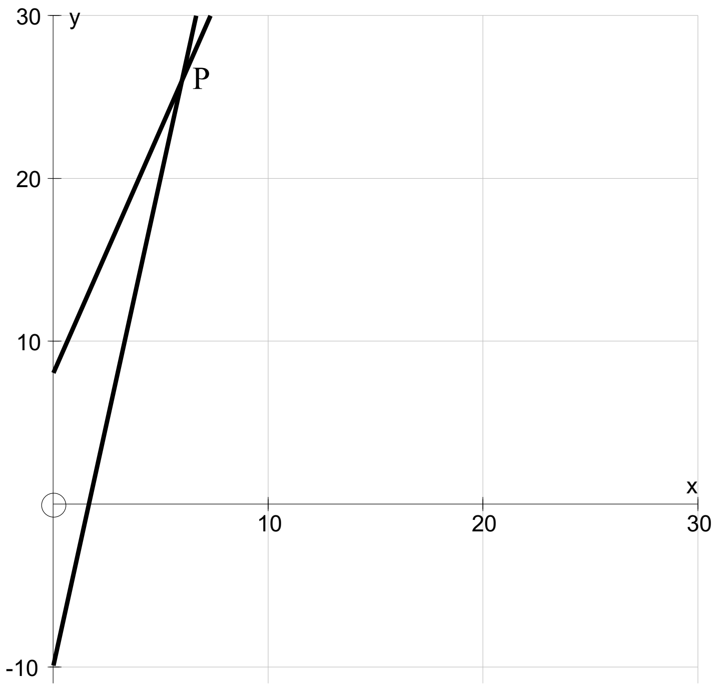
The graph is decreasing. As you increase the x-values from the left to the right side, the corresponding y-values decrease.

4. a) Let the son's present age equal x and the father's present age equal y.

The man was 6 times x (the son's age) 2 years ago.
So $y - 2$ (2 years before the father's present age) = $6(x - 2)$ (6 times the son's age two years ago)
$y - 2 = 6x - 12$
$\boldsymbol{y = 6x - 10}$
Also: $y + 4 = 3(x + 4)$
$\boldsymbol{y = 3x + 8}$ $(12 - 4 = 8)$

b) To plot points, we must choose values of x and find the corresponding values of y in the equations.

Try $x = 0$ and $x = 1$
For $y = 6x - 10$: $x = 0$ gives $y = 10$;
$x = 1$ gives $y = 6(1) - 10 = 4$
For $y = 3x + 8$: $x = 0$ gives $y = 8$;
$x = 1$ gives $y = 3(1) + 8 = 11$
So for $y = 6x - 10$ we have the co-ordinates (0;10) and (1;4).
For $y = 3x + 8$ we have the co-ordinates (0;8) and (1;11)



c) See point P.

d) When these two graphs are equal (at the point of intersection), the values for y and x are the same on both graphs.

So we can say $3x + 8 = 6x - 10$ (both are equal to the same y)
To solve this equation for x:
$3x + 8 + 10 - 3x = 6x - 10 + 10 - 3x$
$18 = 3x$
$6 = x$
$x = 6$
Substitute $x = 6$ in any equation to get the value of y.
$x = 6$ gives $6(6) - 10 = 26$ or $3(6) + 8 = 26$
So: $y = 26$.

Lesson 5

1. $x = 5$
2. $x = 5$
3. $x = 6$
4. $x = 8{,}49$
5. $x = 13$
6. $x = 12{,}21$
7. $x = 7$
8. Side of rhombus = 10
   Therefore, perimeter of rhombus = 40

9. The base of the rectangular house is a rectangle. The square of the diagonal should be equal to the sum of the squares of the other two sides, that is, the square of the length plus the square of the width.
   $16^2 + 12^2 = 256 + 144 = 400$

   The supervisor expects the diagonal to be 20 metres long.

The use of the diagonal is very important here, because if the diagonal is exactly 20 metres long, then the supervisor knows that the base is a rectangle. The internal angles are all right angles.

Lesson 6

1. a) Kaiser Chiefs: $\frac{260°}{360°} = \frac{26}{36} = \frac{13}{18}$

   Swallows: $\frac{58°}{360°} = \frac{29}{180}$

   Roland Pirates: $\frac{32°}{360°} = \frac{16}{180} = \frac{4}{45}$

   Sundown's: $\frac{10°}{360°} = \frac{1}{36}$

   b) $\frac{13}{18} \times 100 = 72{,}2\%$
   $\frac{29}{180} \times 100 = 16{,}1\%$
   $\frac{4}{45} \times 100 = 8{,}88\%$
   $\frac{1}{36} \times 100 = 2{,}77\%$

   c) $\frac{72}{100} \times 200 = 144$
   $\frac{16}{100} \times 200 = 32$
   $\frac{9}{100} \times 200 = 18$
   $\frac{3}{100} \times 200 = 6$

   Did you notice that the number of points scored was double the percentage? This is because the number of points is out of 200 and the percentage is out of 100.

2. a) Morocco
   b) Ethiopia
   c) Morocco; Botswana; Kenya; South Africa; Lesotho; Ethiopia
   d) South Africa is ranked fourth in these six countries mentioned.

3. a) Approximately 3 million more people
   b) Answers will differ here but the reasoning is that if in the seven years from 2001 to 2008, there were 3 more million people living with HIV, that by 2015 (another 7 years), there will be at least 27 million or more, unless major interventions change the spread of HIV.
   c) The number of new infections decreased slightly. This could be due to the higher awareness campaigns and efforts by the government and non-governmental agencies to alert people to the risks.
   d) The number of AIDS related deaths appear to have stayed the same during the 7 years. Reasons will differ here but there is a good chance that this number will not increase or decrease rapidly if the number of new infections is not escalating and more and more people with HIV are receiving proper treatment.

Feedback from Activities

Lesson 1

Activity 1

1. There were 2 cups of flour out of the total of 4 cups, so the fraction of flour is $\frac{2}{4}$. If we simplify this fraction we get $\frac{1}{2}$. Do you remember how to simplify fractions? Look back to Unit 1, lesson 3 if you don't remember.

   Look again at Seca's drawing. Do you agree that half of the ingredients consist of flour? Yes, there are 2 cups of flour and 2 cups of other ingredients.

2. Milk is $\frac{1}{4}$ of the ingredients (1 of the 4 cups or parts).

3. There are 2 cups of flour for every cup of milk so the amount of flour is 2 times the amount of milk.

Activity 2

1. 3 buckets of sand: 1 bucket of cement: 2 buckets of stones
   Ratio = 3:1:2
   The ratio is 3 to 1 to 2.
2. Sand was 3 out of 6 buckets = $\frac{3}{6}$ = $\frac{1}{2}$. Did you manage to simplify the fraction? If you didn't, look at the box in the margin.
   Cement was 1 out of 6 = $\frac{1}{6}$. This fraction can't be simplified further.
   Stones were 2 out of 6 = $\frac{2}{6}$ = $\frac{1}{3}$.

When we simplify fractions we divide the top (numerator) and bottom (denominator) numbers by the same number to make them smaller. We simplify $\frac{3}{6}$ by dividing the numerator and denominator by 3 to get $\frac{1}{2}$. Ratios can also be simplified by dividing all of the numbers by the smallest number in the ratio itself.

Activity 3

3. The matches are divided in the ratio 4:16. 4 and 16 divided by 4 gives the simplified ratio of 1:4.

The larger group has four times as many matches as the smaller group.

Let's check: 4×4 (group 1) = 16 (group 2).

The total number of matches is 20. 4 are in group 1 so the fraction in group 1 is $\frac{4}{20}$ and the fraction in group 2 is $\frac{16}{20}$. If we simplify these fractions we get $\frac{1}{5}$ for group 1 and $\frac{4}{5}$ for group 2. What did we divide by? $\frac{4}{4}$ of course. You can see that we simplified the fractions in the same way that we simplified the ratio, by dividing by the smallest number, 4.

Activity 4

We must first convert all the fractions to have the same denominator. We chose the denominator 12. Can you remember why?

$\frac{1}{4} \times \frac{3}{3}$ (to get to 12) = $\frac{3}{12}$ and $\frac{2}{6} \times \frac{2}{2} = \frac{4}{12}$

So the ratio is $\frac{3}{12} : \frac{1}{4} : \frac{2}{6}$ = 3:3:4 (in simplified form).

Activity 5

The amounts of ingredients Thabo combined made a cake for 4 people. 8 people is twice as many as 4 people so to make a cake for 8 people Thabo would need twice the amount of ingredients.

So that is:
4 cups flour
2 cups milk
2 cups sugar

Activity 6

1. Did you get 4:2:2 for the ratio?
   You can see that the total number of cups Thabo had to add to make a cake for 8 people was 8 (4 + 2 + 2 = 8). So he needed twice the number of cups for 8 people than he needed for 4 people.

2. Did you get 2:1:1? Have you noticed that the simplified ratio is the same as the ratio of ingredients he needed for 4 people? Remember we said that the simplified ratio does not give us the actual amount of each ingredient added. This ratio is only used to compare the amounts of the different ingredients added by seeing how many times more/less of one ingredient we have than another.

   Look again at the ratio 4:2:2. We had twice as much flour as milk and sugar:
   Twice (2 ×) 2 = 4

   When we look at the simplified ratio 2:1:1 the ratio still tells us that we had twice as much flour as milk or sugar.
   Twice (2 ×) 1 = 2.

3. The total number of cups is 8 so our fractions are $\frac{4}{8}$; $\frac{2}{8}$; $\frac{2}{8}$.

4. Simplified, these fractions are $\frac{2}{4}$; $\frac{1}{4}$; $\frac{1}{4}$.
   The ratio 4:2:2 simplified is 2:1:1

Activity 7

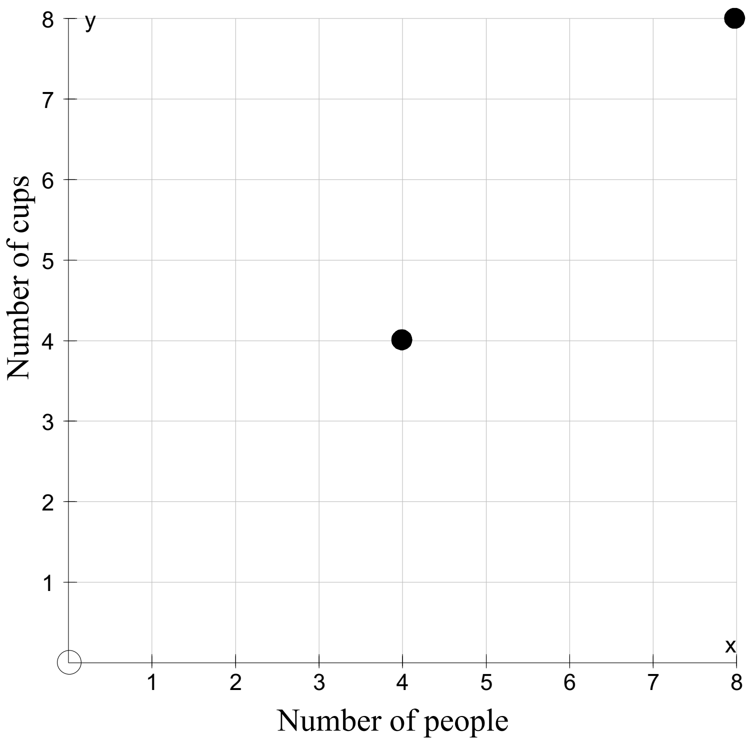
1. a) 6 cups flour; 3 cups milk; 3 cups sugar
   b) 8 cups flour; 4 cups milk; 4 cups sugar
   c) 10 cups flour; 5 cups milk; 5 cups sugar

2. a) 6: 3: 3 = 2: 1: 1
   b) 8: 4: 4 = 2: 1: 1
   c) 10: 5: 5 = 2: 1: 1

3. a) 4 cups
   b) 8 cups
   c) 12 cups
   d) 16 cups
   e) 20 cups

Activity 8

Did your graph look like this?



Activity 9

1. a) 20 kg = 2 × 10 kg
   So the patient needs 2 re-hydrate mixtures per day.

   b) 50 kg = 5 × 10 kg
   So the patient needs 5 re-hydrate mixtures per day.

2. a) 20 % of 40 is equal to $\frac{20}{100}\times 40 = 8$ kg of food per day.

   b) 20 % of 60 is equal to $\frac{20}{100}\times 60 = 12$ kg of food per day.

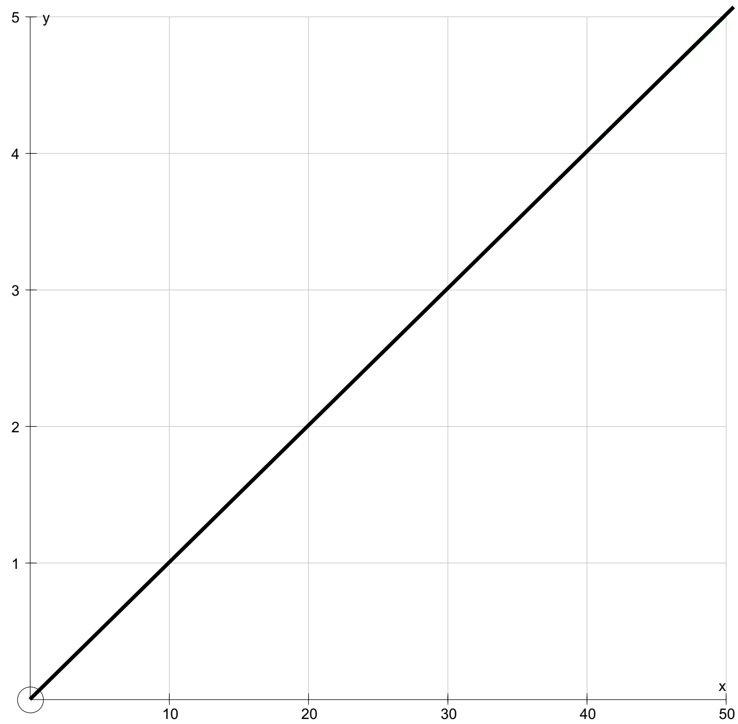
   c) 20 % of 100 is equal to $\frac{20}{100}\times 100 = 20$ kg of food per day.

Activity 10

1. a)

Weight of patient	Number of re-hydrate mixtures
10 kg	1
20 kg	2
30 kg	3
40 kg	4

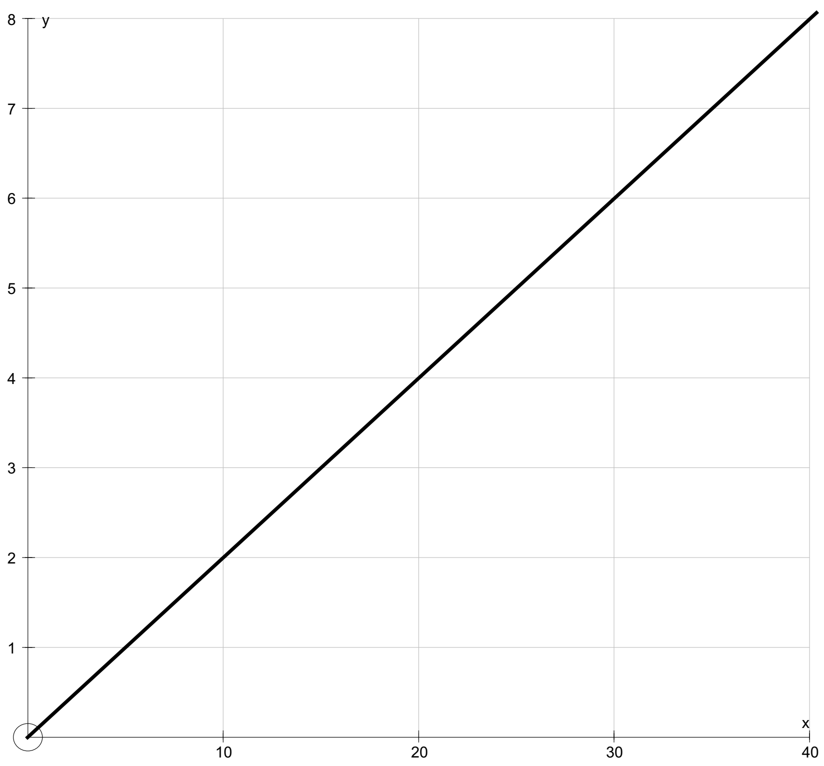
b)



2. a)

Weight of baby buffalo	Kilograms of food per day
10 kg	2 kg
20 kg	4 kg
30 kg	6 kg
40 kg	8 kg

b)



Activity 11

ZK655,37 buys R1, so the rate is written as 655,37:1

It would be more helpful to know how many Rands Barbara gets for one Kwacha. So we change the ratio into unit form to get the first ratio amount to a unit of 1:

In order to do that we divide both sides by 655,37:

So for 1 ZK: $\frac{655,37}{655,37} : \frac{1}{655,37} = 1:0,0015...$

Each 1ZK buys R0,0015... (keep this amount on your calculator)

So for 1 000 000 ZK: $1 \times 1\,000\,000 : 0,0015... \times 1\,000\,000$
$= 1\,000\,000 : 1\,525,855$

Barbara would be able to buy R1 526,00 with her ZK1 000 000. Exchange rates are also an example of direct proportion. As the number of Rands increased or decreased, the number of Zambian Kwachas increased or decreased in direct proportion to the Rands. For example, if R1 = ZK700, then R2 = ZK1400. The ratios are 1:700 and 2:1 400. The simplified ratio therefore remains 1:700.

Activity 12

a) As the bus had to stick to the main road, the route that Chimy's tour would have followed is the route indicated by the thick line.

b) Map distance: 8 cm
Scale 1 cm: 5 km
Travelling distance: 1 cm × 8 : 5 km × 8
8 cm : 40 km

c) Approximately 11 - 15 cm. Whatever the answer was that you measured in cm, this value needs to be multiplied by 5 in order to convert the distance to kilometres. So for example, if you measured 12 cm, the actual distance will be 12 × 5 = 60 km.

d) 1 cm : 5 km
1 cm × 38 : 5 km × 38
38 cm : 190 km
So the approximate total distance the bus will travel during its three-day tour of the park is 190 km.

Activity 13

1. 1 cm : 5 km
1 cm × 2,5 : 5 km × 2,5
2,5 cm : 12,5 km

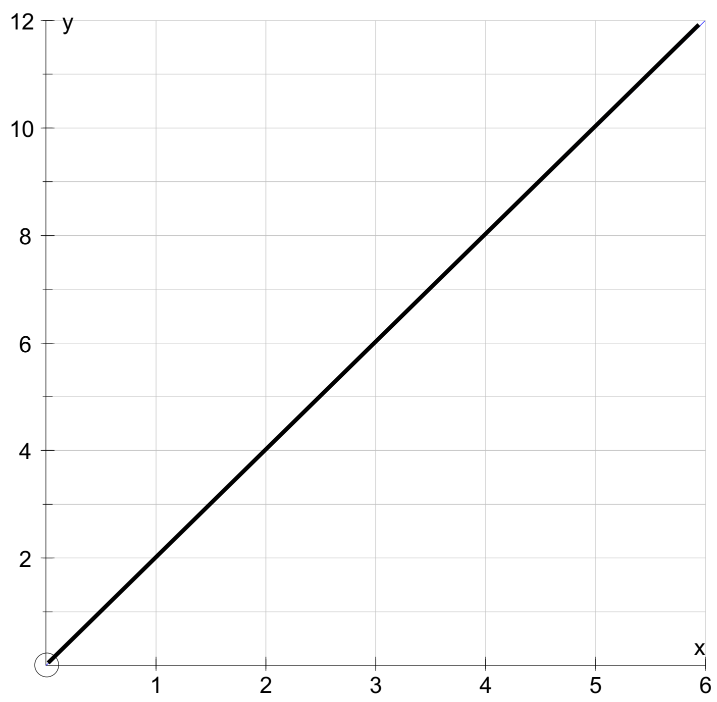
2. 5 km : 1 cm
5 × ? = 50
5 × 10 = 50
Therefore measurement on map is 10 cm.

Lesson 2

Activity 1

(1 ; 2), (2 ; 4), (3 ; 6), (5 ; 10)

Activity 2



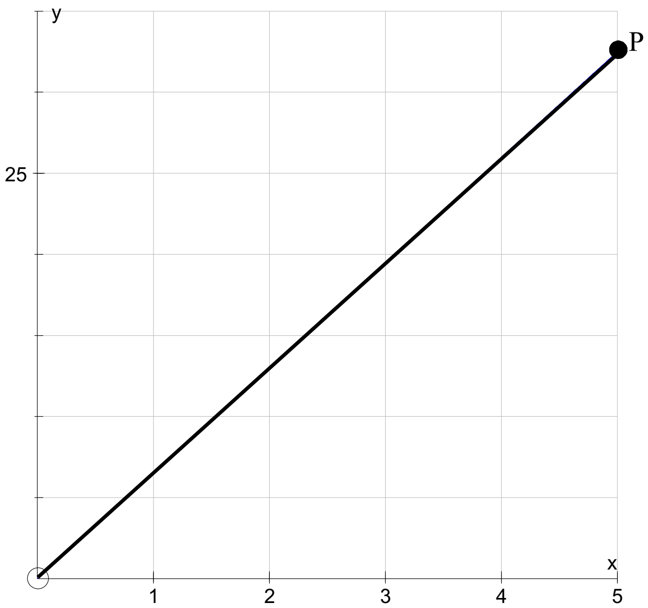
Activity 3

1.

US Dollars ($)	1	2	3	4
Rands (R)	6,5	13	19,5	***26***

2. Yes. Constant ratio is 6,5

3.



4. See point P on the graph above.

Activity 4

1. $x = 5$; $y = 2$; $z = 1$
2. $x = 4$; $y = 3$; $z = 1$
3. $x = 30$; $y = 1500$; $z = 1$

Activity 5

1. Direct proportion with constant ratio of 1 : 3
2 Inverse proportion with constant product of 60
3. Direct proportion with constant ratio of 1 : 10
4. Inverst proportion with constant product of 400

Activity 6

1. 8 × 1 400 = 11 200 kg. This is the mass of weight the ferry can carry. So for cars of 1 120kg each, we calculate 11 200 (total mass of cars ferry can carry) divided by 1 120 (the mass of each car). This is equal to 10. So the ferry can carry 10 cars of mass 1 120 kg each. We used the constant product of 11 200 kg so this is an example of inverse proportion.

2. 7 litres : 280 km

An increase in the number of litres should result in an increase in the number of km you can travel, so this is an example of direct proportion.

Let us find the constant ratio:
7 : 280 = 1*l* : 40 km

So your motorbike travels 40 km on 1 litre of petrol.
Therefore for 20 litres, we say:
1 litre : 40 km = 1 × 20 : 40 × 20 = 20 : 800
Your motorbike can travel approximately 800 km on a full tank of petrol.

3. 5 men lay 2 000 bricks in a day.

More men should therefore result in more bricks being laid.
This is direct proportion so we need to look for the constant ratio.
5 men : 2 000 bricks = 1 man : 400 bricks
1 man : 400 bricks = 1 × 9 : 400 × 9 = 9 : 3 600

So 9 men can lay 3 600 bricks in a day.

Activity 7

1. $\text{Average speed} = \dfrac{\text{Distance travelled}}{\text{Time taken}}$

$\text{Average speed} \times \text{Time taken} = \text{Distance travelled}$

$\text{Distance travelled} = \text{Average speed} \times \text{Time taken}$

2. $\text{Average speed} = \dfrac{\text{Distance travelled}}{\text{Time taken}}$

$\text{Time taken} \times \text{Average speed} = \text{Distance travelled}$

$\text{Time taken} = \dfrac{\text{Distance travelled}}{\text{Average speed}}$

Activity 8

1. $$\begin{aligned}\text{Time taken} &= \frac{\text{Distance travelled}}{\text{Average speed}}\\ &= \frac{600}{120}\\ &= 5 \text{ hours}\end{aligned}$$

2. $$\begin{aligned}\text{Average speed} &= \frac{\text{Distance travelled}}{\text{Time taken}}\\ &= \frac{80}{2}\\ &= 40 \text{ km/h}\end{aligned}$$

3. Distance travelled = Average speed × Time taken

$= 5 \times 1{,}5$

$= 7{,}5$ km

Activity 9

1. $$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

So I need to know the total distance travelled and the total time of the journey to calculate the average speed of the journey.

Time taken for first stage of journey: $\frac{160}{40} = 4$ hours

Time taken for second stage of journey: $\frac{120}{60} = 2$ hours

Total time: 6 hours
Total distance: 160 km + 120 km = 280 km

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$
$$= \frac{280}{6}$$
$$= 46{,}\vec{6}$$
$$\approx 47\text{km/h}$$

This sign, ≈ means "approximately equal to". We use this to show that we have rounded off. Here it is appropriate to round off to the nearest whole number.

2. Time taken for outbound journey: $\frac{200}{50} = 4$ hours

Time taken for return journey: $\frac{200}{80} = 2{,}5$ hours

Total time: 6,5 hours
Total distance: 200 km + 200 m = 400 km

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$
$$= \frac{400}{6{,}5}$$
$$= 61{,}538$$
$$\approx 62\text{km/h}$$

Activity 10

1. Total distance travelled: 182 km
Distance travelled in first 3 hours: Average speed for those 3 hours × time (3 hrs)

$= 30 \times 3$

$= 90$ km

Distance for remainder of journey: $182 - 90\text{ km} = 92\text{ km}$
Average speed for remainder of journey: 23 km/h

Time for remainder of journey: $\frac{92}{23} = 4$ hours

Total time: 3 + 4 = 7 hours

Average speed: $\frac{\text{Total distance travelled}}{\text{Total time taken}}$

$= \frac{182}{7}$

$= 26$ km/h

2. Time for walking 3 km at 6 km/h: $\frac{3}{6} = 0{,}5$ hours

Time for cycling 6 km at 12 km/h: $\frac{6}{12} = 0{,}5$ hours

Total time: 0, 5 + 0,5 = 1 hour
Total distance: 3 + 6 = 9 km

Average speed: $\frac{\text{Total distance travelled}}{\text{Total time taken}}$

$= \frac{9}{1}$

$= 9$ km/h

3. Distance of section of road: Average speed $\times$ Time taken

$= 80 \times \frac{25}{60}$

$= 33{,}333...$

≈ 33 km

New time on road: 25 + 10 = 35 minutes

Average speed: $\frac{\text{Total distance travelled}}{\text{Total time taken}}$

$= \frac{33}{\frac{35}{60}} = 33 \times \frac{60}{35}$

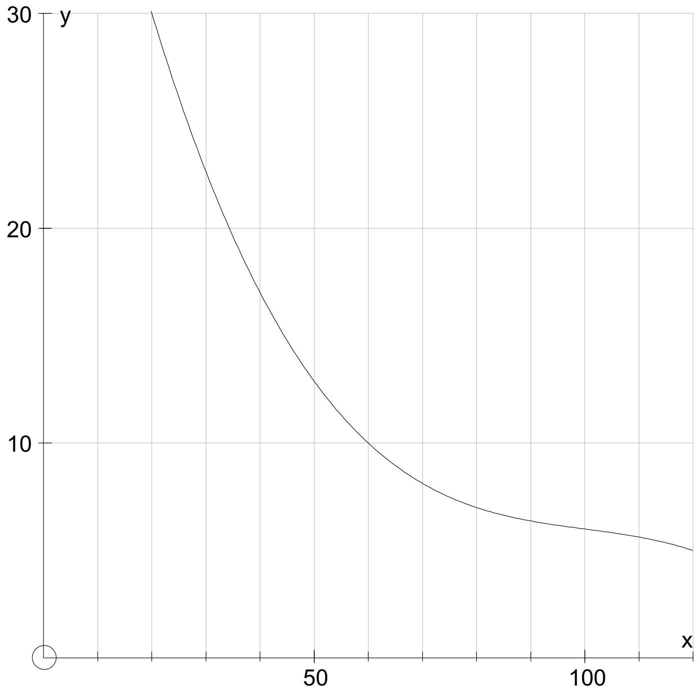
$= 56{,}571...$

≈ 57 km/h

The new speed limit is probably 60 km/h.

Activity 11

You are able to join the points as the average speed is not restricted to whole numbers.



Activity 12

1. a) (11; 1,2), (12; 1,28), (13; 1,35), (14; 1,42), (15; 1,53)
   b) (11; 42), (12; 43), (13; 45), (14; 50), (15; 53)
   c) Age is the independent variable and height is the dependent variable.
   d) His height varies directly according to his age (although not in proportion).

2. The length of the side of the square is the independent variable and the area is the dependent variable.

Lesson 3

Activity 1

1. $SI = Prn$

$$= 50000 \times 12\% \times 5$$
$$= 50000 \times 0{,}12 \times 5$$
$$= 30000$$

2. $SI = Prn$

$$= 75000 \times 14\% \times 4$$
$$= 75000 \times 0{,}14 \times 4$$
$$= 42000$$

$$75000 + 42000 = 117000$$

Activity 2

1a) $SI = Prn$

$= 5000 \times 0,2 \times 2$

$= 2000$

1b) Total amount = Simple interest + principal loan

$= 2000 + 5000$

$= 7000$

1c) Term of loan: 2 years = 24 months

$7000 \div 24 =$ R291,67 per month

3 years = 36 months

2. $SI = Prn$

$= 12000 \times 15\% \times 3$

$= 12000 \times 0,15 \times 3$

$= 5400$

$12000 + 5400 = 17400$

$17400 \div 36 =$ R483,33 per month

Activity 3

1a) 10% of R4 800 = $\frac{10}{100} \times 4800 = 480$

1b) R4 800 – R480 = R4 320

1c) $SI = Prn$

$= 4320 \times 0,21 \times 2$

$= 1814,40$

1d) Total amount= Simple interest + principal loan

$= 1814,40 + 4320$

$= 6134,40$

1e) Term of loan: 2 years = 24 months

$6134,40 \div 24 =$ R255,60 per month

1f) R6134,40 – R4 800 = R1 334,40

2a) 20% of R2 650 = R530

R200 per month for 18 months = R3 600

R3 600 + R530 = R4 130

2b)

Cash price:	R2 650
Hire-purchase price:	R4 130
Difference:	R4 130 – R2 650 = R1 480

Activity 4

1a) R8 000 × 36 = R288 000

1b) Simple interest = Total loan amount − principal loan amount

Simple interest = R288 000 − R230 000 = R58 000

1c)
$$SI = Prn$$
$$58000 = 230000 \times r \times 3$$
$$58000 = 690000r$$
$$\frac{58000}{690000} = r$$
$$r = 0{,}08$$
$$r = 8\%$$

2a) Cost of Mushi lounge suite on hire-purchase is:

R526,68 × 36 = R18 960,48

Simple interest = R18 960,48 − R9999,00 = R8961,48

$$SI = Prn$$
$$8961{,}48 = 9999 \times r \times 3$$
$$8961{,}48 = 29997r$$
$$\frac{8961{,}48}{29997} = r$$
$$r = 0{,}298...$$
$$r = 0{,}3$$
$$r = 30\%$$

2b) Cost of Bwona lounge suite on hire-purchase is:

R620,55 × 24 = R14 893,20

Simple interest = R14 893,20 − R12 100,00 = R2 793,20

$$SI = Prn$$
$$2793{,}20 = 12100 \times r \times 2$$
$$2973{,}20 = 24200r$$
$$\frac{2973{,}20}{24200} = r$$
$$r = 0{,}122...$$
$$r = 0{,}12$$
$$r = 12\%$$

2c) If one buys with cash the Mushi lounge suite is cheaper. But the Bwona lounge suite is the better deal on hire-purchase as it works out cheaper when you pay it off on credit (R14 893,20) compared to the Mushi suite (R18 960,48). This is because the interest rate charged on the hire-purchase of the Bwona suite is less and the term of the loan is also shorter.

Activity 5

1.

	Using simple Interest	Using compound Interest
End of 1st year	$20000+0{,}12\times 20000=22400$	$20000+0{,}12\times 20000=22400$
End of 2nd year	$22400+0{,}12\times 20000=24800$	$22400+0{,}12\times 22400=25088$
End of 3rd year	$24800+0{,}12\times 20000=27200$	$25088+0{,}12\times 25088=28098{,}56$
End of 4th year	$27200+0{,}12\times 20000=29600$	$28098{,}56+0{,}12\times 28098{,}56=31470{,}39$
End of 5th year	$29600+0{,}12\times 20000=32000$	$31470{,}39+0{,}12\times 31470{,}39=35246{,}83$

2. Total investment amount using simple interest: R32 000
   Total investment amount using compound interest: R35 246,83
   Difference: R35 246,83 – R32 000 = R3 246,83

3. The compound interest option.

Activity 6

1. $A=P+P\times r\times n$

   $A=5400+5400\times 0{,}21\times 1{,}5$

   $A=\text{R}7101$

2. $A=P+P\times r\times n$

   $A=50000+50000\times 0{,}15\times 5$

   $A=\text{R}87500$

Activity 7

1. $A=P(1+r)^n$

   $A=5400(1+0{,}21)^{1{,}5}$

   $A=\text{R}7187{,}40$

2. $A=P(1+r)^n$

   $A=50000(1+0{,}15)^5$

   $A=\text{R}100567{,}86$

Activity 8

1. a) $A=P(1+r)^n$

   $A=9000(1+0{,}14)^4$

   $A=\text{R}15200{,}64$

   b) $A=P+P\times r\times n$

   $A=9000+9000\times 0{,}18\times 4$

   $A=\text{R}15480$

Option b would provide the better option.

2. a) $A = P + P \times r \times n$

$A = 180000 + 180000 \times 0,15 \times 4$

$A = \text{R}288000$

b) $A = P(1+r)^n$

$A = 180000(1+0,12)^4$

$A = \text{R}283233,48$

Option b is the better option.

3. $A = P(1+r)^n$

$A = 550000(1+0,11)^{20}$

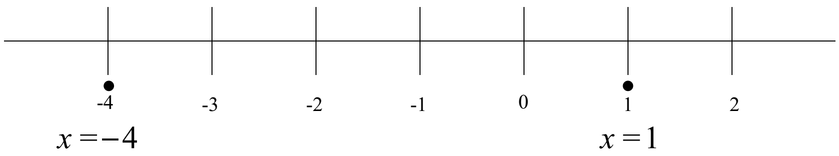
$A = \text{R}4434271.35$

Compound interest: R4 434 271,35 – R550 000 = R3 884 271,35

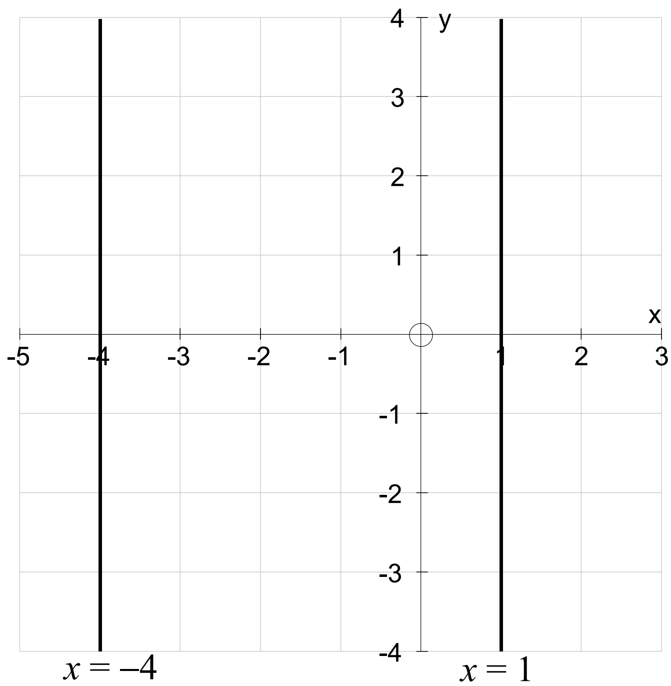
Lesson 4

Activity 1

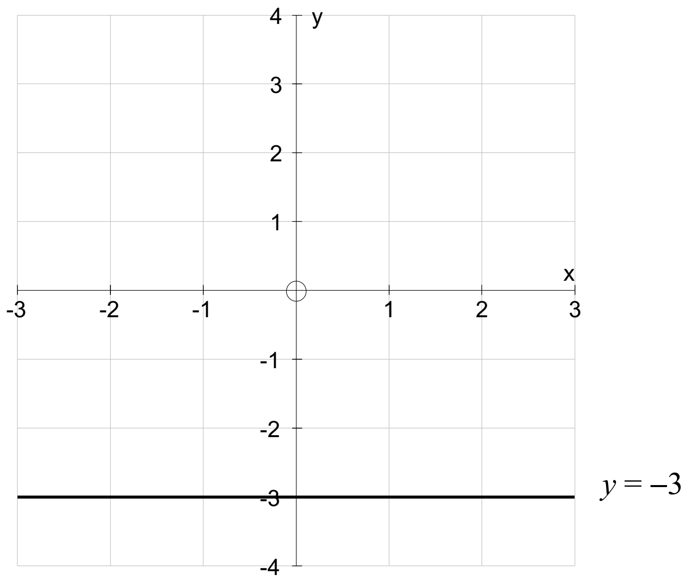
a)



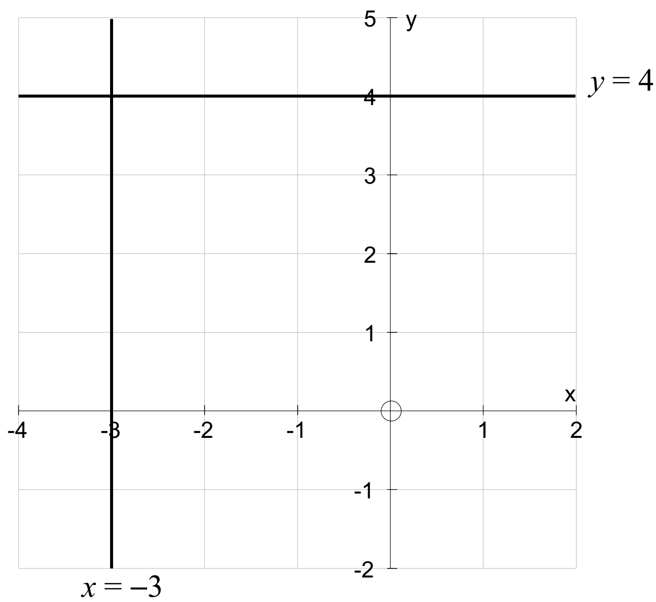
b)



Activity 2



Activity 3



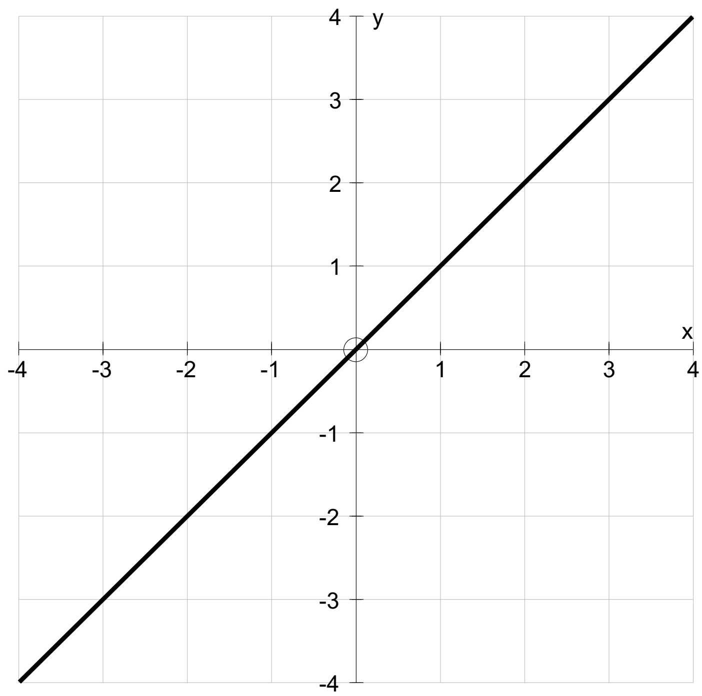
Activity 4

Write out the equations of the following graphs:

1. $x = -1$

2. $y = 1$

Activity 5

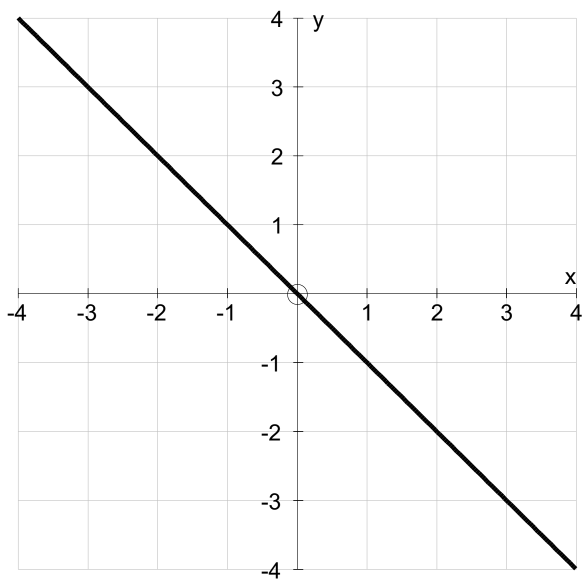


Activity 6

If you chose –2 as your first *x* value you saw that your *y* co-ordinate was also –2. If you then moved across the *x*-axis from left to right (increasing *x* values) and chose a larger number for *x*, for example the value of 2, then you would notice that the corresponding *y* value was also 2. So *y* increased from –2 up to 2 as *x* increased from –2 to 2. So the *y* value increased as the *x* value increased. We call this type of graph an *increasing graph* or function.

Activity 7

(2; –2); (–2; 2); (3; –3)



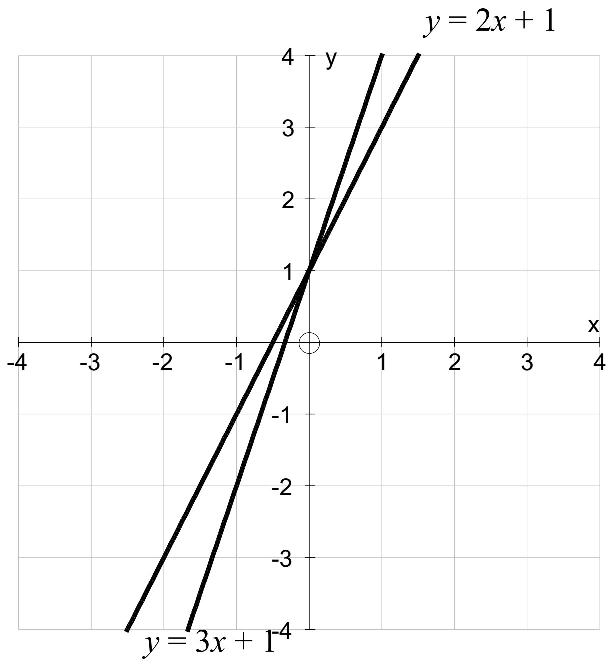
Activity 8

When x is –3, $y = 3$
When x is 0, $y = 0$
When x is 2, $y = –2$

The y value is decreasing (getting smaller) as the x value is increasing (getting bigger).

(Do you remember that: $–1 > –2 > –3$?)

Activity 10



Point of intersection is: (0 ; 1)

Activity 12

1. When we substitute (8 ; 38) into equation 1: $y = 6x – 10$
   Does 38 = 6(8) – 10? Yes, 38 = 48 – 10

   When we substitute into equation 2: $y = 4x + 6$
   Does 38 = 4(8) + 6? Yes, 38 = 32 + 6

2. When we substitute (2 ; 6) into equation 1: $y = 6x – 10$
   Does 6 = 6(2) – 10? No, 6 12 – 10

   When we substitute into equation 2: $y = 4x + 6$
   Does 6 = 4(2) + 6? No, 6 8 + 6

Lesson 5

Activity 3

a) $\hat{F}$

b) EG or f

c) $f^2 = e^2 + g^2$ or $EG^2 = FG^2 + EF^2$

d)
$$f^2 = e^2 + g^2$$
$$f^2 = 8^2 + 6^2$$
$$f^2 = 64 + 36$$
$$f^2 = 100$$
$$f = 10$$

Activity 4

The first thing to do is to write down the condition that the sides of any triangle must satisfy for it to be a right-angled triangle. We know that a right-angled triangle will satisfy the equation from the theorem of Pythagoras.

For ΔABC: $a^2 + c^2 = b^2$

LHS (left-hand side) = $7^2 + 10^2$
=49 + 100
= 149

RHS (right-hand side) = 12^2
=144

But, $149 \neq 144$
So LHS $\neq$ RHS
$\therefore$ ΔABC isn't a right-angled triangle.

For ΔDEF: $d^2 + f^2 = e^2$

LHS = $9^2 + 12^2$
=225

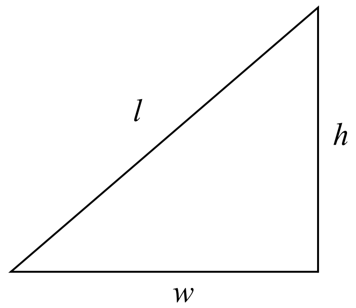
RHS = 15^2
= 225

LHS = RHS
$\therefore$ ΔDEF is a right-angled triangle.

Activity 5

1.

Let the height of the wall of the house be h and the width of the garden be w. Let the length of the ladder be l. From the theorem of Pythagoras, we know that $l^2 = h^2 + w^2$



Taking the square root on both sides of the equation, we get:

$l = \sqrt{h^2 + w^2}$

Therefore, the length of the ladder should be

$l = \sqrt{h^2 + w^2}$

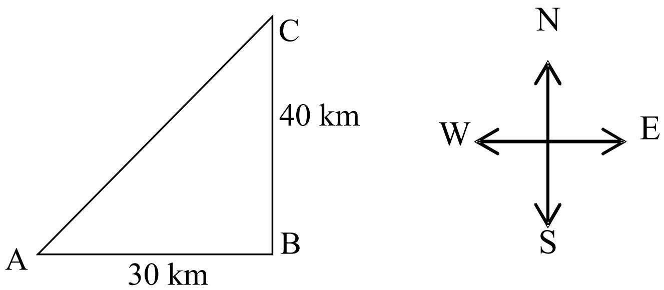
$l = \sqrt{2^2 + 1{,}5^2}$

$l = \sqrt{6{,}25}$

$l = 2{,}5m$

Activity 6

a)



b) AC = 50 km

c) AC = 50 km but A to B to C = 30 + 40 = 70 km. So AC is 20 km shorter.

Lesson 6

Activity 1

The proportional amounts are given as 'pieces' of the pie chart with these percentages:
14% : 33% : 25% : 28%

To simplify, we divide all of them by the lowest percentage (14%). Do this on paper or with your calculator.
Simplified ratio = 1 : 2,36 : 1,79, : 2

Activity 3

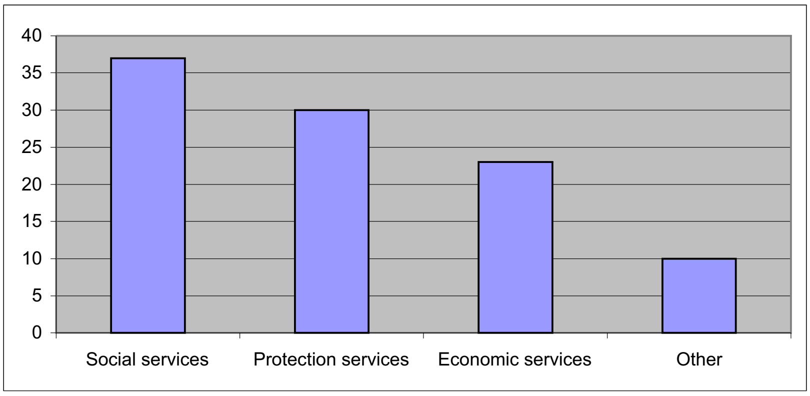
1. Economic services: 28% – 23% = 5% increase
   'Other': 14% – 10% = 4% increase

2. Protection services: 30% – 25% = 5% decrease
Social services: 37% – 33% = 4% decrease

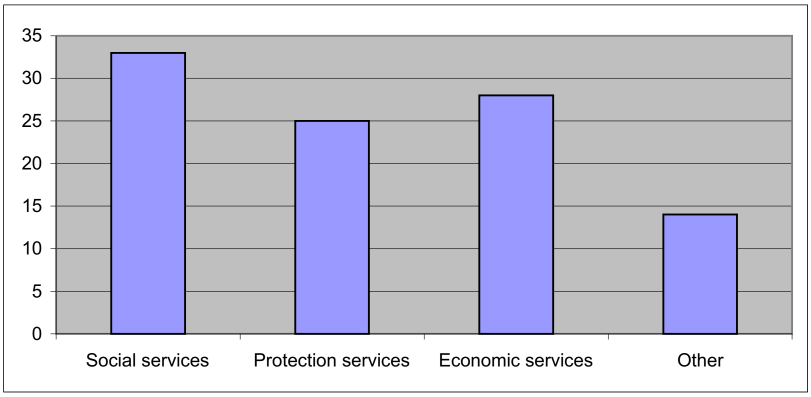
3. These changes appear to be reasonable although it would also be favourable to see social services not decreasing too much.

Activity 4

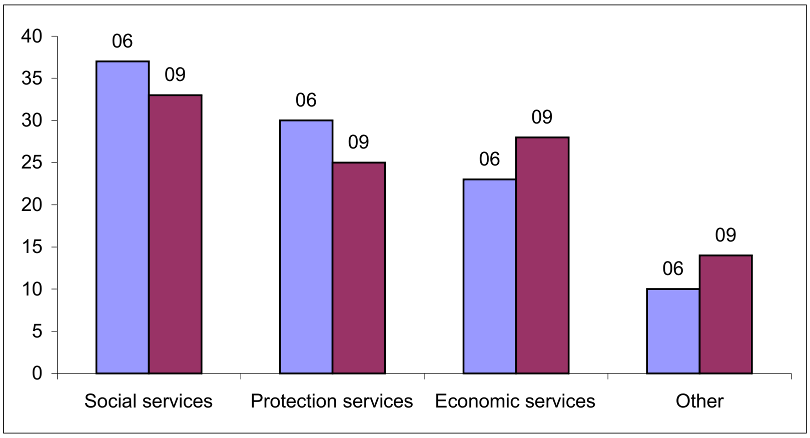
Your graph should look like this:
Use a ruler to make sure that the top of each bar is in line with the correct percentage on the *y*-axis.



Activity 5



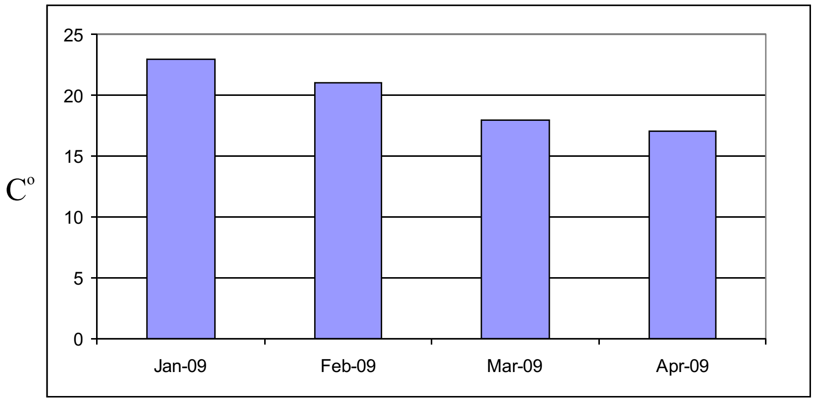
Activity 6



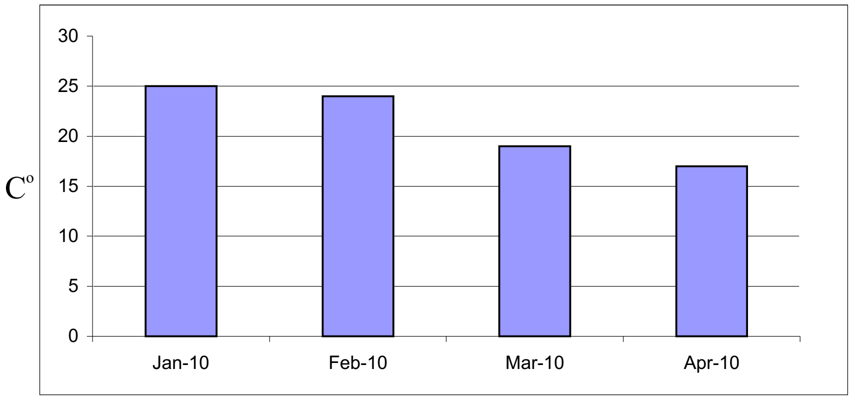
Note that the graph shows quite clearly the increase and decrease of spending on each thing over the 3 years.

Activity 7

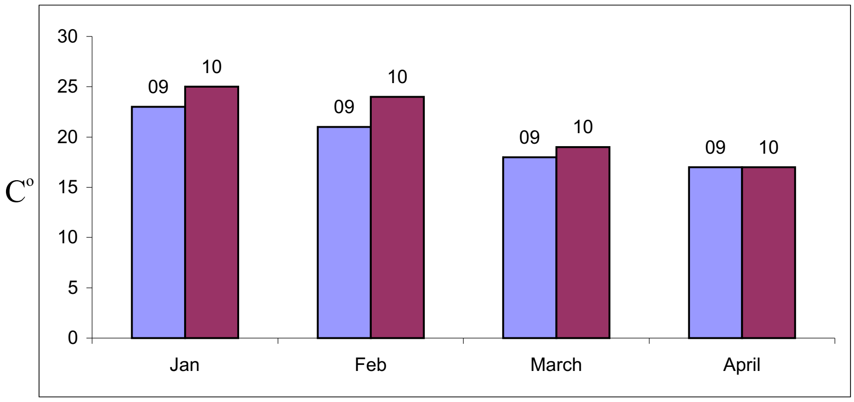
1. Did your graph look like this?



2. a) Did your graph look like this?



b) Did your graph look like this?



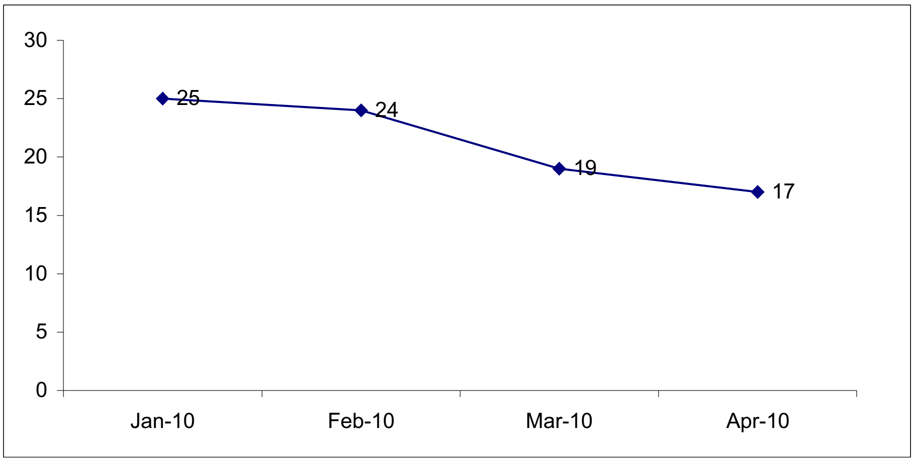
3. Change in January, 2009 to 2010:
   25°C – 23°C = 2°C higher in 2010.
   Change in February: 24°C – 21°C = 3°C higher in 2010.
   Change in March: 19°C – 18°C = 1°C higher in 2010.
   Change in April: 17°C – 17°C = no change.

   Did you see from your graph that the temperature in February changed the most, and the temperature in April changed the least?

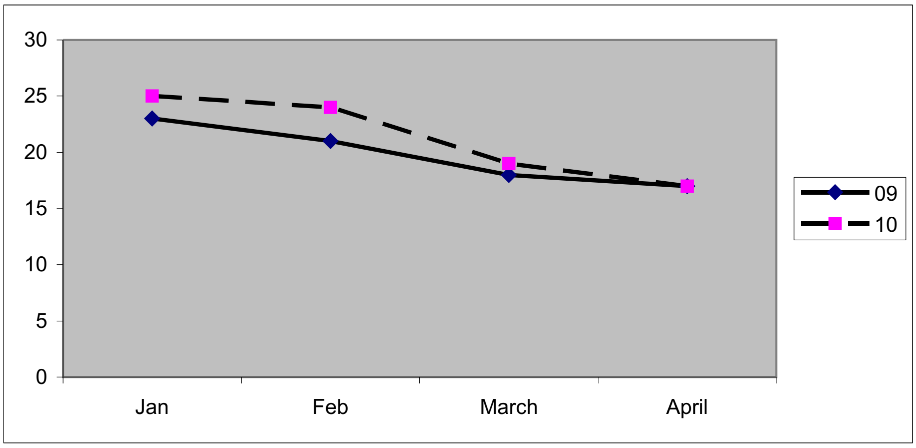
4. Did you see from your graph that there was a general increase in temperature from 2009 to 2010?

Activity 8

1.



2.



Activity 9

1. We can see that the pollution was worst on 6 December.

2. The most deaths occurred on 8 December.

3. 925 deaths occurred on this day. Did you get that?
There are 10 blocks from 0 to 250, so each block is $\frac{250}{10} = 25$ deaths.

   Check that this is so by counting 25 for each little block until you have gone up to 10 blocks, where 250 is.

   The highest number of deaths is seven little blocks above 750.
$7 \times 25 = 175$ deaths above $750 = 925$ deaths altogether for this day.

4. Pollution was highest during the period from 4 to 10 December.

5. The most deaths occurred between 5 and 10 December.

6. The graph shows clearly that during the period when pollution was the highest, the most deaths occurred. Can we say that the number of deaths is related to the level of pollution?

Activity 10

Did you notice that the scale for *'deaths per day'* has been made much smaller on the second graph by changing the right-hand y-axis? Each block is now equal to a greater number of deaths, so the actual number of deaths looks much smaller. Now the correlation between pollution and deaths is not so obvious any more.

Activity 11

1. Did you get close to the following amounts?
   Social services = 130°
   Economic services = 83°
   Protection services = 110°
   "Other" = 36°

2. All these amounts are fractions of the total number of degrees in the circle (360°).
   We write the fractions for these amounts like this (we have rounded off to two decimal places):
   Social services = $\frac{130}{360} = \frac{13}{36} = 0{,}36$
   Economic services = $\frac{83}{360} = 0{,}23$
   Protection services = $\frac{110}{360} = \frac{11}{36} = 0{,}31$
   "Other" = $\frac{36}{360} = \frac{1}{10} = 0{,}10$
   Did you use your calculator to work out the decimal fractions?

3. To show these fractions as percentages we multiply the fraction or decimal fraction by 100. For example, 0,36 × 100 = 36% for social services. Round off to the nearest whole number.

   You should get 36% for social services, 23% for economic services, 31% for protection services and 9% for 'other'.

   These values are close to the actual percentages given in the pie chart. The values aren't exactly the same because the drawing of the pie chart, or our measurements, may not be completely accurate.