# Mathematics

Topic 2: Functions, Graphs and Algebra

NASCA Mathematics

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# Sub-Topic 1: Manipulation, simplification and factorisation of algebraic expressions

This sub-topic may sound very scary because of all the big words in its title. But the truth is that it is really quite simple. This topic is all about changing Maths “sentences” from one form into another. You can think of it like translating sentences from one language to another. The sentence will look different, but it will still say the same thing.

Being able to change Maths sentences from one form to another is an essential skill. Sometimes you will need to change the form of a Maths sentence in order to make sense of the situation, simplify what has been given to you or solve a particular problem.

## Unit 1: Simplifying algebraic expressions

#### Learning Outcomes

By the end of this unit, you should be able to:

1. State the different terms used to describe the different parts of an algebraic expression;
2. Identify the different parts of algebraic expressions; and
3. Collect like terms in order to simply algebraic expressions.

#### Introduction

**Algebraic expressions** are really just Maths sentences that contain constants and variables.

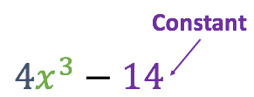
1. A **constant** is a number we know the value of. Examples of constants are “3”, “452” and . The value of these numbers does not change. Their value is **constant**.
2. **Variables** are those parts of an algebraic expression whose value we don’t know. We might be able to calculate their value if we can solve for them in an equation. We normally use letters to represent variables.

Here is an example of a very simple algebraic expression. In this expression, the variable is but it is being cubed (multiplied by itself three times). The number in front of the variable is called the **coefficient** of the variable.



When we write coeffects in front of variables we mean that we multiply the coefficient by the variable. So, this expression actually means

Here is another algebraic expression.



In this expression there are two **terms** – one contains a variable and the other is just a constant. We know the value of the constant. We do not know the value of the variable.

Terms are always separated by + or – signs which we call **operators**. Expressions with only one term are called **monomials**. Expressions with two terms are called **binomials**.

|  |  |
| --- | --- |
| Monomial | Binomial |
|  |  |

Here is an expression with **three** terms. We call algebraic expressions with three terms **trinomials**.



This trinomial has two variables – an and a . The term has a coefficient of 3 and the term has a coefficient of 4.

Had here is an expression with **four** terms. One of the terms has an and a variable.



We call algebraic expressions with **more than three terms polynomials**.

### Activity 1: Getting to Know Algebraic Expressions

|  |  |
| --- | --- |
| Bullseye | Purpose This activity will help you to know, identify and name the different parts of algebraic expressions. |
| Stopwatch | Suggested Time You will need about 10 minutes. |
| Pencil | What You Will Need  * A pen or pencil * Some blank paper or a notebook |

#### Tasks

1. On a piece of paper write an algebraic expression with the following characteristics.
   1. 2 terms; variable is and coefficient of is 6
   2. 3 terms; one term has a and another term has a
   3. 4 terms; one term has a and another term has a ; the coefficient of the other term is -5
2. How many terms are in these algebraic expressions?
3. Now try and write this expression in the simplest way possible:

?

**Hint**: There are three terms all with an . No matter what value the variable has, the in each term will always have the same value so you could treat as another variable, like .

#### Guided Reflection

Let’s look at question 1 first. There were many possible expressions you could have written for each part. Here are some examples.

1. . There are two terms and the coefficient of the variable, , is 6 But would also be right. It also has two terms and the coefficient is 6. The variable is still even if it is being squared or cubed.
2. . There are three terms and two different variables, and . The coefficient of the term with the variable is 1 and the coefficient of the term with the variable is -1. If you see no coefficient, always assume it is 1. As nothing was said about coefficients or constants would also be right.
3. . There are four terms but now there are three variables, , and . We know that the coefficient of must be -5 but we could have chosen any other letter we liked to represent this variable. For example, would have been just as correct.

Question 2 asked you to state how many terms there were in each expression.

1. There are two terms separated by a + sign.
2. Here there is only one term. Even though there are two different variables, terms are always separated by a + or – sign. We could re-write this one term as . Because we know the value of the 2 constants (3 and 2), we are able to multiply these together to get 6.
3. Here there are two terms separated by the – sign. We can simplify the first term like we did in part b) above to get .
4. Here there are three terms. Term 1 and 2 are separated by a – sign and term 2 and 3 by a plus sign. We could simplify this expression to .
5. Here there are three terms. As we have seen, this is exactly the same expression as in d).

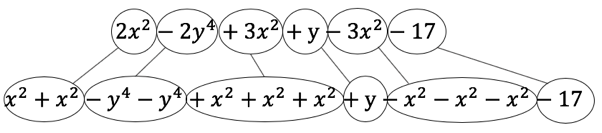
Question 3 was quite a tough one. We saw in part 1 and 2 of the activity that sometimes we can write algebraic expressions more simply.

because

Things are simple when everything is being multiplied (or divided). But what happens when we have more than one term. How would we simplify

?

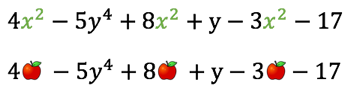
Here we have two different variables in the expression: and . We also have three terms that all have in them. We call these **like terms**. They are exactly the same, except for their coefficients. We could expand the whole expression as follows:



Now can you see what to do with all the like terms? Add them up. We have a total of two terms in the expression, so we can simplify it as follows:

Even though the term and the term both have the variable , they are NOT like terms and cannot be added together. One term is and the other is just . Think about the difference in value between and .

Here is another example showing another way we can think about like terms in algebraic expressions. See if you can simplify this expression.



The simplified expression is .

We call simplifying expressions in this way **collecting like terms**.

### Activity 2: Collecting Like Terms

|  |  |
| --- | --- |
| Bullseye | Purpose This activity will give you practice simplifying algebraic expressions by collecting like terms. |
| Stopwatch | Suggested Time You will need about 10 minutes. |
| Pencil | What You Will Need  * A pen or pencil * Some blank paper or a notebook |

#### Tasks

Simplify the following expressions as much as possible.

#### Guided Reflection

How did you do with these questions?

1. . This was an easy one. One term contained two ’s and the other term contained three ’s. So, we have five ’s altogether.
2. . Be careful not to collect the and terms together. Even though they are both cubes, they have different variables, so they are NOT like terms.
3. . Be careful not to collect the and terms together. One is a squared term and the other is to the power one. They are NOT like terms.
4. . Like terms do not only have to contain one variable. They can contain many as long as **all** the variables in the terms are the same and they are being raised to the same power in every term.
5. . Watch out for cases where the variables are not in the same order in each term. Order does **not** matter. We can write as in the same way as .

## Unit 2: Expanding and simplifying algebraic expressions

#### Learning Outcomes

By the end of this unit, you should be able to:

1. Multiply a monomial by any algebraic expression;
2. Multiply two or more binomial expressions together; and
3. Multiply a binomial and a trinomial expression together.

#### Introduction

In Unit 1, we learnt about the basics of algebraic expressions and how to simply them by collecting the like terms. This is a foundational skill that we will continue to use in this unit but in this unit, we will focus on how to multiply different algebraic expressions together.

How would you work out the value of this expression? Do the calculation remembering the correct order of operations (e.g. BODMAS).

There are two ways to do this calculation:

1. First work out what is inside the brackets and then multiply this by 2: .
2. Multiply each term inside the bracket by the term outside the bracket: .

With this in mind, how would you simply this expression?

**Note**: We don’t have to write the multiplication sign between the 2 and the open bracket.

Because all the terms inside the bracket are **unlike**, we cannot use method 1 above so we have to use method 2. We get the following:



We multiply the term outside the brackets by every term inside the brackets. Can we simplify this expression any further?

Try this example on your own first. Simply

After we multiply the term outside the brackets by each term inside the brackets, we get

Are there any like terms? Can we simplify the expression any further?

What about this example? Try it on your own first. Remember to multiply the term outside the brackets with each term inside the brackets and then see if there are like terms.

After we have multiplied away the brackets, we have

We can see that we have some like terms ( terms). Collecting like terms, we can simplify the expression.

We call this process of multiplying away the brackets in an algebraic expression **expanding the expression**.

### Activity 1: Expanding and Simplifying

|  |  |
| --- | --- |
| Bullseye | Purpose This activity will give you practice expanding and simplifying algebraic expressions. |
| Stopwatch | Suggested Time You will need about 15 minutes. |
| Pencil | What You Will Need  * A pen or pencil * Some blank paper or a notebook |

#### Tasks

Expand and simplify the following expressions.

#### Guided Reflection

How did you do with these questions?

1. . No problems here. There are no like terms to collect.
2. . Remember to multiply the term outside the brackets by each term inside the brackets. This will result in . After doing this, there are like terms to collect.
3. . We follow the same process as in question b) and then simplify by collecting all the like terms.

It is traditional that we write our final answer in decreasing powers of one of the variables. In this case, there is only one variable; we write the terms in decreasing powers of .

1. . After multiplying all three sets of brackets away, there were many like terms to collect.

### Activity 2: Expanding Binomials Part 1

|  |  |
| --- | --- |
| Bullseye | Purpose In this activity, you are going to discovery how to multiply two binomial expressions together. Remember that binomial expressions contain two terms. |
| Stopwatch | Suggested Time You will need about 15 minutes. |
| Pencil | What You Will Need  * A pen or pencil * Some blank paper or a notebook |

#### Tasks

1. Expand and simplify this expression by multiplying each term outside the brackets with each term inside the brackets.
2. Now expand and simplify this expression by evaluating each bracket first.
3. Expand and simplify this expression.
4. Now, expand and simplify this expression by first making it look like the equivalent expression in part 3).
5. Now expand and simplify this expression using the same method as in part 4).

#### Guided Reflection

How did you do with these questions?

1. . This was an easy question because all the terms were constants.
2. 70. We get the same answer as in part 1 which means that

In other words, if we multiply two binomial expressions together, it is the same thing as multiplying each term in the first binomial with each term in the second binomial.

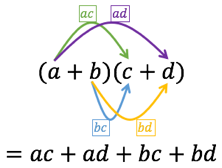
1. . There are no like terms to collect so we cannot simplify the expression any further.
2. . Just like we saw in parts 1) and 2), we can re-write the given expression as follows:

In other words, we multiplied each term in the first binomial with each term in the second binomial.

1. . The first thing we can do is to re-write the expression to unpack the first binomial.

There are no like terms to collect so we cannot simplify the expression any further.

We can formalise the process of multiplying two binomials together as follows:



You do not have to unpack the first binomial before you start multiplying each of its terms of each of the terms in the second binomial, but you can if you like.

### Activity 3: Expanding Binomials Part 2

|  |  |
| --- | --- |
| Bullseye | Purpose In this activity, you are going to practice multiplying two binomials together. |
| Stopwatch | Suggested Time You will need about 20 minutes. |
| Pencil | What You Will Need  * A pen or pencil * Some blank paper or a notebook |

#### Tasks

Expand and simplify the following expressions. All questions sourced from Everything Maths Grade 10 Products Exercise 1.4 ([https://www.siyavula.com/read/maths/grade-10/algebraic-expressions/01-algebraic-expressions-05)](https://www.siyavula.com/read/maths/grade-10/algebraic-expressions/01-algebraic-expressions-05).

#### Guided Reflection

If you managed all of these questions, you are doing really well. Let’s go through each one.

1. . Remember we need to multiply each term in the first binomial with each term in the second binomial to get

.

Then we need to collect the like terms to simplify the expression:

*.*

1. . Once again, we multiply each terms in the first binomial with each term in the second binomial and then collect and like terms.

Remember that we like to write our final answer with the terms in descending powers of one of the variables. In this case, is the only variable.

1. . In this case, the terms cancelled out.
2. This question gives the same answer as question 3). Can you see why? Remember that the – sign in front of the first binomial is really a -1. You can multiply each term in the first binomial by this -1 first or you can multiply each term in your final simplified expression by -1. Here are both solutions.

OR

1. . The first thing you need to realise is that . Then the question becomes simply.
2. . The only tricky part of this question was all the – signs.
3. . Once again, we need to see that .
4. . The only thing slightly tricky about this question was the – signs again.
5. . This question was slightly different to all the others. Here we had a **binomial** multiplied by a **trinomial**. However, the very same principle applies. Multiply each term in the binomial by each term in the trinomial and then simplify, like this.
6. . This question was just like question 9). We need to multiply each term in the binomial by each term in the trinomial and then simplify.

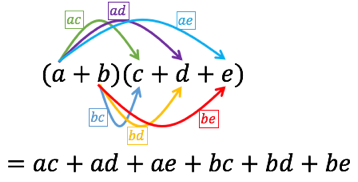
Remember to write your final answer in descending powers of .

1. . This question might have looked hard. The best way to treat this question was to think of it as two questions in one. First multiply binomials 1 and 2 together and then the question looks like questions 9) and 10).

Did you answer this question differently? Did you multiply different binomials together first? Can you see that it does not matter in what order we multiple the brackets together?

Questions 9) and 10) showed us how to multiply a binomial by a trinomial. We can formalise the process of multiplying a binomial by a trinomial together as follows:

We multiply each term in the binomial with each term in the trinomial.



What do you think will be the process needed to multiply two trinomials together? See if you can expand and simplify this expression:

Here is the answer:

### Activity 4: Expanding Binomials and Trinomials

|  |  |
| --- | --- |
| Bullseye | Purpose In this activity, you are going to practice expanding and simplifying different algebraic expressions. |
| Stopwatch | Suggested Time You will need about 20 minutes. |
| Pencil | What You Will Need  * A pen or pencil * Some blank paper or a notebook |

#### Tasks

1. Expand and simplify the following expressions. All questions sourced from Everything Maths Grade 10 Products Exercise 1.4 ([https://www.siyavula.com/read/maths/grade-10/algebraic-expressions/01-algebraic-expressions-05)](https://www.siyavula.com/read/maths/grade-10/algebraic-expressions/01-algebraic-expressions-05).
2. If , what is the value of ?
3. If
   1. Which of these values of will make positive?
   2. Which of these values of will make positive?

#### Guided Reflection

TBC

## Unit 3: Factorising Algebraic Expressions

#### Learning Outcomes

By the end of this unit, you should be able to:

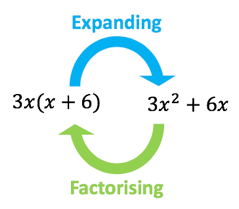
1. Factorise algebraic expressions by taking out a common factor;
2. Factorise algebraic expressions by grouping in pairs;
3. Factorise algebraic expressions that are the difference of two squares; and
4. Factorise algebraic expressions that are trinomials;

#### Introduction

In Unit 2 we saw how to **expand** algebraic expressions by multiplying, for example, binomials and trinomials together. When we did this, we removed all the brackets in the original expression. In this unit, we will learn how to **factorise** algebraic expressions – how to put the brackets back.

Here is a simple illustration. If you are given the expression , it is simple to expand this to get .

Factorisation is the process of starting with and getting back to .



We call this process factorisation because it means that we write the algebraic expression as the **product** of its **factors** (the factors are multiplied together). Therefore, to understand factorisation, we need to make sure we understand what **factors** are.

### Activity 1: Factors and Divisibility

|  |  |
| --- | --- |
| Bullseye | Purpose In this activity, you are going to learn about algebraic factors. |
| Stopwatch | Suggested Time You will need about 20 minutes. |
| Pencil | What You Will Need  * A pen or pencil * Some blank paper or a notebook * An Internet connection |

#### Tasks

1. Watch the video called [*Intro to factors and divisibility*](https://www.khanacademy.org/math/algebra/polynomial-factorization/modal/v/factors-and-divisibility-in-algebra). It is 05:30 long.

(https://www.khanacademy.org/math/algebra/polynomial-factorization/modal/v/factors-and-divisibility-in-algebra).

Now, using what you learnt in the video, answer the following questions.

1. What are the factors of:
   1. (what do you notice about all the factors of ?)
2. Is a factor of ?
3. Which of the following are NOT factors of ?
4. Which of the following ARE factors of ?
5. If one factor of is , what is the other factor?

HINT: times *what* will expand to ?

#### Guided Reflection

We saw in the video that just as we can write natural numbers like as the product of their factors (e.g. ), we can also write algebraic expressions as the product of their factors. For example, we can write as the product of its factors .

Let’s have a look at the questions now.

1. Question 2 asked for the factors of various numbers.
   1. . These are the factors of because
   2. . These are the factors of because

Did you notice that all the factors of 128 are powers of . We can also write as or .

* 1. are all factors of because

We know that and are factors of because . But we can also write as the product of its **prime factors** (remember prime numbers only have 1 and themselves as factors).

In the same way, we can also write algebraic expressions as the product of its indivisible factors

Any product of these indivisible factors will also be a factor of the original expression. So is a factor of and is also a factor of .

* 1. are all possible factors of . Another way we can say this is that is **exactly divisible** by any of these numbers.

1. Question 3 asked if is a factor of ? The answer is no. is not exactly divisible by .

We land up with something other than 1 in the denominator (the bottom of the fraction). There is one too many ’s in which is why we are NOT left with a simple monomial after division. The factors of are . There is no product of these factor that is equal to .

1. . We can write as the product of its indivisible factors: . is the only one in the list that is not the product of these indivisible factors. All the others are. Therefore, all the others are factors of .
2. and . is not a factor because 10 is not a factor of 15 and is not a factor of .
3. . needs to be multiplied by in order to get because

Have another look at the result we got from question 6 in Activity 1. We were told that was one factor of and were asked to find the other factor. We discovered that the other factor was because

is the **highest common factor** of each of the terms in . In other words, it is the biggest factor that is a factor of all of the terms.

is also a factor common to all three terms but it is NOT the highest factor. Do the next activity to learn more about factorising algebraic expressions by finding the highest common factor.

### Activity 2: Common Factors

|  |  |
| --- | --- |
| Bullseye | Purpose In this activity, you are going to learn how to factorise expanded algebraic expressions by taking out a highest common factor. |
| Stopwatch | Suggested Time You will need about 20 minutes. |
| Pencil | What You Will Need  * A pen or pencil * Some blank paper or a notebook * An Internet connection |

#### Tasks

1. Watch the video called [*Factoring polynomials: common factor*](https://www.khanacademy.org/math/algebra/polynomial-factorization/factoring-polynomials-1-common-factors/v/factoring-and-the-distributive-property-3). It is 06:00 long.

(https://www.khanacademy.org/math/algebra/polynomial-factorization/factoring-polynomials-1-common-factors/v/factoring-and-the-distributive-property-3).

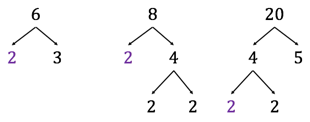
Now, using what you learnt in the video, answer the following questions.

1. What are the greatest common factors of the following lists of terms?
2. Factorise the following algebraic expressions by taking out the highest common factor. Some questions sourced from Everything Maths Grade 10 Factorisation Exercise 1.5 (<https://www.siyavula.com/read/maths/grade-10/algebraic-expressions/01-algebraic-expressions-06>.
3. Factorise this algebraic expression by taking out a common factor: . Hint: What common to both terms?

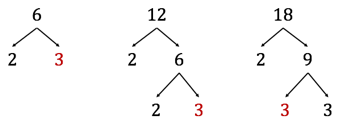
#### Guided Reflection

In the video, we saw how to factorise an algebraic expression by taking out a highest common factor. We saw that the highest common factor of was because this was the biggest factor that was a factor of each of the terms.

1. Question asked you to identify the highest common factor of each list of terms.
   1. . The biggest factor that is a factor of all three terms is 3.
   2. 2. The biggest factor that is a factor of all three terms is 2. Remember that it can help to split each term into its prime or indivisible factors first, like this.



* 1. 3. The biggest factor that is a factor of all three terms is 2. Remember that it can help to split each term into its prime or indivisible factors first, like this.



is also a common factor but it is not the HIGHEST common factor.

* 1. . First look at the coefficients. The highest common factor of is . Then look at the variables. The highest common factor of is .
  2. . First look at the coefficients. The highest common factor of is . Then look at the variables. The highest common factor of is .

1. Question 3 asked you to factorise the expressions by taking out the highest common factor. The first thing you need to do is identify the highest common factor and write it down. Then you need to open up brackets and write what is left in each term. Remember, you can always check yourself by expanding your factorised expression to see if you get back to where you started.
   1. . The highest common factor of both terms is . To work out what goes into the brackets…

AND

* 1. . This was a slightly trickier question because of the – sign.

First look at the coefficients. The highest common factor of and is . The reason we take out as the common factor and not just + is that this will make the first term in the brackets) rather than . We normally like the first term in an algebraic expression to be positive.

But, if we take out as the common factor, we need to change the sign on the second term to make sure that, if we were to expand the expression again, we would get back to .

Now look at the variables. The highest common factor is . So, the overall highest common factor we are taking out of each term is .

To complete what goes into the brackets…

AND

So, to factorise:

* 1. . Once again, we need to take out a negative highest common factor so that the first term inside the brackets is positive. The highest common factor of both terms is . To complete the brackets…

AND

* 1. . In this case, there are no common factors of the coefficients. The highest common factor of the variables is .
  2. . In this question, there were four terms. In principle, there is no limit to the number of terms you can take a common factor out of. However, the more terms there are, the less likely there will be a common factor.
  3. . In this question, there were no common factors of all four terms. But one could take out a common factor out of the and terms and another common factor out of the and terms.

and : Common factor is . Terms inside the brackets are ()

and : Common factor is . Terms inside the brackets are .

If there are no common factors of all the terms in an expression, look for common factors of some of the terms. You will see why this can be so useful in the next activity.

1. . This question asked you to factorise by taking out a common factor.

The first term has two factors: and .

The second term has two factors: and .

The highest common factor is .

To find out what is left of each term after taking the common factor out…

AND

### Activity 3: More Advanced Common Factors

|  |  |
| --- | --- |
| Bullseye | Purpose In this activity, you are going to learn how to factorise expanded algebraic expressions by taking out more advanced highest common factors. |
| Stopwatch | Suggested Time You will need about 30 minutes. |
| Pencil | What You Will Need  * A pen or pencil * Some blank paper or a notebook * An Internet connection |

#### Tasks

1. Watch the video called [*Factoring polynomials: common binomial factor*](https://www.khanacademy.org/math/algebra/polynomial-factorization/factoring-polynomials-1-common-factors/v/factoring-a-common-binomial-factor). It is 01:40 long.

(https://www.khanacademy.org/math/algebra/polynomial-factorization/factoring-polynomials-1-common-factors/v/factoring-a-common-binomial-factor)

1. Factorise the following expressions by taking out a common factor. Some questions sourced from Everything Maths Grade 10 Factorisation Exercise 1.5 (<https://www.siyavula.com/read/maths/grade-10/algebraic-expressions/01-algebraic-expressions-06>.

#### Guided Reflection

We saw in the video that common factors do not always have to be monomial terms. They can be binomials, trinomials or any polynomial. So long as the term is a common factor, you can take it out to factorise the expression. Let’s work through the questions now.

1. You were asked to factorise by taking out a common factor in each case.
   1. . This was a simple question to start off with. You need to recognise the common factor of .
   2. . In this question, there were three terms all with the common factor of .
   3. . In this question, the common factor was a trinomial and was common to three terms.
   4. . You may have initially missed that the common factor was actually . If you had, you would have factorised the expression in two steps rather than one (which does not really matter):

(

If so, you may only then have noticed the common factor of in the second bracket. You could have taken it out at this point to get.

* 1. . There was no binomial or trinomial common factor here just to keep you on your toes!! The common factor was .
  2. . At first it may have seemed that there were no common factors at all. But remember in Activity 2, we said that sometimes we need to take a common factor out of only some of the terms in an expression? This is such a case.

If you take the highest common factor out of the first two terms, you get Now we can see that we have created a common factor with the third term of

If you take this new common factor out of both terms, you are able to fully factorise the expression:

(take out a common factor from the first two terms)

(take out a common factor of )

* 1. . This question did not start out with a common factor because is not the same as in the same way that . But we can create a common factor by changing the sign in front of the second term and then also changing the signs inside the term. This is the same thing as saying that and

So:

(take out the common factor of )

* 1. . Like question f) we need to take out the common factor from the first two terms. But this does not immediately create the common factor we need. We also need to change the sign in front of the third term.

(take out the common factor of from the first two terms)

(change the sign in front of the last term to change the order of the terms inside the last bracket to create a common factor of )

(take out the common factor of form both terms)

* 1. . It might be tempting to take out the common factor of form the first three terms to get . But now there is nothing else we can do, and the expression is still not fully factorised (not everything is inside brackets).

However, if we take a common factor out of the first two terms and another common factor out of the last two terms, we create a new common factor of .

(watch out for the minus sign in the second bracket!)

Now, you may have noticed that there were like terms we could have collected in the original expression. If we collect these like terms our expression becomes . This is a trinomial expression and we have another way of factorising trinomials which we will learn about in a later activity.

* 1. . Answering this question correctly the first time takes experience, so don’t worry if you had some trouble with it or had to try a few methods before finding the right one.

You may have been tempted to take out the from the first three terms to get . But then there is nothing more you can do, and you would still have terms outside of brackets.

There are two other options. You could take a common factor of out of the first two terms and a common factor of out of the last two terms: .

Or you could take the common factor of out of terms one and three and the common factor of out of terms two and four: .

In both cases, you will create a new common factor – either or .

Can you see that, sometimes, there are a few different ways to factorise an expression? Your goal is most often to make sure that you factorise the given expression fully i.e. that there are no terms outside of brackets.

* 1. . Here the common factor is easy to spot – it is . But the other factor of ) is not fully factorised, as we shall see in the next activity.

### Activity 4: The Difference of Two Squares

|  |  |
| --- | --- |
| Bullseye | Purpose In this activity, you are going to learn how to factorise algebraic expressions that are the difference of two squares. |
| Stopwatch | Suggested Time You will need about 60 minutes. |
| Pencil | What You Will Need  * A pen or pencil * Some blank paper or a notebook * An Internet connection |

#### Tasks

1. Have a look at the following statements:

* is a square number because . We say that is the square root of .
* is a square number because . is the square root of .
* is a square number because . is the square root of .
* is a square number because . is the square root of .

Write the square roots of the following terms.

1. Watch the video called [*Difference of squares intro*](https://www.khanacademy.org/math/algebra/polynomial-factorization/modal/v/difference-of-squares-intro). It is 05:00 long.

(<https://www.khanacademy.org/math/algebra/polynomial-factorization/modal/v/difference-of-squares-intro>)

1. Now watch the video called [Factoring difference of squares: leading coefficient 1](https://www.khanacademy.org/math/algebra/polynomial-factorization/modal/v/factoring-to-produce-difference-of-squares). It is 02:30 long.

(<https://www.khanacademy.org/math/algebra/polynomial-factorization/modal/v/factoring-to-produce-difference-of-squares>)

1. Factorise the following expressions as fully as possible. Some questions sourced from Everything Maths Grade 10 Factorisation Exercise 1.6 (<https://www.siyavula.com/read/maths/grade-10/algebraic-expressions/01-algebraic-expressions-06>.

#### Guided Reflection

1. In this question, we just needed to write the square roots of the given terms.
   1. . This first one was quite easy. We need to find the square root of each part of the term.

The square root of is .

The square root of is (finding the square roots of exponents is easy – you just divide the exponent by 2!).

The square root of is .

* 1. . There was a coefficient and three variables, but the process is the same – find the square root of each part of the term.
  2. . In this case, we had a whole bracket being squared. Here it is the bracket that is being squared NOT the terms inside the bracket so, like before, we just divide the exponent by 2 to find the square root.
  3. . This case might have been confusing because there were perfect squares INSIDE the brackets as well. Once again, just focus on the bracket not on what’s inside and divide the exponent on the brackets by 2 to find the square root.
  4. . is not a perfect square. Perfect squares have nice whole number square roots but we can still find the square root of 2. It is an irrational number (a number that is an endless and never repeating decimal fraction) so we leave the answer written as because this is the best way we have of writing this number.
  5. . Like e) is not a perfect square but we can still find the square root – just put everything inside the square root sign.

1. Numbering should be 4 After watch those two videos, you should have had a good idea about how to factorise expressions that contained the difference of squares. Let’s go through the questions together.
   1. . Both and are square and they are separated by a – sign. So, we have the difference of two squares. All we need to do to factorise the expression is to follow the template of . It does not matter if the bracket with the + sign is first or no, although it is good to be consistent in your work. Just remember that the signs in the brackets **have** to be different.
   2. . In this question it does not really matter if you take out the common factor of first or not. As a general rule, thought, we like to always start factorising an expression by taking any common factors out first. So, you could have answered this question in either of these ways:

(take out the common factor)

(factorise the difference of squares)

OR

(factorise the difference of squares)

(take out the common factor of from BOTH brackets)

* 1. . Like we said in part b), it is always a good idea to take out any common factors first. In this case, the common factor was which was, itself, a difference of squares which could be factorised further:

(take out the common factor)

(factorise the difference of squares)

* 1. . You needed to be care with this one. After taking out the common factor of , you may have been tempted to factorise . But this is the **sum** of squares. We can only factorise the **difference** of squares.
  2. . In this question, we had to create the common factor of by first changing the sign in front of the second term:

(remember that this is legal because we are really just taking out a common factor of from the second term - and )

(take out the common factor)

(factorise the difference of squares)

* 1. . In this question, you need to recognise that it is the whole binomial that is being squared to create a difference of two squares. If it makes it easier for you, you can replace the whole binomial with another variable like A:

Let then

(remember to replace with again)

You could also write the final answer without the inner brackets:

* 1. . Once again, we have a difference of squares where one of the squares is a binomial. Again, you can replace the whole binomial with another variable if you like.

(we have not used another variable but have used square a round brackets to make things easier because the – signs still need to be taken into account)

(it is always a good idea to simplify whatever is inside the factors if possible)

(take out the common factor from the first factor)

(we can also take out a ffrom the second factor to simplify the answer a bit more)

**Note**: Later on, we will see how we could also factorise this expression by first expanding it and then factorising the resulting trinomial expression.

* 1. . Once again, we have a binomial as one of the squares.

(factorise the difference of squares)

(simplify each product)

* 1. . Here both squares are binomials, but we still just need to follow the basic template.

(factorise the difference of squares)

(simplify each product)

* 1. . Even though both terms are not **perfect** squares, we can still factorise it using the difference of squares method. We just need to realise that the square root of is and apply the template as normal. Numbering wrong from this point – this should be j)
  2. . In this question, the only thing we can do to factorise the expression is to take out the common factor. After this, we are still not left with a **difference** of squares.
  3. . The first thing you needed to realise in this question was the need to group the four terms into two pairs in order to first take out a common factor from each pair. There are two ways to pair the terms:

(pair terms 1 and 3 and terms 2 and 4)

(factorise the difference of squares in the first pair and take out the common factor from the second pair)

(factorise the difference of squares in the second pair)

(take out the common factor)

OR

(pair terms 1 and 2 and terms 3 and 4, watching out for the sign in the second bracket)

(take out the common factors from each pair)

(take out the common factor)

* 1. . This question also required grouping of terms in order to take out common factors. Here is just one of the options. No matter how you grouped, you should still get the same final product of factors.

(here we grouped terms into pairs in the order in which they were given – remember to watch out for the signs inside the brackets)

(take out the common factors)

(take out the common factor)

Quadratics are trinomial algebraic expressions with the general form of . The highest exponent is 2. We call the term the **leading** term. They are everywhere, as you will see when we start the sub-topics on solving equations and functions. You will see that, in order to solve equations of this form, we need to be able to factorise these kinds of expressions. Because this kind of factorising is so common and so useful, we have some techniques to make the process as easy as possible.

### Activity 5: Factorising Quadratic Trinomials

|  |  |
| --- | --- |
| Bullseye | Purpose In this activity, you are going to learn how to factorise algebraic expressions that are quadratics. |
| Stopwatch | Suggested Time You will need about 60 minutes. |
| Pencil | What You Will Need  * A pen or pencil * Some blank paper or a notebook * An Internet connection |

#### Tasks

1. Let’s start with a simple investigation.
   1. Expand the following expressions, remembering to collect any like terms. Organise your answers in a table with column 1 containing the original expression and column 2 containing the expanded expressions.
   2. Are these original expressions all in factorised form i.e. are they the product of factors?
   3. What kind of trinomial does each expression expand into?
   4. What relationship can you see between the original expressions and the term in each quadratic trinomial?
   5. What relationship can you see between the original expressions and the constant term in each quadratic trinomial?
   6. What relationship can you see between the original expression and the term, and especially its coefficient, in each quadratic trinomial?
   7. Using what you have discovered, factorise
   8. Using what you have discovered, factorise .
2. Now watch the video called [*Factorising quadratics*](https://www.youtube.com/watch?v=_lSGP8wYKC4) to learn a good method for deciding what the factors of a quadratic trinomial are. It is 03:00 long. While you watch the video, make sure that you pause it and answer the questions asked in the video.

(<https://www.youtube.com/watch?v=_lSGP8wYKC4>)

1. The video in task 2) showed you one way to work out what the correct factors of a quadratic trinomial are. There is another method you might like to use, called the “Cross Method”. Watch the video called [Factorisation (4) Cross Method](https://www.youtube.com/watch?v=WHh1pWIMsbo). It is 05:45 long.

(<https://www.youtube.com/watch?v=WHh1pWIMsbo>)

1. Using either method demonstrated in the videos, factorise the following expressions. Some questions sourced from Everything Maths Grade 10 Factorisation Exercise 1.8 (<https://www.siyavula.com/read/maths/grade-10/algebraic-expressions/01-algebraic-expressions-06>.

#### Guided Reflection

1. Let’s work through the investigation together to see what we can discover.
   1. Here are all the expressions in expanded form:

|  |  |
| --- | --- |
| Original | Expanded |
|  |  |
|  |  |
|  |  |
|  |  |

* 1. All the original expressions are in factorised form. Each of them is the product of 2 factors.
  2. Each expression expands into a quadratic trinomial because each of them have the general form . In every case, though, .
  3. The term in each quadratic trinomial is just the product of the terms in each bracket.
  4. The constant term in each quadratic trinomial is just the product of the second term in each bracket:
  5. The coefficient of the term in each quadratic trinomial is the sum of the factors of the last term in the quadratic trinomial:
  6. If we want to factorise , using what we have discovered, the first thing we need to do is check if it is a quadratic trinomial. It is in the form .

Next, we need to set up our 2 sets of brackets: .

The term comes from the product of the first terms in each bracket. So, these terms must be and : .

We know that the factors of the last term in the quadratic trinomial must add up to the coefficient of the term. We have a few options for the products of 24:

|  |  |
| --- | --- |
| Factors | Sum of factors |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

From looking at the table, it is clear that the factors of that will add to give us the coefficient of the middle term are .

Now we can complete our 2 factors: .

It is always wise, to quickly expand your final factorised form to make sure that it is the same as what you started with: .

* 1. Now let’s factorise the other expression .

In this case the first terms in both brackets are still going to be and : .

But now, the last term is **not** . It is . Therefore, we need a new table of factors.

|  |  |
| --- | --- |
| Factors | Sum of factors |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

From looking at the table, it is clear that the factors of that will add to give us the coefficient of the middle term are .

Now we can complete our 2 factors: .

Quickly expanded your final factorised form to make sure that it is the same as what you started with: .

NOTE: Did you notice that, if the last term in the quadratic trinomial is positive, the signs of the factors will be the same – both + or both -? If the last term is negative, the signs of the factors will be different.

1. Remember that, when factorising quadratic trinomials, there is no one correct method. You are free to use whatever method makes most sense to you. Numbering wrong from this point – this should be 4)
   1. . The factors of are and . These are the first terms in each bracket. The factors of that add up to are . These are the second terms in each bracket.
   2. . The factors of are and . These are the first terms in each bracket. The factors of that add up to are . These are the second terms in each bracket.
   3. . The factors of are and . These are the first terms in each bracket. The factors of that add up to are . These are the second terms in each bracket.
   4. . The factors of are and . These are the first terms in each bracket. The factors of that add up to are . These are the second terms in each bracket.
   5. . The factors of are and . These are the first terms in each bracket. The factors of that add up to are . These are the second terms in each bracket.
   6. . This question was a little bit trickier because there was another variable in the expression. But our normal method still works. The factors of are and . These are the first terms in each bracket.

The factors of are as follows:

or

or

or

Remember, that we want a pair of factors that add up to the coefficient of the middle term. In this case we can think of as the coefficient of . The pairs of factors that will work are .

* 1. . In this question, we need to recognise that needs to be treated as a single variable. If it helps, you can replace with another variable.

Let : . Now the expression is easy to factorise as .

But we need to replace with again: .

* 1. . There are two ways we can approach this question. We can either start by taking a common factor of 3 out of each term or we can expand the brackets and then start to factorise. Let’s look at both methods.

Common factor first:

(take out a common factor of 3)

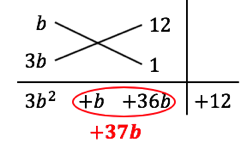
(simplify the bracket – we have an ordinary quadratic inside the brackets)

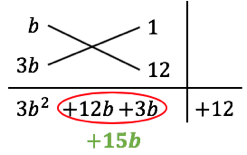
(factorise the quadratic)

Expand first:

We have a quadratic **but** the coefficient of the leading term, is not 1, it is 3 The factors of are . So, we can start writing our factors as .

But now, when we expand the brackets, the second term in the first bracket is going to be multiplied by . In this case, we need to list and explore which factors of 12 are going to work out. Here the cross method can be very useful.





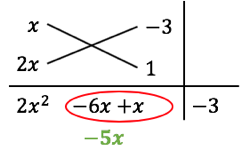
We can now see that our factors are going to be . However, there is a common factor of 3 in the second bracket which we need to take out to give us the final answer of

There is another method we can use when the leading coefficient is not 1 but, most of the time, factorising these expressions is still a matter of trial and error.

* 1. . Here is another question where the leading coefficient is not 1. At least in this case, we only have 2 options for the factors of 1 - and . We can see, by inspection, that we need to use .
  2. . We already know that the best place to start factorising an expression is to take out any common factors.

(take out the common factor of 3)

(factorise the trinomial, using the cross method to help you work through the various options)



### Activity 6: Factorising More Advanced Quadratic Trinomials

|  |  |
| --- | --- |
| Bullseye | Purpose In this activity, you are going to learn how to factorise more advanced quadratic expressions. |
| Stopwatch | Suggested Time You will need about 60 minutes. |
| Pencil | What You Will Need  * A pen or pencil * Some blank paper or a notebook * An Internet connection |

#### Tasks

1. Factorise the following expressions. Some questions sourced from Everything Maths Grade 10 Factorisation Exercise 1.8 (<https://www.siyavula.com/read/maths/grade-10/algebraic-expressions/01-algebraic-expressions-06>.
2. Watch the video called [Factorising quadratics by grouping](https://www.khanacademy.org/math/algebra/polynomial-factorization/factoring-quadratics-2/v/factoring-trinomials-by-grouping-4) to learn about another method of factorising quadratics where the leading coefficient is not 1.

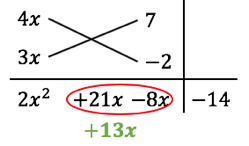
(<https://www.khanacademy.org/math/algebra/polynomial-factorization/factoring-quadratics-2/v/factoring-trinomials-by-grouping-4>)

1. Using any method(s) you like, factorise the following expressions. Some questions sourced from Everything Maths Grade 10 End Of Chapter Exercises 1.11 (https://www.siyavula.com/read/maths/grade-10/algebraic-expressions/01-algebraic-expressions-09).

#### Guided Reflection

1. All of these questions were quadratics were the leading coefficient was not 1.
   1. The choices for the factors of are . The choices for the factors of are . We just need to try the various combinations to figure out which one works. As you gain more experience, this process will get easier and quicker.
   2. . The choices for the factors of are . The choices for the factors of are or or or . We just need to try the various combinations to figure out which one works. Remember to use the cross method if this helps.
   3. . As always, it is useful to take out a common factor first. If you don’t, you need to work with the factors of for which there is more than one option. Taking out the common factor simplifies this situation.
2. This question gave you an opportunity to practice many of your new factorising skills.
   1. . This was a nice easy question to start with.
   2. . In this expression, there are two terms with powers of 2. You can decide which terms is the leading term. Normally, we make the leading term. The factors of are and the factors of are . Trial and error on the grouping method will help us figure out what the factors are.
   3. . You need to be careful with this question because the leading coefficient is not 1 nor is it positive. Instead of working with the factors of , take out a common factor of first to make the leading coefficient positive.

(take out a common factor of )



You could also have used the grouping method to factorise the trinomial:

We want to factorise . Therefore, we are looking for products of that are 13 units apart. Testing out some possibilities, we find and .

* 1. . Take out the common factor first. However, you should also notice that your leading term is and the middle term is . You can either create factors like ( or you can make a substitution.

Let . Then

(take out the common factor)

(factorise the quadratic)

(replace with )

(factorise the difference of squares in the second bracket.

* 1. . As usual, it is best to start by taking out the common factor and then to work from there.

(take out the common factor of , making sure that the signs INSIDE the bracket are correct)

What is inside the bracket may not immediately look like a quadratic. Firstly, the terms are in the wrong order and the leading term is . However, the other term with the variable in has an exponent exactly half the leading term’s so it does still fit the quadratic form. You can either work with or you can make a substitution.

Let . Then

(write the terms in the normal order)

(factorise the quadratic)

## Unit 4: Simplifying Algebraic Expressions Containing Fractions

#### Learning Outcomes

By the end of this unit, you should be able to:

1. Simplify algebraic expressions containing fractions;
2. Multiply and divide algebraic fractions; and
3. Add and subtract algebraic fractions.

#### Introduction

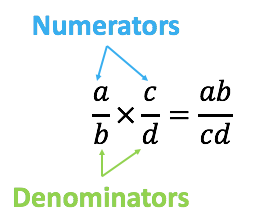
So far in this sub-topic you have learnt all about algebraic expressions, how to expand them and how to factorise them. In this unit you are going to combine all these skills together to allow you to add and subtract algebraic expressions that are in the form of fractions, like this one

You will also get some practice manipulating algebraic fractions where the coefficients are fractions.

In order to get started, there are four basic rules of working with fractions that you need to know. You probably know them already, but let’s go through them.

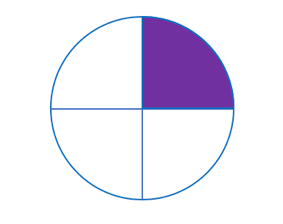
#### Multiplication of Fractions:

When we multiply fractions together, we simply multiply the numerators and the denominators together.

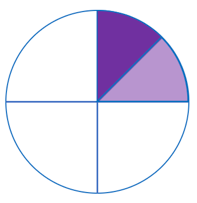


Here is an example to illustrate the point:

Remember, when we say , for example, what we mean is “what is three groups **of** four?” e.g. three groups of four chocolates or three groups of four cups. So, when we write we mean what is “half a group of one quarter?” or “what is one half of one quarter?”



If we cut one quarter in half, we get one eighth.



Here is another example: or what is one half of four.

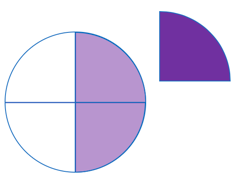
Try this one on your own.

Did you get ?

1. Division of Fractions:

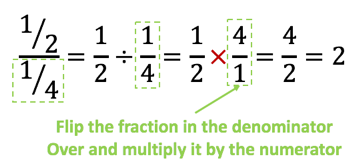
We know that and that we are asking “how many times does 2 go into 4?” The answer is clearly 2. What about ? Here we are asking “how many times does 4 go into 2?” If you think about, the answer of makes sense. Four goes into two half a time.

Now what about ? How many times does one quarter go into one half?



Can you see that the answer is 2? We can fit two quarters into one half.

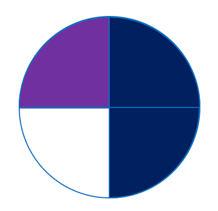
When we divide one fraction one fraction by another, we have an easy way to calculate the answer. We flip the fraction in the denominator over and multiply it by the fraction in the numerator.



So, in general,

#### Addition of Fractions:

Adding fractions can be quite tricky. Let’s start with a simple example. What is ? In the picture below, we can see that the answer is .

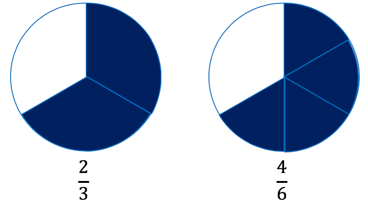


What we really do when working this out is to convert the one half into two quarters and then add up the number of quarters.

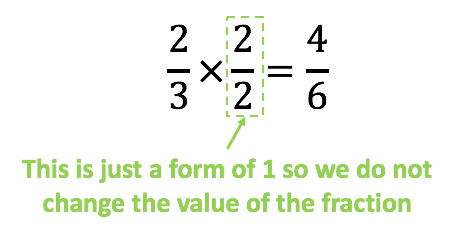
We say that four is the **Lowest Common Denominator (LCD)** – the smallest number that all the denominators can go into. In this case the LCD is 4 because it is the smallest number that both four and two can go into.

What about ?

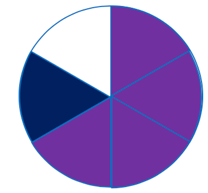
Here, we need to convert two thirds into sixths. Six will be the LCD. If one third is the same as two sixths, two thirds is the same as four sixths.



Mathematically, we change the form of the fraction WITHOUT changing its value by multiplying the numerator and denominator by the same number (just another form of 1) and we choose the form of one that will create the denominator we want, which in this case is 6.



Now both denominators are the same and we can easily add the fractions.



We can only add fractions that have the same denominators. If they do not have the same denominators, we have to change their form so that they do have the same denominator. We convert the fractions to all have the lowest common denominator.

Look at this example:

We have to get all the denominators the same. What is the smallest number that 7, 3 and 6 all go into? This is 42 (3 goes into 6 and ).

So, in general

#### Denominators may never be zero:

Because we can’t divide by zero, the denominators of algebraic fractions may never be zero. We need to state what values any variables in the denominator **may not** have.

For example, we know that . But this also means that and .

Similarly, we know that . But none of the denominators may be zero. Therefore, , and .

Finally, we know that but this also means that .

Now that we have covered the three basic rules of working with fractions, let’s see how to deal with algebraic fractions.

### Activity 1: Multiplying Algebraic Fractions Part 1

|  |  |
| --- | --- |
| Bullseye | Purpose In this activity, you are going to learn how to simplify algebraic fractions. |
| Stopwatch | Suggested Time You will need about 35 minutes. |
| Pencil | What You Will Need  * A pen or pencil * Some blank paper or a notebook |

#### Tasks

1. Simplify the following algebraic fraction using these steps:
   1. Factorise the numerator and denominator.
   2. “Cancel” any common factors between numerator and denominator.
   3. Simplify the numerator and denominator, if possible.
   4. Write down the restrictions on the variables.
2. Simplify the following algebraic fraction using the following steps:
3. Factorise the numerators and denominators.
4. Multiply the numerators and denominators.
5. “Cancel” any common factors between numerator and denominator.
6. Simplify the numerator and denominator, if possible.
7. Write down the restrictions on the variables.
8. Simplify the following algebraic fraction using the following steps:
9. Flip and multiply the denominator.
10. Factorise the numerators and denominators.
11. Multiply the numerators and denominators.
12. “Cancel” any common factors between numerator and denominator.
13. Simplify the numerator and denominator, if possible.

#### Guided Reflection

Let’s work through each of these examples to see how well you did.

1. You were asked to simplify .

Step 1 was to factorise the numerator and denominator:

(group and take out common factors in the numerator and take out a common factor in the denominator)

Step 2 was to “cancel” any common factors in the numerator and denominator. We are not really cancelling anything. We are simply recognising that these fractions are simply forms of 1.

Step 3 was to simplify the numerator and denominator if possible. They are in their simplest form.

NOTE: You cannot “cancel” the in the numerator and denominator because you have two terms in the numerator. You can only cancel when you have a single term in the numerator and denominator.

Lastly, we need to make sure that we write down all the restrictions on the variables. The best way to figure this out is to look at the fully factorised form of the denominator which is:

.

This tells us that , and (as this will make ).

1. You were asked to simplify .

Step 1 was to factorise the numerators and denominators.

Step 2 was to multiply the numerators and denominators.

(the 6 in the denominator comes from multiplying the 2 and the 3)

Step 3 was to “cancel” any common factors.

( because

Step 4 was to simplify if possible, but the fraction is already in its simplest form.

Finally, we need to note the restrictions on the variables. Look at the fully factorised denominator: .

, .

1. You were asked to simplify .

Step 1 was to flip and multiply the denominator.

Now the question is the same as question 2) and we can follow the same steps.

Factorise the numerators and denominators:

Multiply the numerators and denominators:

“Cancel” common factors:

Finally, we need to note the restrictions on the variables. Look at the fully factorised denominator: .

, , , .

Wow! That whole algebraic fraction was actually equivalent to one!

### Activity 2: Multiplying Algebraic Fractions Part 2

|  |  |
| --- | --- |
| Bullseye | Purpose In this activity, you are going to practice simplify algebraic fractions. |
| Stopwatch | Suggested Time You will need about 60 minutes. |
| Pencil | What You Will Need  * A pen or pencil * Some blank paper or a notebook |

#### Tasks

1. Simplify the following algebraic fractions, assuming that all denominators are not zero.

#### Guided Reflection

1. TBC

### Activity 3: Adding Algebraic Fractions Part 1

|  |  |
| --- | --- |
| Bullseye | Purpose In this activity, you are going to learn how to add algebraic fractions. |
| Stopwatch | Suggested Time You will need about 20 minutes. |
| Pencil | What You Will Need  * A pen or pencil * Some blank paper or a notebook |

#### Tasks

1. Use the following steps to add these algebraic fractions:
2. Factorise the denominators.
3. Find the LCD.
4. Write the expression as a single fraction over the LCD.
5. Expand and simplify the numerator.
6. Factorise the numerator, if possible.
7. “Cancel” any common factors between numerator and denominator.
8. Simplify, if possible.

#### Guided Reflection

1. You were asked to add these algebraic fractions together:

Step 1 was to factorise all the denominators in order to find the LCD.

Step 2 was to find the LCD. A good way of doing this is simply to list all the factors in the denominators. In this case we have in all three denominators and in the first and last denominator. The LCD is the product of these unique factors i.e. the LCD is .

Now we need to write the fractions all over the LCD. An easy way to think about this is that you have to multiply a numerator by every factor in the LCD that is not part of its denominator.

In the case of the first fraction, there are no factors in the LCD which are not in the denominator, so we don’t multiply the numerator by anything.

In the case of the second fraction, there is a missing in the denominator, so we need to multiply the numerator by

There is no part of the LCD missing from the third denominator. So, the expression becomes:

NOTE: It is a very good idea to write all the numerators in their own sets of brackets to make sure that you deal with any – signs correctly.

Now we need to expand and simplify the numerator:

(Watch out for the – sign)

Next, we need to factorise the numerator:

We can now “cancel” any common factors between the numerator and denominator:

We cannot simplify the answer any further.

### Activity 4: Adding Algebraic Fractions Part 2

|  |  |
| --- | --- |
| Bullseye | Purpose In this activity, you are going to learn how to add algebraic fractions. |
| Stopwatch | Suggested Time You will need about 45 minutes. |
| Pencil | What You Will Need  * A pen or pencil * Some blank paper or a notebook |

#### Tasks

1. Simplify the following algebraic fractions:

#### Guided Reflection

1. TBC

So far, we have covered how to simplify, multiply and add algebraic expressions that are fractions. But what about algebraic expressions that have fractions as coefficients? Well, we use the same basic fraction rules to deal with these situations as well.

#### Expansions

When we expand algebraic expressions with fraction coefficients, we really just need to apply our rule. Here is an example:

We know that we need to multiply each term in the first bracket with each term in the second bracket. Let’s do each multiplication separately.

After collecting the like terms, we get

If we wanted to, we could add these fractions over an LCD of .

Try this example on your own first.

Here is the full solution.

(expand each of the brackets)

(simplify the fractions)

(collect like terms)

1. Factorisation

When we factorise expressions with fractions, we need to think about the factors that, when multiplied together, will result in the fraction. Here is an example.

We can see that this is a difference of two squares because and . So, we can factorise this expression as

Here is another example:

Because there are four terms, we should start looking for terms to group and take a common factor out of these groups. If we grouped together, we could take out a common factor of and get

Can we take out a common factor from the first two terms so that we also have left?

Well if we took out a common factor of , we would get because .

Now we have . You should be able to complete this question to get a final answer of or, even better, .

Try factorising this expression on your own first:

Here is the worked solution:

(take out the common factor of – remember that )

(factorise the difference of two squares)

Exercises from [https://www.siyavula.com/read/maths/grade-10/algebraic-expressions/01-algebraic-expressions-07](https://www.siyavula.com/read/maths/grade-10/algebraic-expressions/01-algebraic-expressions-07%20and%20Everything%20Maths%20Gr10) and Everything Maths Gr10 (pg34)

### End of Sub-Topic Test

1. Factorise and simplify the following, assuming any denominators are .

#### Solutions

1. We had to simplify various algebraic expressions.

(if we group the first two and second two terms, we can take a common factor out of each pair which creates a new common factor)

(we took out the common factor of but there is still a common factor of in the second factor)

* 1. or

(start by factorising the difference of squares to create a new common factor of )

(take out the common factor)

(simplify the expression)

(the first three terms are a perfect square trinomial. If we factorise them, we will create a difference of two squares)

(now we have a difference of two squares)

(we can create a common factor of if we take out of the second bracket)

(we have the difference of two squares. If it makes it easier, you can temporarily let )

(this is actually a quadratic trinomial. If it makes it easier, you can temporarily let )

(you need to remember to replace with again)

(the first thing we should do is collect all the like terms)

(now we need to find a good way to group the terms. Let’s group the first two and last two terms)

(remember to change the sign in the second bracket. Now we can take out common factors)

(there is almost another common factor. If we take out of both terms in the first bracket, we will create a term because , what we started with)

(now we have a common factor of )

(we almost have a difference of squares in the second factor. We just need to take the out of this bracket as well. We will have in the second bracket)

(we need to factorise the numerator by taking out the common factor. Remember, we cannot “cancel” unless there is a single term in both the numerator and denominator. Factorising the numerator creates this single term where all the + and – signs are in brackets)

(now we can “cancel” the 2’s in the numerator and denominator)

(We need to factorise the numerator and denominator before we can do anything else)

(Now we can “cancel” the factors in the numerator and denominator. Remember, this is not really cancelling. We ae simply recognising that )

(Factorise the numerator by grouping)

(We need to factorise whatever we can in order to “cancel”)

(As usual, we need to factorise whatever we can to see if there are any factors that will “cancel”)

(We cannot cancel yet. We still need to deal with the division. Remember, a shortcut is to change this to a multiplication and turn the fraction upside down)

(We can go further by taking a out of the numerator to create identical factors)

(Always check that you have not changed the value of the factor by multiplying out again - )

(Now let’s deal with the division sign)

(Wow! What a result)

(These fractions are being added so we need to first find the LCD. The LCD is )

(The LCD is )

(Expand and simplify the numerator)

(Factorise the numerator, just in case this creates factors we can cancel)

(Factorise the numerators and denominators to see if we can simplify any of the fractions but also to help us find the LCD)

(The first fraction can be simplified)

(The LCD is )

(Expand and simplify the numerator)

(Factorise the numerator to see if there are any factors we can “cancel”)

# Sub-Topic 2: Solving Algebraic Equations

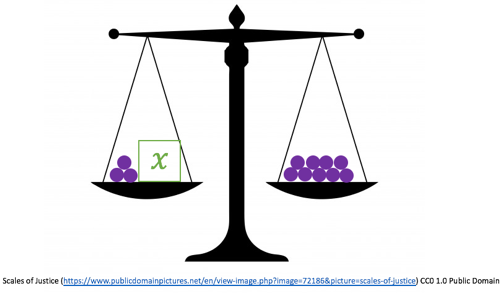
This sub-topic is all about solving equations. In many respects, Maths is actually all about solving equations. Solving equations is how we find the answers we are looking for.

For example, in Physics, Engineering and Economics, people spend much of their time trying to capture the real world in the form of equations to solve. These equations come in different forms and have different variables. Some are very simple like . Some are more complicated, like this one used to calculate the flow rate of a gas through a rocket exhaust.

We have equations that describe how a ball rolls down a hill or how the planets move around the sun. We have equations that describe how the population of lions in a game reserve can be expected to change. We even have equations for how likely it is that the economy will grow or shrink.

No matter how simple or complicated the equation looks, all equations are basically solved in the same way

When we solve an equation, we are finding out what all the possible values of a variable are that will make the equation true or that will make the left-hand side of the equation equal to the right-hand side. You can think of equations like a scale.



For example, in the equation , what value can we replace with that will make the equation true or that will “balance the scales”? The solutions to an equation are sometimes called the **roots** of the equation.

The golden rule of solving any kind of equation is this:

**You can do whatever you like to an equation so long as you do the same thing to both sides of the equation.**

In other words, you **have** to keep the scales balanced.

All the skills you learnt in Sub-Topic 1 are only really useful because they help you to solve equations.

## Unit 1: Solving linear Equations

#### Learning Outcomes

By the end of this unit, you should be able to:

1. Solve linear equations.

#### Introduction

Linear equations are the simplest kinds of equations. In these equations, the highest power of the variable being solved is 1. Just because linear equations are the simplest to solve mathematically does not mean that they cannot be extremely important or useful.

For example, is a linear equation. The biggest exponent on the variable is 1 but this equation describes how matter and energy are related.

### Activity 1: The Basics of Solving Linear Equations Part 1

|  |  |
| --- | --- |
| Bullseye | Purpose In this activity, you are going to learn the basics of solving linear equations. |
| Stopwatch | Suggested Time You will need about 50 minutes. |
| Pencil | What You Will Need  * A pen or pencil * Some blank paper or a notebook * An Internet connection |

#### Tasks

1. Watch the videos called [Same thing to both sides of equations](https://www.khanacademy.org/math/algebra/one-variable-linear-equations/modal/v/why-we-do-the-same-thing-to-both-sides-simple-equations) (02:30) and [Representing a relationship with an equation](https://www.khanacademy.org/math/algebra/one-variable-linear-equations/modal/v/representing-a-relationship-with-a-simple-equation) (02:30) to learn about the basic principles of solving linear equations.

(<https://www.khanacademy.org/math/algebra/one-variable-linear-equations/modal/v/why-we-do-the-same-thing-to-both-sides-simple-equations>)

(<https://www.khanacademy.org/math/algebra/one-variable-linear-equations/modal/v/representing-a-relationship-with-a-simple-equation>)

1. Have a look at this equation: . You can probably work out the answer in your head.
   1. What do you have to do to the left-hand side (LHS) of the equation to get on its own?
   2. What do you have to do the right-hand side (RHS) of the equation to keep it balanced?
   3. What value of makes the equation balanced or satisfies the equation?
2. Have a look at this equation: .
   1. What do you have to do to the LHS of the equation to get on its own?
   2. What do you have to do the RHS of the equation to keep it balanced?
   3. What value of makes the equation balanced or satisfies the equation?
3. Now, solve the following equations for . (i.e. work out what value x must be to make the equation balanced):
   1. Once you have had a go, watch the video called [Linear equations 1](https://www.khanacademy.org/math/algebra-home/alg-basic-eq-ineq/alg-old-school-equations/v/algebra-linear-equations-1) to see if you got the correct answers.

(<https://www.khanacademy.org/math/algebra-home/alg-basic-eq-ineq/alg-old-school-equations/v/algebra-linear-equations-1>)

1. Have a look at this equation: .
   1. What do you have to do to the left-hand side (LHS) of the equation to get on its own?
   2. What do you have to do the right-hand side (RHS) of the equation to keep it balanced?
   3. What do you have to do to the LHS of the equation to get on its own?
   4. What do you have to do the right-hand side (RHS) of the equation to keep it balanced?
   5. What value of makes the equation balanced or satisfies the equation?
2. Have a look at this equation:
   1. What do you have to do to the left-hand side (LHS) of the equation to get on its own?
   2. What do you have to do the right-hand side (RHS) of the equation to keep it balanced?
   3. What do you have to do to the LHS of the equation to get on its own?
   4. What do you have to do the right-hand side (RHS) of the equation to keep it balanced?
   5. What value of makes the equation balanced or satisfies the equation?
3. Now solve these equations for .
   1. Once you have had a go, watch the video called [Linear equations 2](https://www.khanacademy.org/math/algebra-home/alg-basic-eq-ineq/alg-old-school-equations/v/algebra-linear-equations-2) to see if you got the correct answers.

(<https://www.khanacademy.org/math/algebra-home/alg-basic-eq-ineq/alg-old-school-equations/v/algebra-linear-equations-2>)

1. Solve the following equations:

#### Guided Reflection

1. We were asked to solve .
   1. In order to get just on the LHS, we have to **divide** by 2.
   2. But whatever we do to the one side of an equation, we must do to the other side of the equation. So, we need to divide the RHS by 2 as well. This gives us:
   3. The solution to the equation is . We can also say that 5 is a root of the equation.

NOTE: Notice the symbol. This symbol means “therefore”. Solving an equation is just like making a logical argument. . Therefore, . You must start every line of your equation solutions with this symbol.

1. We were asked to solve .
   1. In order to get just on the LHS, we have to **multiply** by 3.
   2. But whatever we do to the one side of an equation, we must do to the other side of the equation. So, we need to multiply the RHS by 3 as well. This gives us:
   3. The solution to the equation is . We can also say that 27 is a root of the equation.
2. We were asked to solve . Numbering wrong from this point – this should be 5)
   1. In order to get 2 on the LHS, we have to **subtract** 4.
   2. But whatever we do to the one side of an equation, we must do to the other side of the equation. So, we need to subtract 4 from the RHS as well. This gives us:
   3. Now in order to get on the LHS, we have to **divide** the LHS by 2.
   4. But whatever we do to the one side of an equation, we must do to the other side of the equation. So, we need to divide the RHS by 2 as well. This gives us:
   5. The solution to the equation is .
3. We were asked to solve . Should be 6)
   1. In order to get on the LHS, we have to **add** 7.
   2. But whatever we do to the one side of an equation, we must do to the other side of the equation. So, we need to add 7 from the RHS as well. This gives us:
   3. Now in order to get on the LHS, we have to **multiply** the LHS by 3.
   4. But whatever we do to the one side of an equation, we must do to the other side of the equation. So, we need to multiply the RHS by 3 as well. This gives us:
   5. The solution to the equation is .
4. Here are the solutions to the equations. Should be 8)
   1. . This was a nice simple one to start with.

(subtract 6 from both sides)

(divide both sides by 3)

* 1. . This was another wimple equation.

(add 2 to both sides)

(multiply both sides by )

* 1. . Even though the first term on the LHS is not the variable, we still solve as normal.

(subtract 3 from both sides)

(divide both sides by )

* 1. . It does not matter if the RHS is zero. We solve as normal.

(subtract 5 from both sides)

(multiply both sides by )

* 1. . For this question we have unknowns on both sides of the equation. But it is easy to get them all onto the LHS if we subtract from both sides.

(subtract from both sides)

(add 9 to both sides)

(collect the like terms on the LHS)

(divide both sides by )

### Activity 2: The Basics of Solving Linear Equations Part 2

|  |  |
| --- | --- |
| Bullseye | Purpose In this activity, you are going to learn the more of the basics of solving linear equations. |
| Stopwatch | Suggested Time You will need about 50 minutes. |
| Pencil | What You Will Need  * A pen or pencil * Some blank paper or a notebook * An Internet connection |

#### Tasks

1. Have a look at this equation:
   1. What do you have to do to get all the terms with on the LHS and all the constant terms on the RHS such that the equation is still balanced?
   2. What do you have to do to solve for ?
2. Have a look at this equation: .
   1. What do you have to do to get all the terms with on the LHS and all the constant terms on the RHS such that the equation is still balanced?
   2. What do you have to do to solve for ?
3. Now solve these equations for .
   1. Once you have had a go, watch the video called [Linear equations 3](https://www.khanacademy.org/math/algebra-home/alg-basic-eq-ineq/alg-old-school-equations/v/algebra-linear-equations-3) to see if you got the correct answers.

(<https://www.khanacademy.org/math/algebra-home/alg-basic-eq-ineq/alg-old-school-equations/v/algebra-linear-equations-3>)

1. Have a look at this equation: .
   1. What do you have to multiply each side by to get the out of the denominator?
   2. What do you have to do to solve for ?
   3. What value is not allowed to have?
2. Have a look at this equation:
   1. What do you have to multiply each term by to get both the and the out of the denominator?8the RHS such that the equation is still balanced?
   2. What do you have to do to solve for ?
   3. What value is not allowed to have?
3. Now solve these equations for .
   1. Once you have had a go watch the video called [Linear equations 4](https://www.khanacademy.org/math/algebra-home/alg-basic-eq-ineq/alg-old-school-equations/v/algebra-linear-equations-4) to see if you got the correct answers.

(<https://www.khanacademy.org/math/algebra-home/alg-basic-eq-ineq/alg-old-school-equations/v/algebra-linear-equations-4>)

#### Guided Reflection

1. We need to solve for in .
   1. Remember that in order to solve linear equations, we need to get all the terms with the unknown on one side of the equation. We usually choose the LHS but this is not essential. In this case, we have to subtract from both sides and add to both sides.
   2. To solve for , we have to collect all the like terms on each side of the equation.

Now we can divide both sides by to solve for .

1. We need to solve for in
   1. This time we have a coefficient that is a fraction. We can leave the fraction, but it is usually easier to multiply each term in the equation by the LCD of all the denominators to get rid of any fractions. As there is only one fraction, finding the LCD is easy. It is 4.

(we can multiply each term by 4 because this is the same thing as multiplying both sides by 4 and then expanding the brackets: )

* 1. Now we can proceed as normal by getting all the terms with on one side of the equation and all the constants on the other side.

1. The equation has an unknown in the denominator.
   1. Like before, we can multiply each term by the LCD of all the fractions in order to get rid of them. Here there is only one denominator, so the LCD is just .
   2. Now we just need to divide both sides by 4 to solve for .
   3. In the original equation, we had in the denominator. Because we may never divide by zero, this means that . We have to write this restriction next to the original equation like this:
2. Now we need to solve for in .
   1. Here we have and in the denominator, but, like before, we can get rid of these fractions by multiplying through by the LCD which is .

(remember that )

* 1. Now we can proceed as normal.
  2. Once again, we had ’s in the denominator. So, and . But the only way for to be equal to zero is if . So, we really only have the one restriction of .

### Activity 3: Solving More Advanced Linear Equations

|  |  |
| --- | --- |
| Bullseye | Purpose In this activity, you are going to learn about solving more complex linear equations. |
| Stopwatch | Suggested Time You will need about 70 minutes. |
| Pencil | What You Will Need  * A pen or pencil * Some blank paper or a notebook * An Internet connection |

#### Tasks

1. Have a look at this equation:
   1. What do you need to do to get rid of the brackets?
   2. What do you need to do to get all the terms with in them onto one side and all the constants onto the other?
   3. Now solve for .
2. Now solve the following equation for .

Once you have had a go watch the video called [Equations with parentheses](https://www.khanacademy.org/math/algebra/one-variable-linear-equations/modal/v/solving-equations-with-the-distributive-property) (06:00) to see if you got the correct answer.

(<https://www.khanacademy.org/math/algebra-home/alg-basic-eq-ineq/alg-old-school-equations/v/algebra-linear-equations-4>)

1. Now work through this excellent [review of solving linear equations](https://www.khanacademy.org/math/algebra/one-variable-linear-equations/modal/a/multi-step-equations-review). Try the four practice problems at the end.

(<https://www.khanacademy.org/math/algebra/one-variable-linear-equations/modal/a/multi-step-equations-review>)

1. Have a look at this equation: .
   1. What is the LCD? HINT: can you factorise any of the denominators first?
   2. Multiply through by the LCD and solve for .
   3. What are the restrictions on ?
2. Solve the following equations, stating any restrictions on the variable.

#### Guided Reflection

1. You were asked to solve .
   1. The first thing we need to do is expand any brackets away. This is going to help us get all the terms with on one side of the equation.
   2. Now we can get all the terms with on one side of the equation and all the constants on the other side. We need to add to both sides.

(add to both sides)

(simply by collecting like terms)

* 1. Now all we need to do is divide both sides by 19 to solve for .

1. At first glance, this may not look like a linear equation because it contains an term. But if we follow our normal process, we will see that it is actually a linear equation.
   1. We have fractions, so we need to find the LCD. Before, we can do that, we need to factorise all of the denominators to see exactly what their factors are. We can factorise the first denominator of because it is the difference of squares.
   2. Now it is clear what the LCD is. It is . We need to multiply through by the LCD.
   3. To find the restrictions on , we have to go back to the original equation and look at the denominators. It is easiest to look at the factorised denominators. These are , and .

So ; . We can see that these two restrictions also cover the denominator. If either factor is equal to zero, the entire denominator will be equal to zero. This is a very important and useful principle that we will come back to in the next sub-unit.

1. You were asked to solve the equations.
   1. . The first thing we need to do, is expand all the brackets.

(expand all the brackets)

(subtract and add to both sides)

(simplify)

(divide both sides by 3 to solve for )

* 1. . In this case, we have fractions. The best way to get rid of the fractions is to multiply each term by the LCD of the denominators. This is 12.

(using brackets helps to make sure that you multiply the entire numerator correctly)

(expand the brackets)

(collect the unknowns and constants on either side of the equation)

* 1. . In this question, you can either expand the brackets first or multiply each side by the LCD first. The solution below, multiplies by the LCD first.

(multiply through by the LCD of 15)

(expand the brackets)

(get the unknowns on one side)

19

* 1. . Here the LCD of the denominators is and .

(multiply through by )

(get the terms with the variable alone on one side of the equation)

(divide both sides by 12 and simplify the answer)

* 1. . We have fractions; therefore, we need to find the LCD. But the best way to do this is to factorise the denominators first.

(the LCD is and therefore, the restrictions are )

(multiply through by the LCD)

## Unit 2: Solving Quadratic Equations

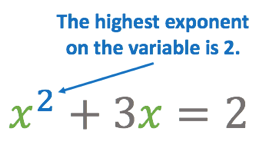
#### Learning Outcomes

By the end of this unit, you should be able to:

1. Solve quadratic equations by factorisation;
2. Solve quadratic equations by completing the square; and
3. Solve quadratic equations by using the quadratic formula.

#### Introduction

Quadratic equations are equations where the highest power on the unknown or variable is two. Here is an example:



In the case of linear equations like , we always had only one answer. Linear equations have only **one** **root**. Because the highest power on the variable in quadratic equations is two, quadratic equations have two possible answers or **two roots**.

There are a few different ways of solving quadratic equations, but most of them involve a very important property called the **zero product law**. Do you remember this equation from Unit 1 in this sub-topic?

In order to solve it, we had to first factorise the denominators in order to find the LCD.

We saw that the LCD was . But we also noted that none of the denominators could equal zero. We had to place restrictions on what values could have. We did this by setting the denominators not equal to zero:

When we do this for the first denominator, we get. In this case, we have two factors whose product may not equal zero. That means that neither factor may be zero. If either of the factors is equal to zero, the product will be equal to zero because anything multiplied by zero is zero.

So, if , this means that **AND** that .

What about this case?

Using the same logic, we can say that either OR .

This is the zero product law:

**If then either or or both .**

Use the zero product law to solve for the variable in each case:

Here are the answers:

1. or
2. or or

The zero product law means that if we can get the product of factors on the LHS and zero on the RHS, we can solve for the variable.

### Activity 1: Solving Quadratic Equations by Factorisation

|  |  |
| --- | --- |
| Bullseye | Purpose In this activity, you are going to learn how to solve quadratic equations using factorisation. |
| Stopwatch | Suggested Time You will need about 75 minutes. |
| Pencil | What You Will Need  * A pen or pencil * Some blank paper or a notebook * An Internet connection |

#### Tasks

1. Look at this equation: .
   1. What do you have to do to get the RHS equal zero?
   2. Can you factorise the LHS to get a product of factors?
   3. Solve for .
   4. Check your solutions.
2. Use factorisation and the zero product law to solve the following equations:

#### Guided Reflection

1. You were given the equation .
2. In order to get the RHS equal to zero we have to subtract 16 from both sides.
3. We can factorise the LHS. It is the difference of two squares.
4. Now we can use the zero product law to solve for .

or

1. To check our solutions, we have to substitute each value into the original equation.
2. All of the equations given were quadratic equations.
3. or . When we have to solve a quadratic equation, we also start by getting the RHS equal to zero.

(get the RHS equal to zero)

(factorise the LHS)

(use the zero product law to solve for )

or

1. or .

(get the RHS equal to zero and organise the terms to make factorisation easier)

or

1. or .

(get the RHS equal to zero)

(factorise the LHS)

or (solve for )

It is OK if you need to add the following steps to your solution, especially while you are still practicing.

or

or

or

1. or .

(get the RHS equal to zero)

(multiply both sides of eth equal by to make the quadratic easier to factorise)

(factorise the quadratic)

or (solve for )

1. or . In this question, we first need to expand the brackets.

(expand the brackets)

(get the RHS equal to zero)

(multiply through by to make the quadratic easier to factorise)

(factorise)

or (solve for )

1. . You needed to recognise that the LHS could be factorised as the difference of two squares.

(factorise)

(solve for )

1. or . This equation has fractions. Just like with linear equations, we have to multiply through by the LCD of the denominators in order to get rid of the fractions.

(multiply through by the LCD of )

(expand the brackets)

(get the RHS equal to zero)

(simplify by collecting like terms)

(factorise)

(factorise)

or (solve for )

1. or . This question makes use of all the skills you have learnt so far in this topic. In this equation, we need to factorise the denominators first to see what the LCD is.

(factorise the denominators to find the LCD. The last numerator was also factorised in case there was a factor to cancel with the denominator)

(take the – sign out of the factor in order to change into a factor)

(multiply through by the LCD)

(expand)

(expand)

(make the RHS equal to zero)

(simplify the LHS)

(multiply through by to make the quadratic easier to solve)

(factorise)

or (solve for )

1. or . In this question, we have to square both sides in order to get rid of the square root sign. Remember that we can do whatever we like to an equation so long as we do the same thing to both sides.

(square both sides of the equation)

(expand)

(make the RHS equal to zero)

(simplify)

(factorise)

or (solve for )

1. . Here is another equation where we have to square both sides. But there is quite a bit of simplification we can do first.

(add 4 to both sides)

(take out a common factor on the RHS)

(divide both sides by 5)

(square both sides)

(expand)

(get the RHS equal to zero)

(simplify)

(multiply through by )

(factorise)

or (solve for )

### Activity 2: Solving Quadratic Equations by Completing the Square

|  |  |
| --- | --- |
| Bullseye | Purpose In this activity, you are going to learn how to solve quadratic equations using a technique called completing the square. |
| Stopwatch | Suggested Time You will need about 75 minutes. |
| Pencil | What You Will Need  * A pen or pencil * Some blank paper or a notebook * An Internet connection |

#### Tasks

1. Take a look at this equation: .
2. Try and solve this equation using the zero product law technique from Activity 1.
3. Why can you not use the zero product law technique?
4. What do you notice about the LHS of the original equation? Can you factorise the original LHS?
5. If you factorise the original LHS, what can you now do to each side of the equation?
6. Can you solve for now?
7. Now watch the video called [Completing the square](https://www.khanacademy.org/math/algebra/quadratics/solving-quadratics-by-completing-the-square/v/solving-quadratic-equations-by-completing-the-square) (14:00).

(<https://www.khanacademy.org/math/algebra/quadratics/solving-quadratics-by-completing-the-square/v/solving-quadratic-equations-by-completing-the-square>)

1. Take a look at this equation: .
2. Can this equation be easily solved by factorisation and the zero product law?
3. Can you solve it by completing the square?
4. Solve this equation by completing the square: . Once you have had a go, watch the video called [Worked example: Solving equation by completing the square](https://www.khanacademy.org/math/algebra/quadratics/solving-quadratics-by-completing-the-square/v/solving-quadratics-by-completing-the-square) (06:20) to see if you are right.

(<https://www.khanacademy.org/math/algebra/quadratics/solving-quadratics-by-completing-the-square/v/solving-quadratics-by-completing-the-square>)

1. Solve this equation using either factorisation or by completing the square.
2. Solve the following equations using any technique.
3. (Hint: Let )

#### Guided Reflection

1. We were asked to solve .
2. We cannot solve for because we cannot factorise the quadratic on the LHS.
3. The original LHS was . We can factorise this as .
4. If we factorise the original LHS we get

Now we can take the square root of both sides of the equation:

1. Now we can solve the equation as follows:

or

1. We were asked to solve .
2. The LHS cannot be factorised so we cannot solve the equation by factorisation.
3. We have to solve the equation by completing the square. To complete the square we need to half the coefficient of and then square this number. This is what we add to both sides of the equation.

(add 1 to both sides in order to complete the square)

(get the perfect square quadratic on its own on the LHS)

(factorise the perfect square and simplify the RHS)

(take the square root of both sides)

or (solve for )

1. We were asked to solve using any method. Because the quadratic does not easily factorise, we will use the completing the square method. Should be 5)

(divide through by 2 to get the leading coefficient equal to 1)

(add to both sides to complete the square - half of is and )

(get the perfect square quadratic on its own on the LHS)

(factorise the perfect square and simplify the RHS)

(take the square root of both sides)

or (solve for )

1. You could use any method to solve these equations. Should be 6)

TBC

* 1. (Hint: Let )
  2. – no real solution!

### Activity 3: The Quadratic Formula

|  |  |
| --- | --- |
| Bullseye | Purpose In this activity, you are going to develop the quadratic formula |
| Stopwatch | Suggested Time You will need about 15 minutes. |
| Pencil | What You Will Need  * A pen or pencil * Some blank paper or a notebook |

#### Tasks

1. In Unit 3 of Sub-topic 1, we said that the general form of a quadratic expression is given by . Therefore, the general form of the quadratic equation can be given as

Use the completing the square method to solve for if .

1. Remember to divide through by so that the leading term coefficient is 1.
2. What number must be added to both sides of the equation so that Can be factorised as a perfect square? Hint: remember you need to halve the coefficient of the term and then square this number.
3. Simplify both sides of the equation so that you can take the square root of both sides.
4. Solve for by getting it alone on the LHS of the equation and simplify the RHS.

#### Guided Reflection

1. We were asked to solve for if by completing the square.
2. In order to complete the square, we need to add to both sides. Half of is and .
3. (subtract from both sides of the equation)

(factorise the perfect square on the LHS and add the fractions on the RHS over the LCD of )

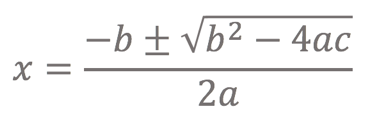
(take the square root of both sides)

(simplify the RHS by taking the square root of the denominator)

1. (subtract from both sides of the equation to get alone on the RHS)

(add the fraction on the RHS over the LCD of )

The expression you have found is called the **quadratic formula** and can be used to solve any quadratic equation. It is really just a general form of the completing the square method.



All you need to do to use it, is to plug the correct values of , and into the equation, like this.

(plug the values into the equation – remember to include all the - signs)

(sometimes you can leave your answer like this)

You can also use a calculator to work out the solutions.

or

or (here we have used a calculator to calculate , correct to three decimal places)

6 or (simplify both solutions)

You can also try and simplify your answers algebraically.

(split 24 into its factors of 6 and 4)

(Take the square root of 4)

(split the fraction into two)

(split 6 into its factors of 2 and 3)

(simplify)

The quadratic formula is especially useful if the quadratic does not factorise easily or if the numbers in the completing the square method are too complicated. The quadratic formula will always work.

### Activity 4: Solving Quadratic Equations using the Quadratic Formula

|  |  |
| --- | --- |
| Bullseye | Purpose In this activity, you are going to practice using the quadratic formula to solve quadratic equations. |
| Stopwatch | Suggested Time You will need about 60 minutes. |
| Pencil | What You Will Need  * A pen or pencil * Some blank paper or a notebook |

#### Tasks

1. Solve the following equations using the quadratic formula.
2. Solve the following equations using any method.
3. One root of the equation is 8. What is the value of and the other root?

#### Guided Reflection

TBC

1. The two roots of differ by 5. Calculate the value of .
2. An equation of the form is written on the board. Saskia and Sven copy it down incorrectly. Saskia has a mistake in the constant term and obtains the solutions -4 and 2. Sven has a mistake in the coefficient of and obtains the solutions 11 and 15. Determine the correct equation that was on the board.
3. (Hint: )

. Here the hint was that )

(using the hint, we can see that we have created a quadratic equation – again we should make a substitution to make things easier)

Let

or

## Unit 3: Solving Exponential Equations

#### Learning Outcomes

By the end of this unit, you should be able to:

1. Solve exponential equations (equations that have the unknown in the exponent).

#### Introduction

In this sub-topic on solving equations, we have dealt with linear equations, where the unknown is in the base and greatest exponent on the unknown was one and quadratic equations, where the unknown was in the base and the greatest exponent on the unknown was two.

**Linear Equation:**

**Quadratic Equation:**

But what happens when the unknown is in the exponent and not the base? In other words, how can we solve equations like this?

We call equations where the unknown is in the exponent **exponential equations**.

Before we tackle how to solve exponential equations, we need to make sure that we remember all the exponential laws from Topic 1.

If any of these seem unfamiliar, you should go back to Unit 1 and Unit 2 in Sub-Topic 1 of Topic 1 before continuing.

### Activity 1: The Basics of Solving Exponential Equations

|  |  |
| --- | --- |
| Bullseye | Purpose In this activity, you are going to discover the basic principles of solving exponential equations. |
| Stopwatch | Suggested Time You will need about 15 minutes. |
| Pencil | What You Will Need  * A pen or pencil * Some blank paper or a notebook |

#### Tasks

When completing these three tasks, it can be a good idea to have a look at eth guided reflection below after completing each one.

1. Look at this equation: .
   1. Can you solve it by inspection (i.e. without doing any calculations on paper)?
   2. If you are struggling think about the question like this. What power do we have to raise 2 to for it to be equal to 32?
   3. What do 2 and 32 have in common?
   4. Is there anything we can do to the RHS of the equation to make it look more like the LHS and to help us solve for ?
2. Now look at this equation: .
   1. Can you solve this equation by inspection?
   2. What do 125 and 25 have in common?
   3. What can we do to both sides of the equation to make them more similar and help us solve for .
   4. Check that your answer is correct by substituting it into the original equation.
3. Now have a look at this equation: .
   1. What do 2 and 4 have in common?
   2. Is there anything we can do to both sides of the equation to make them look more similar that would help us solve for ?
   3. Check that your answer is correct by substituting it into the original equation.

#### Guided Reflection

1. We were given the equation .
2. You may have been able to see that because .
3. In other words, we have to raise 2 to the power 5 for the LHS to equal the RHS.
4. In the original equation, both 2 and 32 are powers of 2.
5. We can change the RHS to be in the form of a power of 2 like this:

.

Now that the bases on both sides of the equation are the same, it is easy to see that is the only value for that will make the equation true i.e. the LHS = RHS. In other words, we could simple equate the exponents to solve for .

1. We were given the equation .
2. This equation was harder to solve by inspection, but you may have still been able to do it and get that .
3. Both 25 and 125 are powers of 5 so maybe we can make both sides of the equation powers of 5.
4. LHS: RHS:

So, we have that

Now, like in question 1, the bases are the same so we can equate the exponents.

1. We should check that this is the correct answer:

LHS:

This is equal to the RHS.

1. We were given the equation .
2. Both 2 and 4 are powers of 2.
3. We can rewrite the equation like this:

(get the bases on both sides of the equation the same)

(simplify where necessary)

(equate the exponents and solve the ordinary linear equation)

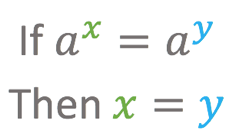
1. Check the solution.

LHS:

RHS:

So, the LHS = RHS.

The general principle we use to solve exponential equations is as follows:



### Activity 2: Solving Exponential Equations

|  |  |
| --- | --- |
| Bullseye | Purpose In this activity, you are going to practice solving exponential equations. |
| Stopwatch | Suggested Time You will need about 45 minutes. |
| Pencil | What You Will Need  * A pen or pencil * Some blank paper or a notebook |

#### Tasks

1. Solve this exponential equation:. Once you have had a go, watch the video called [Solving exponential equations using exponent properties](https://www.khanacademy.org/math/algebra2/exponential-growth-and-decay-alg-2/solving-exponential-equations-using-properties-of-exponents/v/solving-exponential-equations-with-exponent-properties) (05:00) to see if you are right.

(https://www.khanacademy.org/math/algebra2/exponential-growth-and-decay-alg-2/solving-exponential-equations-using-properties-of-exponents/v/solving-exponential-equations-with-exponent-properties)

1. Solve the following exponential equations:

Once you have had a go, watch the video called [Solving exponential equations using exponent properties (advanced)](https://www.khanacademy.org/math/algebra2/exponential-growth-and-decay-alg-2/solving-exponential-equations-using-properties-of-exponents/v/solving-exponential-equations-with-exponent-properties-advanced) (07:00) to see if you are right.

(https://www.khanacademy.org/math/algebra2/exponential-growth-and-decay-alg-2/solving-exponential-equations-using-properties-of-exponents/v/solving-exponential-equations-with-exponent-properties-advanced)

1. Solve the following exponential equations. Some questions sourced from Everything Maths Grade 10 Exponential Equations Examples and Exercise 2.3 (https://www.siyavula.com/read/maths/grade-10/exponents/02-exponents-03)
2. (Hint: )
3. (Hint: Let )
4. (Hint: )
5. (Hint: you will need to use the quadratic formula)

#### Guided Reflection

1. You were asked to solve several exponential equations. Should be 3)
2. . This was a nice simple question to start off with. We just need to remember that .

(the bases are the same on both sides so we can equate the exponents and solve the linear equation)

1. 2. Both 27 and 81 are powers of 3, so we can get both sides to be powers of 3.

(we can get both sides to have bases of 3)

(the bases are the same so we can equate the exponents and solve the linear equation)

2

1. . Once we realise that the question is easy.

(the bases are the same so we can equate the exponents and solve the linear equation)

1. . We have to do a little bit of simplification before we can start solving for .

(the first thing we need to do is multiply both sides by )

1. . We were given a hint that reminded us that .

(when we apply the hint, we see that we have a common factor on the LHS of )

(take out the common factor)

(divide both side by 16)

(solve the exponential equation as normal)

1. or . This question looked just like a factorised quadratic equation where we would use the zero product law.

or (use the zero product law to create two equations)

or (solve each exponential equation as normal)

or

1. or . This question was very similar to f).

or (use the zero product law to set up two exponential equations)

or (divide through the first equation by 2)

or

or

1. . The hint for this question was to let . That means that .

(let – now we have a standard quadratic equation)

or (solve the quadratic equation by factorisation)

or (replace with )

The solution can be discarded because there are no real values for that will make . In fact, can never be less than or equal to zero.

1. . The hint give was that

(by applying the hint, we see that we can multiply through by )

(now we have an easy equation to solve)

1. We have no way yet of solving for .

( because if it does )

(the first thing we need to see is that which can be factorised as the difference of squares)

(factorise the LHS)

(cancel the common factors on the LHS)

(rearrange the quadratic equation into the standard quadratic form)

(make a substitution to make solving the quadratic equation easier)

(use the quadratic formula or completing the square to solve the quadratic equation)

or

or (substitute back for )

or (use a calculator to work out the RHSs to three decimal places)

Because can never be negative, we only have one possible solution - . But we have no way of solving for using our current method of getting the bases on both sides of the equation to be the same. It seems that we are stuck. However, mathematicians have devised a way of solving exponential equations like this using logarithms. We will explore logarithmic equations in the very next unit.

## Unit 4: Solving Logarithmic Equations

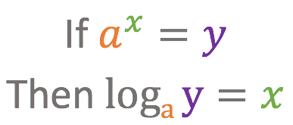
#### Learning Outcomes

By the end of this unit, you should be able to:

1. Solve logarithmic equations.

#### Introduction

The last question in the previous unit resulted in an equation that we could not solve for . It was . In Topic 1, however, we learnt that we can rewrite exponents in the form of logarithms. Remember that



So, in our case, if then we know that . Now we have all by itself and we can solve for it. We no longer need to try and get the bases on both sides of the equation to be the same.

Logarithmic equations, therefore, give us a clever way of solving exponential equations when it is too hard or complicated to get the bases on both sides of the equation to be the same by giving as a way to get the unknown we are solving for out of the exponent and on its own on one side of the equation.

In many cases, we still need to solve for using a calculator. Some calculators have a button that allows you to calculate the logarithms of any base. Some calculators only have a or a button. The button always means and the button always means where . is called the natural logarithm because it occurs naturally in the world and universe.

If your calculator only has a button, you can still easily calculate the value of any logarithm with any base using this rule:

Let’s see how this works now.

### Activity 1: Logarithms as Useful Tools.

|  |  |
| --- | --- |
| Bullseye | Purpose In this activity, you are going to learn how to solve simple exponential equations using logarithms. |
| Stopwatch | Suggested Time You will need about 30 minutes. |
| Pencil | What You Will Need  * A pen or pencil * Some blank paper or a notebook * An Internet connection |

Tasks

1. We are asked to solve for if .
   1. Get the equation in the form of a logarithm with the unknown, , on its own on one side of the equation.
   2. Use the rule that says that to write the logarithm with base 10.
   3. Calculate the value of using your calculator.
   4. Now watch the video called [Solving exponential equations using logarithms: base-2](https://www.khanacademy.org/math/algebra2/exponential-and-logarithmic-functions/solving-exponential-equations-with-logarithms/v/solve-exponentials) (05:10) to see if you are right.
2. Calculate if correct to three decimal places.
3. Calculate if correct to three decimal places.
4. Calculate if .
   1. Write the expression in logarithmic form.
   2. Get on its own.
   3. Solve for using your calculator.
5. Calculate if .
6. Now calculate the value of for the last equation in the previous unit. Remember, .

Guided Reflection

1. You should have got that .
2. . This equation is already in logarithmic form with the unknown on its own.
3. . We first need to divide both sides of the equation by two.
4. . We were asked to solve for if .
   1. It is not possible for us to get the bases on both sides of the equation to be the same. First we need to write the expression in logarithmic form.
   2. Now we need to isolate .
   3. Now we can use our calculator to solve for . We always work from the inside out.

Evaluate first ()

Add three ()

Divide by two ( to three decimal places)

1. . This question is slightly different because the unknown is now part of the logarithm.

( implies and we have simply re-written the logarithm in exponential form to get on its own.

1. We are now ready to tackle our problem from the previous unit.

Did you notice that question 4) in the previous activity was a bit different? In all the other questions, we had to use the logarithmic form to get the unknown on its own but in this question, we needed to move back to the exponential form to get the unknown on its own. This happened because the unknown was part of the logarithm.

All the other questions were actually exponential equations where we had to use the logarithmic form to help us solve them. Question 4) was a proper logarithmic equation with the unknown in the logarithm. Let’s take a closer look at these.

### Activity 2: Basic Logarithmic Equations

|  |  |
| --- | --- |
| Bullseye | Purpose In this activity, you are going to learn how to solve simple logarithmic equations. |
| Stopwatch | Suggested Time You will need about 10 minutes. |
| Pencil | What You Will Need  * A pen or pencil * Some blank paper or a notebook |

Tasks

1. In the previous activity, we solved for in by writing the equation in exponential form to isolate the unknown.
2. What is the value of ?
3. What is the value of ?
4. What is the value of
5. Use these facts to solve WITHOUT rewriting the equation in exponential form.

Guided Reflection

1. We have to solve .
2. This makes sense because .
3. If then . Therefore, .

(remember )

(remember )

(remember means )

In the previous activity, we got our logarithmic equation to the point where . Because both sides were we could just say that .

In general, if then .

This statement is the key to solving most logarithmic equations. To solve logarithmic equations, we also need to make use of the three logarithmic laws we learnt about in Topic 1. Here is a reminder of what these are.

**Law 1**: (we made use of this law in the previous activity)

**Law 2**: (the logs must have the same base)

**Law 3**: (the logs must have the same base)

### Activity 3: Solving Logarithmic Equations with the Logarithmic Laws

|  |  |
| --- | --- |
| Bullseye | Purpose In this activity, you are going to learn how to solve simple logarithmic equations using the logarithmic laws. |
| Stopwatch | Suggested Time You will need about 20 minutes. |
| Pencil | What You Will Need  * A pen or pencil * Some blank paper or a notebook |

Tasks

1. Use to logarithmic laws to solve for in the following equations. You may rewrite the expressions as exponential equations or us the fact that if then .
2. .
3. ()
4. .
5. .

Guided Reflection

1. We were asked to solve several logarithmic equations.
2. .

(used law 2)

(using the fact that to get the logs the same on both sides)

Or we can go straight to rewriting the logarithm in exponential form to get or

1. .

()

(law 2)

(if then )

1. .
2. .

## Unit 5: Solving Simultaneous Equations

#### Learning Outcomes

By the end of this unit, you should be able to:

1. Solve two linear equations simultaneously using substitution or elimination; and
2. Solve a linear and quadratic equation simultaneously using substitution.

#### Introduction

Consider this situation. A community theatre put on a play and sold 1,000 tickets. They sold the adult tickets for R8.50 each and the child tickets for R4.50 each.



Image source: <https://pixabay.com/en/ticket-entry-admit-one-stamp-red-153937/>

Altogether, they sold R7,300 worth of tickets but did not keep track of how many adult and child tickets were sold. They need this information for a report. Is there a way they can work out how many of each kind of ticket was sold?

Or what about this situation? Sampson invested R30,000. Part of it was invested at a simple interest rate of 5% and another part at a simple interest rate of 8%. If he earned R2,100 in interest, how much was invested at each rate?

### Activity 1: Simultaneous Equations

|  |  |
| --- | --- |
| Bullseye | Purpose In this activity, you are going to learn what simultaneous equations are and how to solve them. |
| Stopwatch | Suggested Time You will need about 20 minutes. |
| Pencil | What You Will Need  * A pen or pencil * Some blank paper or a notebook |

#### Tasks

1. Go back to the first scenario about the ticket sales above.
   1. If the total number of tickets sold was 1,000, set up an equation for the number of adult tickets and the number of child tickets sold? Here’s a hint. You can let the number of adult tickets sold be and the number of child tickets sold be .
   2. How many variables are there? What sort of equation is this– is it linear or quadratic? Can we solve for either of the variables?
   3. If each adult ticket was sold for R8.50 and each child ticket was sold for R4.50 and R7,300 was made altogether, can you set up another equation for this information?
   4. What sort of equation is this– is it linear or quadratic? How many variables are there? Can we solve for either of the variables?
   5. So, we have two variables, but we also now have two equations. Take a look at the first equation again. Rearrange it to get one of the variables on its own. Can you make a substitution into the second equation to get rid of one of the variables? Can you solve for one of the variables now?
   6. If you have solved for one of the variables, what can you do to solve for the other variable?
   7. How many of each type of ticket were sold?
2. Go back to Sampson’s question.
   1. How many unknowns do you need to find?
   2. Set up two equations like you did in question 1) to express the relationship between these variables and the other information given?
   3. Solve for the variables using the same method you used in question 1).

#### Guided Reflection

1. Let’s go through the first scenario and see how we can work out how many of each ticket was sold.
2. If we let the number of adult tickets sold be and the number of child tickets sold be then we can say that . We don’t have to use and as our variables. We could also use (for adult) and (for child).
3. There are two unknowns in the equation. It is a linear equation because the greatest power on the unknowns is one. We cannot solve for either of the variables because we do not have enough information.
4. If each adult ticket was sold for R8.50, then the total money made from selling adult tickets was (the price per ticket multiplied by the number of tickets sold). Similarly, the amount of money made by selling child tickets was . But the total amount of money made was R7,300. Therefore, we can say that .
5. This is also a linear equation with two variables.
6. The first equation can be rewritten as . Now we have an expression for that we can insert into the second equation in place of .

Now we have an equation with only one variable which we can easily solve.

(expand the brackets)

(subtract 8,500 from both sides and collect the like terms)

(divide both sides by to solve for )

1. Now that we have a solution for , we can use our rearranged first equation to solve for .
2. 600 adult tickets and 400 child tickets were sold.
3. The second scenario was actually very similar to the first.
4. We need to find two unknowns – the amount of money invested at 5% and the amount of money invested at 8%.
5. Let’s let the amount of money invested at 5% be and the amount of money invested at 8% be . We know that the total amount of money invested was R30,000. Therefore, we know that . Let’s call this equation (1).

We also know that the total interest earned was R2,100. So, we can say that . Remember that . Let’s call this equation (2).

1. From equation (1) we can say that . We can insert this expression for into equation (2).

(substitute (1) into (2))

(expand and simplify)

(solve for )

So, R20,000 was invested at 8%, which means that or R10,000 was invested at 5%.

In both these situations, there were **two** **unknowns** to find. This means that we needed **two** **equations**. We call these equations **simultaneous equations**, because their solutions are values of the unknowns that satisfy both equations **at the same time** or **simultaneously**.

In general, you need the same number of equations as the same number of variables you are trying to solve for. If you have **three** variables, you will need **three** equations.

In Activity 1, we used a method called **substitution**.

1. We used the simplest equation to create an expression for one variable in terms of the other.
2. We substituted this expression in place of this variable into the other equation to create an equation with only one variable.
3. We solved for this other variable.
4. We substituted this solution into the original rearranged equation to find the first variable.

### Activity 2: Solving Simultaneous Equations

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| Bullseye | Purpose In this activity, you are going to practice using the substitution method to solve simultaneous equations. |
| Stopwatch | Suggested Time You will need about 30 minutes. |
| Pencil | What You Will Need  * A pen or pencil * Some blank paper or a notebook * An Internet connection |

#### Tasks

1. Solve these equations simultaneously: and .
2. If the sum of two numbers is 70 and they differ by 11, find the numbers.
3. When you have answered questions 1) and 2), watch the video called [Solving linear systems by substitution (old)](https://www.khanacademy.org/math/algebra-home/alg-system-of-equations/alg-solving-systems-of-equations-with-substitution/v/solving-linear-systems-by-substitution) (09:30) to see if you solved them correctly.

(<https://www.khanacademy.org/math/algebra-home/alg-system-of-equations/alg-solving-systems-of-equations-with-substitution/v/solving-linear-systems-by-substitution>)

1. Solve for both variables in these equations:
2. and
3. and
4. and
5. and
6. and
7. and
8. and (Hint: you don’t have to substitute for or – you could also substitute for or )

#### Guided Reflection

1. You were asked to solve some simultaneous equations. Should be 4)
2. . Here the second equation is easy to rearrange in terms of one of the variables. Let’s call this equation (1).

Equation (1):

Substitute this into the other equation, which we will call equation (2):

Equation (2):

Substitute this back into equation (1):

1. . The first equation already has one of the variables on its own.

(1)

Substitute (1) into (2):

Substitute back into (1):

1. . Neither equation is easiest to rearrange.

(1)

(2)

Rearrange (2):

Substitute into (1):

Substitute back into (2):

1. . The first equation only had one fraction so will probably be easier to rearrange.

(1)

Substitute into (2):

Substitute back into (1):

1. . This was a tricky question that gave some unusual answers. Both equations are given as “” so we really just need to put the RHS of each one equal to each other.

We can already see that the LHS = RHS for ANY value of . So can be any real number. But what about ?

Look at the second equation .

When , .

When is very big (e.g. 1,000,000), the denominator of the fraction gets very big which means that the value of the fraction gets very small. This means that gets closer to 2 as gets bigger.

Because the denominator has , the same thing happens when gets very negative (e.g. 1,000,000).

So is never smaller than and never bigger than 2. Because can equal , we write the value of like this: .

1. .

(1)

(2)

Rearrange (2):

Substitute into (1):

Substitute back into (2):

1. . In this case, you needed to make a slightly different substitution to normal.

(1)

(2)

Rearrange (1):

Substitute for into (2):

Substitute into (1):

In the last question in Activity 2, you were asked to solve and simultaneously. In doing so, you may have ended up with one or both of these equations:

(1) (2)

If you had, you might have had trouble working out what substitution you could make in order to create an equation with only one variable. If this happens, go back to the original equations and see if there are other kinds of substitutions you can make like in this case.

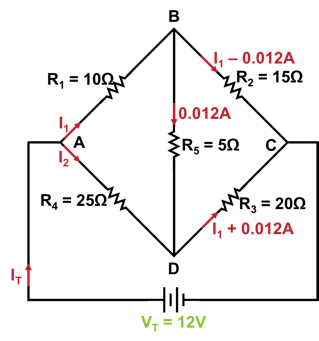
However, there is another method of solving simultaneous equations that you could try.

### Activity 3: Solving Simultaneous Equations by Elimination

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| Bullseye | Purpose In this activity, you are going to learn how to solve simultaneous equations using the elimination method. |
| Stopwatch | Suggested Time You will need about 45 minutes. |
| Pencil | What You Will Need  * A pen or pencil * Some blank paper or a notebook * An Internet connection |

#### Tasks

1. A student studying to be an electrician is trying to work out what the currents and are in this circuit.



He was been able to create the two simultaneous equations he needs but got stuck trying to solve them. His two equations are:

(1)

(2)

1. Can you solve these equations simultaneously using the substitution method? Which equation would be easiest to rearrange?
2. What could you multiply equation 1 and 2 by in order to get the coefficients of the same in both equations?
3. Multiply through equation 1 by 15 and equation 2 by 10.
4. Make sure that you write the equations one under the other. Now what can you do to get rid of the variable?
5. Solve for .
6. Solve for .
7. Here are some more simultaneous equations:
8. Write these two equations one under the other. What can you do to the equations to get eliminate the variable ?
9. Solve the equation.
10. Solve for the other variable.
11. Solve these simultaneous equations by elimination (multiplying one or both of the equations through to get the coefficients of one of the variables to be the same so that you can add or subtract the equations to eliminate that variable).
12. Once you have tried the questions in question 3), watch the video called [Simultaneous Equation by Elimination](https://www.youtube.com/watch?v=UDRTu2cInFU) (06:20) to see how you did.

(<https://www.youtube.com/watch?v=UDRTu2cInFU>)

#### Guided Reflection

1. This example shows us that simultaneous equations have some very real practical uses.
2. We could solve these equations using the substitution method although it is not clear which of the two equations would be simpler to rearrange and substitute for. In either case, we are going to land up with pesky fractions.
3. If we multiplied through equation 1 by 15 and multiplied through equation 2 by 10, we would have in both equations. At this point, we could rearrange one of the equations and substitute for into the other. But there is an even easier way.
4. (3)

(4)

1. If we subtract equation (4) from equation (3), we would get rid of (or eliminate) the terms.

(3)

- (4)

1. We can substitute out answer for into any of the four equations. Let’s use equation 1.
2. We can also use the elimination method to solve these simultaneous equations.
3. In this case, we don’t have to multiply one or both equations through by anything in order to make the coefficients of the variable we are trying to eliminate the same. All we need to do is **add** the equations to eliminate the .

+

1. Now we can solve for one of the variables

(divide through by )

1. Substitute this answer into equation 1 or 2. Let’s use equation 2.

So far, in every question we have tackled, both equations have been linear. But what happens if one fo the equations is a quadratic?

### Activity 4: Solving Simultaneous Equations Where One is a Quadratic

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| --- | --- |
| Bullseye | Purpose In this activity, you are going to learn how to solve simultaneous equations where one of the equations is a quadratic. |
| Stopwatch | Suggested Time You will need about 45 minutes. |
| Pencil | What You Will Need  * A pen or pencil * Some blank paper or a notebook * An Internet connection |

Tasks

1. Have a look at these simultaneous equations:

(1)

(2)

1. Which equation do you think should be rearranged and which variable would be the simplest to substitute?
2. Rearrange equation 2 in order to substitute for in equation 1.
3. Solve for . Why do you get two answers?
4. Solve for . How many answers do you get for ?
5. Solve for and : ;
6. Is one of the equations a quadratic equation? How many solutions do you expect?
7. Choose an equation to rearrange and substitute for one of the variables in the other equation. Does this create a quadratic equation?
8. Solve for the variable.
9. Solve for the other variable.
10. Solve the following simultaneous equations.
11. It takes3 hours for a boat to travel 27km upstream. The same boat can travel 30km downstream in 2 hours. What is the speed of the boat and the current? Remember that speed = distance/time.
12. A 25% alcohol solution means that a quarter of the mixture is alcohol. Therefore, 8 litres of a 25% alcohol solution contains 2 litres of alcohol. A chemist needs 18 litres of a 50% alcohol solution, but she only has a 30% alcohol solution and a 60% alcohol solution. How much of each of these other mixtures must she mix together to get 18 litres of a 50% alcohol solution?

Guided Reflection

1. You should have noticed that one of the equations was a quadratic equation.
2. Equation 2 is a linear equation and so would be simpler to rearrange. Also, there is an term in equation 1 so it would be simpler to substitute for into this equation. Generally, it is best to rearrange the linear equation to make a substitution into the quadratic equation for the variable that is NOT squared.

Now we can substitute in place of in equation 1.

1. To solve for , we need to solve the quadratic equation as normal.

(the quadratic is easy to factorise, so solving by factorisation will be quickest)

or

We get two answers because we solved a quadratic equation.

1. To solve for , we need to substitute **both** answers for into equation 2.

We write our final answer as ordered pairs to make it clear which answers belong to each other.

So, the solution is and .

As we will see when we learn about graphs, these answers, represented as ordered pairs are actually the coordinates of where the graphs of and cross each other or intersect.

1. In this question,
2. It does not look like either of the equations is a quadratic equation, therefore we expect to only get a single solution.
3. The equations given are:

(1)

(2)

Rearrange (2): (3)

Substitute equation (3) into equation (1):

1. (multiply through by the LCD of )

(it looks like we have a quadratic equation after all, therefore we do expect two solutions)

or

1. Substitute these answers into equation (3):

Solution is and )

1. TBC
2. TBC
3. Let be the number of litres of the 30% solution and be the number of litres of the 60% solution.

The total amount of solution needed is 18 litres: (1).

The total amount of alcohol in the solution is 9 litres (50%): (2)

From (1): (3)

Substitute (3) into (2):

Substitute into (3):

The chemist will need 6 litres of the 30% solution and 12 litres of the 60% solution.