Mathematical Literacy

Unit 4

Further Building

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This Study Unit is the property of the learner to whom it is given.

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Unit 4

Further building

A course for adults at secondary level by distance education

1. Number patterns

Introduction

In Unit 1, we looked at the origins of number. People have been interested in number throughout time. People like patterns and surprising outcomes. There is an interesting pattern in the nine-times table: 9... 18... 27... 36... 45

Do you notice that if you add the two digits of any multiple together they always add up to 9?

For example:	18	→	$1+8=9$
	27	→	$2+7=9$
	36	→	$3+6=9$
	45	→	$4+5=9$

In this lesson you will explore the following number patterns:

1. Consecutive numbers
2. Constant differences
3. Square numbers
4. Fibonacci sequence
5. Leap years

Adding consecutive numbers

Consecutive: one after the other

There is a story about a young man called Gauss. One day, he was naughty in class. His teacher said, 'As punishment you will stay in at break and add up all the numbers from one to a hundred.'

'That's easy, sir. The answer is 5 050.'

The teacher was astounded because Gauss was right. How did he do it? He wrote down the numbers from 1 to 100

1 2 3 4 5 6 ... 100

Then he wrote them backwards.

1	2	3	4	5	6	... 100
100	99	98	97	96	95	... 1

Then he added them like this

1	2	3	... 100
100	99	98	... 1
101	101	101	... 101

Now there are a hundred 101's

The total is $101 \times 100 = 10\ 100$ but he had added the numbers twice, so if he divided 10 100 by 2 the answer is 5 050.

When young Gauss grew up he became a famous mathematician.

ACTIVITY 1

Try Gauss's technique for adding the numbers 1 to 10.

1. Write down the numbers from 1 to 10.
   1... 2... 3 to 10
2. Underneath write the numbers backwards from 10 to 1.
3. Add them up vertically and multiply by 10.
4. Finally divide by 2.
5. What is your answer? Check on your calculator to see if this technique works.

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Patterns with constant differences

Vusi buys the following piece of wrapping paper to wrap her daughter's birthday present.

There a number of number patterns we can study in the wrapping paper, such as the flowers, the faces, the eyes and the petals.

ACTIVITY 2

1. Write down how many:

   a) flowers there are in one row
   b) petals there are in one row

2. How many rows are there on the paper?

3. Write down how many:

   a) flowers there are on the paper
   b) petals there are on the paper

4. Complete the following table:

Number of rows	Number of flowers	Number of petals
1 row		
2 rows		
3 rows		

5. Can you work out how many flowers there would be in 4 rows?

6. Can you work out how many petals there would be in 4 rows?

7. Can you work out how many flowers there would be in 10 rows?

8. Can you work out how many petals there would be in 10 rows?

9. Can you see a relationship between the number of rows and the number of flowers? If so, write this down in words.

10. Can you see a relationship between the number of flowers and the number of petals? If so, write this down in words.

11. If we represent the number of rows with an r, the number of flowers with an f and the number of petals with a p, choose the equation from the list below that:
    $p = 24 \times r$
    $p = 8 \times f$
    $f = 3 \times r$

    a) represents the relationship between the number of rows and the number of flowers.
    b) represents the relationship between the number of flowers and the number of petals.
    c) represents the relationship between the number of rows and the number of petals.

12. If Vusi bought a total of 3 pieces of the same wrapping paper, show your calculations to work out;

    a) how many rows would there be?
    b) how many flowers would there be?
    c) how many petals would there be?

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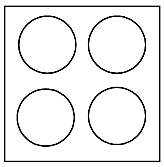
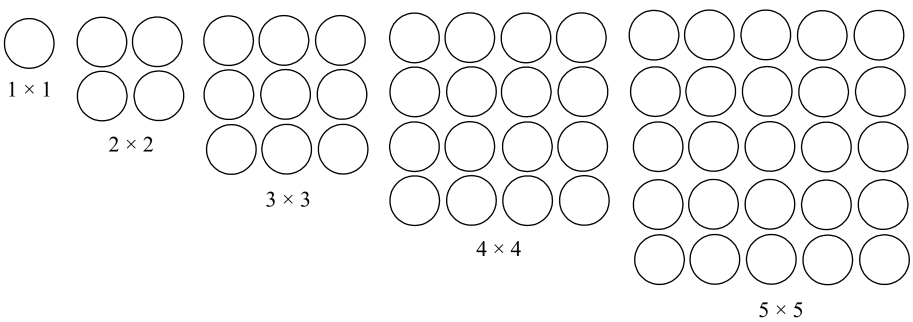
Square numbers

A square number can be shown in a square.



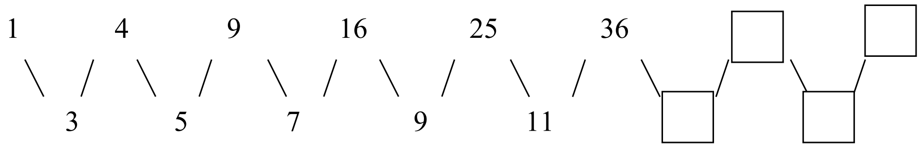
Square numbers are easy to find.



ACTIVITY 3

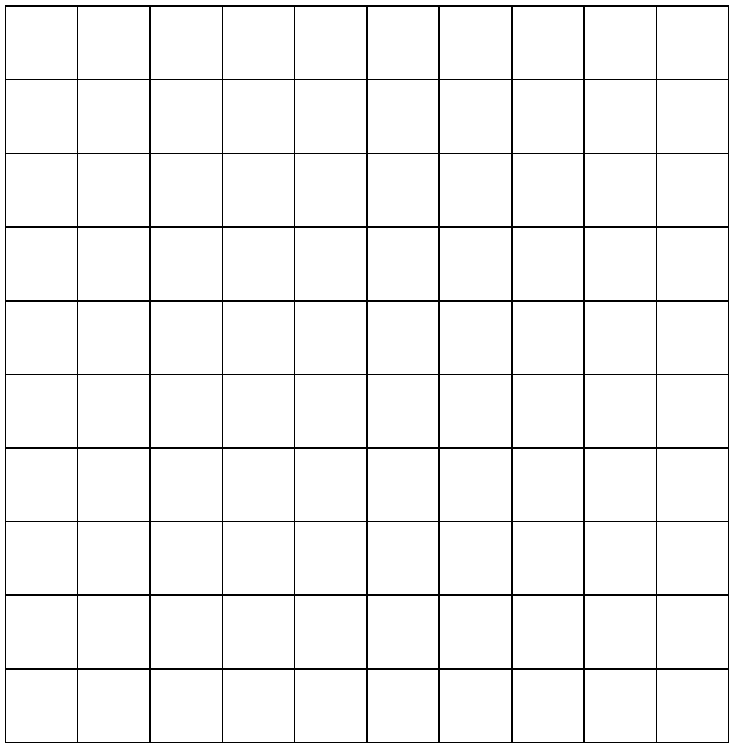
1. Here are the first 5 square numbers. Copy the numbers down and complete the list for the first 10.
   1 4 9 16 25

2. a) Write the difference between each pair of consecutive square numbers.

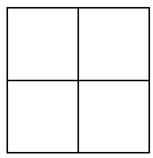


   b) What do you notice about these differences?

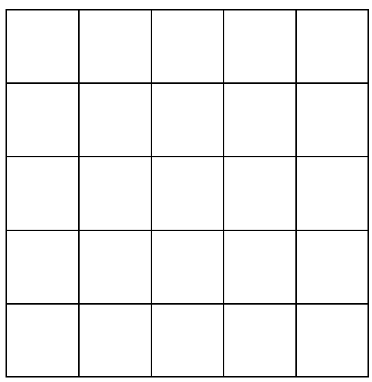
3. a) How many small squares are there in the block below?



   b) How many squares like this can the block in a) be divided into?



   c) How many squares like this can the block in a) be divided into?



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Fibonacci sequence

A famous mathematician, Fibonacci lived in Italy in 1200. He wrote this problem about rabbits. This was the problem.

Imagine that you own a pair of baby rabbits. They grow up in one month, and produce a pair of babies when they are two months old. Then they produce another pair every month.

Meanwhile the second pair grow up. They don't produce babies in the first month but when they are two months old they produce a pair.

If you start with one pair of babies on 1 January, how many pairs of rabbits will you have on the first day of every month?

See of you can work this out:

1 January	-	1^{st} pair of rabbits
1 February	-	pair of rabbits are one month old
1 March	-	pair of rabbits produce a new pair
1 April	-	pair of rabbits produce a 2^{nd} new pair
1 May	-	1^{st} pair of rabbits produce a 3^{rd} new pair
	-	2^{nd} pair of rabbits produce a 1^{st} new pair

Try to develop this family over a few more months

1 June	-
1 July	-
1 August	-

The Fibonacci sequence begins:

1 1 2 3 5 8 13 21 34 55 89 144...

ACTIVITY 4

1. Look at the Fibonacci sequence. Try adding together any pair of consecutive numbers in the sequence e.g. 1 + 1 or 21 + 34. What do you get?

2. Try adding together the 1st and the 3rd numbers. Then the 1st, 3rd and 5th.
What do you notice?

   Then add the 1st, 3rd, 5th and 7th. Do these numbers appear in the sequence?

3. Again looking at the Fibonacci sequence:
   - Add the first 2 numbers
   - Add the first 3 numbers
   - Add the first 4 numbers
   - Add the first 5 numbers.

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Do the numbers in your answers appear in the Fibonacci sequence?

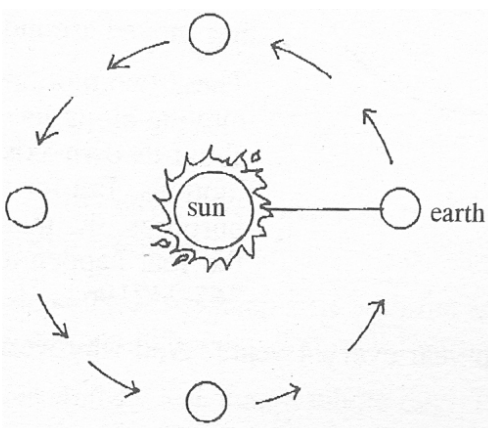
Leap years

Most people know that there is a leap year every four years. But what about the year 2000? Is this a leap year? Yes it is. But is every century a leap year? No it isn't. 1800 and 1900 were not leap years. Why not?

Well, in order to answer these questions we have to find out some information about the earth going round the sun, about the earth rotating on its own axis, and the number of days in the year.

Look back at Unit 1 Lesson 6, or go to the library and find out more about the rotation of the earth. For the purposes of this problem, we will give you the following information.

The length of the **year** depends on the **time it takes for the earth to travel around the sun**.



The distance from the earth to the sun is roughly 149 600 000 kilometres.

The path is an ellipse, but for this problem we shall consider it as being roughly a circle with a radius of 149 600 000 kilometres. To work out the distance travelled, we need to work out the circumference of a circle.

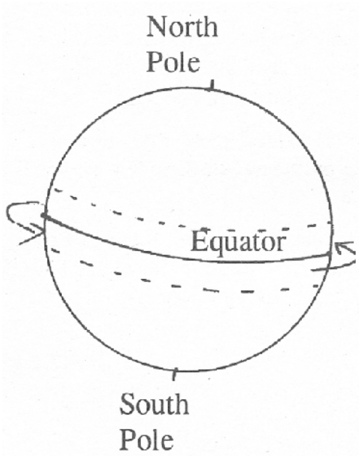
$$\begin{aligned} \text{Circumference of a circle} &= \pi d = 2\pi r \\ &\approx 2 \times 3{,}142 \times 149\ 600\ 000 \text{ km} \\ &\approx 940\ 086\ 400 \text{ km} \end{aligned}$$

Therefore the distance travelled by the earth around the sun is approximately: 940 086 400 km

The average speed is roughly 106 194 km/h.
This means that in 1 hour the earth travels 106 194 km.
Therefore in 1 day (i.e. 24 hours) the earth travels:
106 194 km × 24 hours
The speed per day is 2 548 656 (kilometres per day)

The time taken: $\dfrac{\text{Distance}}{\text{Speed}} = \dfrac{940\ 086\ 400}{2\ 548\ 656} \approx 368{,}86$ days.

Therefore 1 year (the time taken for the earth to travel around the sun) ≈ 369 days (approx.)



(This answer does not work out exactly to the answer you might expect because of our approximations. We call this 'round-off error'. The symbol ≈ means 'approximately equal to').

The second fact you need to understand if you want to know about leap years is this. The **day** depends on the amount of **time it takes for the earth to rotate about its own axis**.

The earth actually takes 23 hours 56 minutes and 4,1 seconds. The remaining 3 minutes 55,9 seconds is accounted for by the distance it has moved around the sun.

These two motions, of the earth moving along its orbit and spinning about its own axis, are independent motions. But for time-keeping purposes, the number of days in the year happen to be 365,242196 ...

Why is there a leap year every 4 years? And why weren't 1800 and 1900 leap years?

The length of a year, that is the time it takes for the earth to travel around the sun, is 365,242196 ... days. This is an awkward number.

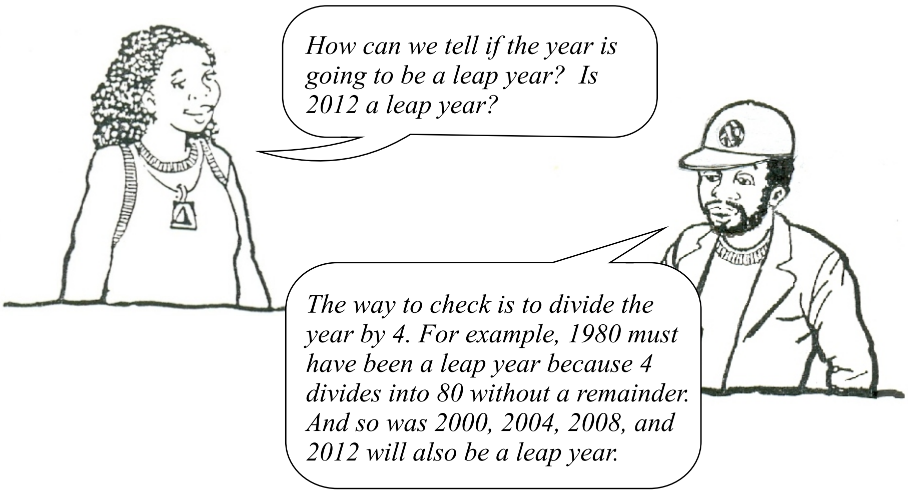
Let's approximate and call it $365\frac{1}{4}$ days for now.

Then 2 years $= 730\frac{1}{2}$ days

and 4 years $= 1461$ days

Now this is equal to: $(4 \times 365) + 1$
$= 1460 + 1$

We need to keep our calendar year in time with the solar year. This is easily adjusted if every four years we add another day. This day happens to be February 29th.



Let's carry on our investigation a bit further. $365\frac{1}{4} = 365,25$ days is not very accurate.

Let's take 365,242196 and calculate what happens over a hundred years and four hundred years.

$100 \times 365{,}242196 = 36\,524{,}2196$ days
and $400 \times 365{,}242196 = 146\,096{,}8784$ days $\approx 146\,097$ days

Well, if we keep our 4-year cycle of leap years we get:
400 years $= 400 \times 365{,}25 = 146{,}100$ days.

We therefore need to use 3 days every 400 years. The method that was chosen was to allow only 1 century in every 4 to be a leap year. Thus 1700, 1800, and 1900 were not. The year 2000 was a leap year. If we continue the pattern, the years 2100, 2200 and 2300 will not be leap years, but the year 2400 will be a leap year.

ACTIVITY 5

1. Answer the following questions:
   a) What is a leap year?
   b) How often does it occur?
   c) When is the next leap year?
   d) Complete the following:
   The solar year (the time it takes for the earth to travel around the sun) is approximately ___.
   The time it takes for the earth to rotate on its own axis is ___.
   e) Is the solar year longer or shorter than 365 days?

2. The next cricket world cup is in 2015. Is that a leap year?
3. Name all the leap years between 2000 and 2020.
4. Explain to a friend why we have a leap year every 4 years.

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Summary

In this lesson we looked at patterns in number. We looked at some phenomena which are interesting mathematically. Of course there are many more interesting patterns.

We showed that the nine times table generates an interesting number pattern. So do the other tables.

The next step is to describe patterns and sequences mathematically. This is dealt with in more detail in the Mathematics course. For further reading look at mathematics books in the library.

We also looked at leap years and why they occur. The calendar year is 365 days. The solar year is 365,24 days. Every 4 years we gain almost a whole day.

In order for the calendar year to keep in step with the solar year, we have to add a day every 4 years. This day is the 29 February. However the gain is slightly less than a whole day. This means that over 400 years there could be an extra three days in the calendar year.

To solve this problem we leave out the extra day in 3 out of 4 centuries. The year 2000 was a leap year. The next three centuries are not leap years. They don't concern us, however, because we aren't going to be around!

Self-assessment checklist:

After this lesson you should be able to:

- use patterns in number to find factors, and help with calculations
- recognise triangular and square numbers
- write down a sequence of triangular or square numbers
- recognise the Fibonacci sequence
- recognise a leap year and calculate other leap years.

SELF-CHECK EXERCISE

1. Use Gauss' method to add up the numbers from 1 to 20.

2. Describe a square. How do you check if a number is a square number?

3. Write down the next four square numbers to continue the pattern below:
   …64 ; 81; 100; ___ ; ___ ; ___ ; ___

4. The Fibonacci sequence begins:
   1 1 2 3 5 ...
   Complete the next five numbers in the sequence.

5. We had our first democratic elections in 1994. Was 1994 a leap year?

6. If you were born on the 29th February 2000, would you be able to celebrate your 40th birthday on the 29 February 2040 or would that not be a leap year?

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2. Financial maths: income tax, accounts and discounts

Introduction

In this lesson we cover three important financial mathematics topics you will come across in your daily life namely income tax, accounts (such as electricity, phone bills etc.) and discounts. These topics will use skills you have learned throughout this level. The situations we have chosen are drawn from everyday life. In everyday life you use many different skills and resources. We will focus specifically on the mathematical skills you use to solve problems, make plans or organise things.

In this lesson you will:
- learn how to read tax tables and calculate basic income tax
- learn about reading and comparing different accounts
- learn how to calculate and compare discounts

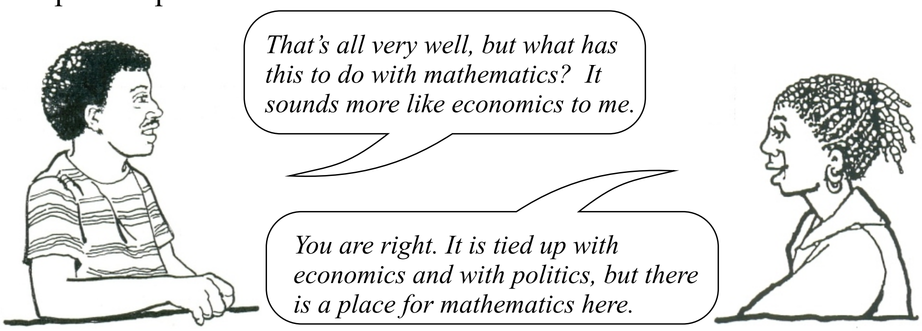
Taxes

In days gone by, kings demanded a portion of the farmers' produce to be paid to them every year. This payment of a portion of one's earnings to an authority is called a tax.

Taxes are used to pay the governments' expenses and other public expenses such as enforcing law and order, building roads, schools and hospitals, and the salaries of public servants.

Sources of tax
There are different kinds of taxes that the government collects.
1. *Income tax* from individuals and companies. The employer has to take off tax which is then sent to the government. Companies also have to pay a proportion of their earnings to the government.
2. *Wealth tax*. This is where people pay tax on their possessions. For example a person pays taxes on the property he or she owns.
3. *Value Added Tax*. This is the tax that is added to the price of the goods that you buy. (We worked with VAT in lesson 3 of Unit 1).
4. *Customs duty*. This is a tax which is paid on goods imported into the country. This raises money for the government and it also protects the manufacture of goods in our own country.

There are also hidden taxes. For example, there is a road tax included in the price of petrol.



Some interesting historical facts

In the 13th century a law was introduced in England that stated that the people must have a say in how they were taxed.

One of the causes of the American revolution, in 1795, was the tax which the British government put on tea in the American colonies. It is important that the burden of taxes should be distributed fairly. Most countries have a sliding scale whereby the low income earners pay 5% of their earnings, but the high income earners pay as much as 50% of their earnings.

Let's look at a tax problem where we need to a apply mathematics.

Example:
My married cousin Mzwai, who is 38 years old, has a job where he earns R26 600 a month.

1. What tax bracket does he fall into?

2. Calculate his year's earnings, then check the tax tables refer to below to find out how much tax he pays.

3. What monthly salary does Mzwai get after tax has been deducted.

Solution:

1. Earnings per month: R26 600
   Earnings per year: R26 600 × 12 = R319 200

   R319 200 is between R305 001 and R431 000. Find this bracket on the tax table below.

2. Therefore the tax on this is R70 650 + 35% of the amount over R305 000 minus the primary rebate of R10 260.

   Tax = R70 650 + 35% of (R319 200 R305 000) – R10 260
   = R70 650 + 35% of R14 200 – R10 260
   = R70 650 + R4970 – R10 260
   = R65 360

   Total earnings after tax: R319 200 – R65 360 = R253 840

3. Monthly salary after tax: R253 840 ÷ 12 = R21 153,33

Tax tables for March 2010 - February 2011

Taxable income			Rates of tax				
R		R	R				R
0	—	140 000			18%	of every R1	
140 001	—	221 000	25 200	+	25%	of the amount over	140 000
221 001	—	305 000	45 450	+	30%	of the amount over	221 000
305 001	—	431 000	70 650	+	35%	of the amount over	305 000
431 001	—	552 000	114 750	+	38%	of the amount over	431 000
552 001		and above	160 730	+	40%	of the amount over	552 000

A rebate is money that is taken off your tax amount.

REBATES
Primary rebate..R10 260
Additional rebate..R5 675

- The rebates for individuals must be deducted from the normal tax.
- The primary rebate is deductible for all individuals although the additional rebate may only be applied for individuals who are 65 years or older.

ACTIVITY 1

Mzwai gets offered a new job at a large corporate company. They offer to pay him an annual salary of R435 000.

1. What is the percentage increase from his previous salary of R319 200 before tax is considered?

2. What annual tax will he pay on his new salary?

3. How much he will earn per month, after tax if he takes the new job?

4. What will the increase be in his monthly salary after taking off the tax?

5. Is this a good increase? Why do you think so?

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If you have more than one job, you need to add all your earnings together and calculate the tax on that amount. We will discuss an example here. For more information on guidelines on calculating your personal tax, visit the South African Revenue Service (SARS) website at http://www.sars.gov.za/

Example:
Bonny, who is 24 years old, has a job where she earns R5600 a month. She has been offered a job for three evenings which will pay her an extra R1500 a week.

1. How much does she currently earn per month after tax has been deducted?

2. How much money will she earn in a year with her extra job?

3. How much tax will she pay annually on both jobs?

4. What will her monthly salary increase to with both jobs?

Solution:

1. Bonny currently annually earns: R5600 × 12 = R67 200
   Her annual tax on this is: 18% of R67 200 – R10 260
   = R12 096 – R10 260
   = R1 836
   Her annual salary after tax: R67 200 – R1836 = R65 364
   Her monthly salary after tax: R65 364 ÷ 12 = R5447

2. Earnings from new job: R1500 a week
   Therefore: R1500 × 52 per year
   R78 000

   Total earnings with both jobs:
   R67 200 + R78 000 = R145 200

3. Tax on R145 200: R25 200 + 25% of earnings over R140 000 minus rebate

$= R25\,200 + 25\%\text{ of }R5200 - R10\,260$
$= R25\,200 + R1300 - R10\,260$
$= R16\,240$

4. Therefore total annual earnings after tax:

$R145\,200 - R16\,240 = R128\,960$

Monthly salary with both jobs: $R128\,960 \div 12 = R10\,746{,}67$

ACTIVITY 2

Use the tax tables to help you to answer the following questions.

1. Mr J, who is 43 years old, earns R13 000 a month.
   a) How much does he earn a year?
   b) What tax bracket does he fall into? How much tax will he pay annually?
   c) How much does he earn annually and monthly after the tax is taken off?

2. Mrs D, who is 68 years old earns R9 000 a month.
   a) How much does she earn a year?
   b) What tax bracket does she fall into? How much tax will she pay annually?
   c) How much does she earn per year and per month after tax is taken off?

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3. Compare the two situations. Who pays more tax, Mr J or Mrs D?

Accounts

Electricity

Electricity is measured in Watt or Kilowatt.

1000 Watt = 1 kW
2000 Watt = 2 kW
500 Watt = $\frac{500}{1000} = 0{,}5$ kW
200 Watt = $\frac{200}{1000} = 0{,}2$ kW

We get charged per kW of electricity that we use and the cost per kW depends on the price set by Eskom or a private service provider, depending on how your electricity is supplied. Study the advert on the next page to see how to calculate the cost per hour and per month of running an appliance, such as a heater.

Example:

COMPARISON OF ELECTRICITY USAGE AND COST

(Based on Cape Town, South Africa, electricity rates: 1st July 2010)

	Electrical rating in Watt (1000W = 1kW)	Monthly cost (@106.37 cents/kWh)	Hourly cost (@106.37 cents/kWh)
Air Conditioner	2500	R1914.66	R2.66
Heater (2 Bars)	1300	R995.62	R1.38
Heater (3 Bars)	2000	R1531.73	R2.13
Heater Fan	2000	R1531.73	R2.13
Heater Oil (Rib Heaters)	2000	R1531.73	R2.13
Econo-Heat Wall Panel Heater	**400**	**R306.35**	**43 cents**
Lighting: Single Bulb (100W)	100	R76.59	11 cents
Lighting: Single Bulb (60W)	60	R45.95	6 cents

CALCULATING ELECTRICITY USAGE

To calculate the monthly cost of an electrical appliance, use the following formula:

Electrical rating in kW x Hours per day x Days per month x Cost per kWh

The following calculation is based on the electricity usage of a 400 Watt Econo-Heat wall panel heater:

1 The electrical rating is **400 Watt**. To convert to kilowatt (kW) divide the rating in Watt by 1000.

400 divided by 1000 = 0.40 kW

2 Electricity usage (kWh) is obtained by multiplying the rating of the appliance (in kilowatt) by the number of hours it is used in a month.

0.40 kW x 24 hours x 30 days = 288 kWh (for a full month)

3 Finally, the monthly cost to run an appliance is obtained by multiplying the electricity usage (kWh) by the cost of one unit of electricity (106.37 cents for City of Cape Town, South Africa domestic low tariff between 450 to 1500 units per month).

288 kWh x 106.37c = R306.35 (in this example, for a full month) = **43 cents per hour**

www.econo-heat.com

ACTIVITY 3

Use the advert above to complete this activity.

Mr J uses his heater fan for approximately 3 hours per evening during June, July and August.

1. What is the electrical rating, in Watt and Kilowatt, of the heater fan?
2. What does it cost Mr J per month in electricity for the heater fan over the three months?
3. What does it cost per hour to run a heater fan? Show your calculations.
4. If a television has an electrical rating of 200 Watt and you watch television approximately 2 hours per day, how much will it cost you in electricity per month for the television?

So watching your television for approximately 2 hours per day in Cape Town costs a little more per month than a loaf of bread!

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Many people prefer to use pre-paid electricity meters so that they can monitor their electricity usage. When you purchase prepaid electricity, you get a slip that looks like the slip below.

Kenilworth, Cape Town,

Date : 28/08/09 Time : 11:28

162.7 units @ 53.90 c/unit

ELEC R 87.72 162.7 units
VAT R 12.28

TOTAL R 100.00
=============
Tender R 100.00
Change R 0.00

Bought online at www.energy.co.za

ACTIVITY 4

Use the slip to answer the questions.

1. On which date was the electricity purchased?
2. Where was the electricity purchased?
3. How many units did the person buy?
4. How much did they pay per unit?
5. How much Value added tax did they pay on the electricity?
6. If the person who bought this electricity uses about 20 units per day, how long can they expect this electricity to last?
7. If you want to buy 200 units, how much will it cost you?

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ACTIVITY 5

Domestic Low
284.0000 kWh @ R0.7737 = R219.73

Here is another pre-paid electricity bill.

1. Compare this bill to the one in Activity 3. Which bill charges a higher rate per unit or kWh? What is that cost?

2. Why do you think their rates might be different?

3. Calculate the total of this bill once VAT has been added.

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ACTIVITY 6

A litre of milk costs R7,50. How long can you run a fridge for R7,50's worth of electricity?

You will need to find out the size of the fridge and the cost of one unit of electricity.

The time taken to use one unit is given $T = \dfrac{1000}{\text{Electrical rate (w) per hour}}$ where w is the wattage.

For the purposes of this problem, let's say 1 litre of milk costs R7,50 and the fridge is a 100 watt appliance. Let's work with the prepaid price above of R0,7737 per unit.

ANSWERS ON PAGE 85

Telephone charges

With cellular coverage spreading across South Africa and Africa, most people now own a cell phone or mobile phone. These can be purchased with contracts that bind you to paying a certain amount for the phone and the calls each month for a period of two years. Or you can choose to buy a phone and a starter pack and then use prepaid to purchase your call time.

This is similar to the electricity system. You either have an account with the municipality or another electricity supplier and you pay that account each month, or you put in a prepaid electricity meter and purchase electricity units as you need them.

Let us compare some cell phone account options. The following table shows some contract packages available from the MTN network. www.mtn.co.za

The 'in-bundle' rate is the cost per minute of a call. When you have used up all your free or "inclusive" minutes, then you start paying the 'out of bundle' rate.

Use the table to do the following activity.

Contract Packages	Monthly Subscription / Airtime Value (incl.VAT)	Inclusive SMSs	In-Bundle Rate (incl.VAT)	Out of Bundle Rate (incl.VAT)	Estimated Inclusive Minutes *
» MTN AnyTime 50	R 50.00	25	R 2.30	R 2.85	21
» MTN AnyTime 100	R 100.00	25	R 2.30	R 2.85	43
» MTN AnyTime 200	R 200.00	25	R 2.30	R 2.85	86
» MTN AnyTime 350	R 350.00	50	R 1.95	R 2.35	179
» MTN AnyTime 500	R 500.00	50	R 1.95	R 2.35	256
» MTN AnyTime 750	R 750.00	100	R 1.60	R 1.75	468
» MTN AnyTime 1200	R 1,200.00	100	R 1.60	R 1.75	750
» MTN AnyTime 1500	R 1,500.00	200	R 1.50	R 1.50	1,000

*The estimated number of inclusive minutes is based on the assumption that all inclusive Airtime Value is used for making local voice calls according to the applicable in-bundle anytime call rate.

*When activating MTN Zone the package in-bundle and out-of-bundle rates will no longer apply. MTN to MTN calls will be charged at R2.50 and MTN calls to all other local networks will be charged at R3.00.

ACTIVITY 7

1. What is the 'in bundle' rate per minute for a call on the MTN AnyTime 350 package?

2. What does the MTN AnyTime 500 package cost? Approximately how many inclusive minutes do you get on this package? Show how they calculate this estimate.

3. What is the difference between the 'in bundle' and the 'out of bundle' cost per minute on the MTN AnyTime 1200 package?

4. Pheladi works out that she usually sends between 40 and 50 sms's per month and uses about 150 minutes of talktime. Which MTN AnyTime package would you recommend for Pheladi. Why?

5. Work out how many sms's and minutes you usually use per month on your cell phone and then decide which package would be best for you.

ANSWERS ON PAGE 87

Discounts

Example

Jakes wants to buy a car which costs R84 000. The garage offers a discount of $12\frac{1}{2}$%. How much will he pay?

This kind of problem can be solved in different ways.

Method 1
We take R84 000 to be the total amount. We then find $12\frac{1}{2}$% of R84 000 and subtract this from the price.

Estimate to get a rough idea. 10% of R84 000 is R8 400, so $12\frac{1}{2}$% is going to be a little more.

Then:

$$12\tfrac{1}{2}\%\text{ of R84 000} = \frac{12,5}{100} \times R84000 = R10500$$

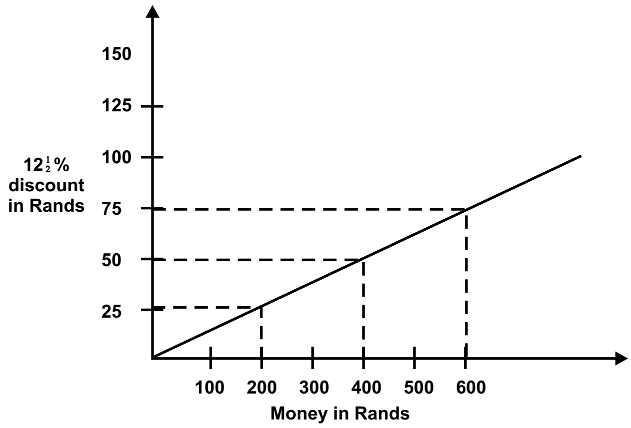
Selling Price = R(84 000 – R10 500) = R73 500

Method 2
We take the market price R84 000 to be 100% and the selling price as

$(100 - 12\frac{1}{2})\% = 87\frac{1}{2}\%$

Therefore the selling price: 87,5 % of R84 000 = R73 500

Method 3
The problem can be solved graphically. A discount of $12\frac{1}{2}$% means a saving of R12,50 on every R100. So we save R125 on R1 000, and R1 250 on R10 000.



ACTIVITY 8

1. A shop has the following notice in its window: 15% off all CD's and DVD's. The DVD's cost R124. How much will Nomi pay for one DVD?

2. Ms L received an account for
   1 year's TV rental: R800
   VAT 14%: R112
   Total: R912
   10% discount if paid within 7 days.
   How much will Ms L pay if she pays within 7 days?

3. Draw a graph to show a discount of 20% on amounts up to R100.

ANSWERS ON PAGE 86

Self-assessment checklist:

After this lesson you should be able to:
- use a tax table to calculate the amount of tax a person should pay
- calculate the amount of electricity an appliance uses and what this could cost
- evaluate phone packages and calculate the cost of calls
- calculate discounts.

SELF-CHECK EXERCISE

1. In most tax systems allowance is made for people who are supporting children. In this case there is a rebate of R100 for each child. This means that you can take off R100 for each child before calculating the tax. If Mrs E who is 45 years old has two children and earns R2 500 a month what tax will she pay per month? (Use the tax table on page 13.)

2. Jakes runs a business from his home and wants to take out a cell phone package. He spends approximately 500 minutes per month on the phone. Which MTN package do you recommend for Jakes and why? (Use the table from activity 7.)

3. A couch originally costs R550. The shop has a sale offering 15% off. What is the sale price of the couch?

ANSWERS ON PAGE 77

3. Introduction to probability and chance

Introduction

Probability is a branch of statistics. It is the study of chance. In other words, probability is about describing and estimating how likely something is to happen. In some situations, you may be able to see that one thing is just as likely to happen as another. In other situations you can make estimates based on what has happened in the past. This lesson will introduce you to both situations. You will need to know the mathematics of lessons 1 and 2 of Unit 3.

You will also need:
- a coin
- a pair of dice
- a set of playing cards

In this lesson you will:
- understand the meaning of probability
- calculate when to play a game of chance
- estimate and calculate simple probabilities

Basic concepts of probability

If you toss a coin into the air, there are two possibilities for the way the coin will land: either 'heads' or 'tails'. In other words, there is one chance in two of the coin landing on the one side or the other side. When a coin is tossed into the air the probability of it being 'heads' is $\frac{1}{2}$ and the probability of it being 'tails' is $\frac{1}{2}$.

Heads and tails: The side with the head on a coin is called 'heads' and the other side is called 'tails'.

Let us take another example. If a dice is tossed, what is the probability that the number '6' will appear on top? A dice has six possible sides (1, 2, 3, 4, 5, 6). Only one of the sides is marked '6'. Therefore the probability of '6' appearing on top of the dice is $\frac{1}{6}$.

Definition of Probability

The probability of an event happening is the ratio of the number of ways the event can happen to the number of equally likely outcomes. That is:

- the probability of an event happening is equal to:

$$\frac{\text{number of ways the event can happen}}{\text{number of equally likely outcomes}}$$

- if the number of ways the event can happen is s and the number of equally likely outcomes is n, then the probability is given by $\frac{s}{n}$.

Example
What is the probability of obtaining an even number when a dice is tossed?

There are six equally likely outcomes when the dice is tossed. These are 1, 2, 3, 4, 5, 6. Therefore $n = 6$. Out of these six outcomes, three of them are even numbers. These are 2, 4 and 6. So $s = 3$. Therefore the required probability is: $\frac{s}{n} = \frac{3}{6} = \frac{1}{2}$

ACTIVITY 1

1. A bag contains six balls, two of which are red and the rest blue. If you pick one ball from the bag without looking at it, what are the probabilities that the ball you picked is
   a) red?
   b) blue?

2. You toss a fair dice. What is the probability of getting a '3'?

3. You draw one card from a set of 52 playing cards, what are the probabilities that:
   a) a king is chosen?
   b) a black card is chosen?
   c) the card you chose is a diamond?

ANSWERS ON PAGE 86

For you to be able to decide when to play a game of chance, you need to know more about probability.

Range of Probability

The probability of an event happening lies between 0 and 1.

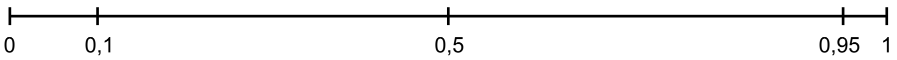
If an event is certain to occur we say that is has a probability of 1. On the other hand if an event cannot occur, we say that it has a probability of 0.

For example, suppose a bag contains only four blue balls and you choose one ball from the bag, what is the probability that the ball you choose is blue? Since all the balls in the bag are blue, it is certain that you will choose a blue ball. Therefore the probability is 1. We can also use the definition of probability to show that the probability of choosing a blue ball is 1. Can you do that? Try it. How many ways can the event happen? And how many possible outcomes do we have? Both answers are 4. Therefore the probability of choosing a blue ball is $\frac{4}{4} = 1$.

Now, what is the probability of obtaining a red ball from the same bag? It is impossible to choose a red ball from a bag which contains only blue balls. Therefore the probability is 0. Try to use the definition of probability to show that the probability of choosing a red ball is 0.

If an event is equally as likely to happen as not, then the probability of it happening is $\frac{1}{2} = 0,5$.

We can show the range of probability on a probability diagram like this:



So for example:

0 means event cannot happen
0,1 means event is unlikely to happen
0,5 means event is just as likely to happen as not
0,95 means event is very likely to happen
1 means event is certain to happen.

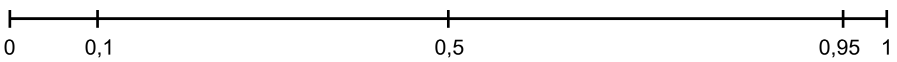
The nearer the probability is to 1, the MORE LIKELY the event is to happen. The nearer a probability is to 0, the LESS LIKELY the event is to happen.

In a game of chance you have to take these statements very seriously. Before you play any game of chance, you should try to calculate the probability of winning. If it is nearer to 1, go for it. Otherwise don't play that game of chance.

ACTIVITY 2

1. A double-headed coin is tossed into the air. Find the probability of the coin coming down:
   a) head
   b) tail.

2. Each of the following statements says how likely an event is to happen. Write the letters of the events in sensible positions on the probability diagram.



   A There is almost no chance for you to win that game.
   B Everybody will die one day.
   C It is very likely that it will rain tonight.
   D It is impossible for you to be at two different places at the same time.
   E It is just as likely as not that your visitor will arrive today.

3. Which of the following numbers could not be probabilities?

   $-0,01; 0,72; 1,2; \frac{3}{8}; \frac{9}{7}; 0,5; 4,0; -\frac{1}{2}$

ANSWERS ON PAGE 87

Estimating Probabilities

So far we have been calculating probabilities of events using the ratio of the number of ways the event can happen and the total number of equally likely outcomes. We can also estimate probabilities based on what has happened in the past. We shall show this with examples.

Example

Suppose Kaiser Chiefs and Orlando Pirates played 10 games in the past three years. Out of these Chiefs beat Pirates on 7 occasions. Use this information to estimate the probability of Chiefs beating Pirates in their next game.

Assuming the present strength of the two teams is the same as in the last three years, then the probability of Chiefs beating Pirates is $\frac{7}{10}$.

Example

You want to buy oranges from a shop. You choose a sample of 30 oranges and you find that 5 of them are bad. What is the probability of buying a bad orange?

Based on your sample the probability of buying a bad orange is $\frac{5}{30} = \frac{1}{6}$.

Now, is there a formula for estimating probabilities? Let's go back to our example on the games between Chiefs and Pirates. Chiefs could have beaten Pirates on all the 10 occasions. However, they beat Pirates on 7 occasions, so we say the probability was $\frac{7}{10}$. Similarly all the 30 oranges could have been bad. But in our case only 5 were bad so we had the probability of $\frac{5}{30} = \frac{1}{6}$.

From these two examples we can say that:
The probability of an event is equal to:

$$\frac{\text{Number of times the event happened}}{\text{Number of times the event could have happened}}$$

ACTIVITY 3

1. Take a fair dice and toss it 60 times. Count the number of times you obtain '3' on top of the dice. Use this information to estimate the probability of obtaining a '3' when you next toss the dice.

2. Patheka tossed a coin 1 000 times. Out of these 1 000 times she had heads 510 times. What is the probability of obtaining a head when the same coin is tossed into the air?

3. The 8:30 train which Jongile takes to work everyday, arrived late 24 times in the last 50 days. There is no way of telling what will happen tomorrow. What is the probability that tomorrow it will arrive late?

ANSWERS ON PAGE 87

The probability of non-occurrence of an event

We said earlier that the probability of obtaining a '6' when a fair dice is tossed is $\frac{1}{6}$. Now, what is the probability of not getting a '6' when a fair dice is tossed? Not getting a '6' means getting any of the other five numbers, 1, 2, 3, 4 and 5. Therefore there are five ways that this can happen. Also there are six equally likely ways of getting an outcome when a fair dice is tossed. Therefore the probability of not getting a '6' when a fair dice is tossed is $\frac{5}{6}$. Now $\frac{5}{6} = 1 - \frac{1}{6}$. That is, the probability of not getting a '6' is 1 minus the probability of getting a '6'.

In general the probability of non-occurrence of an event =
1– the probability of the occurrence of the event.

ACTIVITY 4

1. A multiple choice question in a test has 5 possible answers, only one of which is right. If a person writing the test chooses an answer at random what is the probability that his answer is wrong?

2. The probability that it will rain today is 0,7. What is the probability that it will not rain today?

ANSWERS ON PAGE 87

Summary

In this lesson, you learnt that:

- the probability of an event is a measure of how likely the event is to happen. If an event is certain to happen the probability is 1. If it is impossible for an event to happen, its probability is 0 and if the event is just as likely to happen as not the probability is $\frac{1}{2}$.
- the nearer the probability is to 1 the more likely it is that the event will happen and the nearer the probability is to 0 the less likely the event will happen
- the probability of an event = $\dfrac{\text{the number of an event}}{\text{number of equally likely outcomes}}$

You also learnt that you can estimate probability based on what has happened in the past.

In that case the probability of an event =

$$\frac{\text{Number of times the event happened}}{\text{Number of times the event could have happened}}$$

Finally you learnt that:

- The probability of an event not taking place = 1 – probability of the event taking place.

Self-assessment checklist:

Are you able to:
- calculate the probability of an event happening
- estimate the probability of an event taking place
- calculate the probability of an event not taking place.

SELF-CHECK EXERCISE

1. A bag contains 8 green, 2 red and 5 blue marbles. A child picks one marble from the bag without looking. What is the probability that the marble is:
   a) green
   b) not green?

2. A card is drawn from a well-shuffled pack of playing cards. What is the probability that the card is:
   a) a black ace
   b) a club
   c) not a king?

3. The letters of the word 'S T A T I S T I C S' are each written on squares of cardboard. They are placed in a big box. If you choose a card from this box at random, what is the probability that you pick a card with:
   a) a 'I'
   b) a 'T' written on it?

4. Suppose you tossed a coin 500 times, each time recording whether it lands heads or tails. Your results were that 300 times you recorded heads and 200 times you recorded tails. Estimate the probability of obtaining:
   a) heads when you next toss the coin
   b) tails when you next toss the coin

ANSWERS ON PAGE 78

4. Shapes in 3 dimensions: surface area and volume

Introduction

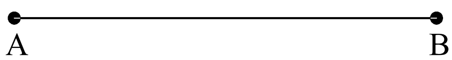
In this lesson you will be learning about three dimensional (3-D) shapes and how to calculate their surface area, volume and capacity. We come across these concepts in our daily lives when we work with any items that need packaging or when we need to know the amount of space that an object occupies or contains **(volume)** or the amount of space inside an object **(capacity)**.

In Unit 1 lesson 5 you learned about society's need for measurement. In Unit 2 lesson 6, you studied a few shapes and learned how to measure lengths. You were also introduced to the concepts of area, volume and capacity. You will need to understand these previous lessons in order to develop measurement further in this lesson.

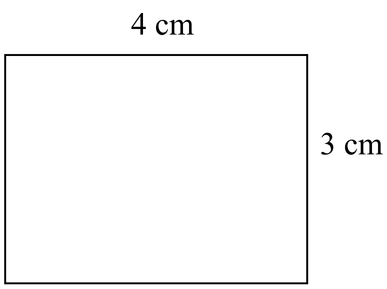
In this lesson you will:
- learn about 3 dimensional shapes
- know and understand the meaning of surface area, volume and capacity
- calculate the surface area of a cube, a rectangular solid and a cylinder
- calculate the volume of a cube, a rectangular solid and a cylinder.

Shapes in 3 dimensions

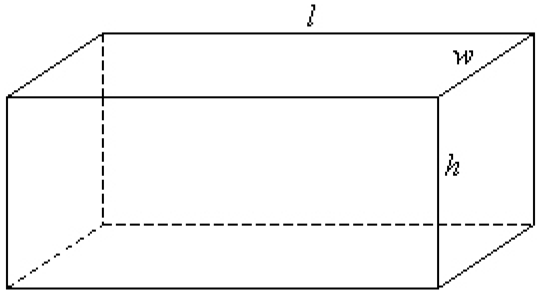
In lesson 5 of Unit 1, you learned about basic measurement of length. When you are measuring a straight line, you are measuring a **one dimensional (1-D) shape** that **only has a length**. For example, the following line AB only has a length. Measure the length with your ruler.



In lesson 6 of Unit 2, you continued learning more about measurement of length in two dimensional shapes. **Two dimensional (2-D) shapes have a length and a width** (or breadth). For example, the rectangle below has a length of 4 cm and a width (breadth) of 3 cm. These are flat shapes.



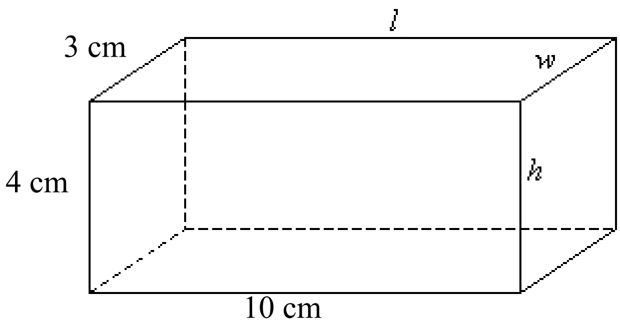
Three dimensional (3-D) shapes have three dimensions (or measurements); a length, a breadth and a height. They are also called solids, even if they are hollow. Some examples of solids are cubes, rectangular solids and cylinders. The box below is a rectangular solid. The three dimensions are length (*l*), width (*w*) and height (*h*).



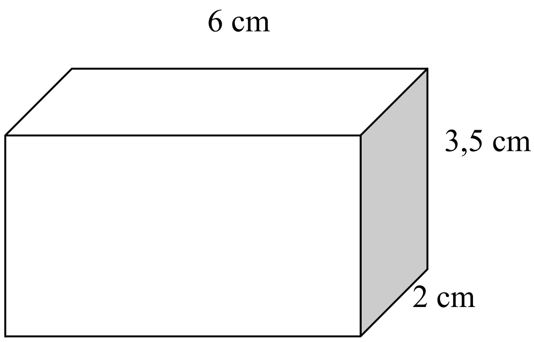
ACTIVITY 1

1. Write down the length, width and height of the following rectangular solids.

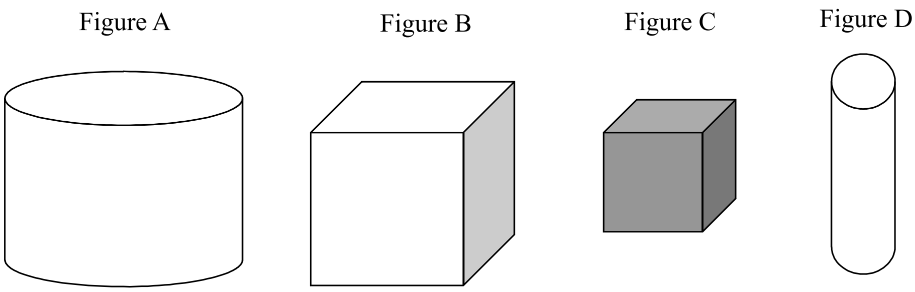
a)



b)



2. Here are four figures; figures A, B, C and D. Write down which figures you think are:
   a) cubes
   b) cylinders

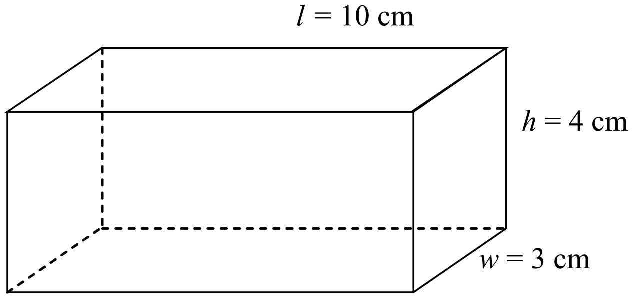


ANSWERS ON PAGE 88

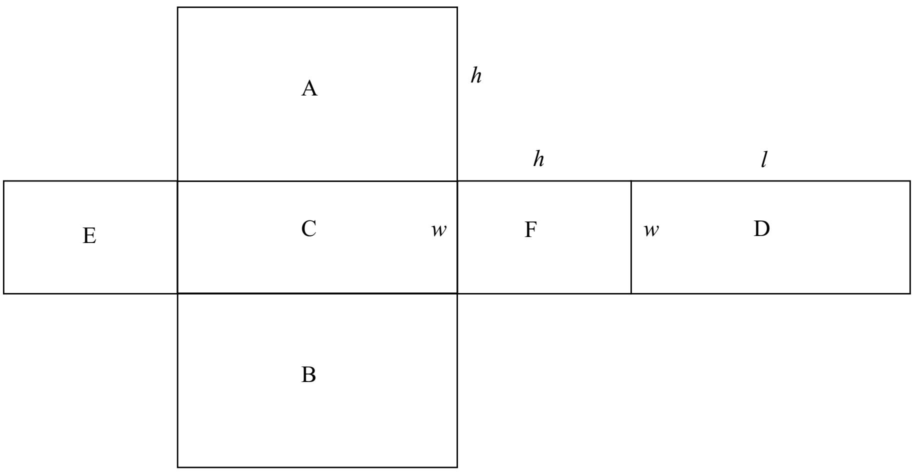
Surface area

In lesson 6 of Unit 2 you learned how to calculate the area of certain 2-D shapes, such as a rectangle, a square, a triangle and a circle. We can also calculate the area of 3-D shapes using the knowledge we gained from Unit 2. The area of 3-D shapes is called the surface area to indicate that the area being calculated is the outside or surface area of the shape.

In order to calculate the surface area of a solid, we need to calculate each 2-D outside surface, called faces, of the solid. The rectangular solid below has 6 faces.



Can you see why? Let us unfold the rectangular solid so that you can see each of the faces. This **'unfolded'** diagram is called a **net or a development** in mathematics.



Now you can clearly see each face. Each face of the solid is a 2-D shape and you already know how to calculate the area of certain 2-D shapes. Can you see that each of these faces is a rectangle? Faces A and B have the same area. Faces C and D have the same area and faces E and F each have the same area. Complete the following activity to calculate the surface area of the rectangular solid.

ACTIVITY 2

Use the two diagrams of the rectangular solid above to complete this activity.

1. The area of face A can be calculated by the formula. Complete:
   Area of face A =
   Area of face B = $l \times h$ = __ × __ = __ cm^2

2. The area of face C can be calculated by the formula . Complete:
   Area of face C = $l \times$ __ = __ $\times$ __ = __ cm^2
   Area of face D =

3. Calculate the area of faces E and F.

4. Now add the areas of all the faces to calculate the surface area of the rectangular solid.

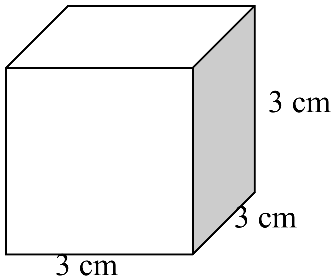
ANSWERS ON PAGE 88

5. Calculate the surface area of the rectangular solid in Activity 1b).

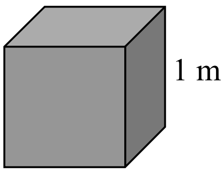
If you have understood the process of calculating the surface area of a rectangular solid, you should not have any problem calculating the **surface area of a cube**. A cube also has 6 faces and each face is a square. Complete the following activity to calculate the surface area of a cube.

ACTIVITY 3

1. All of the dimensions of a cube are equal. The area of one face of the cube below is 9 cm^2.
   Calculate the surface area of the cube.



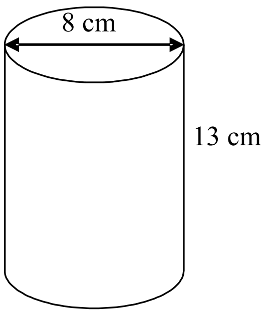
2. Calculate the surface area of the following cubes:
   a)

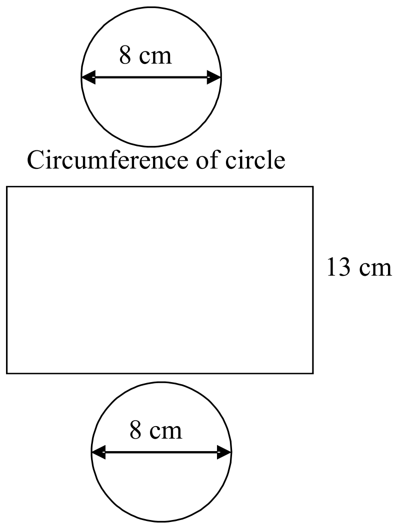


   b) A cube that has one side equal to 10 cm.

ANSWERS ON PAGE 88

Finally, let us look at calculating the **surface area of a cylinder**. Examine the two diagrams of a cylinder below. The first diagram is the 3-D view of the solid and the second diagram is the 2-D view of the unfolded cylinder (the net).





The cylinder is made up of a rectangle and two circles. So there are three areas we need to add together in order to calculate the surface area of the cylinder. Do you remember how to calculate the area of a circle?

Area of circle $= \pi r^2$
The diameter of each of these two circles is 8 cm so the radius of each circle is 4 cm.

$$\begin{aligned} \text{Area of circle} &= \pi r^2 \\ &= \pi \times 4 \times 4 = \pi \times 16 = 3{,}142 \times 16 \\ &= 50{,}272 \text{ cm}^2 \end{aligned}$$

When we unfold the cylinder into a net, we 'roll' out the circles to get the length of the rectangle. Do you remember that this length around the circle is called the circumference? If not, revise lesson 6 of Unit 2. The circumference of a circle is calculated by:

$$\begin{aligned} \text{Circumference of circle} &= \pi \times \text{diameter} \\ &= \pi \times 8\text{cm} \\ &= 25{,}136 \text{ cm} \end{aligned}$$

So the length of the rectangle is 25,136 cm. The width of the rectangle is equal to the height of the cylinder which is 13 cm. So the area of the rectangle is:

$$\begin{aligned} \text{Area of rectangle} &= l \times w \\ &= 25{,}136 \times 13 \\ &= 326{,}768 \text{ cm}^2 \end{aligned}$$

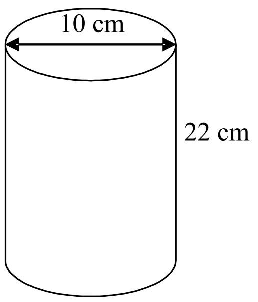
Total surface area of the cylinder:

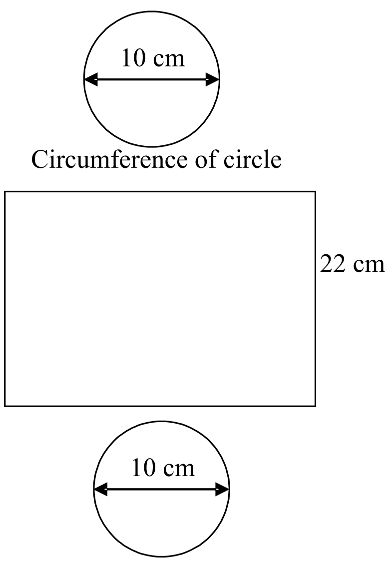
$$\begin{aligned} &\text{Area of circle + Area of circle + Area of rectangle} \\ &= 50{,}272 + 50{,}272 + 326{,}768 \\ &= 427{,}312 \text{ cm}^2 \approx 427 \text{ cm} \end{aligned}$$

ACTIVITY 4

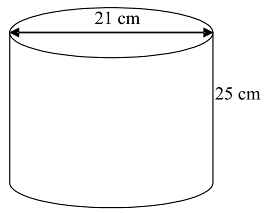
Calculate the surface area of the following cylinders, correct to the nearest cm^2. Use $\pi = 3{,}142$

a)





b)



ANSWERS ON PAGE 89

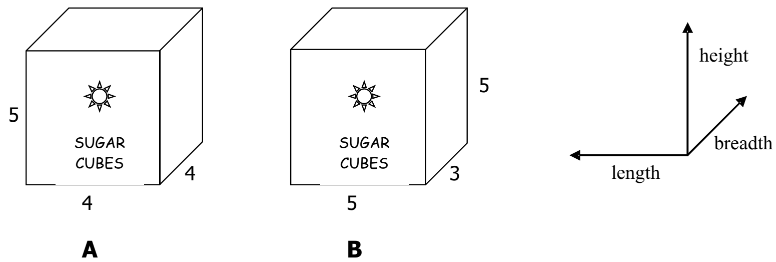
You should now understand and be able to calculate the surface area of a cube, a rectangular solid and a cylinder. We will now move on to the volume of a 3-D shape.

Volume

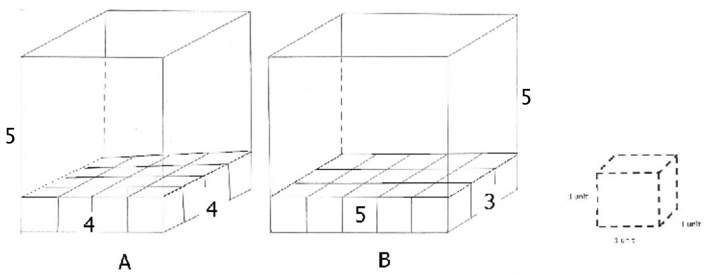
The **volume** of an object is the amount of space that it contains or occupies. Try the following activity.

ACTIVITY 5

Here are two boxes made to hold sugar cubes. Their measurements are in the same units. Which box is bigger?



a) Which box is: longer, broader, taller?
b) Which has the larger face?
c) Which has the larger base?
But which box holds more sugar cubes?
To find out, we will fill each box with sugar cubes like this.



For box A:
d) Put in one layer. How many cubes?
e) How many layers will it hold?
f) How many cubes will it hold?

For box B:
Repeat the above three steps.
g) Which box holds more cubes?
h) So which box is larger - box A or box B?

Remember in the last section we learned a standard way of measuring areas. Remember we used a unit of area.

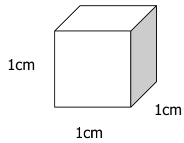
ANSWERS ON PAGE 89

Let us find a standard way of measuring volume by finding the unit of volume.

Unit of volume

The number of cubes gives a measure of the volume of each box, or the amount of space in each box.

This cube has a volume of 1 cubic centimetre, or 1 cm^3.



Other units of volume in the shape of a cube are cubic metres, m^3, and cubic millimetres, mm^3.

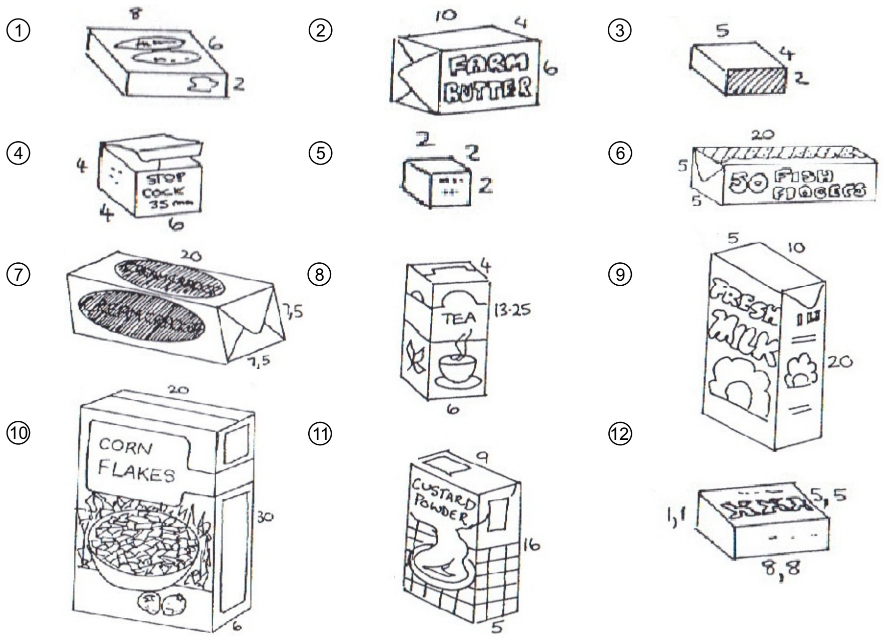
From Activity 5, if the measurements were in centimetres, then:

The volume of box A $= 4 \times 4 \times 5$ cubic centimetres

$= 80\ cm^3$

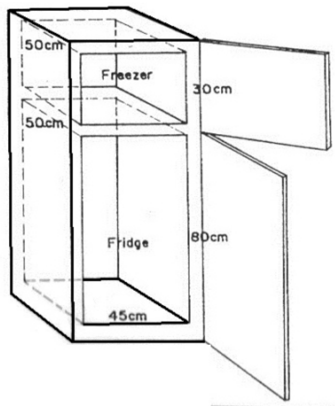
ACTIVITY 6

1. Find the volume of each box. All measurements are in centimetres. Remember to give units in your answers.



2. When fridges and freezers are advertised, we are often told their cubic capacity. This means the amount of space inside them. The larger the capacity, the more food they will hold. Calculate the capacity of:
   a) the freezer
   b) the fridge

   Remember to include units in your answer.



ANSWERS ON PAGE 90

Volume of a cube

The cube below has sides of length 3 cm. There are 3 layers and each layer contains 3cm × 3cm (9 cm^2) blocks. So the volume of the cube is 3cm × 3 cm × 3 cm = 27 cm^3. This means that this cube holds 27 blocks that are each 1 cm^3.

So the volume of any cube is:
$s \times s \times s = s^3$

ACTIVITY 7

ANSWERS ON PAGE 90

Calculate the volumes of the cubes in Activity 3.

Volume of a rectangular solid

As you discovered in Activities 5 and 6 above, the volume of a rectangular solid is equal to the number of cubic centimetre blocks that the solid can contain. It may be easier to remember if you use the following formula:

Volume of rectangular solid is: $l \times w \times h$

ACTIVITY 8

ANSWERS ON PAGE 90

Calculate the volumes of the rectangular solids in Activity 1, question 1.

Volume of a cylinder

If you can identify the base of a 3-D solid, then it is easier to calculate the volume of the solid. The bases of the 3-D shapes we have dealt with (the cube, rectangular solid and cylinder) are simple to identify as they are the 2-D face that defines the solid. In other words:

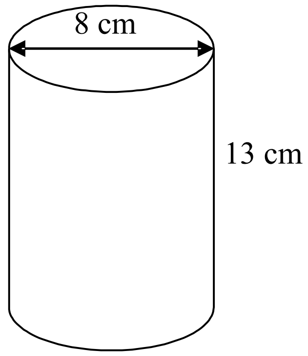
- the base of the cube is a square ($s \times s$)
- the base of the rectangular solid is a rectangle ($l \times w$)
- the base of a cylinder is a circle (πr^2)

To calculate the volume of the solid, you calculate the base area and multiply it by the height of the solid. This is the equivalent of finding the number of cubic units in one layer and then multiplying that number by the number of layers (the height of the solid). So a general formula for the volume of solids is:
V = base area × height

If we apply this to finding the volume of a cylinder we get:

Volume of cylinder = base area × height
= area of circle × height
= $\pi r^2 \times h$

Work through the following example and then complete Activity 9.



The volume of this cylinder is $= \text{base area} \times \text{height}$

$= \text{area of circle} \times \text{height}$

$= \pi r^2 \times h$

$= 3{,}142 \times 4 \times 4 \times 13$

$= 653{,}536$

$= 654\ \text{cm}^3$ (to the nearest cubic centimetre)

ACTIVITY 9

Calculate the volumes of the rectangular solids in Activity 4.

ANSWERS ON PAGE 90

Summary

In this lesson you learned that three dimensional (3-D) shapes are shapes that have 3 dimensions and that they are called solids. You learned about three specific solids; the cube, rectangular solid and cylinder. You learned how to calculate the surface area and volume of these solids.

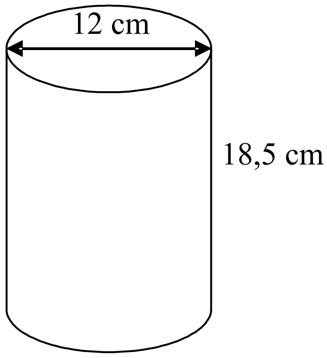
After this lesson you should be able to:

- know that a 3-D shape is also called a solid
- know and understand the meaning of surface area, volume and capacity
- calculate the surface area of a cube, a rectangular solid and a cylinder
- calculate the volume of a cube, a rectangular solid and a cylinder.

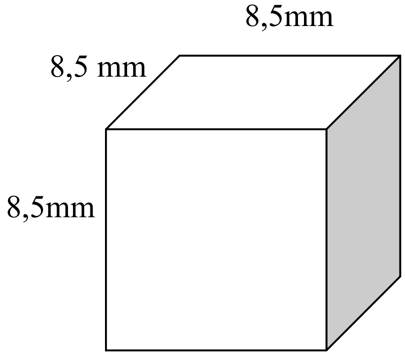
SELF-CHECK EXERCISE

1. Calculate a) the surface area and b) the volume of the following 3-D shapes.

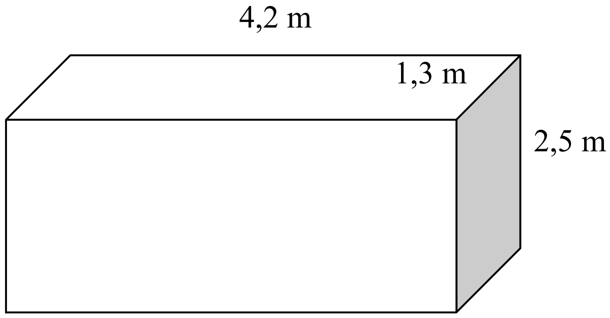
i)



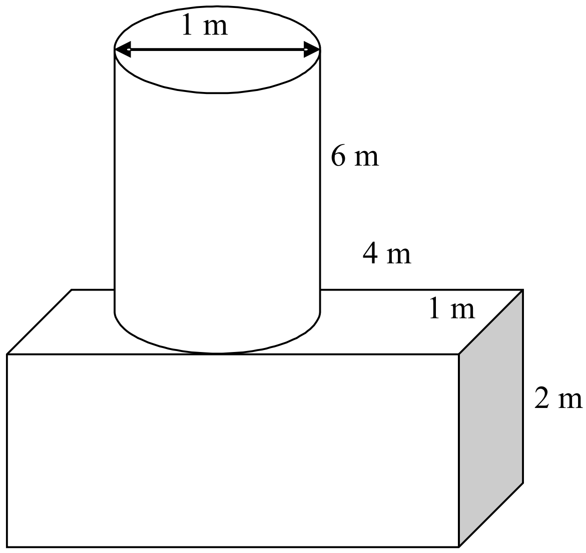
ii)



iii)



iv)



Hint: when you calculate the volume of two 3-D shapes joined together, find the total of the volumes of each of the shapes. Remember though that surface area is the exposed outside surface of the shape (the outside area that you can paint) so think carefully about what to do with the area where the cylinder is joined to the rectangular solid.

ANSWERS ON PAGE 78

5. Plans

Introduction

When someone draws a plan to build a house, she first makes a scale drawing so they can decide how they want their house to look. This will give her an idea of what the house will look like, before it is built. She may want to compare the sizes of the rooms or look at different ways of placing the rooms before she decide which part of the house will be where.

Do you remember that we looked at scale in relation to reading and drawing maps in Unit 3, lesson 1? If you understood that lesson, you should have no problem working through this lesson.

In this lesson you will:

- learn what a plan is
- draw a plan
- interpret and do calculations from a plan

Drawing plans to scale

Thandi wants to know how many concrete blocks she can fit in a row in the fenced area where she is building. She wants to know how many blocks she needs to make the bottom row of the outline of the building. The length of Thandi's fenced plot is 20 m and the width is 10 m. She wants to leave a 1 m pathway round the building to grow flowers and to make place for people to walk. Each concrete block is 1 m long and half a metre wide. If Thandi has already made 45 blocks, how many more would she have to make to complete the outline of her house out of concrete blocks?

Think about how you can solve a problem like this. First make a drawing of the fence and the building. We call this drawing a *plan*. Then work out the number of blocks that will make up the walls of the building. Work out the solution to this problem step by step. We will go through the steps together in the following activities.

ACTIVITY 1

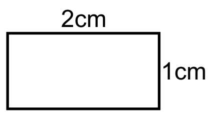
First draw a plan of the building, the pathway and the fence.

Thandi wants to see how much space there will be between the fence and the house, compared to the space inside the house. To compare these different spaces she must draw the house and fence to scale (in proportion).

Let's draw the plan to scale before we work out the number of blocks needed for the building.

Seca took a piece of Thandi's notebook and showed her how to work out a scale for the house so that the drawing will fit on the piece of paper. The piece of paper in Thandi's notebook is 30 cm long and 21 cm wide. It is the same size as the page you are reading now. Use your ruler to see if this is true. So the scale drawing or plan can't be more than 30 cm long and 21 cm wide.

We have to show a fence 20 m long and 10 m wide. If we change these metre lengths into centimetre lengths we will fit the drawing on the paper if we make the ratio 1 cm : 1 m. Then the length of the rectangular fence on the drawing will equal 20 cm and the width will equal 10 cm.
Draw this now.



This drawing is smaller than yours if you used 20 cm and 10 cm. Here the fence is 2 cm long and 1 cm wide to save space. Can you see that we have made both the length and the width the same number of times smaller? By doing this we made sure that the width and the length of the drawing are in the right proportion for the real fence. This is what it means to draw the fence to scale.

The length and width here are also both the same number of times smaller than the real fence.

How many times smaller than the real fence is your scale drawing? For every metre you have drawn 1 cm. There are 100 cm in a metre so your scale is 1 cm for every 100 cm (1 m). We write this mathematically as 1:100 cm. It means that your scale drawing is 100 times smaller than the fence is in real life. So you can work out the size in real life by multiplying your scale drawing by 100. This helps people who are reading maps when they want to see how big something is in real life.

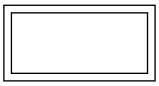
What about our scale drawing? This scale drawing is ten times smaller than yours.

Check your thinking: 2 cm is ten times smaller than 20 cm and 1 cm is ten times smaller than 10 cm.

If your drawing is 100 times smaller than the real fence and our drawing is 10 times smaller than yours, then this scale drawing is 1 000 times smaller than the real fence. 10 × 100 = 1 000. We would write this scale mathematically as 1:1 000 cm.

ANSWERS ON PAGE 90

Let's draw in our building. We must change every measurement from metres to centimetres so that the drawing stays in proportion. So the distance of the building from the fence becomes 1 cm. Now you can draw in the outside line of the building.



This drawing was not as easy as yours. You had to draw 1 cm all the way around your building, but this one is ten times smaller or $\frac{1}{10}$ of the length of your 1 cm. Have another look at your ruler. Can you see the ten points in between your centimetres? Since we need $\frac{1}{10}$ of your centimetre we need one of the 10 parts that make up the centimetre. What are these parts called? Remember 1 cm = 10 mm. So the width of the pathway in this drawing is 1 mm.

Interpreting plans

Let's find out what the width of the real path is, using this scale. The scale is 1:1 000, so the real pathway will be a thousand times wider than the drawing above, 1 mm × 1 000 = 1 000 mm. 1 000 mm = 1 m. This gives the correct width for the real pathway.

ACTIVITY 2

What real length and width is left inside the fence for the building? Have a look at your drawing again. Measure the length of the lines of the building and use your scale to find their real length. Notice that there is 1 m between the fence and the building on both sides. This means that the building is 2 m shorter than the fence and also 2 m narrower than the fence.

ANSWERS ON PAGE 91

ACTIVITY 3

Now let's check how many concrete blocks Thandi will be able to fit along the width and length of the outline of the building. Remember, each block is 1 m long and half a metre wide. Remember also that the building is 18 metres long and 8 metres wide. Draw the blocks onto your plan to the correct scale and count them.

What other way can you get the number of blocks that will fit?

ANSWERS ON PAGE 91

ACTIVITY 4

Think what would happen if Thandi had the width ($\frac{1}{2}$m) facing the outside.

ANSWERS ON PAGE 92

Calculations with plans

If you have a scale and a drawing, can you work out the size of something in the drawing as it is in real life?

ACTIVITY 5

Try this! In a drawing on a big piece of paper a fence is drawn as 40 cm long. The scale of the drawing is 1:20. What is the fence's real length a) in centimetres and b) in metres?

ANSWERS ON PAGE 92

Summary

In this lesson you have learnt about plans (also called scale drawings) and how to draw and interpret them. In the following unit you will build on this knowledge and understanding in a lesson on plans and elevation that involve more technical drawing.

Self-assessment checklist:

Are you able to:

- recognise a plan
- draw a plan
- interpret plans
- do calculations using plans.

SELF-CHECK EXERCISE

Part A

1. You are given a scale drawing of a house with a scale of 1:500 cm. How many times smaller than the real house is the scale drawing?

2. If the kitchen length in your scale drawing is 5 cm, work out the real length of the kitchen. Remember that 100 centimetres = 1 m.

Part B

1. If Thandi places the half-metre side of the bricks facing the outside when she makes the building, how many bricks does she need?

2. Make a scale drawing of 1 cm = 100 cm (1:100) for a fence for a plot of 12 m length and 8 m width and place the building 1 m from the fence. Try and work out the area of the floor of the building.

ANSWERS ON PAGE 79

6. Plans and elevations

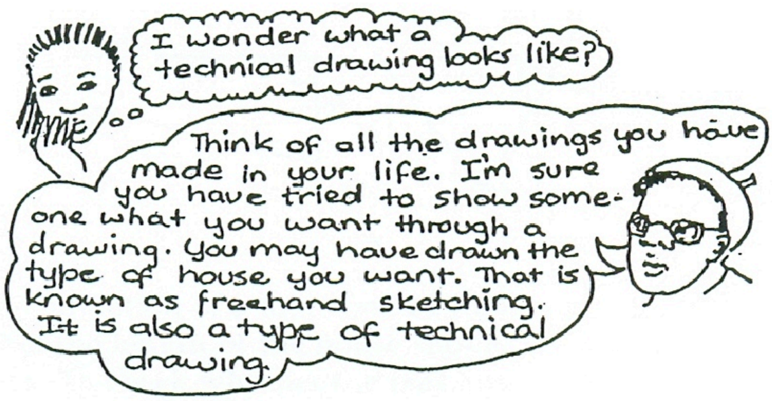
Introduction

If you want to be a builder and to draw plans like an engineer, this lesson will help with that. In the previous lesson we looked at drawing scale drawings (plans) that help you, for example, when you make alterations to your house.

This lesson is about drawings made by many engineers and other people who do practical work. If we want to draw or interpret these drawings, we have to understand a kind of drawing called **technical drawing**. Some people call it an **engineering drawing** or a **building drawing**. Plans and elevations are part of technical drawing. Computers have also become powerful tools for this type of drawing.

Interpret: understand

A technical drawing is a way of communicating. Instead of words, people use symbols. This language can be understood by everyone, no matter what language they speak if they can read the drawing.



Communication through plans is called a **graphical language**. Graphical means 'something that is drawn'. You have been using this language for many years already. You have drawn so many things in your life you may not be able to count them: bicycles, tables, roads, when giving directions to a friend, and so on. We will now improve those skills. You will learn more symbols in order to communicate with many people who understand technical drawing.

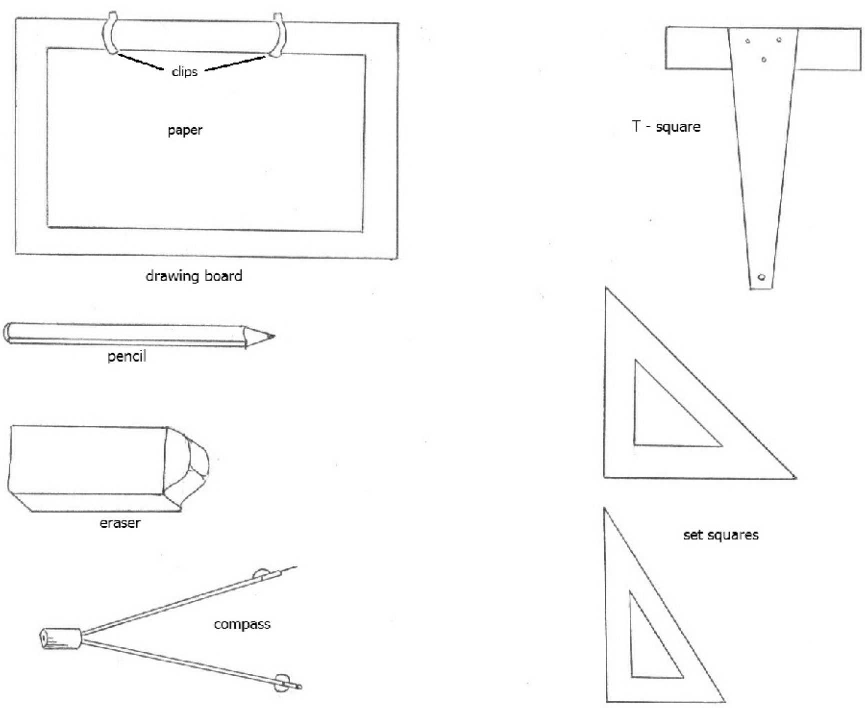
For this lesson you will need to remember geometric shapes and scale drawing of maps.

You will need practise to develop your drawing skills. This is not difficult. If you draw a circle using your compass, it is very difficult the first time and you may make many mistakes. The second time it gets easier, the third time easier still, and so on. Eventually it becomes so easy you do not need to think about the process.

In this lesson you will:
- give exact information about objects using drawings
- understand and use dimensioning
- use projections to give information
- draw plans and elevations of objects
- use developments to make copies of objects.

Who makes technical drawings?

A person who makes technical drawings is called a draughtsperson. The different types of equipment used by a draughtsperson are a drawing board, set squares, a pair of compasses, a pen, a tee square, a pencil and a rubber. A draughtsperson uses these instruments to give information about the exact sizes of things, how they work, how they fit together and what materials are used. Below are sketches of these instruments.



Try to go to a place where you can see some of these instruments, if you have not seen them before. You will find them in the office of a draughtsperson, architect or engineer, or in a shop that sells drawing equipment or in a technical college. Many draughting people use draughting machines. These make drawing an easy process.

We have so far spoken about draughtspersons. Are there other people who use technical drawings? Yes, of course. A draughtsperson is one who draws plans. These plans may be drawings for buildings, machines, and so on.

Even those who draw plans for legal and parliamentary documents are called draughtspersons.

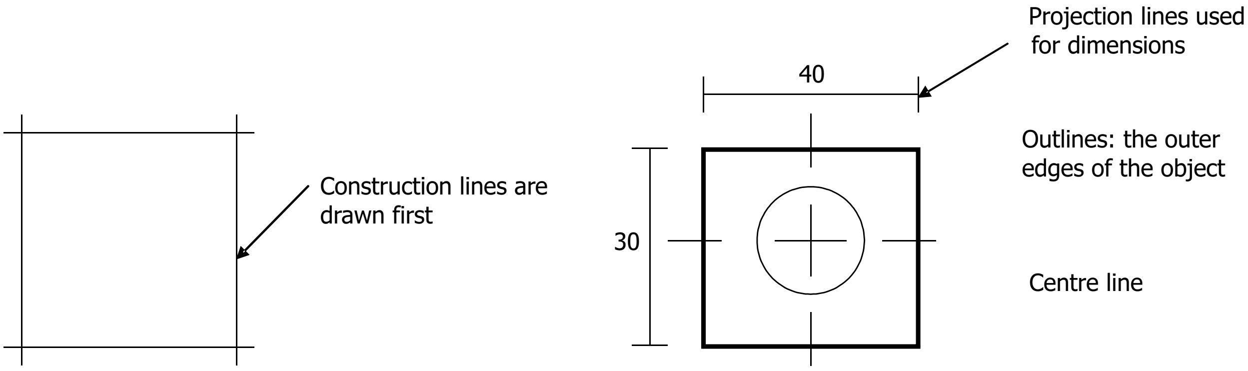
ACTIVITY 1

Write down six different acitivities where people could use technical drawings. Write down what they might design.

ANSWERS ON PAGE 92

Lines, shapes and forms

There are different types of lines used in technical drawing. Some of these lines you may have used or seen many times before. These lines are used for different purposes. Here they are:



Then other lines are drawn in.

Construction lines are very thin, light lines. These are the first lines used in construction. They are drawn so thinly that it is usually difficult to see them.

Outlines are dark, thick lines. They are used to show the exact shape of an object. Outlines are usually drawn along part of a construction line.

Projection and dimension lines are thin lines in between a construction line and an outline. They are used for giving measurements.

Centre lines are thin, long broken lines. They are used to show the exact centre of what is being drawn.

Dotted lines show hidden detail of parts of an object which you cannot see from the outside. They may be outside but still hidden from the side you are looking at.

Impressions: ideas

Fragile: something that can easily break

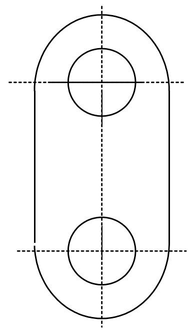
Dense: lots of detail

Lines are used to create shapes. Types of lines give different impressions about the object. Heavy lines can make an object look heavy and solid. Faint, thin lines can suggest a light, fragile object that can easily break. Sometimes light and shade on an object are shown by drawing lines on one (dark) part of the object, and leaving the other part clear (light). Circles are used to attract the eye towards the centre. That is why numbers are usually circled in very dense drawings.

The drawing on the previous page shows a drawing using different lines. The construction lines are barely visible, especially after the outline has been drawn in.

ACTIVITY 2

Copy the following drawing. Do not use tracing paper. Use construction lines, outlines, projection, dimension and centre lines in your construction. Do not worry about the actual size of the object, as long as the sides are in proportion. Choose a scale for your drawing so that it will fit on your page.



Dimensioning

What does the word dimension mean? It simply means measurement. The main reason why draughtspersons have dimensions in technical drawings is to give exact sizes of the things they have drawn. Dimensioning is important. Imagine going to buy a carpet without knowing the dimensions of your room! Very unwise.

Technical drawings are dimensioned in different ways:

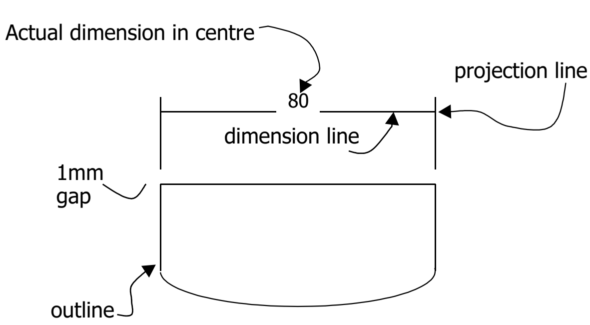
- Dimension lines are used to tell you exact distances between points. The units of measurement used in most drawings are millimetres. Do you remember scale drawings of maps and plans? Unit 3 and the previous lesson have the background information for understanding dimensioning. You will need that background.

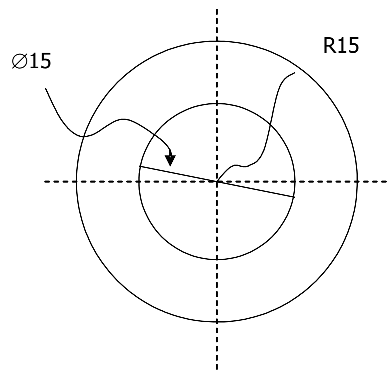
- Symbols and signs are used to give information about the measurements used. The most common symbols are:

  ∅ which means diameter

  R which means radius

These symbols are put in front of the number, for example, *R*.40 on a drawing simply means the radius of the object is 40 mm. The following diagrams show you the basic steps in a drawing with simple dimensions.





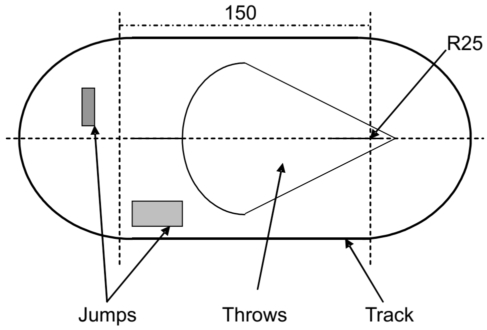
Note: People who use these kinds of drawings put dimensions in their drawings as simply as possible.

ACTIVITY 3

Look at the two diagrams on the previous page. Use them as a guideline. Then try to draw something on your own and put in the dimensions.

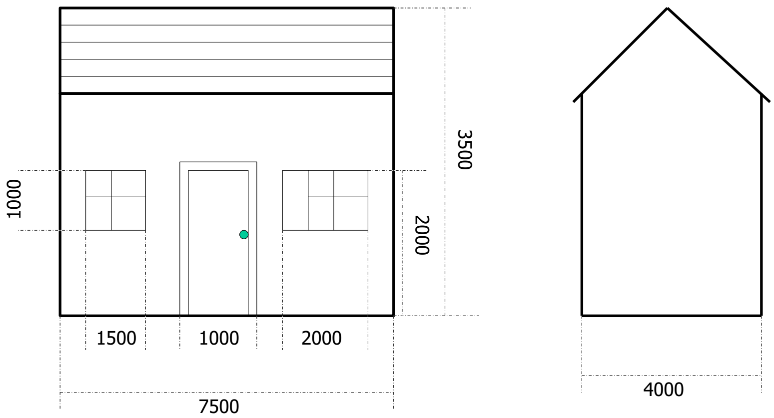
ANSWERS ON PAGE 92

Here is an athletics sports-field. The owner of the company that constructs the stadium has asked you to draw the sports-field, making l = 150 metres, and b= 50 metres. Do not forget the different types of lines. Draw the field as neatly as you can. You can assume that the rounded ends are semi-circles.



ACTIVITY 4

We also use technical drawings to get information about a building or object. Look at the drawing of a house below and then answer these questions.



1. How wide is the front part of the house?
2. How high is the house?
3. How high from the floor is the bottom of the windows?
4. How wide are the windows?
5. How high is the door?
6. How wide is the door?

ANSWERS ON PAGE 93

Projections

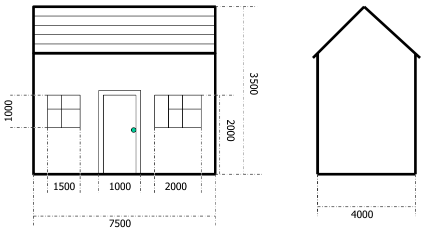
Most objects that we make are not flat. They have length, width and height. Drawing something like that is not easy. The art of making drawings of these complicated objects on a flat surface, like paper, is called *making a projection*.

In this lesson we will learn about two types of projections, called *orthogonal* and *isometric projections.*

Orthogonal projections

Engineers use orthogonal projections when they want to show the way an object actually looks.

Let us take a look again at the house in Activity 4.



These are the shapes you will see if you are standing in front and at the side of the house. As you can see, the drawing of the front of the roof does not show that it is going up at an angle. The angle of the roof is shown on the drawing of the side of the house.

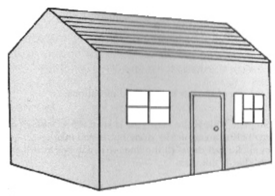
The word orthogonal means drawings which are at right angles to each other. Orthogonal projections are the working drawings of engineers. This is the method that you will have to master in technical drawing.

ACTIVITY 5

Make an orthogonal projection of your home with all dimensions. Explain your drawing in full to a friend.

Isometric projections

Isometric projections show people what objects look like from more than one side. Look at the drawing below and compare it with the orthogonal projection of the same house in the previous section on orthogonal projections.

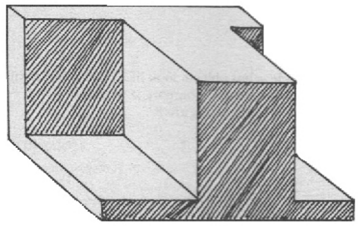


You will note that this drawing shows each side in the same drawing: the front, the end and the top. Observe that you cannot see any right angles in the drawing. Isometric drawings show shapes without true angles.

Plans and elevations

So far we have only covered the two main methods used in technical drawing. Let us now look at the terms used in technical drawing.

Projections that are drawn are known as **plans** and **elevations.** There are two systems used, an American and a British system. We will deal with the easier one, the American system, which is called the *third angle projection*. Look at the following drawing.



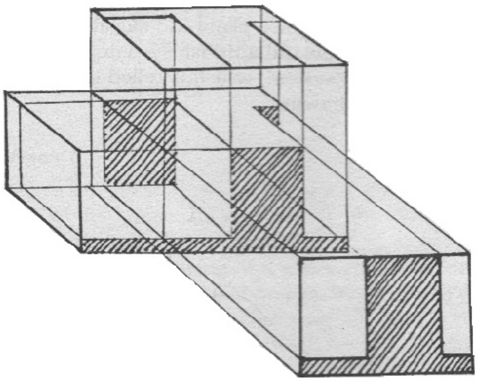
This drawing can be represented with its plan and elevations.

(a) The plan
There can only be one plan. This is the view that is seen when looking at the object from above. If you take an object, look at it from above and draw all the lines that you see as if they are on the same plane; that is the plan.

(b) Front elevation
The front elevation is the side of the object that gives the most information about that object. In a drawing of a house the front of the house is usually given as the front elevation.

(c) End elevation
The end elevation is the view from the side. The tricky part is that you draw the lines as if they are on the same plane.

The following drawing shows you what we mean.



We have drawn the elevations and the plan of the same block. Look at how they seem to be on a sheet of paper.

Remember, the front elevation is not always the front part of an object. For example, the front elevation of a car is its side, because the front elevation is the side that shows the most information about the object.

ACTIVITY 6

Make a drawing of a car of your choice. Show its plan and elevations. Label your drawing.

ANSWERS ON PAGE 93

Developments

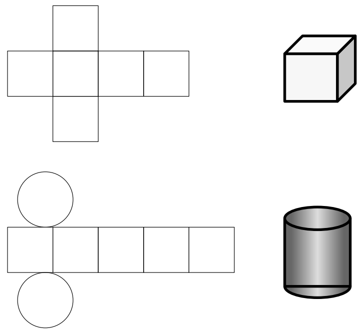
A development is a true shape of a flat piece of material that is used to make an object. Developments of objects are models of the object. They are also called nets of the objects (remember we touched on these in lesson 4 of this unit?)

We make the development using the following steps:

1. A drawing of the object is made on the material (paper, cardboard, metal, fabric) that you want to use to make the object or model of the object. This drawing should give the exact surface area of the object.
2. The drawing is cut out along its edges.
3. The cut-out sheet is then folded to make the shape of the object.

Note:
$C = d$ or $2r$.
C = circumference
d = diameter
r = radius

Here is an example of how we would make developments of a cube and a cylinder.



Look at the cylinder. There is no problem with finding the width of the rectangle, which will be the height of the cylinder you'd like. The length of the rectangle should be equal to the circumference of the circle.

In most cases we make the length a little longer than necessary. We use the extra length as joining tabs which we glue together, or we can add in extra flaps.

Summary

Let us see whether you can remember the main points of the lesson. We have said that plans and elevations are just a part of what is known as technical or engineering drawing. These are drawings that are made, for example, by engineers. People who specialise in making drawings are called draughtpersons. The drawings should have dimensions that give the necessary measurements to help anyone who wants to build the object.

There are two main projections that are used by engineers. These are orthogonal and isometric projections. In order to understand a drawing further, plans and elevations of the object are given. A plan is a drawing that shows how the object looks from above. Elevations give the sides of the object. The system that we have chosen is the American system, which is easier to understand.

For more information on how the object looks in reality, developments are made. These are made from flat sheets of material, either paper or metal after a net has been drawn.

Self-assessment checklist:

After this lesson you should be able to:

- indicate dimensions on drawings
- read dimensions from a technical drawing
- draw orthogonal projections of objects
- draw isometric projections of objects
- draw plans and elevations of objects
- make developments of simple geometric shapes.

SELF-CHECK EXERCISE

1. Look at the plan and front elevation of a mini-van car.

   a) Write down which figure shows the plan.

   b) Write down which figure shows the front elevation.

   c) The plan of this car shows that it was designed to seat ____________ people.

   d) The scale of this plan is 1 : 40 cm or 1cm : 40 cm
   Use the dimensions shown on the plan and elevation below to calculate the actual:
   i) length of the car (in metres)
   ii) width of the car (in metres)
   iii) height of the car (in metres)

Figure A

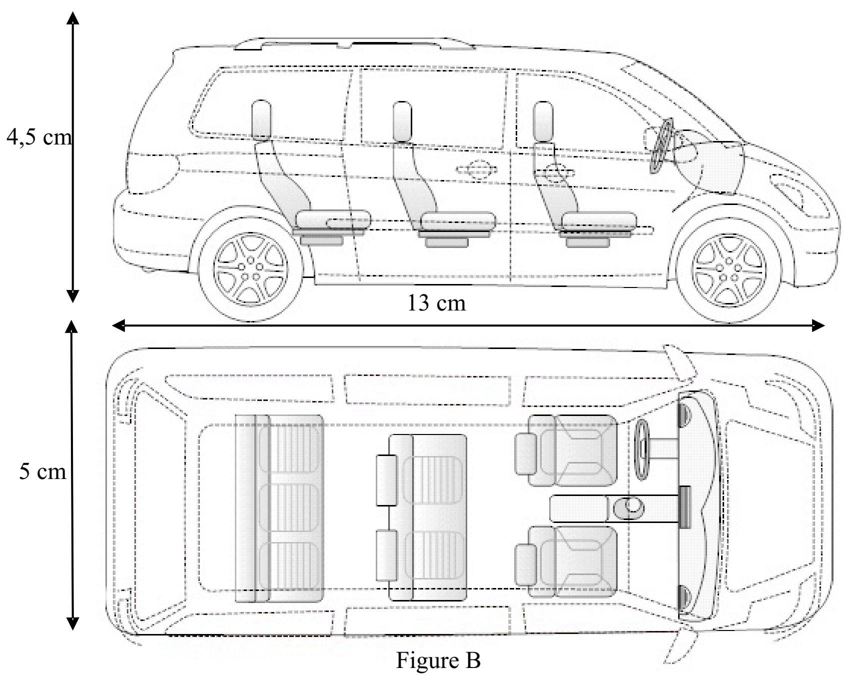
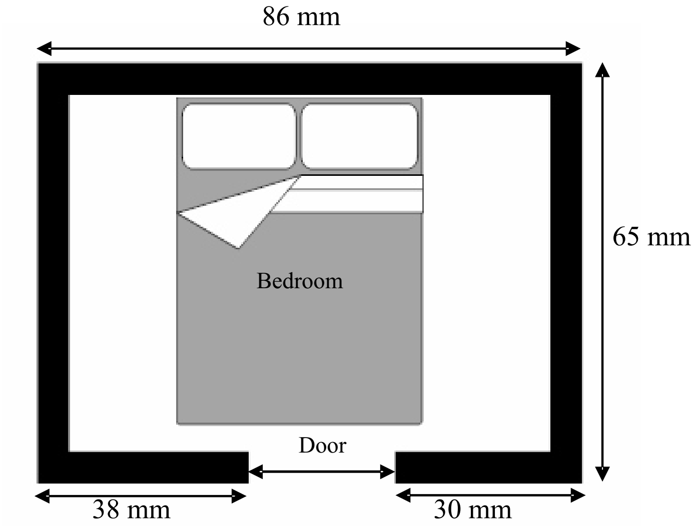


Figure B

2. Below is the plan of Anna's bedroom. The scale is 1 : 50 mm.



Give the following actual dimensions of Anna's bedroom in metres:

a) the length
b) the width (or breadth)
c) the width of the door opening
d) the area of the bedroom

ANSWERS ON PAGE 80

7. Averages and spread

Introduction

The general idea of an average is quite familiar to you from everyday life. Have you heard people say they are above average height? Why do they say that? Because they feel most people are shorter than they are. You may feel your shoe size is about average because there are always plenty of shoes to choose from in your size. Somebody may also say he earns below average because he thinks many of his friends earn more money than he earns. There are many other examples of averages that you come across every day.

This lesson is a direct continuation of lesson 7 of Unit 2 and lesson 6 of Unit 3. In this lesson we will describe three common types of average. These are the mean, the median and the mode. We will also deal with the range. The range is used to describe the spread of data.

In this lesson you will:

- understand averages in statistics
- find the mean, median and mode of a set of data
- find the mean, median and the mode from a simple frequency table
- understand what we mean by spread or dispersion in statistics
- find the spread or range of a set of data.

Averages

In Units 2 and 3 you learned to represent a set of data on a frequency table and on a pie chart. A single value is also very often used by statisticians to represent a set of data. This value summarises the data and it is called the average.

For example, if statistics show that farmworkers in Limpopo are paid amounts ranging from R300 a week to R3500 a week, the statistician may look for the wage that most of the farmworkers receive. If most of them are paid about R1 000,00, he will say that the average wage in that part of the country is R1 000,00 per week. An average should be a representative of a set of data. It should show what is generally true about the data so we take it as the central or the most frequent occurring value.

The mean

The mean is sometimes called the *arithmetic mean*. It is the average that is used most often. We can find the mean by finding the total of the set of data and dividing this total by the total number of data.

That is:
mean = (sum of data) ÷ (number of data items)
mean = (total when the results in a set of data are added) ÷ (number of results in the set of data)

Example
8 people who do similar jobs but in different places, have the following weekly wages: R669, R867, R1 315, R512, R585, R615, R979, R710.
Calculate the mean wage that the people earn.

Solution

Sum of data:	669 + 867 +1 315 + 512 + 585 + 615 + 979 + 710 = 6252
Number of data items:	8
Mean wage:	$\frac{6252}{8} = 781{,}5$
	R781,50

ACTIVITY 1

Find the mean of:
1. a) 5, 7, 3, 13
   b) 305, 307, 303, 313
   c) 12 km, 15 km, 25 km, 18 km, 15 km
   d) The rainfall in June for 5 years for a certain place was 48 mm, 23 mm, 62 mm, 47 mm and 56 mm. What is the mean rainfall for that place in the month of June?

ANSWERS ON PAGE 93

The median

A simple way to find an average of a set of data is to arrange the data in order of magnitude (size) and pick the middle value. This order can be from the smallest to the largest or from the largest to the smallest. To make things easier for us we shall stick to the order from smallest to the largest.

The median is the middle value when the values are arranged in order of size.

If the number of observations is odd there will be only one middle value. This value is the median. For example, the median of 4, 5, 7, 9, 14 is 7. But if the number of observations is even, then there will be two middle numbers. To find the median we add these two numbers and divide the total by 2.

Example
Find the median wage of the 8 people who do similar jobs but in different places in the example in the last section.

Solution
We had the following data:
R669, R867, R1 315, R512, R585, R615, R979, R710

When we arrange the data in order of magnitude we get:
R512, R585, R615, R669, R710, R867, R979, R1 315

We have underlined the two middle amounts.

So the median wage = $\frac{669 + 710}{2} = \frac{1379}{2} = R689,50$

(We add the two middle amounts and divide them by 2 to get the median.) Here is another example.

Example
In a week a shop owner sold 39, 48, 36, 49, 42, 46 and 51 cans of Coke each day. Find the median daily sale of cans of coke.

Solution
When we arrange the amounts in order of size, we get:
36, 39, 42, 46, 48, 49, 51
The median is 46 because this is half-way along the list. There are three values before it and three values after it.

ACTIVITY 2

1. These are the masses in kilograms of five one-year old babies:
   8, 10, 13, 9, 11
   Find the median mass of the babies.

2. Find the median of the following sets of numbers:
   a) 8, 3, 4, 7, 6, 4, 8, 4, 2, 5
   b) 7, 6, 2, 6, 4
   c) 93, 91, 91, 90, 85, 82, 80

ANSWERS ON PAGE 94

The mode

In a set of data, the mode is the value or number that occurs most often.

Example
These are the ages in years of 10 workers of a company:
25, 27, 30, 25, 36, 40, 36, 44, 36, 50.
What is the *modal* age?

Note:
Modal - is the adjective of mode.

Solution
The modal age is 36 years because this is the age that occurs most often. Three workers are 36 years old.
Note that it is possible to have more than one mode for a set of data.

Example
If the ages of 11 workers in a firm are:
25, 25, 27, 30, 25, 36, 40, 36, 44, 44, 36, 50
then there are 3 workers each aged 25 and 3 workers each aged 36. In such a case we have two modes: 25 years and 36 years.

Bi - two

A set of data with more than one mode is said to be 'multimodal'. A set of data with two modes is 'bimodal'.

ACTIVITY 3

1. Find the mode or modals of the following sets of numbers:
   a) 3, 5, 7, 6, 8, 7, 1, 3, 5, 7, 2
   b) 12, 11, 13, 11, 14, 12, 13, 13
   c) 7, 6, 5, 5, 19, 7

ANSWERS ON PAGE 94

So far you have learnt how to find the three kinds of average, namely the mean, the median and the mode from a set of data. We said:

- The mean of a set of values is the sum of all the values divided by the number of values.
- The median is the middle value when the values are arranged in order of size.
- The mode is the value that occurs most often.

In the next section you will learn to find these averages from a simple frequency table. Before we do that let's take an example that considers all the three averages.

Example
The monthly wages of ten junior staff members of a big company are as follows: R5000, R8000, R7000, R7000, R6000, R10 000, R7000, R6000, R9000, R4000.

Find:
a) the mean monthly wage
b) the median monthly wage and
c) the modal monthly wage for the junior staff members

Solution

a) Mean wage = $\dfrac{\text{sum of all the wages}}{\text{number of the staff members}}$

$$\frac{5000+8000+7000+7000+6000+10000+7000+6000+9000+4000}{10}$$

$= \dfrac{69000}{10}$

= R6900

The hypotenuse is the name given to the side of a right-angled triangle that is opposite the right angle.

b) Arrange the wages in order of size:
4000, 5000, 6000, 6000, 7000, 7000, 7000, 8000, 9000, 10 000.

The median wage = $\dfrac{7000+7000}{2} = \dfrac{14000}{2}$ = R7000

c) Clearly R7000 appears most often. Therefore the modal wage is R7000.

ACTIVITY 4

1. Find the mean, median and the mode for each set of data:
   a) 8, 9, 12, 12, 11, 16, 9, 7, 12, 12, 13

   b) Test marks of 60%, 75%, 60%, 89%, 87%

   c) 107, 98, 100, 102, 100, 99, 100, 102

2. The weekly rent for each of 8 occupants of a house is as follows:
   R350, R325, R300, R325, R325, R300, R300
   Find:
   a) the mean
   b) the median and
   c) the mode of the weekly rents

3. Nine sacks of potatoes have a mean weight of 25,2 kg. A tenth sack weighs 27,3 kg. What is the mean of all ten sacks?

ANSWERS ON PAGE 94

Averages from frequency tables

In lesson 7 of Unit 2 you learned how to draw a frequency table for a set of data. You are now going to learn how to use a frequency table to get the averages.

The mode

Example
Let us consider the example on the size of family of the 20 school children in lesson 4. We had a frequency table like this one:

Size of family	Frequency
1	3
2	7
3	6
4	2
5	1
6	1
Total	20

Can you tell the modal size of family from this table?

We said that the mode is the most frequently occurring value. This means that the value with the highest frequency is the mode. Therefore from the table the mode is the family of 2.

Do you see how easy it is to find the mode from a frequency table? All you do is look for the value with the highest frequency. In fact, whenever you want to find the mode of a set of data, you can draw a frequency table for the data and pick the value with the highest frequency.

ACTIVITY 5

The frequency table of marks scored by a student is given below.

Marks	Frequency
2	3
3	5
5	2
Total	10

ANSWERS ON PAGE 95

Find the modal mark.

The median

The median is the middle number when the data are arranged in order of size. Remember that if the number of observations is odd there will be only one middle value. This value is the median. For example, the median of 4, 5, 7, 9, 14 is 7.

But if the number of observations is even, then there will be two middle numbers. To find the median we add these two numbers and divide the total by 2.

In order to make things easier for us we can take the median to be the $\frac{n+1}{2}$ *th* value.

Example
Let us go back to our "size of family" example.

Size of family	Frequency
1	3
2	7
3	6
4	2
5	1
6	1
Total	20

The total frequency, $n = 20$ is even so there will be two middle numbers. To find the two middle numbers we can say the median is the $\frac{n+1}{2} = \frac{21}{2} = 10{,}5th$ value after arranging the data according to size. This means we take the average of the 10^{th} and 11^{th} numbers by adding them together and dividing by 2.

From the table, the frequency of 1 is 3 and the frequency of 2 is 7, and so on. So if you arrange the data in order of size you will get:
1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3, 4, 4, 5, 6

The 10th value is 2. Therefore the median is $\frac{2+3}{2} = \frac{5}{2} = 2{,}5$.

ACTIVITY 6

20 people gave the size of the shoes they wear. The results are set out in this table:

Shoe size	Frequency
5	3
6	4
7	6
8	3
9	3
10	1
Total	20

Find a) the modal shoe size
b) the median shoe size

ANSWERS ON PAGE 95

The mean

Let's use an example to illustrate how we can find the mean from a frequency table.

Example
The manager of a small business drew up a frequency table for the weekly salaries of her workers and got the following table.

Monthly salary (in Rand)	Number of workers/frequency
5 000	1
3 000	2
1 025	3
760	2
Total	8

Find the mean weekly salary for the workers.

Solution
Remember we said the mean of a set of data is given by:

$$\text{Mean} = \frac{\text{sum of all the results}}{\text{total number of results}}$$

The total number of results is the total frequency. In this case the frequency is 8.

How can we find the total of all the results? We need to multiply each value by its frequency to get the total for each group. This will give us the total earned by all the workers for each different salary. Therefore to find the solution we need to create another column with the heading: 'salary × frequency'. The total of such a column is the total of all the results. Our new table is:

Salary	Frequency	Salary × frequency
5 000	1	5000
3 000	2	6 000
1 025	3	3 075
760	2	1 520
Total	8	15 595

Therefore the mean weekly salary is:

$$\frac{15595}{8} = \text{R1 949,38}$$

ACTIVITY 7

Tubes of toothpaste are made by a machine in a factory and do not always all have the exact correct weight. This table shows us the number of grams overweight of a sample of 45 tubes of toothpaste. Find the mean and the median amount overweight.

Grams overweight	Frequency
0	4
1	8
2	12
3	11
4	6
5	3
6	1
Total	45

ANSWERS ON PAGE 96

Measures of spread

We have seen that an average tells us something general about a set of data. However, an average fails to give the whole story of the set of data. The average only tells us about the central value. We need some additional measure which will tell us how the individual observations are spread among themselves. These measures may also give us an idea of how the observations spread around the average. These measures are called *measures of spread*.

The range

The range of a set of data is the difference between the highest value and the lowest value. The range is a very common measure. This is illustrated in everyday life by such statements as 'the price of a kilogram of mutton ranges from R6,99 to R10,99'.

Example
Find the range of the workers' salaries in the last example.

Solution
The highest salary is R5 000.
The lowest salary is R760.
The range = R5 000 – R760
= R4 240.

ACTIVITY 8

1. Find the range for these clothes sizes of 10 women:
   14, 12, 12, 16, 10, 8, 12, 10, 12

2. Find the range in the set of data:
   6, 5, 2, 14, 6

ANSWERS ON PAGE 96

Summary

In this lesson you have learned about three averages.

The median is the middle value of a set of data arranged in order of size. The mode is the value that occurs most frequently in a set of data. It is easily found after drawing a frequency table for the set of data. In that case the mode is the value with the highest frequency.

The mean is the: $\frac{\text{total of the values in a set of data}}{\text{number of values in a set of data}}$

You have also learned to calculate the mean from a simple frequency table.

You know that the range of a set of values is the difference between the highest and lowest values. That is:
range = highest value – lowest value
The range is a measure of the spread of a set of data.

The methods of calculating these are very simple. But you may find it hard to remember all the different names used for them. The best way to solve this problem is for you to practice the activities in this lesson, and then make up your own examples of statistics and calculate the mean, median, mode and range. Also try to read newspaper reports about statistical information, and see if you can find the mean, median, mode and range of the sets of data.

Self-assessment checklist:

After this lesson you should be able to:
- calculate the mean, median and mode of a set of data
- use frequency tables to help calculate averages of data
- calculate the range of a set of data.

SELF-CHECK EXERCISE

1. Find the mean, median, mode and range for the following sets of data:
   a) 53, 58, 50, 61, 55, 50, 52, 61, 50
   b) 5, 23, 26, 25, 21, 23, 25

2. The price of the same weight of a sliced loaf of bread in 30 different shops was found to be as follows:

Price of loaf	Number of shops/frequency
R8,10	4
R8,30	6
R8,40	5
R8,90	5
R9,00	2
R9,20	3
R9,30	5

Find the mean, mode, median and range for the price of bread.

3. The goal record for a team is as follows:

Goals	0	1	2	3	4
Matches/frequency	5	5	4	2	2

If 18 matches were played altogether, find:

a) the number of matches in which two goals were scored
b) the total number of goals scored in these 18 matches
c) the mean number of goals per match
d) the range of goals scored by the team

ANSWERS ON PAGE 80

8. Revision and consolidation

Introduction

The objective of this final lesson is to revise and consolidate the skills you learnt in Unit 4 and to help prepare you for the exam. This is a 'test-yourself' lesson and the answers are not contained in this unit but in a separate booklet entitled 'Revision and consolidation answer booklet'. Section A is revision of the whole unit combined to see that you are able to understand and integrate the various topics dealt with in the unit. Section B is in an examination form and the questions are taken from previous Mathematical Literacy tests and exams. Try to stick to the time given in Section B to ensure that you are working fast enough. If you find that you are not able to finish, continue working through problems in the units so that your calculation and interpretation speed will improve. This will also help you improve your confidence, accuracy and timing in the examinations.

Summary of unit

In this unit we covered the following knowledge and skills:

- Patterns, relationships and representations including:
  - Number patterns
- Finance
  - Tax: Income tax
  - Accounts
  - Discounts
- Measurement
  - Surface area and volume of 3-D shapes
- Data handling
  - Averages and spread
  - Probability and chance

Section A

1. Complete the following number patterns:

   a) –12 ; –6 ; _____ ; 6 ; 12 ; _____ ; _____

   b) 21 ; 27 ; 33 ; _____ ; _____ ; _____ ;

   c) 1 ; 4 ; 9; 16 ; _____ ; _____ ; _____ ; 64 ; _____

   d) _____ ; _____ ; ______ ; 10 ; 15 ; 20 ; 25 ; _____

2. Write down the square numbers between 10 and 50.

3. Which of the following was a leap year?
   A. 2003
   B. 2010
   C. 1998
   D. 2004

4. Look for a pattern to complete the following:

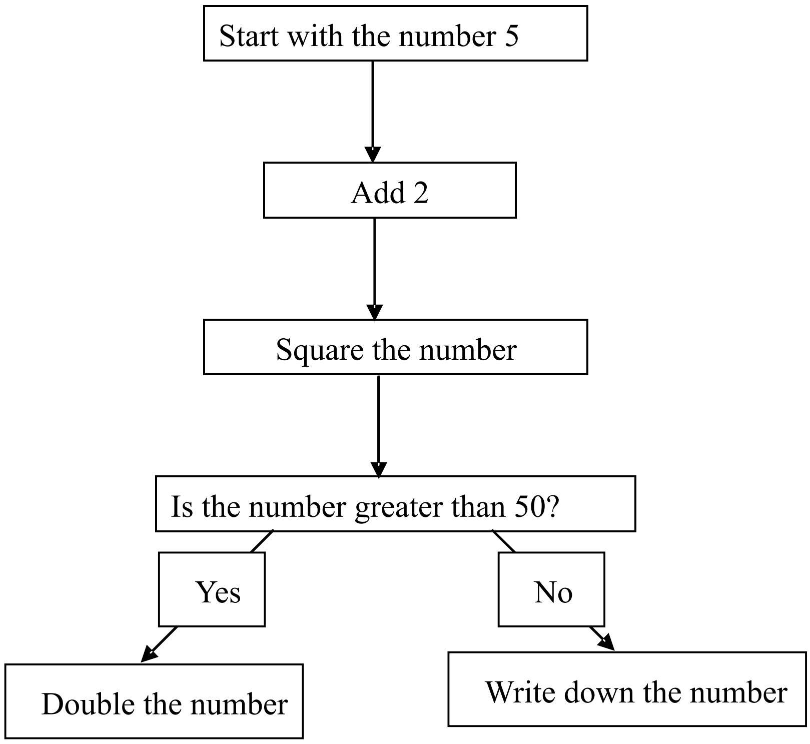
a)

2	$=$	2^1
2×2	$=$	2^2
$2 \times 2 \times 2$	$=$	2^3
$2 \times 2 \times 2 \times 2$	$=$	_____
______________	$=$	2^5

b)

5	$=$	5^1
_________	$=$	5^2
$5 \times 5 \times 5$	$=$	______

5. Follow the instructions in the flow diagram:



6. Write down the year that you were born and whether or not it was a leap year.

7. a) Use a calculator to calculate the following:
   - i) 11×24
   - ii) 11×25
   - iii) 11×26
   - iv) 11×27
   - v) 11×28

   b) Do you notice a pattern? Write down the pattern in words.

c) Check if your pattern works for these examples:

i) 11×34
ii) 11×35
iii) 11×36
iv) 11×37
v) 11×38

d) Use your pattern to show how to get the answer for 11×46. Don't use a calculator.

8. Below is the tax table for March 2010 to February 2011. Use the table to answer the questions that follow:

Taxable income			Rates of tax				
R		R	R				R
0	—	140 000			18%	of every R1	
140 001	—	221 000	25 200	+	25%	of the amount over	140 000
221 001	—	305 000	45 450	+	30%	of the amount over	221 000
305 001	—	431 000	70 650	+	35%	of the amount over	305 000
431 001	—	552 000	114 750	+	38%	of the amount over	431 000
552 001		and above	160 730	+	40%	of the amount over	552 000

REBATES

Primary rebate..R10 260
Additional rebate...R5 675

- The rebates for individuals must be deducted from the normal tax.
- The primary rebate is deductible for all individuals although the additional rebate may only be applied for individuals who are 65 years or older.

a) Bathsheba is 44 years old and works as a senior lecturer at a university. She earns R28 200 per month before tax has been deducted.

i) Calculate Bathsheba's annual salary.
ii) How much tax does Bathsheba pay per year?
iii) If Bathsheba earns another R125 000 that year from writing mathematics textbooks, how much will her annual tax then be?

b) Subashni retired at the age of 66 and now works four days a week at a local bookshop. She earns R1 450 per week at the bookstore. What will Subashni's tax be for one year?

c) Lawrence who is 25 years old, works as a security officer at the supermarket during the day. He earns R87 000 per year.

He takes on an additional job in the evenings working for a community watch and earns another R1 200 per week for that job.

i) How much tax would Lawrence pay per year if he only had the job at the supermarket?
ii) What would his monthly salary be at the supermarket after tax has been deducted?
iii) Calculate the annual tax that Lawrence will pay if he takes on the additional job for that tax year?
iv) Calculate the monthly salary that Lawrence will get for both jobs once tax has been deducted.
v) By what percentage does Lawrence's monthly salary, after tax deductions), increase when he takes on the second job?

9. This receipt was issued when prepaid electricity was purchased:

………Ekurhuleni Service provider…….

RECEIPT; 17865000
Name: Thabile Morena
Meter: 57890000876
Date: Mon 6 June 2011: Time: 16h45

Units purchased: 556.60kWh
Cost per unit: …………..

Amount electricity: R438,60
VAT: _______

TOTAL: R500,00
689 998 879 345 467 509 210 333
For queries call: 0860 222 456 788
CHURCH STREET SPAR

a) Where did this person buy the prepaid electricity?
b) Who is the service provider of this electricity?
c) How much did the person pay for the electricity itself?
d) Calculate the cost per unit of the electricity (to three decimal places).
e) How much was the VAT on this prepaid purchase?
f) This person uses approximately 25 units of electricity per day. How long can she expect this prepaid purchase to last?

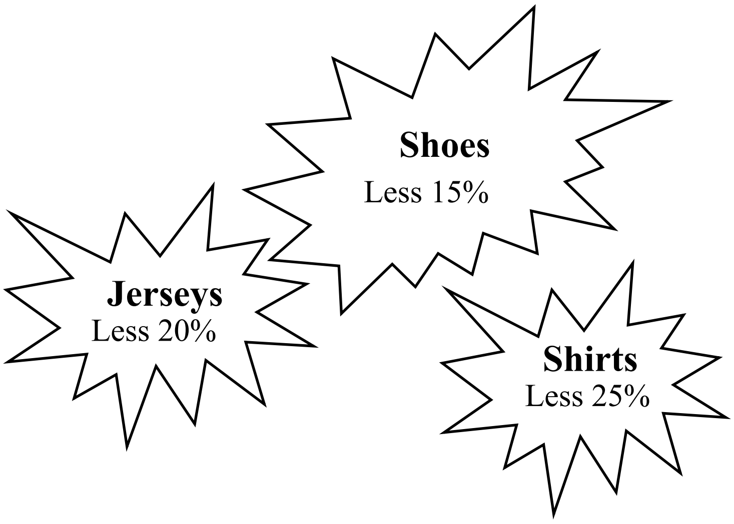
10. Joey's laptop has an electrical rating of 30 Watts. If she uses her laptop for around 8 hours per day for 5 days a week, how much does the electricity for her laptop cost her in a year if she pays 0,7677 c per unit of electricity?

11. Two contract options available from Cell C are listed below:

Costs	Cell C casual chat 100	Cell C casual chat anytime
Monthly fee	R115	R130
Contract length	24 months	24 months
Included minutes	100 minutes off-peak	50 minutes anytime
Peak time costs	Per minute	Per minute
Cell C to Cell C	R1,80	R1,80
Other mobile	R2,70	R2,70
National landline (Telkom)	R2,30	R2,30
SMS per message sent	R0,80	R0,80

a) What is the charge per minute of calls to another Cell C phone in peak time if you have the casual chat 100 package?
b) What is the charge per minute of calls to other mobile phones in peak time if you have the casual chat anytime package?
c) How much does it cost to send an sms during peak time?
d) Sibongile has a casual chat 100 package. She discovers that a single call to a Telkom phone cost her R34,50 during peak time. How long was this call?
e) How could Sibongile have saved that money taking into account the package she has?

12. The following adverts appear in shop window:



a) Annette buys the following:
1 jersey marked R150
A pair of shoes marked R260
Two shirts each marked R120

i) Calculate the discounted price she pays for each item.
ii) How much does Annette save by buying these items on the sale?

b) Complete the table on the next page of marked and discounted prices that Paulina will pay for the sale items she buys for her 3 children.

Item	*Marked*	*Discounted price*
Jersey	R180	
Jersey	R150	
Jersey	R99	
Shoes	R250	
Shoes	R180	
Shoes	R145	
Shirt	R80	
Shirt		R90
Shirt		R112,50
Total		

13. Playing dice have six sides. Each side has a number on it from 1-6.
    a) What is the probability of throwing a 1?
    b) What is the probability of throwing an odd number?
    c) If you have two dice, what is the probability of throwing a score of 12 if you add the two numbers?

14. You draw one card from a set of 52 playing cards. What are the probabilities that:
    a) you draw an Ace?
    b) you draw the King of diamonds?
    c) you draw a red card?

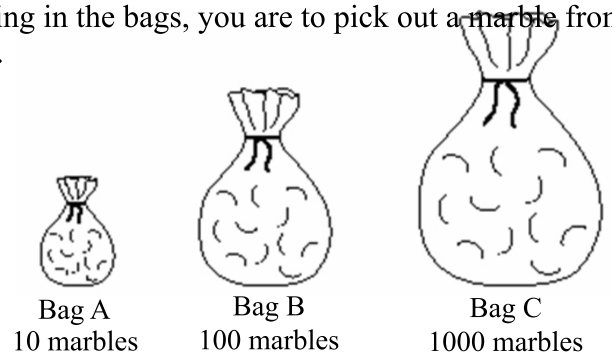
15. a) What is the range of probability?
    b) Which of the following numbers cannot be probabilities?
    $0{,}5; 1{,}4; \frac{3}{2}; -1; \frac{3}{4}; 0{,}99$

16. A multiple choice question has four possible answers. What is:
    a) the probability of the person choosing the correct answer by guessing?
    b) the probability of getting the answer wrong?

17. There is a $\frac{1}{10}$ chance that it will rain today. What is the probability that it will not rain today?

18. You buy apples from a wholesale shop. You sample 10 of the apples and find that 2 of them are rotten. What is the probability of buying a rotten apple from that batch?

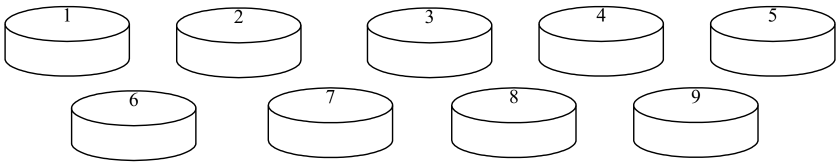
19. There is only one blue marble in each of these bags. Without looking in the bags, you are to pick out a marble from one of the bags.



Which bag would give you the greatest chance of picking the blue marble?

20. The nine disks shown below are put in a bag and mixed. Sharif draws one disk from the bag. What are the chances Sharif draws:

a) a nine?
b) an even number?
c) a square number?

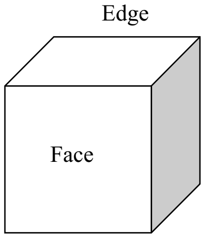


21. Here is a paper clip.

How many paper clips can fit on this line?

22. Study the following cube.

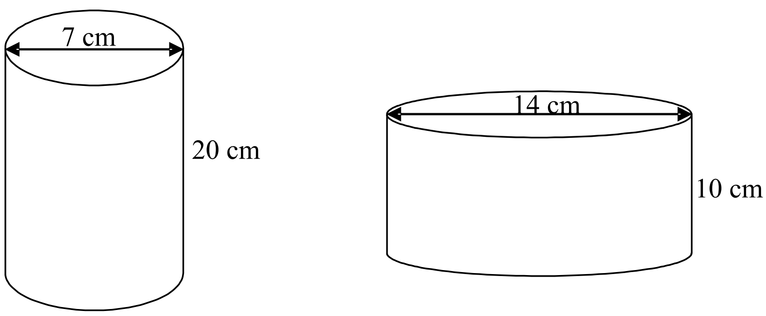
a) How many faces does the cube have?
b) How many edges does the cube have?
c) If the length of one side (or edge) of the cube is 3 cm, calculate:
   i) the surface area of the cube.
   ii) the volume of the cube.



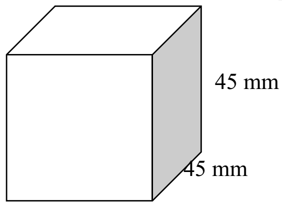
23. Johanda put a box on a shelf that is 90,4 centimetres long. The box is 31,3 centimetres long. What is the longest box she could put on the rest of the shelf . Calculate the answer to the nearest centimetre.

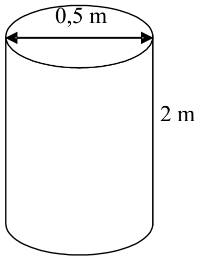
24. Compare the following two cylinders.

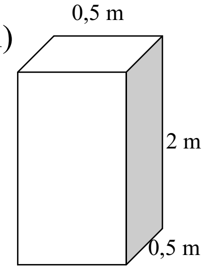
a) First calculate the volume of each cylinder and then state which one has the bigger volume.
b) Now calculate the surface area of each cylinder and state which one has the larger area.



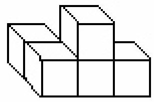
25. Draw a net of each of the following shapes and then calculate:
   a) the surface area of each shape.
   b) the volume of each shape.

i) 

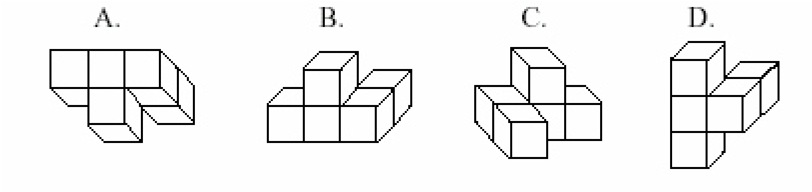
ii) 

iii) 

26. This figure will be turned to a different position.



Which of these could be the figure after it is turned?



27. Your rectangular kitchen floor needs to be retiled.
   a) If the length of the kitchen floor is 3 m and its perimeter is 10 m, what is the area of your kitchen floor in square metres?
   b) Using a scale of 1 cm: 1 m, draw a plan to scale of your kitchen floor indicating the length and width.

28. The plane below is a French plane called the Latecoere 28-3.

Here are some views of the plane.

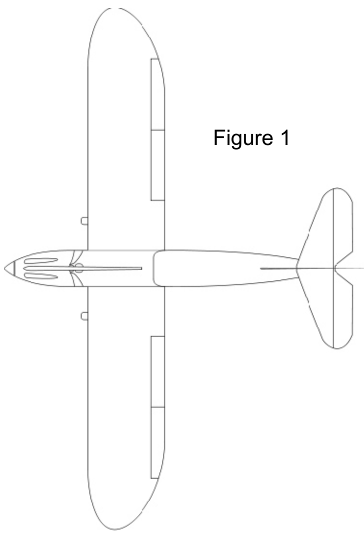


Figure 1

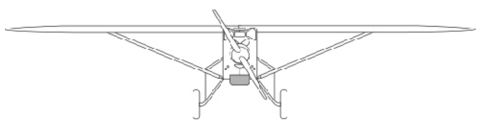


Figure 2

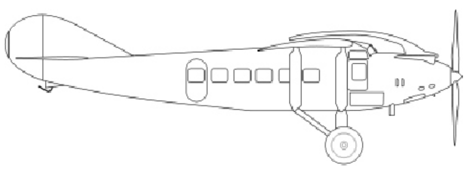


Figure 3

State which figure is:
a) the front view of the plane
b) the side view of the plane
c) the top view of the plane

29. The Volkswagen Beetle is a popular car that has been around for a number of years!

Two views of the Beetle are provided below. Answer the questions that follow.

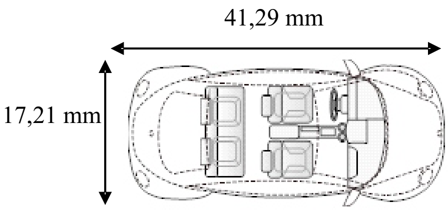


Figure 1

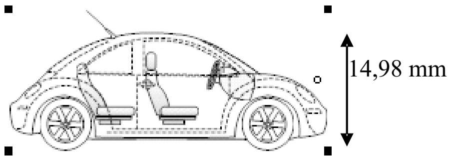
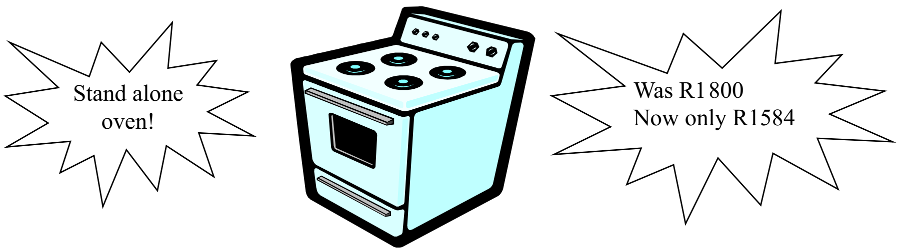


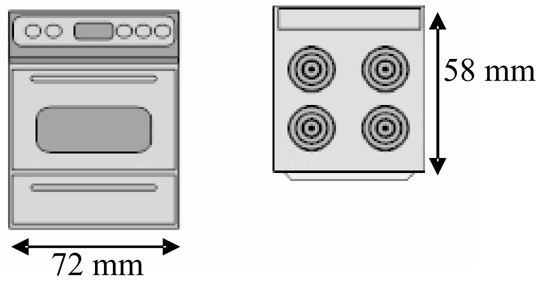
Figure 2

a) Write down which figure shows the plan of the beetle.
b) Write down which figure shows the front elevation, or side view.
c) From the plan, how many people do you think the car is designed to seat?
d) If the scale for the plan is 1: 100 mm (or 1 mm: 100 mm), calculate the actual:
    i) length of the car (in millimetres and metres)
    ii) width of the car (in millimetres and metres)
    i) height of the car (in millimetres and metres)
e) The maximum boot capacity is 769 litres. Write down in your own words what this means.

30. Stibile wants to buy a new oven. She finds one on special but is not sure if it will fit into the space in her kitchen.



The manager gives her a scale drawing of the oven to take home so that she can compare the floor area of the oven and the floor area she has available in her kitchen.



a) If the scale on the plan above is 1 mm: 1 cm, calculate the actual floor area that the oven will cover in centimetres.
b) Stibile measures her available kitchen floor space and finds that it is 0,75 m long and 0,6 m wide. Calculate the available kitchen floor space in metres.
c) How much is this in centimetres?
d) Will the oven fit in Stibile's available floor space?
e) If Stibile buys the oven for R1584, what percentage discount is she getting on the original price of R1800.
f) Stibile is given the option to buy the oven on hire purchase at a simple interest rate of 18% over 24 months with no deposit. If she takes this option, how much will she end up paying for the oven?

31. Make a scale drawing of 1 cm = 100 cm (1:100) for a fence of 11 m length and 7 m width and place a rectangular building 1 m from the fence. Calculate the area of the floor of the building.

32. The ages of the people in the Samonenga family are as follows: 62; 33; 34; 64; 30; 36
a) Determine the mean.
b) Determine the mode.
c) Determine the median.
d) Determine the range.

33. The table below shows the number of talk time minutes Yvonne used in one week on her cell phone.

Day	*Number of minutes*
Monday	13 min
Tuesday	27 min
Wednesday	8 min
Thursday	14 min
Friday	39 min
Saturday	14 min
Sunday	4 min

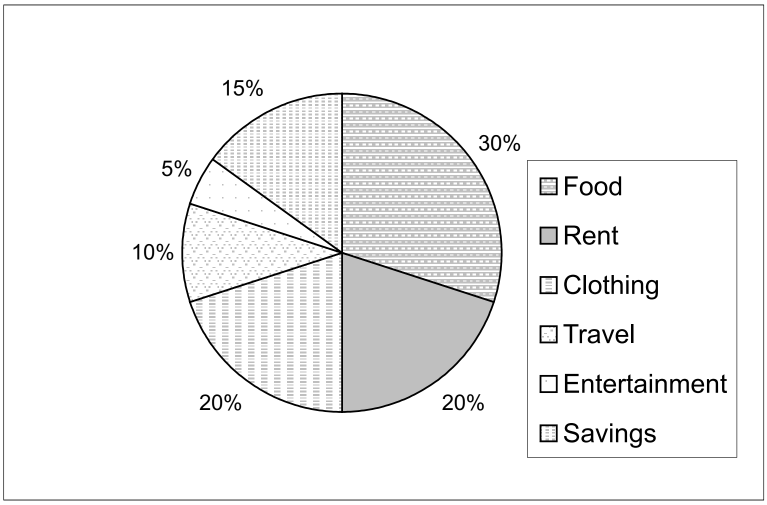
a) Determine the mean number of minutes that Yvonne used in this week.
b) Determine the mode.
c) Determine the median.
d) Determine the range.

34. Below is a frequency table showing the marks scored by a class of 30 students in a short assessment out of 5.

Marks	**Frequency**
0	1
1	3
2	8
3	12
4	4
5	2
Total	30

a) Determine the mode of the marks.
b) Determine the median of the marks.
c) Determine the mean mark out of 5 in this class of students.

35. The pie chart below indicates how a family spend their monthly income on the various items they need:



a) Draw a bar graph of this data.
b) Determine the mode percentage spent.
c) Determine the median percentage.
d) Determine the range.

36. A farmer wants to study the way the birds in a certain area breed. He counts the eggs found in 20 different nests of birds at the start of the breeding season and obtains the following data:
0 1 0 3 1 0 0 0 2 2
1 2 0 3 2 0 1 2 3 1

a) Rank this data from smallest to biggest.
b) Determine the mean.
c) Determine the mode.
d) Determine the median.
e) Determine the range.

37. Make up your own data set of 5 numbers that meets the following conditions:

A mode and a median of 4.
A range of 7.

Section B

Time: 3 hours
Marks: 140

QUESTION 1 **[42]**

1.1 Calculate the following:

1.1.1 $\frac{2}{5}$ of 300 (2)

1.1.2 $2{,}1(8{,}2 \div 1{,}9) + 4{,}8$ (1)

1.1.3 $9 \times 2 - 7 + 3(4)$ (1)

1.1.4 $3\frac{1}{2} - 4\frac{3}{5}$ (2)

1.2 Calculate the product of –3 and –7. (2)

1.3 Total 13, 17 and 14. (1)

1.4 Write one fifth as a decimal. (1)

1.5 The two equal angles of an isosceles triangle are each 55^0. What is the size of the third angle? (2)

1.6 A box of matches costs 60c. How much money do you need to buy 11 boxes of matches? (2)

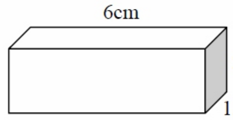
1.7 For every question you answer correctly at a quiz, you get 3 points. How many questions do you need to answer correctly to get 39 points? (2)

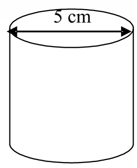
1.8 A student gets 13 out of 25 for a short test. What is the student's percentage? (2)

1.9 Write down an expression for 'a certain number decreased by 4' if the certain number is x. (1)

1.10 Use the diagrams and formulae below to answer the questions.

<u>Formulae:</u>
Area rectangle: $l \times w$
Volume of a rectangular solid: $l \times w \times h$
Circumference of a circle: $2 \times \pi \times r$
Area of a circle: $\pi \times r^2$
Volume of a cylinder: $\pi \times r^2 \times h$
Where $\pi = 3{,}14$





1.10.1 Calculate the surface area of the box. (4)
1.10.2 Calculate the surface area of the cylinder. (4)
1.10.3 Calculate the volume of the box. (2)
1.10.4 Calculate the volume of the cylinder. (2)

1.11 The table below shows the currency cross rates for 20/08/2007.

Currency	$	R	€	£	¥
1 US ($) =	1	7,3597	0,7412	0,5036	113,7100
1 Rand =	0,1359	1	0,1007	0,0684	15,4504
1 Euro(€) =	1,3492	9,9297	1	0,6795	153,4175
1 UK(£) =	1,9857	14,6142	1,4718	1	225,7939
1 Japan(¥) =	0,0088	0,06472	0,0065	0,0044	1

1.11.1 How many South African rand will you get for 1$? (1)
1.11.2 How many Euro will you get for R1? (1)
1.11.3 Calculate how many Japanese Yen you would receive for R600? Answer to the nearest Yen. (2)
1.11.4 If you have R600 to spend on a hotel in the United Kingdom (UK) per day, how many Pounds is this? (2)

Records: writes down.

1.12 Bayani sells small wire bead items at the side of the road. He records his earnings for a week. Use the information to answer the questions that follow.

	Earnings
Monday	R48,50
Tuesday	R172,00
Wednesday	R189,00
Thursday	R112,50
Friday	R138,50
Saturday	R402,00
Sunday	R56,00

1.12.1 Calculate his mean earnings per day. (3)
1.12.2 Determine his median earnings for the week. (2)

QUESTION 2 [36]

In January 2006 Luka, who was 36 years old worked for a hotel chain. Luka's gross salary was R97 575,00.

ANNUAL DEDUCTION TABLES

Remuneration			Persons under 65			Persons over 65		
			SITE	PAYE	TOTAL SITE +PAYE	SITE	PAYE	TOTAL SITE + PAYE
91991	-	92290	4500.00	6635.13	11135.13	0.00	6635.13	6635.13
92291	-	92590	4500.00	6710.13	11210.13	0.00	6710.13	6710.13
92591	-	92890	4500.00	6785.13	11285.13	0.00	6785.13	6785.13
92891	-	93190	4500.00	6860.13	11360.13	0.00	6860.13	6860.13
93191	-	93490	4500.00	6935.13	11435.13	0.00	6935.13	6935.13
93491	-	93790	4500.00	7010.13	11510.13	0.00	7010.13	7010.13
93791	-	94090	4500.00	7085.13	11585.13	0.00	7085.13	7085.13
94091	-	94390	4500.00	7160.13	11660.13	0.00	7160.13	7160.13
94391	-	94690	4500.00	7235.13	11735.13	0.00	7235.13	7235.13
94691	-	94990	4500.00	7310.13	11810.13	0.00	7310.13	7310.13
94991	-	95290	4500.00	7385.13	11885.13	0.00	7385.13	7385.13
95291	-	95590	4500.00	7460.13	11960.13	0.00	7460.13	7460.13
95591	-	95890	4500.00	7535.13	12035.13	0.00	7535.13	7535.13
95891	-	96190	4500.00	7610.13	12110.13	0.00	7610.13	7610.13
96191	-	96490	4500.00	7685.13	12185.13	0.00	7685.13	7685.13
96491	-	96790	4500.00	7760.13	12260.13	0.00	7760.13	7760.13
96791	-	97090	4500.00	7835.13	12335.13	0.00	7835.13	7835.13
97091	-	97390	4500.00	7910.13	12410.13	0.00	7910.13	7910.13
97391	-	97690	4500.00	7985.13	12485.13	0.00	7985.13	7985.13
97691	-	97990	4500.00	8060.13	12560.13	0.00	8060.13	8060.13
97991	-	98290	4500.00	8135.13	12635.13	0.00	8135.13	8135.13
98291	-	98590	4500.00	8210.13	12710.13	0.00	8210.13	8210.13
98591	-	98890	4500.00	8285.13	12785.13	0.00	8285.13	8285.13
98891	-	99190	4500.00	8360.13	12860.13	0.00	8360.13	8360.13
99191	-	99490	4500.00	8435.13	12935.13	0.00	8435.13	8435.13
99491	-	99790	4500.00	8510.13	13010.13	0.00	8510.13	8510.13
99791	-	100090	4500.00	8585.13	13085.13	0.00	8585.13	8585.13

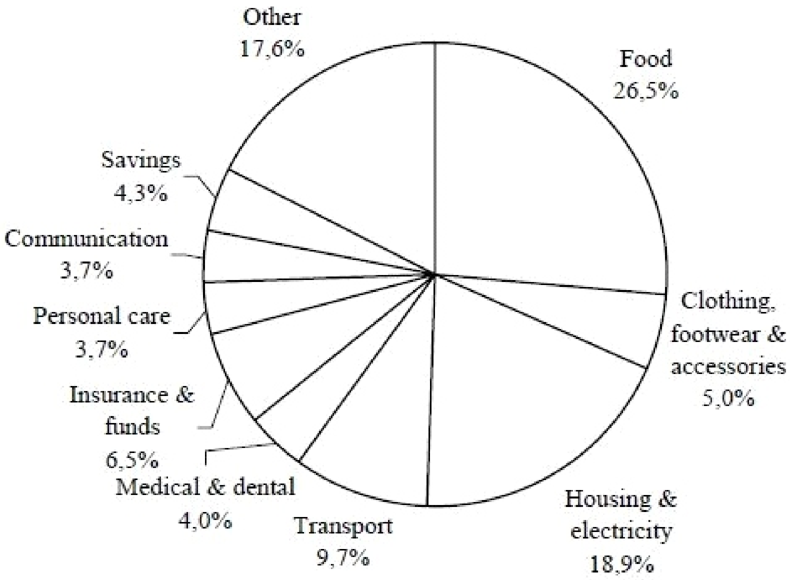
2.1 Use the table on the previous page the SARS tax document to determine how much tax (SITE + PAYE) Luka paid in 2006. (3)

2.2 All employees contribute 1% of their monthly salary (before tax) to the Unemployment Insurance Fund (UIF). How much did Luka contribute to the UIF each month in 2006? (4)

2.3 Now show how to calculate that the monthly salary Luka took home was R7009,51. (3)

According to market researchers, people in Luka's income bracket typically spend their money as shown in the pie chart below.

Top 10 items of expenditure for the 'emerging middle class'



2.4 Assume that Luka's money is spent as shown in the graph and calculate to the nearest Rand how much of each month's take-home salary of R7009,51 is spent on the following:

- Food
- Clothing, footwear and accessories
- Housing and electricity
- Transport (8)

2.5 Luka's employee offers Luka a salary increase of 5% for 2007. What would Luka's gross salary (total salary before tax) be after the increase? (3)

Rates applicable to individuals

TAXABLE INCOME R		R	RATES OF TAX R				R
0	—	100 000			18%	of each R1	
100 001	—	160 000	18 000	+	25%	of the amount above	100 000
160 001	—	200 000	33 000	+	30%	of the amount above	160 000
220 001	—	300 000	51 000	+	35%	of the amount above	200 000
300 001	—	400 000	79 000	+	38%	of the amount above	300 000
400 001		and above	117 000	+	40%	of the amount above	400 000

Tax Rebates

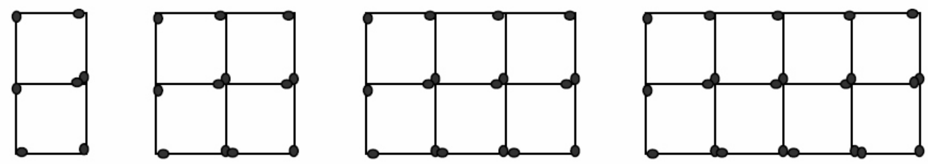
- Primary rebate R7 200
- Additional rebate (for person 65 years and older) R4 500

2.6 The tax formula table applicable to 2007 is shown below. Use this formula and the salary you calculated in 2.5 to show that Luka's new monthly take-home salary after paying tax and UIF contributions will be R7501,31. (7)

2.7 Calculate the percentage increase in take-home salary from January 2006 to January 2007 and explain in terms of taxes why you think this is greater than the 5% increase that the employer gave Luka. (8)

QUESTION 3 **[10]**

3.1 Matches are used to make the figures below:



If the pattern of two rows of squares continues.

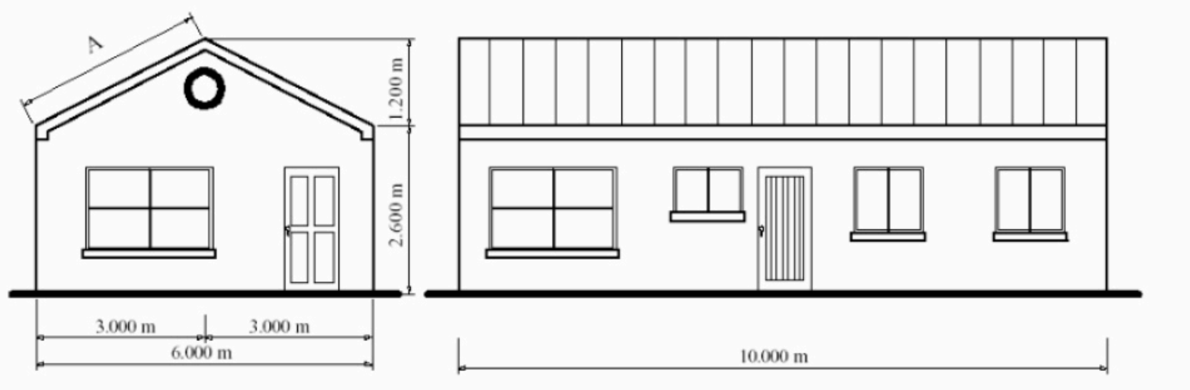
3.1.1 Find the values for A, B and C in the table below. (3)

Area	2	4	6	8	10	12
Number of matches	7	11	16	A	B	C

3.1.2 What is the difference between the first and second area as the number of matches increases? (1)
3.1.3 What is the difference between the first and second number of matches as the area increases? (1)
3.1.4 Is this an example of direct or inverse proportion? Give a reason for your answer. (2)
3.1.5 Write down the number of matches that are needed to make an area of 14. (1)
3.1.6 With 42 matches, what do you think the area will be? (2)

QUESTION 4 **[15]**

Below are front and side elevations of a house. Use them to answer the questions that follow.



4.1 The roof of this house is made of corrugated iron sheets. Use the Theorem of Pythagoras $c^2 = a^2 + b^2$ to calculate the length of the roof sheets, indicated by A in the diagram. (3)

4.2 Suggest a reason why all the measurements are given to three decimal places. (1)

4.3 The floor slab of the house is a block of cement which sits directly beneath the house.

For this particular house, the floor slab is 150 mm thick.

4.3.1 Determine the surface area of the top of the floor slab. (3)
4.3.2 Convert 150 mm to a measurement in metres. (1)
4.3.3 Calculate the volume of the floor slab. (3)
4.3.4 Once mixed with sand and water, five bags of cement will produce enough cement to fill a volume of 1 m^3. Calculate how many bags of cement are needed to produce the floor slab for this house. (2)
4.3.5 It costs R69,99 for a bag of cement. Calculate the cost of the cement needed for this floor slab. (2)

QUESTION 5 **[9]**

The table below is an extract from the Vodacom tariff tables for the 4U and Top Up 135 cell phone packages. Use the information in the table to answer the questions.

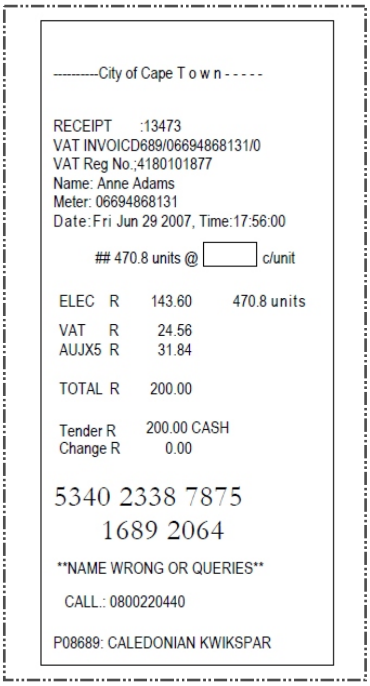
Package	**4U**	**TopUp 135**
Domestic calls		
Vodacom to other mobile networks (Off Peak)	R1,30	R1,25
Vodacom to other mobile networks (Peak)	R2,75	R2,65
Vodacom to Telkom (Peak)	R2,99	R2,35
Vodacom to Vodacom (Peak)	R2,58	R1,99
Vodacom to Vodacom/Telkom (Off Peak)	R1,12	R1,08

5.1 What is the charge for a Vodacom to Telkom during *Off Peak* time if you have a Vodacom 4U package? (1)

5.2 Elsie has a TopUp 135 package. She makes a call to her husband's MTN cell phone during *Peak* time. If the call lasts 3 minutes, how much does it cost? (2)

5.3 A Vodacom TopUp 135 customer is shocked to find that single call has cost her R28,20. The call was made during *Peak* time to a Telkom number. How long was this call? (3)

5.4 How much would the customer in question 5.3 have saved by making the same call during *Off Peak* time? (3)


---------City of Cape T o w n - - - - -

RECEIPT :13473
VAT INVOICD689/06694868131/0
VAT Reg No.:4180101877
Name: Anne Adams
Meter: 06694868131
Date:Fri Jun 29 2007, Time:17:56:00

470.8 units @ ☐ c/unit

ELEC R 143.60 470.8 units

VAT R 24.56
AUJX5 R 31.84

TOTAL R 200.00

Tender R 200.00 CASH
Change R 0.00

5340 2338 7875
1689 2064

NAME WRONG OR QUERIES

CALL.: 0800220440

P08689: CALEDONIAN KWIKSPAR

QUESTION 6 **[20]**

A customer buys some prepaid electricity.
The following receipt is issued.

6.1 What is the name of the business that sold the electricity? (1)

6.2 The time of the purchase was given as 17:56:00. Write this as a time using am/pm notation. (2)

6.3 How much did the customer pay for the electricity itself? (2)

6.4 Show by calculation that the VAT (of 14%) is based on both the electricity and the AUJX5 amounts. (3)

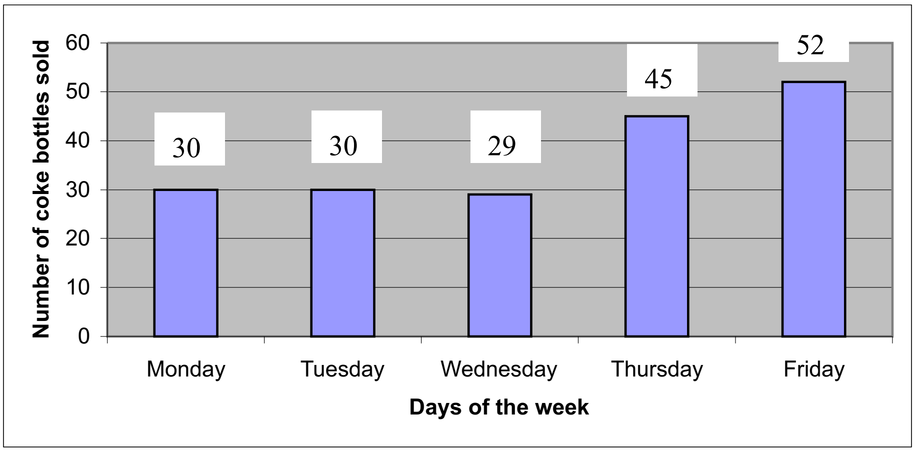
6.5 Refer to the line marked ##. The number of units bought has been shown but the cost per unit has been left out. Determine the cost per unit in cents. (3)

6.6 The household which bought the electricity uses about 35 units per day during winter. For how many days would the amount of electricity bought on this slip last? (3)

6.7 Estimate the total amount that the family can expect to pay for electricity for the month of August, including VAT and the AUJX5 charge. (6)

QUESTION 7 **[8]**

The graph shows the number of bottles of coke Maggie sells each day of the week at her Spaza tuck shop at a local high school.



7.1 How many bottles of coke did Maggie sell on Monday? (1)

7.2 Determine the mean number of bottles of coke Maggie sold this week. (3)

7.3 Determine the median of the number of bottles sold this week. (2)

7.4 What is the mode of the number of bottles of coke sold this week? (1)

7.5 Calculate the range of the number of bottles of coke sold this week. (1)

Feedback to self-check exercises

Lesson 1

1. 1 2 3 4 ...19 20
   20 19 18 17 ...2 1
   $21 \times 20 = 420$; $420 \div 2 = 210$

2. 1 3 6 10 15 21 28 36 45 55

3. A square has equal sides. If you multiply a number by itself you get a square number e.g.
   $2 \times 2 = 4$ or $3 \times 3 = 9$.

4. 121; 144; 169; 196

5. 1 1 2 3 5 8 13 21 34 55

6. No, 1992 and 1996 were but not 1994.

7. Yes, you will be able to celebrate it on the 29th February as 2040 is a leap year.

Lesson 2

1. R2 500 × 12 = R30 000 per year.
   First subtract R200 off this amount for the rebates for her children.
   R30 000 – R200 = R29 800
   Her annual tax: 18% of R29 800 – R10 260
   R5 364 – R10 269
   Mrs E will not pay any tax per month as she does not earn enough to qualify to pay tax.

2. His best two options are probably the MTN AnyTime 750 or the AnyTime 1200. Let's see which one works out cheapest with 500 minutes of talk time. The AnyTime 750 package gives him 468 inclusive minutes (free) before he has to start paying the 'out of bundle' rate of R1,75 per minute. With 500 minutes, he would need to pay the 'out of bundle' rate for 32 minutes. So his monthly talk time costs would be:

   R750 (for the package) + R1,75 × 32 (out of bundle talk time) = R806,00

   The AnyTime 1200 package will work out more expensive at R1200 per month. On both packages he will still get 100 free sms's included in the monthly package. So at this stage the AnyTime 750 package is his best option.

3. 15% of R550 = R82,50
   R550 – 82,50 = R467,50

Lesson 3

1. Using: Probability of an event = $\frac{\text{number of ways the event can happen}}{\text{number of equally likely outcomes}}$

   a) $\frac{8}{15}$

   b) $\frac{7}{15}$

2. a) 2 black aces in a pack (clubs and spades); $\frac{2}{52} = \frac{1}{26}$

   b) 13 clubs in a pack: $\frac{13}{52} = \frac{1}{4}$

   c) 52 cards 4 kings = 48 cards: $\frac{48}{52} = \frac{12}{13}$

3. a) $\frac{2}{10} = \frac{1}{5}$ b) $\frac{3}{10}$

4. Using: $\frac{\text{Number of times the event happened}}{\text{Number of times the event could have happened}}$

   a) $\frac{300}{500} = \frac{3}{5}$ b) $\frac{200}{500} = \frac{2}{5}$

Lesson 4

1. Calculate a) the surface area and b) the volume of the following 3-D shapes.

   a) i) Area of 1 circle $= \pi \times 6 \times 6 = 113{,}122 \text{ cm}^2$
   Length of rectangle = circumference of circle
   $= \pi \times 12 = 37{,}704 \text{ cm}$
   Area of rectangle $= 37{,}704 \times 18{,}5 = 697{,}524 \text{cm}^2$
   Total surface area $= 113{,}122 + 113{,}122 + 697{,}524$
   $= 923{,}768 \text{ cm}^2$

   ii) Area of one face of cube $= 8{,}5 \times 8{,}5 = 72{,}25 \text{mm}^2$
   Total surface area of cube $= 6 \times 72{,}25 = 433{,}5 \text{mm}^2$

   iii) Area of face of $l \times w$ $= 4{,}2 \times 1{,}3 = 5{,}46 \text{ m}^2$
   Area of face of $l \times h$ $= 4{,}2 \times 2{,}5 = 10{,}5 \text{ m}^2$
   Area of face of $h \times w$ $= 2{,}5 \times 1{,}3 = 3{,}25 \text{ m}^2$
   Total surface area: $= 2(5{,}46) + 2(10{,}5) + 2(3{,}25)$
   $= 38{,}42 \text{ m}^2$

   iv) *Surface area for cylinder:*
   Area of 1 circle $= \pi \times 0{,}5 \times 0{,}5 = 0{,}7855 \text{ m}^2$
   Length of rectangle = circumference of circle
   $= \pi \times 1 = 3{,}142 \text{m}$
   Area of rectangle $= 3{,}142 \times 6 = 1{,}5 \text{ m}^2$
   Surface area for rectangular solid:
   $2(4 \times 1) + 2(1 \times 2) + 2(4 \times 2) = 2(4) + 2(2) + 2(8) = 28 \text{ m}^2$
   Total surface area of object:
   Includes the outside surfaces, i.e. the surfaces on the outside of the object that you can paint. The bottom circle of the cylinder cannot be painted. So we do not add this into the surface area of the cylinder and we subtract that area from the surface area of the rectangular solid.

Therefore we get:
$0{,}7855 + 1{,}5 + 28 - 0{,}7855 = 29{,}5 \text{ m}^2$

b) i) Volume of cylinder = Area of base × height
= Area of circle × height
$= \pi r^2 \times h$
$= 113{,}122 \text{ cm}^2 \times 18{,}5 \text{ cm}$
$= 2092{,}76 \text{ cm}^3$

ii) Volume of cube = Area of base × height
= Area of square × height
$= s \times s \times s$
$= 72{,}25 \text{ mm}^2 \times 8{,}5 \text{ mm}$
$= 614{,}13 \text{ mm}^3$

iii) Volume of rectangular solid = Area of base × height
= Area of rectangle × height
$= l \times w \times h$
$= 4{,}2 \text{ m} \times 1{,}3 \text{ m} \times 2{,}5 \text{ m}$
$= 13{,}65 \text{ m}^3$

iv) Total volume of object = Vol.cylinder + vol.rectangular solid
Volume of cylinder = Area of base × height
= Area of circle × height
$= \pi r^2 \times h$
$= \pi \times 0{,}5 \times 0{,}5 \times 6$
$= 4{,}713 \text{ m}^3$

Volume of rectangular solid = Area of base × height
= Area of rectangle × height
$= l \times w \times h$
$= 4 \text{ m} \times 1 \text{ m} \times 2 \text{ m}$
$= 8 \text{ m}^3$

Total volume of object $= 4{,}173 \text{ m}^3 + 8 \text{ m}^3$
$= 12{,}17 \text{ m}^3$

Lesson 5

Part A

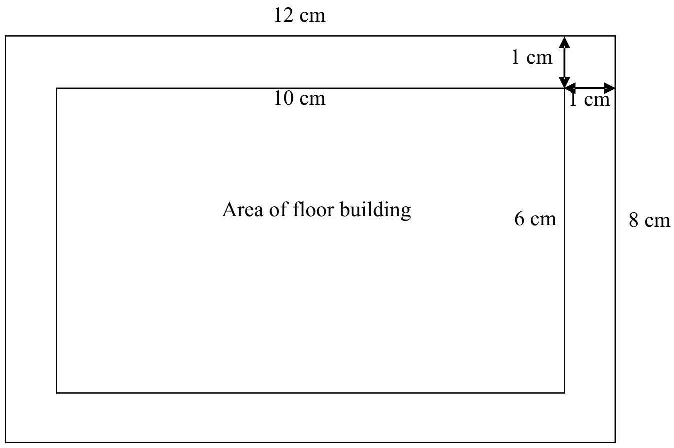
1. 500 times smaller
2. Scale 1 : 500
   5 cm of kitchen length is 1 × 5 cm
   So real length is 500 × 5 = 2 500 cm = 2,5 m

Part B

1. Number of blocks for each length: 18 × 2 = 36
   2 lengths = 2 × 36 = 72 blocks
   Each width needs 6 metres worth of blocks.
   1 metre needs 2 blocks

So each width needs: 6×2 blocks = 12 blocks
2 widths = 2×12 blocks
Total blocks = 2 lengths + 2 widths
= 72 + 24
= 96 blocks.

2. 1 cm represents 100 cm = 1 m
So 12 m 12 cm; 8 m 8 cm; 1 m 1 cm.



Area of floor of building : length × width
= 10 m × 6 m
= 60 m^2

Lesson 6

1. a) Figure B
b) Figure A
c) 7 people
d) i) Scale: 1 cm : 40 cm = 13 cm : 40 × 13 cm = 520 cm = 5,2 m
ii) 2 m
iii) 1,8 m

2. a) Scale: 1 mm : 50 mm = 86 mm : 50 × 86 mm = 4300 mm = 4,3 m
b) 3,25 m
c) 38 mm + 30 mm = 68 mm
86 mm (length of room) – 68 mm = 18 mm
18 × 50 mm = 900 mm = 0,9 m
d) 4,3 m × 3,25 m = 13,975 m^2

Lesson 7

1. a) Mean = 54,4
Median = 53
Mode = 50
Range = 61 – 50 = 11

b) Mean = 21,14
Median = 23
Mode = 23 and 25
Range = 26 – 5 = 21

2. Mean = R8,69
   Median = R8,65
   Mode = R8,30
   Range = R9,30 – R8,10 = R1,20

3. a) 4
   b) 27
   c) 1,5
   d) 4

Feedback from Activities

Lesson 1

Activity 1

1. 1 2 3 4 5 6 7 8 9 10
2. 10 9 8 7 6 5 4 3 2 1
3. 11 11 11 11 11 11 11 11 11 11
   $10 \times 11 = 110$
4. $\frac{110}{2} = 55$
5. The answer is 55.

Activity 2

1. a) 3 b) 24
2. 3 rows
3. a) 9 b) 72

Number of rows	Number of flowers	Number of petals
1 row	*3*	*24*
2 rows	*6*	*48*
3 rows	*9*	*72*

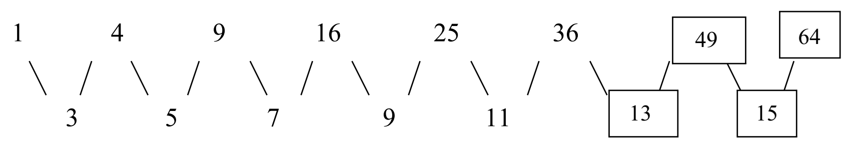
5. 12 flowers
6. 96 petals
7. 30 flowers
8. 240 petals
9. The number of flowers is equal to three times the number of rows.
10. The number of petals is equal to eight times the number of flowers.

11. a) $f = 3 \times r$
    b) $p = 8 \times f$
    c) $p = 24 \times r$

12. a) 9 rows
    b) 27 flowers
    c) 216 petals

Activity 3

1. 1 4 9 16 25 36 49 64 81 100
2. a)



   b) The difference is always odd and it goes up by 2 each time.

3. a) 100
   b) 25
   c) 4

Activity 4

1. 1 + 1 = 2   By adding the consecutive numbers
   1 + 2 = 3   you get the next number.
   2 + 3 = 5

2. 1 + 2 = 3   By adding consecutive odd numbered
   1 + 2 + 5 = 8   terms you get the next number.
   1 + 2 + 5 + 13 = 21
   1 + 2 + 5 + 13 + 34 = 55

3. 1 + 1 = 2   The answer is one less
   1 + 1 + 2 = 4   than a Fibonacci number.
   1 + 1 + 2 + 3 = 7
   1 + 1 + 2 + 3 + 5 = 12
   1 + 1 + 2 + 3 + 5 + 8 = 20

4. 3; 5; 8
   Fibonacci numbers are often found in nature, as seen in this example.

Activity 5

1. a) A leap year has an extra day. This keeps the calendar year in time with the solar year.
   b) It occurs every 4 years.
   c) The next leap year is 2012 and after that 2016 and then 2020.
   d) 365,24 days
   23 hours 56 minutes 4 seconds
   e) longer

2. No, 2012 and 2016 are leap years, not 2015.

3. 2004, 2008, 2012, 2016
   NOTE: you do not include 2000 and 2020 as the question asked for the leap years ***between*** those years.

Lesson 2

Activity 1

1. R435 000 – R319 200 = R115 800
   To calculate what percentage of R319 200 this is we say:

$$\frac{115\ 800}{319\ 200} \times \frac{100}{1} = 36{,}278 = 36\%$$

2. R435 000 is in the R431 001 – R552 000 tax bracket. So his tax will be:
   R114 750 + 38% of (R435 000 – R431 000) – R10 260
   = R114 750 + R1520 – R10 260
   = R106 010

3. Annual salary after tax: R435 000 – R106 010
   = R328 990
   Monthly salary: R328 990 ÷ 12
   = R27 415,83

4. R27 415,83 – 21 153,33 = R6262,50 per month more

5. This certainly is a good offer. If a person stays in the same company, there is usually an annual increase of between 8 and 12% on your salary. So this is a good salary increase.

Activity 2

1. a) R13 000 × 12 = R156 000

   b) Tax bracket: R140 001- R221 000
   Annual tax on R156 000:
   R25 200 + 25% of (R156 000 – R140 000) – R10 260
   = R25 200 + R4000 – R10 260
   = R18 940

   c) R156 000 – R18 940 = R137 060 per year or R11 421,67 per month

NOTE: Mrs D is older than 65 so she qualifies for the primary and additional rebates.

2. Mrs D (68 years old) earns R9 000 a month.
   a) R108 000

   b) Tax bracket: R0 - R140 000
   Annual tax on R108 000:
   18% of R108 000 – (Primary + additional rebate)
   = R19 440 – (R10 260 + R5 675)
   = R3 505

   c) R108 000 – R3 505 = R104 495 per year or R8707,92 per month

3. Mr J earns more but he also pays more tax. His monthly salary before tax is deducted is R4000 more than Mrs D's salary. But after tax deductions, his salary is only R2713,75 more per month than Mrs D's salary. This is because he is in a higher tax bracket than Mrs D and Mrs D benefits from the rebate for individuals 65 or older.

Activity 3

1. 2000 Watt or 2 kW

2. 2 kW × 3 hours per day × 30 days = 180 kW
   180 kW × 106,37 c = 19 146,6 c = R191,47 per month

3. 3 hours per day for 30 days is equal to 90 hours in a month.
   Monthly cost of R191, 47 ÷ 90 hours = 2,127444… = R2,13 per hour.

4. 200 Watt = 0,2 kW
   0,2 kW × 2 hours per day × 30 days = 12 kW per month
   12 kW × 106,37 c = 1 276,44 c = R12,76 per month.

Activity 4

1. 28 August 2009
2. Kenilworth, Cape Town
3. 167,2 units
4. 53,90 c/unit
5. R12,28
6. Approximately 30 units per day.
   162,7 units ÷ 20 per day = 8,135 = 8 days.
   This electricity will last the person approximately 8 days using 20 units per day.
7. 200 × 53,90 c/unit = R107,80
   Remember to add VAT of 14%:

   14% of R107,80 = $\frac{14}{100} \times 107{,}80 = 15{,}092$

   R107,80 + R15,092 = R122,89

Activity 5

1. This second charges R0,7737 per unit. The first one only charged R0,5390 per unit.
2. The first bill was from 2009. Perhaps this bill is from a more recent date. Or perhaps it also depends on where you purchase your prepaid electricity. Some areas have cheaper rates and you are also given different rates according to your average usage of electricity per month.
3. 14% of R219,73 = R30,7622. So the total bill is R250,49.

Activity 6

The number of units of electricity available for R7,50.
R7,50 ÷ R0,7737 = 9,69 units

The time taken to use one unit is given by T = $\frac{1000}{\text{Electrical rate (W) per hour}}$ where W is the wattage.

Therefore T = $\frac{1000}{100}$ hours

For 1 unit you get 10 hours.
Therefore for 9,69 units you get 96,9 hours.
There are 24 hours in a day so 96,9 ÷ 24 = 4 days.
For the price of a litre of milk you can run your fridge for 4 days.

Activity 7

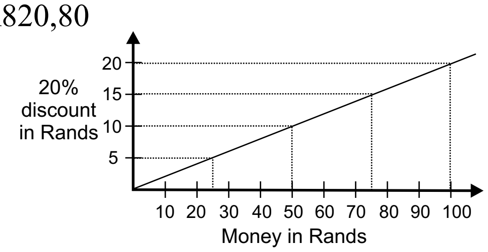
1. R1,95
2. R500 and you get 256 inclusive minutes.
   R500 ÷ R1,95 = 256,4 ≈ 256 minutes.
3. The 'out of bundle' cost is 15 c more per minute than the 'in bundle' cost per minute.
4. Probably the MTN AnyTime 350. She can send 50 free sms's and get 179 inclusive minutes. If she took the MTN AnyTime 200, she would only get 25 free sms's and 86 minutes inclusive talktime. For the other approximately 65 minutes of talking, she would pay R2,85 per minute which would amount to R185,25. This would cost her more than the R350 which is what she would pay on the MTN AnyTime 350.

Activity 8

1. 100%= R124
   Therefore 85% $= \frac{85}{100} \times 124 = \text{R}105{,}40$

2. 100% = R912
   90% = (100 – 10)% = R912 – R91,20
   = R820,80

3.



Lesson 3

Activity 1

1. If you pick one ball from the bag, it can be any of the six balls. Therefore there are six equally likely outcomes (i.e. $n = 6$)
   a) Two of these outcomes are red (i.e. $s = 2$). Therefore the probability that the ball is red is $\frac{s}{n} = \frac{2}{6} = \frac{1}{3}$.

   b) The rest are blue. So we have 6 – 2 blue balls. Therefore the required probability is $\frac{s}{n} = \frac{4}{6} = \frac{2}{3}$.

2. We have six possible outcomes when we toss a fair dice (i.e. $n = 6$). There is only one way of getting a '3', (i.e. $s = 1$). Therefore the probability is $\frac{s}{n} = \frac{1}{6}$.

3. There are 52 cards so $n = 52$.
   a) Out of these 52 cards, 4 are kings. Therefore the probability of obtaining a king is $\frac{4}{52} = \frac{1}{13}$.

   b) Half of the cards are black. That is $\frac{1}{2} \times 52 = 26$. The required probability is therefore $\frac{26}{52} = \frac{1}{2}$.

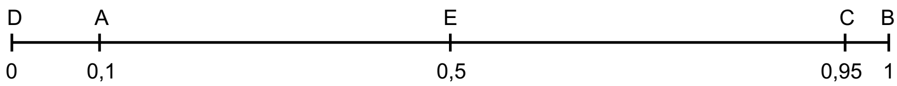
   c) How many of the cards are diamonds? 13 of them are diamonds. Therefore $s = 13$. This gives the required probability to be $\frac{13}{52} = \frac{1}{4}$.

Activity 2

1. a) Since both sides of the coin are heads, it is certain that the coin will come down ‘heads’. Therefore the probability of the coin coming down ‘heads’ is 1.

   b) None of the sides is ‘tails’. Therefore it is impossible for the coin to land ‘tails’ up. Hence the probability of the coin coming down ‘tails’ up is 0.

2. A From the statement, we can say that it is very unlikely for you to win the game. Therefore the probability of winning the game is nearer to 0.
   B It is certain. Everybody will die one day. The probability is therefore 1.
   C Chances of rain are high. Therefore the probability that it will rain is nearer to 1.
   D Is an impossible event. Therefore the probability is 0.
   E The probability of the person arriving today is the same as not arriving today. Hence the probability is 0,5.
   These give us the following positions on the probability line diagram.



3. Probability of any event lies between 0 and 1 inclusive, therefore the following cannot be probabilities:
   –0,01 (less than 0)
   1,2 (more than 1)
   $\frac{9}{7}$ (more than 1)
   4,0 (more than 1), and
   $-\frac{1}{2}$ (less than 0).

   As a guide, each time you calculate probabilities, check to see whether they lie between 0 and 1. If not then you must be wrong, you need to go over your work.
   The others: 0,72; $\frac{3}{8}$ and 0,5 can be probabilities.

Activity 3

1. How many times did you get '3' appearing on top of the dice? Divide this number by 60. This will give you the probability of obtaining a '3' when this same dice is tossed again. Your answer should be around $\frac{1}{6}$.

2. Probability of getting heads equals:

$$\frac{\text{Number of times she got 'heads'}}{\text{Number of times she could have got 'heads'}}$$

$$\frac{510}{1000} = 0,51$$

3. The train could have arrived late in all the 50 days. However, it arrived late on 24 days. Therefore the probability of arriving late tomorrow is $\frac{24}{50} = \frac{12}{25}$ $(= 0{,}48)$.

Activity 4

1. Probability of getting the right answer is $\frac{1}{5}$ since there is only one right answer amongst the five. Therefore the probability of getting a wrong answer is $1 - \frac{1}{5} = \frac{4}{5}$.

2. The probability that it will not rain
= 1 – probability that it will rain
= 1 – 0,7
= 0,3

Lesson 4

Activity 1

1. a) l = 10 cm; w = 3 cm; h = 4 cm
   b) l = 6 cm; w = 2 cm; h = 3,5 cm

2. a) Figures B and C
   b) Figures A and D

Activity 2

1. Area of face A = $l \times h = 10\text{cm} \times 4\text{cm} = 40\text{cm}^2$
   Area of face B = $l \times h = 10\text{cm} \times 4\text{cm} = 40\text{cm}^2$

2. Area of face C = $l \times w = 10\text{cm} \times 3\text{cm} = 30\text{cm}^2$
   Area of face D = $l \times w = 10\text{cm} \times 3\text{cm} = 30\text{cm}^2$

3. Area of face E = $h \times w = 4\text{cm} \times 3\text{cm} = 12\text{cm}^2$
   Area of face F = $h \times w = 4\text{cm} \times 3\text{cm} = 12\text{cm}^2$

4. Total surface area of rectangular solid
= 40 + 40 + 30 + 30 + 12 + 12 = 164 cm^2

5. l = 6 cm; w = 2 cm; h = 3,5 cm
Area of 3 different rectangle faces: $l \times h = 6\text{cm} \times 3{,}5\text{cm} = 21\text{cm}^2$
$l \times w = 6\text{cm} \times 2\text{cm} = 12\text{cm}^2$
$h \times w = 3{,}5\text{cm} \times 2\text{cm} = 7\text{cm}^2$

Total surface area = 21 + 21 + 12 + 12 + 7 + 7 or 2(21) + 2(12) + 2(7)
= 80 cm^2

Activity 3

1. Area of one square face: 9 cm^2
Total surface area of cube = 9 + 9 + 9 + 9 + 9 + 9
= 6 × 9 cm^2
= 54 cm^2

2. a) Area of one square face: $s \times s = 1\text{m} \times 1\text{m} = 1\text{m}^2$

Total surface area $= 6 \times 1\text{m}$

$= 6\ \text{m}^2$

b) Area of one square face: $s \times s = 10\text{cm} \times 10\text{cm} = 100\text{cm}^2$

Total surface area $= 6 \times 100\ \text{cm}^2$

$= 600\ \text{cm}^2$

Activity 4

a) Area of circle $= \pi r^2$

$= \pi \times 5 \times 5 = \pi \times 25 = 3{,}142 \times 25$

$= 78{,}55\ \text{cm}^2$

Circumference of circle $= \pi \times \text{diameter}$

$= \pi \times 10\text{cm}$

$= 3{,}142 \times 10$

$= 31{,}42\ \text{cm}$

So the length of the rectangle is 31,42 cm. The width of the rectangle is equal to the height of the cylinder which is 22 cm. So the area of the rectangle is:

Area of rectangle $= l \times w$

$= 31{,}42 \times 22$

$= 691{,}24\ \text{cm}^2$

Total surface area of the cylinder:

Area of circle + Area of circle + Area of rectangle

= 78,55 + 78,55 + 691,24

$= 848{,}34\ \text{cm}^2 \quad \approx 848\ \text{cm}^2$

b) Total surface area of the cylinder:

Area of circle + Area of circle + Area of rectangle

= 346,4055 + 346,4055 + 1649,55

$= 2342{,}361\ \text{cm}^2 \quad \approx 2342\ \text{cm}^2$

Activity 5

a) B is longer

A is broader

A and B have the same height

b) A has a face of area: $5 \times 4 = 20$

B has a face area: $5 \times 5 = 25$

so B has the larger face.

c) A's base has area: $4 \times 4 = 16$

B's base has an area: $3 \times 5 = 15$

so A has the larger base area

For box A:

d) 16 cubes

e) 5 layers

f) $16 \times 5 = 80$ cubes

For box B:

g) 15 cubes; 5 layers; $15 \times 5 = 75$ cubes

h) So box A is larger than box B because it holds more cubes.

Activity 6

1.
   1. $8 \times 6 \times 2 = 96\text{ cm}^3$
   2. $10 \times 4 \times 6 = 240\text{ cm}^3$
   3. $5 \times 4 \times 2 = 40\text{ cm}^3$
   4. $4 \times 4 \times 6 = 96\text{ cm}^3$
   5. $2 \times 2 \times 2 = 8\text{ cm}^3$
   6. $20 \times 5 \times 5 = 500\text{ cm}^3$
   7. $20 \times 7{,}5 \times 7{,}5 = 1\ 1125\text{ cm}^3$
   8. $4 \times 13{,}25 \times 6 = 318\text{ cm}^3$
   9. $20 \times 10 \times 5 = 1\ 000\text{ cm}^3$
   10. $30 \times 20 \times 6 = 3\ 600\text{ cm}^3$
   11. $16 \times 9 \times 5 = 720\text{ cm}^3$
   12. $8{,}8 \times 5{,}5 \times 1{,}1 = 53{,}24\text{ cm}^3$

2.
   a) $50 \times 45 \times 30 = 67\ 500\text{ cm}^3$

   b) $80 \times 50 \times 45 = 180\ 000\text{ cm}^3$

You have seen that:
Volume of a box or a rectangular prism: length × width × height
We can therefore say **volume = base area × height**
since area = length × width

Volume: = base area × height
= length × breadth × height

Activity 7

1. $V = 3\text{cm} \times 3\text{ cm} \times 3\text{ cm} = 27\text{ cm}^3$

2.
   a) $V = 1\text{m} \times 1\text{ m} \times 1\text{ m} = 1\text{ m}^3$

   b) $V = 10\text{ cm} \times 10\text{ cm} \times 10\text{ cm} = 1000\text{ cm}^3$

Activity 8

1.
   a) $V = 10\text{ cm} \times 3\text{ cm} \times 4\text{ cm} = 120\text{ cm}^3$

   b) $V = 6\text{ cm} \times 2\text{ cm} \times 3{,}5\text{ cm} = 42\text{ cm}^3$

Activity 9

a) $V = \text{Area of base} \times \text{height} = \pi \times 5 \times 5 \times 22 = 3{,}142 \times 550 = 1728{,}1\text{cm}^3$

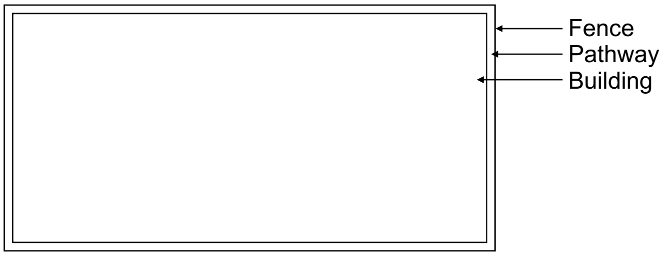
b) $V = \text{Area of base} \times \text{height} = \pi \times 10{,}5 \times 10{,}5 \times 25 = 3{,}142 \times 2756{,}25 = 8660{,}14\text{cm}^3$

Lesson 5

Activity 1

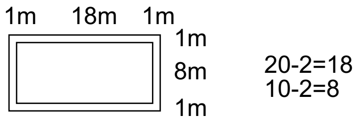
Seca showed Thandi how to draw a plan.

The plan looked like the one in the picture below.



Activity 2

Have a look at the drawing below. It shows the length of the building using subtraction.



Remember that in your drawing both the length and the width are 100 times smaller than the real length and width of the building. The area of the building is length × breadth = 100 × 100 = 10 000 times bigger than your scale drawing. This means that your scale of 1:100 is really the area of the full-size building and ground.

Did you measure 18 cm for your length and 8 cm for the width?
Don't forget to change your measurement to metres.

Your scale is 1:100 so in real life the measurements are 100 times what you measured. We get the length 1 800 cm and the width 800 cm.

Do you remember how to change cm to m? We divide the number of centimetres by 100 (as 100cm = 1m). If we change our measurements to metres we get 18 metres and 8 metres.

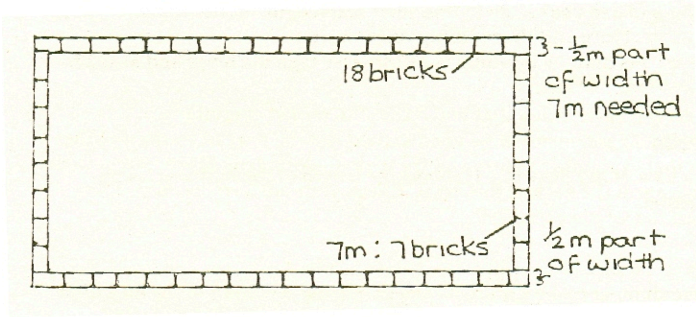
Activity 3

Did you use this mathematical formula to work out the number of blocks?
Number of blocks = perimeter of the space ÷ length of each block.
Did you make the perimeter equal 18 + 18 + 8 + 8 (the two widths + the two lengths) = 52 m? If you did, then you are wrong. Look again at your drawing of the plan.

Try to find the reason why the perimeter of 52 m is wrong before you look at the feedback below.

Have you noticed that the corners of the building are part of the length and also of the breadth? If you put the blocks along the lengths of the building, then the $\frac{1}{2}$ metre widths of the blocks at each end will be part of the width of the building. This means that at each corner there is already half a metre for the width. The two corners on one side of the house will make up one metre ($\frac{1}{2} + \frac{1}{2} = 1$)

Have a look at the drawing below.



So the number of blocks Thandi needs for the width is 1 m less than the width of the building. She needs blocks for 7m. So the total number of blocks she needs is 18 + 18 + 7 + 7 = 50.
Thandi has already made 45 blocks which means she has only 5 more to make.

Activity 4

She would need more blocks, the walls of the building would be thicker and the space inside the house would be smaller.

Activity 5

Every 1 cm we draw on the scale drawing is 20 cm of real fence, so 40 cm on the paper is 40 × 20 = 800 cm of real fence. The actual fence is a) 800 cm and b) 8 m long.

Lesson 6

Activity 1

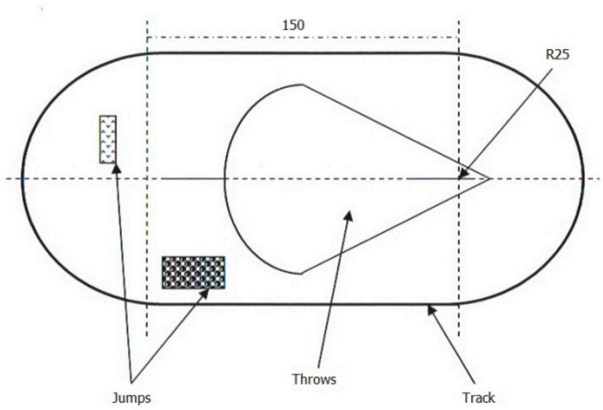
Here are ten different professionals who use technical drawings. Five of these are engineers. These are not the only ones. You may have mentioned others. That's good.

Practical People	**What they do**
Architect	Designs houses, buildings, schools, offices, towns
Builder	Constructs buildings designed by architect
Carpenter	Makes furniture and other wooden objects
Plumber	Builds and repairs water pipes in buildings
Watchmaker	Repairs watches and other small equipment

Type of engineer	**What they design**
Agricultural	Farm machinery
Electrical	Electrical equipment like generators
Marine	Ships, ship engines and ship equipment
Mechanical	Petrol and diesel engines, factory equipment
Aeronautical	Aeroplanes, aeroplane engines and other parts

Activity 3

Your drawing should look something like this one:



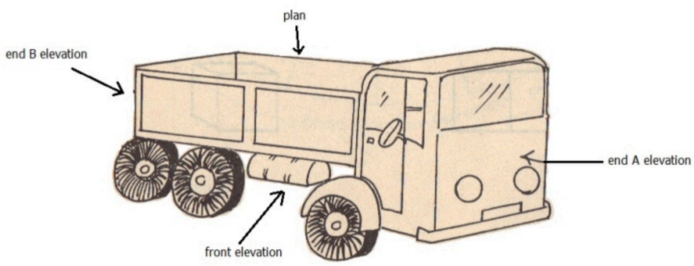
Activity 4

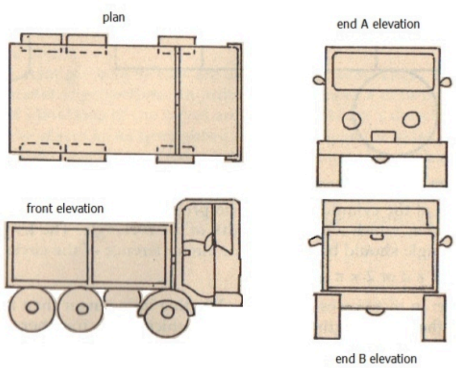
The first thing to remember is that the dimensions of the house are given in millimetres. Therefore, the answers would be as follows:

1. 7,5 metres
2. 3,5 metres
3. 1 metre
4. 1,5 and 2 metres
5. 2 metres
6. 1 metre

Activity 6

We have chosen a truck.





From these diagrams of elevations it is clear why the side of the truck is chosen as front elevation. Without any doubt, the different sides tell you the most about the truck.

Lesson 7

Activity 1

1. a) Mean = $\frac{5+7+3+13}{4}=\frac{28}{4}=7$

   b) Mean = $\frac{305+307+303+313}{4}=\frac{1228}{4}=307$

   c) Mean = $\frac{12+15+25+18+15}{5}$ km = $\frac{85}{5}$ km = 17 km

   d) The mean rainfall for the place is given by: $\frac{\text{Sum of rainfall}}{n}$
   Where n is the number of values
   (in this case the number of different years, which is 5).

   $\frac{48+23+62+47+56}{5}=\frac{236}{5}=47{,}2\text{mm}$

Activity 2

1. First arrange the mass in order of magnitude (size):
   8, 9, 10, 11, 13.
   The middle mass is 10. Therefore the median mass is 10 kg.

2. a) Arrange the numbers in order of size:
   2, 3, 4, 4, 4, 5, 6, 7, 8, 8.
   Note that we write down every number in the list, even if it is repeated.
   The middle numbers are 4 and 5.
   Therefore the median is:

   $\frac{4+5}{2}=\frac{9}{2}=4{,}5$

   b) Arrange in order of size: 2, 4, 6, 6, 7
   The median is 6.

   c) Here the numbers are already arranged from the largest to the smallest. Do you remember we said we could arrange the numbers from largest to smallest and take the middle as the median? Therefore looking at the numbers we see that the median is 90.

Activity 3

a) The mode is 7, since this is the value which occurs most often. If you can't see clearly from the data which value occurs most often then you must arrange them in order of size. In this way it is clearer to see how many times each of them appears. If we arrange the data in order of size, we get:
1, 2, 3, 3, 5, 5, 6, 7, 7, 7, 8
From this we see that 7 is the mode.

b) First arrange the numbers in order of size:
11, 11, 12, 12, 13, 13, 13, 14
The mode is therefore 13.

c) In order of size we get:
5, 5, 6, 7, 7, 19
5 appears 2 times and 7 appears 2 times.
Therefore we have 2 modes. These are 5 and 7.

Activity 4

1. a) Mean $= \frac{8 + 9 + 12 + 12 + 11 + 16 + 9 + 7 + 12 + 12 + 13}{11}$

$= \frac{121}{11} = 11$

To find the median we need to arrange the data in order of magnitude.
7, 8, 9, 9, 11, 12, 12, 12, 12, 13, 16.
The median is 12 and the mode is also 12.

b) The mean is: $\frac{60 + 75 + 60 + 89 + 87}{5} = \frac{371}{5} = 74,2$

In order of size we have 60, 60, 75, 87, 89
Therefore the median is 75 and the mode is 60.

c) Mean $= \frac{107 + 98 + 100 + 102 + 100 + 99 + 100 + 102}{8} = \frac{808}{8} = 101$

Arrange them to get 98, 99, 100, 100, 100, 102, 102, 107

The median is: $\frac{100+100}{2} = \frac{200}{2} = 100$

The mode is also 100.

2. The mean weekly rent:

$\frac{R350 + 325 + 300 + 300 + 325 + 325 + 300 + 300}{8} = \frac{2525}{8}$

= R315,63

Put them in ordered form to get 300, 300, 300, 300, 325, 325, 325, 350.
The median is: $\frac{300+325}{2} = \frac{625}{2}$
= R312,50

The mode is R300.

3. Let *T* be the total weight of the 9 sacks of potatoes, then since the mean is 25,2, we have

$25,2 = \frac{T}{9}$

(Remember, the mean is the total weight divided by the number of different weights.)

This gives $T = 25{,}2 \times 9 = 226{,}8$
Therefore the total weight for the 10 sacks is:
$226{,}8 + 27{,}3 = 254{,}1$ kg.

So the mean weight for the 10 sacks of potatoes is:

$$\frac{254{,}1}{10} = 25{,}41 \text{ kg.}$$

Activity 5

3 has the highest frequency. In other words the student gets the mark 3 the most times. Therefore the mode is 3.

Activity 6

a) Clearly the shoe size with the highest frequency is 7. Therefore the modal shoe size is 7.

b) The median is the $\frac{n+1}{2}$ *th* number.

$$\frac{20+1}{2} = \frac{21}{2} = 11{,}5$$

So the median is the 10th and 11th values added together and divided by 2.

Make sure you understand that this does not mean shoe size 10. It means the 10th and 11th number of the set of data, if you write out all the shoe sizes of the 20 people. After arranging all the shoes according to size, we have 3 of size 5, followed by 4 of size 6 and then 6 of size 7, and so on. Therefore the 10th and 11th shoe size is 7. So the median shoe size is 7. If you are not sure of this, write out the 20 shoe sizes for yourself.

Activity 7

Grams overweight	Frequency	Grams overweight × frequency
0	4	0
1	8	8
2	12	24
3	11	33
4	6	24
5	3	15
6	1	6
Total	45	110

Mean overweight $= \frac{110}{45} = 2{,}44$ grams

The median overweight is the $\frac{n+1}{2}$ *th value.*

$$\frac{45+1}{2} = \frac{46}{2} = 23rd$$

From the table, the 23rd value after arranging the weights in order of magnitude is two grams. So the median overweight is 2 g.

Activity 8

1. The largest clothes size is 16.
   The smallest clothes size is 8.
   Therefore the range = 16 – 8 = 8.

2. The highest figure is 14.
   The lowest figure is 2.
   The range = 14 – 2 = 12.