NASCA Mathematics Materials Draft 1

## Topic 3: Measurement, Euclidean Geometry, Analytical Geometry and Trigonometry

## Euclidean Geometry

### Unit 1: Circles, perpendicular lines through the centre, chords and midpoints

#### Learning Outcomes

By the end of the unit, you should be able to:

* Identify and name the parts of a circle.
* Make the following conjectures through investigation:
* The line from the centre of the circle perpendicular to a chord bisects the chord (Conjecture 1)
* the line drawn from centre of a circle to the midpoint of chord is perpendicular to the chord (Conjecture 2)
* The perpendicular bisector of a chord passes through the centre of the circle (Conjecture 3)
* Apply Conjectures 1, 2 and 3 to solve simple geometry problems

### Circle Geometry

**Introduction**

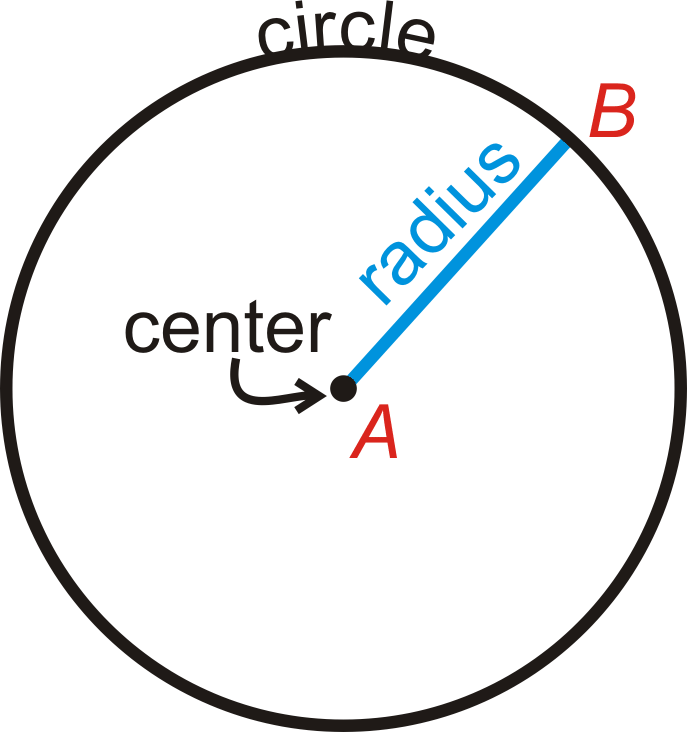
The circle is one of the most familiar geometric figures. Many everyday objects like bicycle wheels are based on circles. Circles have been important figures throughout history. The ancient Greeks believed that the sun, planets, and other celestial objects went around the earth in circles.

You need to know the following terms with respect to circles:

segment chord

*O* diameter

A **circle** is the set of all points in the plane that are the same distance away from a specific point, called the **center**. The center of the circle In Figure 1 is point *A*. We call this circle “circle *A*,” and it is labeled ⨀*A*. The distance from the centre of the circle to any point on its outer rim (commonly known as the circumference) is called the radius, and is labelled . In Figure 1, is the radius of ⨀*A*.



Circumference

(outer rim)

***r***

**Figure 1: Circle**

**Important Circle Parts**

**Radius:** The distance from the centre to any point on the circumference of the circle

**Chord:** A line segment whose endpoints are on a circle.

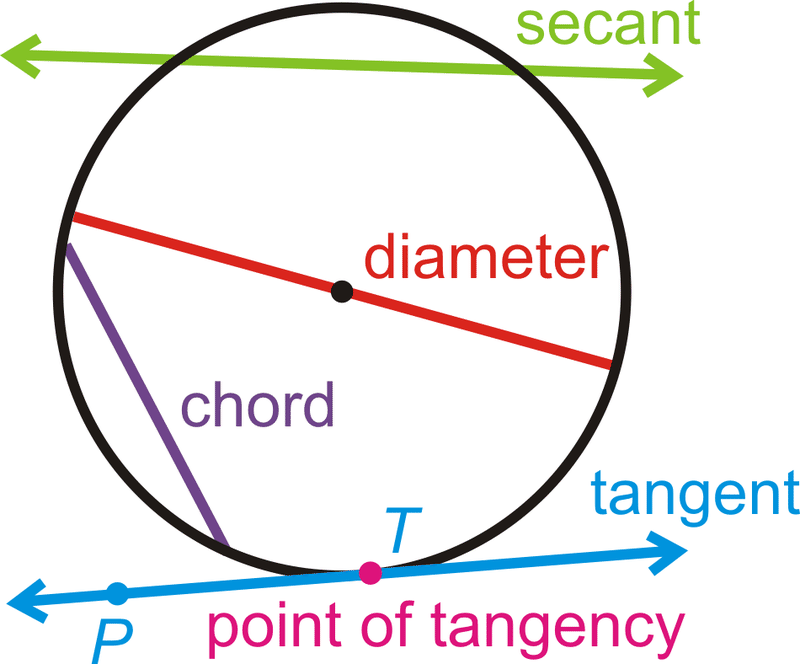
**Diameter:** A chord that passes through the center of the circle. The length of a diameter is two times the length of a radius.

**Secant:** A line that intersects a circle in two points.

**Tangent:** A line that intersects a circle in exactly one point.

**Point of Tangency:** The point where a tangent line touches the circle.

The tangent ray *TP*−→ and tangent segment *TP*⎯⎯⎯⎯⎯⎯⎯ are also called tangents.



**Figure 2: Parts of a circle**

# Arcs and segments of a circle

An arc of a circle consists of two points on the circle and the continuous (unbroken) part of the circle between the two points. The two points are called the endpoints of the arc.

Some special types of arcs and their notation are defined below:

|  |  |  |
| --- | --- | --- |
| **Semicircle** | **Minor Arc** | **Major Arc** |
| A semicircle is an arc of a circle whose endpoints are the end points of a diameter. | A minor arc is an arc of a circle that is less than a semicircle of the circle. | A major arc is an arc of a circle that is greater than a semicircle of the circle |
| Three letters are used to name a semicircle. The first and the last letters are the endpoints and the middle letter is any other point on the arc. | A minor arc may be named with either two or three letters. We will name a minor arc with the letters of the two endpoints of the arc. | To avoid confusion, we will always use three letters to name a major arc. The first and the last letters are the endpoints and the middle letter is any other point on the arc. |
| CB is a diameter  arc CEB and arc CDB are semi circles of ⨀A | Minor arc CB of  ⨀A. | Major arc CDB of ⨀A |

**Figure 3: Arcs of a circle**

In Figure 4, points B and C divides the circle into two arcs. The shorter arc (shaded blue in Figure 4) is called the *minor arc* and the longer one (shaded red in Figure 4) is called the *major arc*. A minor arc is named after its endpoints. In Figure 4, arc BC is the minor arc.



**Figure 4: Segments of a circle**

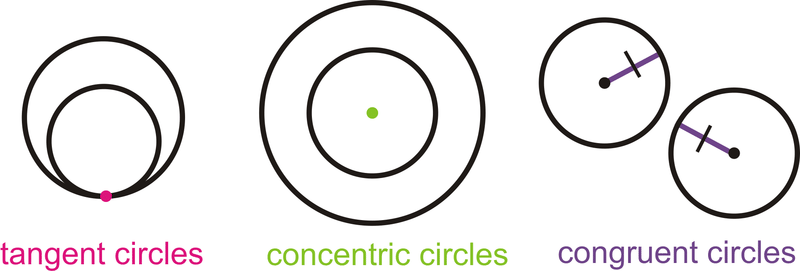
A segment of a circle is area enclosed between a chord an arc. In Figure 4, the area of the circle enclosed by chord BC and minor arc BC is called a **minor segment**. The area of the circle enclosed by chord BC and major arc BC is called a **major segment**

**Some Special Circles**

**Tangent Circles:** Two or more circles that intersect at one point.

**Concentric Circles:** Two or more circles that have the same centre, but different radii.

**Congruent Circles:** Two or more circles with the same radius, but different centres.



**Activity 1: Identifying parts of a circle, segments and special circles**

**Purpose:**

To recognize parts of a circle in a given diagram.

To enable students to identify the major and minor segments in a circle

To make students aware of the special kinds of circles.

**Resources:** A pen or pencil, eraser, ruler, compass and paper

**Suggested time:** 25 minutes

**Task 1:**

1. Study the given figure and then write down the parts of ⨀*A* that best fit each of the following descriptions:
   1. A radius
   2. A chord
   3. A tangent
   4. A point of tangency
   5. A diameter
   6. A secant



1. Study the given figure, and then do the following:
   1. Shade in the minor segment in a red colour
   2. Darken the major arc and chord that forms the major segment using your blue pen.



1. Use your compass and a pencil to draw the circles as indicated in step 1 and 2.

Step1: Draw a circle with a radius of 3 cm. Mark the centre of the circle A.

Step 2: Place your compass needle at point A, and then draw another circle with a radius of 5cm.

Choose from the following options the kind of circles you have drawn:

1. Tangent circles
2. Concentric Circles
3. Congruent circles

**Check your understanding**

Watch the You-tube video found at this link: <https://www.youtube.com/watch?v=Bxxl_cujLbY>

#### Guided reflection on Activity 1

|  |
| --- |
| 1. **Parts of a circle:**   What can you say about the lengths of AD, AC and AB.  Do you see that these are radii of ⨀*A*, and therefore have the same lengths.  Did you see that the diameter BC is equal BA + AC. What can you deduce from this observation?  The diameter of a circle is equal to two times the radius of a circle.  Compare lines GJ and EF  Do you see that line GJ cut ⨀*A* at two points (namely: H and I), and that line EF touches ⨀*A* only at one point (namely: D). We therefore call line GJ a secant and line EF a tangent. We call D the point of tangency of tangent EF to ⨀*A.*  What is the name of the largest chord in ⨀*A?*  Did you notice that BC, LK and HI are chords of ⨀*A,* and that the longest chord is BC. Chord BC is called the diameter, and since it is the longest, it the largest chord of⨀*A.* Note that the diameter is always the largest chord in any circle.   1. **Arcs and segments of a circle**   How did you identify the minor segment in a circle A?  Did you see that you need find area of the circle enclosed by chord BC and minor arc BC.   1. **Some Special Circles**   Why do you think you have drawn a pair of concentric circles?  The two circles that have the same centre, but different radii. |

**Answers to Task 1**

* 1. AC or AD or AB
  2. BC or LK or HI
  3. EF
  4. D
  5. BC
  6. GJ

2.

2.1



2.2



3.



B: Concentric circles

**Activity 2 Using Inductive reasoning to make conjectures**

**Inductive reasoning** is a kind of reasoning in which a conclusion is based upon experimentation and observation of patterns in several specific cases. It is one of the methods used for discovering geometric relationships in a circle.

A **conjecture** is a statement believed to be true based on observations. Making a conjecture does not mean that you are correct or incorrect. In order to ensure that your conjecture is always true you must prove that it true for every case through using deductive reasoning.

**Purpose:**

* To discover the relationship between a chord and the line drawn from the centre of the circle perpendicular to the chord through investigation.
* To discover the relationship between a chord and the line drawn from the centre of the circle to the midpoint of the chord through investigation.
* To discover the relationship between the perpendicular bisector a chord and centre of the circle through investigation.

**Resources:** A pen or pencil, eraser, ruler, compass and paper

**Suggested time:** 60 minutes

**Task 2: Investigating chords in a circle**

Here is a definition of a chord: A chord in a circle is a segment with endpoints on the circle.

You will need a ruler and compass in this Investigation. Use computer software if possible.

1. **Investigation 1**



Step 1 Draw a circle. Mark the centre O.

Step 2 Draw any chord AB other than a diameter.

Step 3 Draw a line from the centre of the circle

perpendicular to chord AB.

Use the letter M to indicate the point of

intersection.

Step 4 Measure the length of AM and MB.

What do you notice?

Step 5 Draw and another circle, and repeat steps 1-4

State your observations as a conjecture.

Remember, a conjecture is a mathematical statement that we think is correct, but is not yet proved.

**Conjecture 1**: The line drawn from the centre of the circle perpendicular to the chord \_\_\_\_\_\_\_\_ the chord.



1. **Investigation 2**

Step 1 Draw a circle. Mark the centre O.

Step 2 Draw any chord AB other than a diameter.

Step 3 Determine the midpoint of chord AB and label

it using the letter M.

Step 4 Draw a line segment joining the centre of

⨀O and the midpoint M of chord AB.

Step 5 Measure . What can you say about OM?

Step 6 Draw another chord CD other than a diameter

in the same ⨀O.

Step 7 Determine the midpoint of chord CD and label it

using the letter N.

Step 8 Draw a line segment joining the centre of ⨀O and the midpoint N of chord CD.

Step 9 Measure . What can you say about ON?

Compare your results in steps 5 and 9. State your observations as a conjecture.

**Conjecture 2**: The line segment drawn from the centre of the circle to the midpoint of a chord is \_\_\_\_\_\_\_\_ to the chord.

**3. Investigation 3**



Step1: Draw a circle. Mark the centre O.

Step 2: Draw chords AB, CD and EF.

Step 3: Construct the perpendicular bisector of each chord.

What do you observe about the perpendicular bisector of each chord.

State your observations as a conjecture.

**Conjecture 3:** The perpendicular bisector of a chord passes through

the \_\_\_\_\_\_.

#### Guided reflection on Activity 2

**CONJECTURE 1**



Given: Circle O with OM perpendicular to AB

What can you conclude about AM and MB? Give a reason for your answer.

AM is equal to MB (which is written as AM = MB) because when a line is drawn

from the centre of the circle perpendicular to the chord bisects the chord as per

our Conjecture 1.

Can you write down your reason in an abbreviated form?

The abbreviated form for your reason is: line from centre ⊥ to chord

**CONJECTURE 2**



Given: Circle O with M the midpoint of chord AB, i.e. AM = BM

What can you conclude about OM and MB? Give a reason for your answer.

OM is perpendicular to AM (which is written as OM ⊥AB) because

a line is drawn from the centre of a circle to the midpoint of

a chord is perpendicular to a chord as per our Conjecture 2.

Can you write down your reason in an abbreviated form ?

The abbreviated form for your reason is: line from centre to midpt of chord

**CONJECTURE 3**

**What have you discovered about the perpendicular bisector of a chord of a circle?**

**It passes through the centre of the circle.**



Can you write down your reason in an abbreviated form?

The abbreviated form for your reason is: perp bisector of chord

**Answers to Task 2**

**Conjecture 1**: The line drawn from the centre of the circle perpendicular to the chord bisects the chord.

**Conjecture 2**: The line segment drawn from the centre of the circle to the midpoint of a chord is perpendicular to the chord.

**Conjecture 3:** The perpendicular bisector of a chord passes through the centre of the circle.

**Activity 3: Basic application of conjectures associated with perpendicular lines through centre, chords and midpoints**

**Introduction**

Our conjectures can be used to do geometry calculations. You need to remember the following from each of your formulated conjectures:

**CONJECTURE 1:** The line drawn from the centre of the circle perpendicular to the chord bisects the chord.



Given : Circle O with OM perpendicular to AB

Conclusion: AM = MB

Reason: line from centre ⊥ to chord

**CONJECTURE 2:** The line segment drawn from the centre of the circle to the midpoint of a chord is perpendicular to the chord.



Given : Circle O with M the midpoint of chord AB, i.e. AM = BM

Conclusion: OM perpendicular to AB

Reason: line from centre to midpt of chord

**Conjecture 3:** The perpendicular bisector of a chord passes through the centre of the circle.



Given : Circle O with MN ⊥AB and AM = BM

Conclusion: MN passes through centre of the circle O.

Reason: perp bisector of chord

When working with Conjectures 1-3, you will be frequently using the theorem of Pythagoras.

Do you recall the Theorem of Pythagoras, which states: In a right-angled triangle the square on the hypotenuse is equal to the sum of the squares on the other two sides.

For example in right-angled , .

For example in right-angled , we have the statement: .

Depending on the given information and the length of the side of the triangle you want to calculate, the statement can also be written as

OR



Can you apply the Theorem of Pythagoras to calculate the length of OC if OA = 15cm, AC = 9cm and ?

Do you see we can use the statement

We then substitute the values of AO and AC:

To find OC, you must find the square root of 144. This then gives OC = 12 cm



**Purpose:**

* To apply the conjectures associated with perpendicular lines through centre, chords and midpoints to do basic geometry calculations.
* To develop the skill of setting out a geometry calculation using geometrical statements supported by reasons
* To encourage students to use deductive reasoning when doing geometrical calculations.

**Resources:** A pen or pencil, eraser, calculator, paper

**Suggested time:** 30 minutes

**How to apply conjectures to do geometric calculations**

When doing geometric calculations, it good practice to:

* Understand the given information for the given diagram
* Insert all the given information onto the diagram if it does not already appear.
* Identify what you must calculate
* State the triangle being considered as you proceed with your calculation
* Give statement and appropriate reason as you develop your calculation
* Give a conclusion

**Example**

In the following worked example you can see how we used these Conjectures 1 and 2 to do geometry calculations. Look at how we state the triangle being considered, provide the reason for each statement, and give a conclusion for each of the calculations.

In circle O, AB = 8 cm, OM = 3cm, AM =BM, OP ⊥ CD and OP = 4cm.

Calculate: (a) The length of OA, the radius of the circle

(b) the length of CD



**SOLUTION:**

**(a)**

O is the centre of the circle and

M is the midpoint of chord AB.

So use Conjecture 2

In any right angle triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides

|  |  |  |
| --- | --- | --- |
| **RTC** | OA |  |
| **Given** | ⨀O*,* AM = MB, AB = 8cm, OM = 3cm |  |
|  | **Calculation** | **Reason** |
| **Step 1** | AM = MB = 4cm | AM = MB, given |
| **Step 2** | OM ⊥ AB | line from centre to midpt of  chord |
| **Step 3** | In AOM, | Pythagoras |

**(b)**

|  |  |  |
| --- | --- | --- |
| **RTC** | **CD** |  |
| **Given** | ⨀O*,* OP ⊥ CD, OP = 4cm |  |
| **RTC** | **Calculation** | **Reason** |
| **Step 1** |  | radii |
| **Step 2** | In OPC, | Pythagoras |
| **Step 3** |  | line from centre ⊥ to chord |
| **Step 4** |  |  |

O is the centre of the circle and

OP is perpendicular to chord CD.

So use Conjecture 1

**Task 3: Applying Conjectures to do geometrical calculations**



1. In ⨀O, C is a midpoint of chord AB.
   1. If radius AO = 5cm, AB = 8cm, find OC.
   2. If radius AO = 13cm, OC = 5cm, find AB.

(c) If OC = 8cm, AC = 15cm, find radius AO.

1. In ⨀O, chord AB = 48 cm , OM ⊥ chord AB, OR ⊥ chord PQ,



OM = 7cm and OR = 5cm.

Calculate the length of PQ.

1. Two concentric circles A have radii 5 cm and 8 cm



respectively. B, C, D and E are on the same straight

line. CD = 6cm. Calculate the length of BE.

#### Guided reflection on Activity 3

|  |
| --- |
| **Question 1(a)**  In Question 1(a), do you know why AC = 4cm?  You are given that C is the midpoint of AB. Therefore AC= CB. This then means that AC = .  But AB is given to be 8cm. Therefore AC = (see Step 1 of solution for 1(a))  Which conjecture do you think will help towards calculating OC?  As you are given ⨀O with C the midpoint of chord AB, the following conjecture should come to mind: The line segment drawn from the centre of the circle to the midpoint of a chord is perpendicular to the chord. Applying the conjecture, you should be able to deduce that OC ⊥ AB and (see step 2 of solution for 1(a))  Now if you join OA, what kind of tringle will be?  Can you see that in you have . Therefore you can say that is a right-angled triangle.  What famous theorem can you apply to a right-angled triangle to calculate the length of a side if you know the lengths of the remaining two sides of the triangle?  Do you recall the Theorem of Pythagoras, which states: In a right-angled triangle the square on the hypotenuse is equal to the sum of the squares on the other two sides.  For example in right-angled , .  **Question 1(b):**  **Can you explain how you will proceed to calculate the length of AB in Question 1(b).**  **Step 1: show that**  by using Conjecture 2.  Step 2: join OA, to obtain a right-angled .  Step 3: Use Theorem of Pythagoras to calculate length of AC.  Step 4: Determine the length of CB using the fact that C is the midpoint of AB.  Step 5: Express AB in terms of AC and CB, and simplify.  **Question 1(c):**  Why can you use the theorem of Pythagoras to calculate the length of AO.  By using Conjecture 2, we can deduce that . Since, we have a right-angled triangle, and know the measure of two sides, namely OC = 8cm and AC =15 cm, we can use the he theorem of Pythagoras to calculate the length of AO.  **Question 2:**  What constructions do you think you should make on the given diagram to help with find the length of PQ?  Do you see that by joining OA you can get a right-angled triangle OMA. Similarly by joining OP you can get another write-angled triangle ORP  Focus on right-angled triangle OMA, and explain why AM = MB = ..  OM is a line drawn from centre of circle O perpendicular the chord AB. As per Conjecture 1, OM bisects AB. Therefore AM = MB = ..  Which theorem will you use to determine the length of AO in ?  Do you see you can use the Theorem of Pythagoras?  Why do you think OP is equal to OA?  Do you see that they are the radii of the same circle?  Did you notice that you can use the Theorem of Pythagoras to calculate the length of PR in ? Use the Theorem of Pythagoras to calculate the length of PR.  Explain why PQ= 2PR  Do you see that PR= RQ by applying Conjecture 1.  **Question 3**  Did you notice how the construction of a perpendicular from the entre of ⨀*A* to chord CD helps to simplify the problem?  Let us say that the perpendicular from the centre of the circle ⨀*A* meets chord CD at F. Briefly write down the steps to calculate the length of AF.  Step1: Determine the length of CF by using Conjecture 1  Step 2: Use the Theorem of Pythagoras to calculate the length of AF in .  Step 3: Use the Theorem of Pythagoras to calculate the length of FE in  Step 4: Use Conjecture 1 to explain why BF = FE.  Step 5: Express BE in terms BF and FE, and simplify |

**Answers to Task 3**

1. (a)

We have a right angle

Use Conjecture 2

|  |  |  |
| --- | --- | --- |
| **RTC** | OC |  |
| **Given** | ⨀O*,* AO = 5 cm and AB = 8 cm |  |
| **Constr** | Join AO |  |
|  | **Calculation** | **Reason** |
| **Step 1** | AC = CB = | AC = CB, given |
| Step 2 | OC ⊥ AB | line from centre to midpt of chord |
|  |
| **Step 3** | In AOC, | Pythagoras |
|  |
|  |
|  |

(b)

|  |  |  |
| --- | --- | --- |
| **RTC** | AB |  |
| **Given** | ⨀O*,* AO = 13 cm, OC = 5cm, AC = CB |  |
| **Constr** | Join AO |  |
| Step 1 | **Calculation** | **Reason** |
| OC ⊥ AB | line from centre to midpt of chord |
| **Step 2** | In AOC, | **Pythagoras** |
| **Step 3** | But CB = AC = 12cm | Given M the midpoint of AB |
| **Step 4** | AB = CB + AC = 12cm + 12 cm = 24 cm |  |

(c)

|  |  |  |
| --- | --- | --- |
| **RTC** | AO |  |
| **Constr** | Join AO |  |
| **Given** | ⨀O*,* AC = 15 cm, OC = 8 cm, AC = CB |  |
|  | **Calculation** | **Reason** |
| **Step 1** | AC = CB = | AC = CB, given |
|  |  |  |
| Step 2 | OC ⊥ AB | line from centre to midpt of chord |
|  |
| **Step 3** | In AOC,      **=** | **Pythagoras** |

|  |  |  |
| --- | --- | --- |
| **RTC** | PQ |  |
| **Given** | ⨀O*,* AM = MB, AB = 48cm, OM = 7cm, QR 5cm, OM ⊥ AB, OR ⊥ PQ |  |
| **Constr** | Join OA and OP |  |
|  | **Calculation** | **Reason** |
| **Step1** | AM = MB = | line from centre ⊥ to chord |
| **Step 2** | In AOM, | **Pythagoras** |
| **Step 3** |  | radii |
| **Step 4** | In ORP, | Pythagoras |
| **Step 5** |  | line from centre ⊥ to chord |
| **Step 6** |  |  |





|  |  |  |
| --- | --- | --- |
| **RTC** | BE |  |
| **Given** | ⨀A*,* AC = 5cm, AE = 8cm, and CD = 6cm |  |
| **Constr** | Draw AF ⊥CD |  |
|  | **Calculation** | **Reason** |
| **Step 1** | CF = FD = | ine from centre ⊥ to chord |
| **Step 2** | In , | **Pythagoras** |
| **Step 3** | ,  = 64-16  = 48    cm | Pythagoras |
| **Step 4** | But FB = FE = cm | line from centre ⊥ to chord  Using Conjecture 1 |
| **Step 5** | **BE = FB +FE =**  cm+ cm  = 2 cm  = 13.86 cm |  |

### Unit 2: Angle at Centre, angles in same segment, equal chord subtending equal angles

#### Learning Outcomes

By the end of the unit, you should be able to:

* Identify in a circle the parts: angle at centre, angle at circle (circumference), a chord (segment) subtending an angle at centre and at the circle
* Make the following conjectures through investigation:
* The angle subtended by an arc at the centre of the circle is double the size of the angle subtended by the same arc at the circle (on the same side as of the chord as the centre) (Conjecture 4)
* The angle subtended at the circle by a diameter is a right angle (Conjecture 5)
* Angles subtended by a chord of the circle on the same side of the chord are

equal (Conjecture 6)

* Discover the converse to conjecture 5 through investigation:
* If the angle subtended by a chord at point on the circle is a right angle, then the

chord is a diameter. (Conjecture 7)

* Make the following conjectures through investigation :
* Equal chords subtend equal angles at the circumference (Conjecture8
* Equal chords subtend equal angles at the centre (Conjecture 9 )
* Equal chords of equal circles subtend equal angles at the circumference (Conjecture 10)
* Apply Conjectures 4-10 to solve simple geometry problems

### Circle Geometry



**Figure 1: Acute angle at centre**

An **angle at the centre** of a circle is an angle whose vertex is at the centre of a circle and whose sides contains radii of the circle. In Figure 1, (which we can labelled as ) is an angle at the centre of circle B.

An angle at the circle (circumference) is an angle whose vertex lies on a circle and whose sides contain a chord of a circle. In Figure 1, (which we can labelled as ) is an angle at the circle (circumference).

We say that minor arc AC (or chord AC) subtends (non-reflex angle) (which we can labelled as ) at the centre and (that is ) at the circle (circumference), which lies in the major segment ADC.

In the diagram in Figure 2, the angle the major arc ADC subtends at the centre of the circle is reflex A possible angle subtended by the major arc ADC at the circle is (that is ) as shown in Figure 2.



**Figure 2: Reflex angle at Centre**

**Activity 1: Identifying angle at centre related to angle at circumference**

**Purpose:**

To recognize an angle subtended by a chord (or arc) at centre and also at the circle.

**Resources:** A pen or pencil, eraser, ruler, compass, protractor and paper

**Suggested time:** 15 minutes

**Task 1:**



Study the given figure and then do the following:

1. Which angle does chord MN subtend at the centre of circle O.
2. Which angles does chord MN subtend at the circumference of circle O.
3. Name the acute angle that minor arc MPN subtends at the centre of circle O
4. Name the angles that minor arc MPN subtends at the circumference of circle O.
5. Which chord subtends and .
6. Name the arc that subtends reflex at the centre.
7. Which arc subtends at the circumference of circumference of circle O.

#### Guided reflection on Activity 1

|  |
| --- |
| 1. **Angle at centre:**   How do you find the angle that a chord of a circle subtends at the centre?  From the endpoints of the chord, which lie on the circle, follow the connecting radii until they meet at the centre of the circle.  How do you find the angle that a chord of a circle subtends at the circumference of a circle?  From the endpoints of the chord, which lie on the circle, follow the connecting chords until they meet at a point on the circle. When the angle formed in this way, we say that the chord subtends an angle at centre and an angle at the circumference of the circle.  How did you find the angles that the major arc subtends at the centre and the circumference?  From the first and last point on the major MRN (or MQN) arc, follow the connecting radii to the centre of the circle B. This then gives you the reflex angle . To find the angle which major MRN (or MQN) arc subtends at the circumference follow the chords that run from the first and last point of the major arc until it points on the minor arc MPN. |

A**NSWERS to Task 1**

1. , , and and
2. and ,
3. MN
4. MQN or MRN
5. MQN or MRN

**Activity 2 Investigation of the relationship between an angle at the centre and the angle at the circumference of a circle**

**Purpose:**

To discover the relationship between an angle at the centre and the angle at the circumference of a circle

**Resources:** A pen or pencil, eraser, ruler, compass, protractor and paper

**Suggested time:** 15 minutes

**Task 2**

In this task you will investigate the relationship between an angle at the centre and the angle at the circle.

Copy this table. Complete the table as you go through steps 1 to 16.

|  |  |  |  |
| --- | --- | --- | --- |
| Case | Measure of angle at centre | Measure of angle at circle | How does the measure of angle at centre compare with the measure of angle at circle |
| Diagram 1 | = |  |  |
| Diagram 2 | = |  |  |
| Diagram 3 | = |  |  |
| Diagram 4 | Reflex = |  |  |
| Diagram 5 (your own) |  |  |  |

In diagrams 1-4, is the angle at centre of ⨀O and is the angle at the circle ( circumference) of ⨀O





Step 1. Use a protractor to measure in diagram 1.

Step 2. Measure in diagram 1.

Step 3. In diagram 1, how does the measure of compare with the measure of

Step 4. Use a protractor to measure in diagram 2.

Step 5. Measure in diagram 2.

Step 6. In diagram 2, how does the measure of C compare with the measure of

Step 7. Use a protractor to measure in diagram 3.

Step 8. Measure in diagram 3.

Step 9. In diagram 3, how does the measure of C compare with the measure of

Step 10. Use a protractor to measure reflex in diagram 4.

Step 11. Measure in diagram 4.

Step 12. In diagram 4, how does the measure of compare with the measure of

Step 13. Construct a circle of your own with an arc subtending and angle at centre and an angle at

the circle (or circumference) . Call this diagram 5.

Step 14. Measure the angle at centre in diagram 5.

Step 15. Measure the angle at the circle (or circumference) in diagram 5. How does the measure of

angle at centre compare with the measure of angle at circle.

Step 16. Study the table and write a conjecture about the measure of the angle subtended by arc at

centre of a circle and the angle subtended by the same arc at the circle.

What kind of reasoning did you use.

**Conjecture 4:** The measure of the angle subtended by arc at centre of a circle is \_\_\_\_\_\_\_\_

(**Angle at Centre Conjecture**)

#### Guided reflection on Activity 2

|  |
| --- |
| 1. **Angle at the centre and the angle at the circumference of a circle**   Can you describe which pairs of angles that you are required to work with in Task 1?  An angle subtended by an arc at centre of a circle and the angle subtended by the same arc at the circumference of a circle.  You were given cases where an angle was subtended by a minor arc at the centre. For each of those cases did you find the angle subtended by the same minor arc at the circumference of the circle lying on the minor arc or major arc?  Major arc?  How do you find the angle that a chord of a circle subtends at the circumference of a circle?  From the endpoints of the chord, which lie on the circle, follow the connecting chords until they meet at a point on the circle. When the angle formed in this way, we say that the chord subtends an angle at centre and an angle at the circumference of the circle.  How did you find the angles that the major arc subtends at the centre and the circumference?  From the first and last point on the major MRN (or MQN) arc, follow the connecting radii to the centre of the circle B. This then gives you the reflex angle . To find the angle which major MRN (or MQN) arc subtends at the circumference follow the chords that run from the first and last point of the major arc until it points on the minor arc MPN.  What do you notice to be common in your response to step 3 and step 6 in your investigation?  The measure of is two times (or double) measure of .  OR  The measure of is half the measure of . |

**Answers to Task 2**

**Conjecture 4:** The measure of the angle subtended by arc at centre of a circle is double (or 2 times) the size of the angle subtended by the same arc at the circumference of a circle.

**Activity 3 Investigation of the relationship between angles subtended by a chord of a circle, on the same side of chord (angles in the same segment)**

**Purpose:**

To discover the relationship between angles subtended by a chord of a circle, on the same side of chord.

**Resources:** A pen or pencil, eraser, ruler, compass, protractor and paper

**Suggested time:** 10 minutes

**Task 3**

In this task you will investigate the relationship between angles subtended by a chord of a circle, on the same side of chord.



In ⨀G on the right, chord AB subtends angles

C, D, E , F and G at the circle .

(a). Without measuring, make a conjecture about the measures of

, , , and

(b). Now use a protractor to measure, , ,and

Use a table like this record your answers:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Measure of | Measure of | Measure of | Measure of | Measure of |
|  |  |  |  |  |

(b) What appears to be true about , , ,and

(c) Measure . Fill in your measurement in the table.

(d) Why do you think the measure of is different from the measure of , or , or

(e) Study your table and make a generalization. What kind of reasoning did you use?

**Conjecture 5**: Angles subtended by a chord at the circle on the same side of the chord are \_\_\_\_\_\_

#### Guided reflection on Activity 3

|  |
| --- |
| 1. Are angles , , ,and subtended on the same side of the a chord AB?   Yes   1. What did you notice about angles subtended by a chord AB at the circle and which are on the same side of the chord?   They are all equal. |

**Answers to Task 3**

The kind of reasoning used is inductive reasoning.

**Conjecture 5**: Angles subtended by a chord at the circle on the same side of the chord are **equal.**

**Activity 4 Investigation of the size of the angle subtended at the circle by a diameter**

**Purpose:**

To discover through investigation that the angle subtended at the circle by a diameter is a right angle

**Resources:** A pen or pencil, eraser, ruler, compass, protractor and paper

**Suggested time:** 10 minutes

**Task 4**

In this task you will investigate the size of angle subtended at the circle by a diameter.

1.1 In diagrams 1-4, COD is the diameter of ⨀O, is the angle at centre of ⨀O, and is the angle at the circle (or circumference) of ⨀O



**Diagram 2**



**Diagram 1**



**Diagram 3**



**Diagram 4**

1.1 Use your protractor to measure in each of diagrams 1-4, and complete the table:

|  |  |
| --- | --- |
| Case | Measure of angle of angle subtended by diameter COD |
| Diagram 1 | = |
| Diagram 2 | = |
| Diagram 3 | = |
| Diagram 4 | = |

1.2 Study your table and complete the following conjecture statement:

**Conjecture 6**: The angle subtended at the circle (or circumference of a circle) by a diameter is a \_\_\_\_\_\_\_\_\_\_\_ angle.

* 1. (a) Do you think that when we have a line that subtended by a chord at a point on the circle is a right angle, then the chord is always a diameter.
  2. Draw a diagram with the given information to verify if your answer to Q1.3(a) is correct or not.

#### Guided reflection on Activity 4

|  |
| --- |
| 1. Does the measure of angle , which is subtended by diameter COD, always a right angle or sometimes a right angle or never a right angle.   Always a right angle   1. What kind of reasoning id you use to make your conjecture 6?   Inductive reasoning   1. Complete the converse statement of Conjecture 6.   Conjecture 7 (Converse of Conjecture 6) :  If the angle subtended by a chord at point on the circle is a right angle, then the chord is a diameter. |

**Answers to Task 4**

**1.1**

|  |  |
| --- | --- |
| Case | Measure of angle of angle subtended by diameter COD |
| Diagram 1 | = 90 |
| Diagram 2 | = 90 |
| Diagram 3 | = 90 |
| Diagram 4 | =90 |

**1.2 Conjecture 6**: The angle subtended at the circle (or circumference of a circle) by a diameter is a **right** angle.

1.3 Yes, it is always a diameter

1.4.



**Activity 5 Investigations with equal chords of a circle**

**Purpose:**

To discover through investigation the following about equal chords in a circle:

1. Equal chords in a circle subtend equal angles at the circumference;
2. Equal chords in a circle subtend equal angles at the centre;
3. Equal chords of equal circles subtend equal angles at the circumference;

**Resources:** A pen or pencil, eraser, ruler, compass, protractor and paper

**Suggested time:** 20 minutes

**Task 5**

**Investigation 1:**



In the given diagram, chords AB and DF are equal. Chord AB subtends at the circumference of the circle, and chord DF subtends at the circumference of the circle. Use your protractor to measure and .

1. Write down the size of

2. Write down the size of .

3. Now complete the following conjecture statement:

**Conjecture 8**: Equal chords in a circle subtend \_\_\_\_\_\_\_angles at the circumference of the circle

**Investigation 2**



In the given diagram, chords AB and CD are equal. Chord AB subtends at the centre of the circle, and chord CD subtends at the centre of the circle. Use your protractor to measure and .

1. Write down the size of

2. Write down the size of .

3. Now complete the following conjecture statement:

**Conjecture 9:** Equal chords in a circle subtend \_\_\_\_\_\_\_angles at the centre.

**Investigation 3**



In the given diagram, circle 0 and circle T are equal circles because they have equal radii.

Chords MN and QR are equal. Chord MN subtends at the circumference of circle O, and chord QR subtends at the circumference of the circle T. Use your protractor to measure and .

1. Write down the size of

2. Write down the size of

3. Now complete the following conjecture statement:

**Conjecture 10:** Equal chords of equal circles subtend \_\_\_\_\_\_\_angles at circumference

#### Guided reflection on Activity 5

|  |
| --- |
| 1. Does equal chords in a circle always/sometimes subtend equal angles at the circumference of the circle   Always.   1. Paul says that equal chords do not always subtend equal angles at the centre. Do you agree with Paul. If you do not agree with Paul, then correct the statement.   I agree with Paul.  Equal chords do not always subtend equal angles at the centre   1. When do we say that two circles are equal circles?   When they have equal radii. |

**Answers to Task 5**

**Investigation 1:**

Equal chords in a circle subtend **equal** angles at the circumference of the circle

**Investigation 2:**

Equal chords in a circle subtend **equal** angles at the centre.

**Investigation 3:**

Equal chords of equal circles subtend **equal** angles at circumference

**Activity 6 Application of conjectures: Angle at Centre, angles in same segment, equal chord subtending equal angles**

**Introduction:**

Our conjectures 4 to 10 discovered in this unit can be used to do geometry calculations. You need to remember the following from each of your formulated conjectures:

**Conjecture 4:**  The angle subtended by arc at centre of a circle is double (or 2 times) the size of the angle subtended by the same arc at the circumference of a circle



*Given*: Circle with centre at O and arc BC

Subtending at the centre

and AC at the circle.

*Conclusion*: = 2 AC .

*Reason*: ∠at centre = 2 ×∠ at circumference

**Conjecture 5:** Angles subtended by a chord at the circle on the same side of the chord are equal.



*Given:* Circle O and chord AB subtending

and on the same side of chord AB.

*OR*

Circle O and arc AB subtendingand

*Conclusion:* =

*Reason:* ∠’s in the same segment

**Conjecture 6**: The angle subtended at the circle (or circumference of a circle) by a diameter is a **right** angle.



*Given:* Circle O and diameter AB subtending

*Conclusion:* =

*Reason:* ∠s in semi circle **OR**

diameter subtends right angle

**Conjecture 7**: If the angle subtended by a chord at point on the circle is a right angle, then the chord is a **diameter**



*Given:* Circle O and chord AB subtending

*Conclusion:* is the diameter

*Reason:* chord subtends 90° **OR**

converse ∠s in semi circle

**Conjecture 8**: Equal chords in a circle subtend equal angles at the circumference of the circle



*Given:* Circle O and equal chords AB and EF

subtending and respectively.

*Conclusion:* =

*Reason:* equal chords; equal ∠s

**Conjecture 9**: Equal chords in a circle subtend equal angles at the centre.



*Given:* Circle O and equal chords AB and CD

subtending and , respectively.

*Conclusion:* =

*Reason:* equal chords; equal ∠s

**Conjecture 10**: Equal chords of equal circles subtend equal angles at circumference



*Given:* Circle O and equal chords MN and QR

subtending and respectively.

*Conclusion:* =

*Reason:* equal circles; equal chords; equal ∠s

**Purpose:**

* To apply the conjectures (4-10) associated with **angle at Centre, angles in same segment, equal chord subtending equal angles** to do basic geometry calculations.
* To develop the skill of setting out a geometry calculation using geometrical statements supported by reasons
* To encourage students to use deductive reasoning when doing geometrical calculations.

**Resources:** A pen or pencil, eraser, ruler, compass, protractor and paper

**Suggested time:** 10 minutes

**How to apply conjectures to do geometric calculations**

When doing geometric calculations, it good practice to:

* Understand the given information for the given diagram
* Insert all the given information onto the diagram if it does not already appear.
* Identify what you must calculate
* State the circle, arc, segment, and chord, being considered as you proceed with your calculation
* Give statement and appropriate reason as you develop your calculation
* Give a conclusion

**Example 1:**



In the diagram, A is the centre of the circle and *B*C = 114°, determine the size of .

**SOLUTION:**

Can you see how Conjecture 4 has been used

|  |  |  |
| --- | --- | --- |
| **RTC** |  |  |
| **Given** | ⨀AandBC = 114° |  |
|  | **Calculation** | **Reason** |
| **Step 1** | *B*C = 2 | ∠at centre = 2 ×∠ at circumference |
| **Step 2** | But *B*C = 114° | given |
| **Step 3** | 2 = 114°  = 57° |  |

**Example 2:**



In the given diagram, O is the centre of the circle and = 54°. Determine the sizes of and .

**Solution:**

Can you see how Conjecture 5 has been used

|  |  |  |
| --- | --- | --- |
| **RTC** |  |  |
| **Given** | ⨀O, and = 54° |  |
|  | **Calculation** | **Reason** |
| **Step 1** | = | ∠’s in the same segment |
| **Step 2** | But = 54° | given |
| **Step 3** | = 54° |  |

|  |  |  |
| --- | --- | --- |
| **RTC** |  |  |
| **Given** | ⨀O, and = 54° |  |
|  | **Calculation** | **Reason** |
| **Step 1** | = | ∠’s in the same segment |
| **Step 2** | But = 54° | given |
| **Step 3** | = 54° |  |

**Example 3**



In the diagram, two equal circles are given. P is the centre of the circle on the left and Q is the centre of the circle on the right. Chord MN is equal to chord RS. . RQV is a chord of ⨀Q. Calculate the size of the following angles:



**Solution:**

Which conjecture we applied here?

Can you see that was calculated in (b) and is used in (c)

|  |  |  |
| --- | --- | --- |
| **RTC** |  |  |
| **Given** | Equal circles ⨀P and ⨀Q, MN= RS,and |  |
|  | **Calculation** | **Reason** |
| **Step 1** | 1. = 2 | ∠at centre = 2 ×∠ at circumference |
| **Step 2** | But | given |
| **Step 3** | = 2(  = |  |
| **Step 1** | 1. = | equal chords; equal ∠s |
| **Step 1** |  | diameter subtends right angle . |
| **Step 2** | **But** |  |
| **Step 3** |  |  |
| **Step 4** | But | From (b) |
| **Step 5** |  |  |

Chord MN= chord RS. Can you see how Conjecture 9 has been used.

Did you notice that RV is a diameter

**Task 6: Applying Conjectures to do geometrical calculations**

In this task make sure that you explain your reasoning. Refer to examples 1-3 to help you with the setting out your solution



1. In ⨀O, AC is an angle at the centre of the circle.
   1. If , what is the size of and
   2. If = 46°, determine the size and the

the size of .

1. In the accompanying figure,



What can you deduce about chord AB and chord BC.

1. In the figure below, O is the centre of the circle. K, L, M and N are points on the circumference of the circle such that LM = MN. LOˆ N=.



* 1. Complete the following statement:

The sum of the angles around a point is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

3.2. Determine, with reasons, the values of the following:

(a) LMˆ N

(b) LKˆ M



1. In ⨀O on the right, O is joined to A, D and C.

AOC is a diameter, AD = AB and 2 = 46˚. Calculate

(a) 1 b) 1

(c) 2 (d)

(e) (f) 3

#### Guided reflection on Activity 6

|  |
| --- |
| 1. For Q1(a) and Q1(b), which conjectures would you use to do the calculations?   **Conjecture 4:**  The angle subtended by arc at centre of a circle is double (or 2 times) the size of the angle subtended by the same arc at the circumference of a circle  **Conjecture 5:** Angles subtended by a chord at the circle on the same side of the chord are equal.   1. For Q2, you must use one of the chord conjectures. Select and state the conjecture you would use.   **Conjecture 9**: Equal chords in a circle subtend equal angles at the centre.   1. How many degrees does the angles around a point add up to: 2. In a circle, what do know about the radii of a given circle?   They are always equal   1. Is the angles opposite equal sides always equal?   Yes.   1. In this unit you had the experience of using your conjectures to solve numerical riders.   How do you feel about your ability to solve numerical riders using your established conjectures?   1. Select a rider in this unit that you experienced difficulty with, but finally succeeded in completing it. Write down the difficulties you experienced and how you managed to overcome the stated difficulties. |

**Answers to Task 6**

1. (a)

We are using Conjecture 5

We are using Conjecture 4

|  |  |  |
| --- | --- | --- |
| **RTC** | and |  |
| **Given** | ⨀O and |  |
|  | **Calculation** | **Reason** |
| **Step 1** | = | ∠at centre = 2 ×∠ at circumference |
| **Step 2** | But = 64° | given |
| **Step 3** | 2 = 64°  = 32° |  |
| **Step 4** | = 32° | ∠’s in the same segment |

1. (b)

|  |  |  |
| --- | --- | --- |
| **RTC** | and |  |
| **Given** | ⨀O and = 46° |  |
|  | **Calculation** | **Reason** |
| **Step 1** | = | ∠at centre = 2 ×∠ at circumference |
| **Step 2** | But = 46° | given |
| **Step 3** | = 2( 46°) = 92° |  |
| **Step 4** | = 46° | ∠’s in the same segment |

1. They are equal

3.1

3.2 (a)

|  |  |  |
| --- | --- | --- |
| **RTC** | LMˆ N |  |
| **Given** | ⨀O, LM = MN, LOˆN= |  |
|  | **Calculation** | **Reason** |
| **Step 1** | *Reflex* | Revolution |
| **Step 1** | *But Reflex*  = 2 LMˆ N | ∠at centre = 2 ×∠ at circumference |
| **Step 2** | But 2 LMˆ N= | given |
| **Step 3** | LMˆ N= |  |

3.2 (b)

|  |  |  |
| --- | --- | --- |
| **RTC** | LKˆ M |  |
| **Given** | ⨀O, LM = MN, LOˆN= |  |
|  | **Calculation** | **Reason** |
| **Step 1** |  | Sum of ∠’s of |
| **Step 1** |  | LMˆ N= from 3.2(b) |
| **Step 2** |  | given |
| **Step 3** | **But** | MN = ML,  sides opp equal ∠s |
|  |  |  |

The sum of the angles of any triangle is always

MN and ML are equal chords

**4.**

(a)

|  |  |  |
| --- | --- | --- |
| **RTC** | 1 |  |
| **Given** | ⨀O, AD = AB, MN, 2 = 46˚ |  |
|  | **Calculation** | **Reason** |
| **Step 1** | 2 = 21 | ∠at centre = 2 ×∠ at circumference |
| **Step 2** | **But** 2 = 46˚ | given |
| **Step 3** |  |  |
| **Step 4** |  |  |

(b)

|  |  |  |
| --- | --- | --- |
| **RTC** | 1 |  |
| **Given** | ⨀O, AD = AB, MN, 2 = 46˚ |  |
|  | **Calculation** | **Reason** |
| **Step 1** | 1 = | ∠’s in the same segment |

(c)

|  |  |  |
| --- | --- | --- |
| **RTC** | 2 |  |
| **Given** | ⨀O, AD = AB, MN, 2 = 46˚ |  |
|  | **Calculation** | **Reason** |
| **Step 1** |  | Diameter subtends right angle. |
| **Step 2** | + |  |
| **Step 3** | + | 1 (from (b)) |
| **Step 4** |  |  |

Use Conjecture 6

(d)

Chords AD =AB

Use Conjecture 8

|  |  |  |
| --- | --- | --- |
| **RTC** |  |  |
| **Given** | ⨀O, AD = AB, MN, 2 = 46˚ |  |
|  | **Calculation** | **Reason** |
| **Step 1** |  | Equal chords, equal ∠’s |

e)

|  |  |  |
| --- | --- | --- |
| **RTC** |  |  |
| **Given** | ⨀O, AD = AB, MN, 2 = 46˚ |  |
|  | **Calculation** | **Reason** |
| **Step 1** |  | ∠’s in the same segment |

(f)

|  |  |  |
| --- | --- | --- |
| **RTC** | 3 |  |
| **Given** | ⨀O, AD = AB, MN, 2 = 46˚ |  |
|  | **Calculation** | **Reason** |
| **Step 1** |  | Diameter subtends right angle. |
|  |  | From (d), |
|  |  |  |

**Summary Assessment**

1. In the diagram, O is the centre of the circle passing through A, B and C.

.



Determine with reasons, the size of:

1.2

****

1. In the accompanying figure, chord AB = chord CD.

What can you deduce about



1. In the accompanying figure, P, Q, R, B,A are points on a circle.

Give reasons why:





1. In the accompanying figure, O is the centre of the

circle passing through points M, N, P and Q.

Calculate, giving reasons:

#### Guided reflection on Summary Assessment

|  |
| --- |
| 1. Use the answers given for the summary assessment to check if your answers were correct. 2. Did you realize why you got some your answers incorrect? 3. Have you been able to correct errors you made? |

**Answers to Summary Assessment**

**1.1**

|  |  |  |
| --- | --- | --- |
| **RTC** |  |  |
| **Given** | ⨀O and . |  |
|  | **Calculation** | **Reason** |
| **Step 1** | = | ∠at centre = 2 ×∠ at circumference |
| **Step 2** | But = 48° | given |
| **Step 3** | = = 2( 48°) = 96° |  |

**1.2**

|  |  |  |
| --- | --- | --- |
| **RTC** |  |  |
| **Given** | ⨀O and . |  |
|  | **Calculation** | **Reason** |
| **Step 1** |  | Sum of ∠’s of |
| **Step 2** | + |  |
| **Step 3** |  |  |
| **Step 4** | **But** | OC=OB, radii of circle |
| **Step 5** |  |  |

1. (a) Angles in the same segment (OR angles subtended by chord AB)

(b) Angles in the same segment (OR angles subtended by arc PQ)

(c)Angles in the same segment (OR angles subtended by arc QR)

(a)

|  |  |  |
| --- | --- | --- |
| **RTC** |  |  |
| **Given** | ⨀O and . |  |
|  | **Calculation** | **Reason** |
| **Step 1** |  | Sum of ∠’s of |
| **Step 2** | + |  |
| **Step 3** |  |  |
| **Step 4** | **But** | OP =ON, radii of circle.  ∠’s opposite equal sides |
| **Step 5** |  |  |

(b)

|  |  |  |
| --- | --- | --- |
| **RTC** |  |  |
| **Given** | ⨀O and . |  |
|  | **Calculation** | **Reason** |
| **Step 1** |  | ∠’s in the same segment |

(c)

|  |  |  |
| --- | --- | --- |
| **RTC** |  |  |
| **Given** | ⨀O, AD = AB, MN, 2 = 46˚ |  |
|  | **Calculation** | **Reason** |
| **Step 1** |  | Diameter subtends right angle. |
|  |  | From (b), |
|  |  |  |

### Unit 3: Cyclic Quadrilaterals

#### Learning Outcomes

By the end of the unit, you should be able to:

* Define a cyclic quadrilateral
* Make the following conjectures through investigation:
* The opposite angles of a cyclic quadrilateral are supplementary (Conjecture 11)
* An exterior angle of a cyclic quadrilateral is equal to the interior opposite angle (Conjecture 12)
* Apply Conjectures 11& 12 to solve simple geometry problems
* Prove that conjectures 11 and 12 are always true, and then accept them as theorems 6 and 7 respectively.
* State the converse of each of theorems 6 and 7.
* State a conditions to prove that a quadrilateral is cyclic.
* Apply the cyclic quadrilateral theorems and converses to solve geometric problems including both calculations and proof.

### Circle Geometry

Remember a quadrilateral is a ……………………………………………………………………………………………….

Now we are going to looks at the relationships between circles and quadrilaterals. In particular, we will begin to define what a cyclic quadrilateral is in Activity 1.

**Activity 1: Definition of a cyclic quadrilateral**

**Purpose:** To develop a definition of a cyclic quad in terms of concyclic points

**Resources:** A pen or pencil, eraser, ruler, compass, protractor and paper

**Suggested time:** 5 minutes



Points A, B, C and D lie on the circumference of ⨀O. When this happens we say that points A, B, C and D are concyclic. Since the four points are concyclic, we call the quadrilateral formed by joining points A, B,C, and D a cyclic quadrilateral.

Write down in your own words what is a cyclic quadrilateral: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Activity 2: Converse of geometrical statement**

**Purpose:** To develop the skill of formulating the converse a geometrical statement

**Resources:** A pen or pencil, eraser, ruler, and paper

**Suggested time:** 15 minutes

To formulate a converse of a statement, the condition and the conclusion should be reversed. Some converses are true statements, others are false.

For example, consider the Conjecture 5 from Unit 2: Angles subtended by a chord at the circle on the same side of the chord are equal

|  |  |
| --- | --- |
| **Condition** | **conclusion** |
| Angles subtended by a chord at the circle on the same side of the chord | **are equal** |

If we swop the condition and conclusion we have the following statement:

|  |  |
| --- | --- |
| **Condition** | **conclusion** |
| If a line segment joining two points subtends equal angles at two other points on the same side of that line segment | then the four points lie on a circle (i.e. they are concylic. |



Below you can see an example of this converse:

**Given:** Points E and F, with EF being the line segment joining them.

Two other points G and H, on the same side of EF,

such that = .

**Conclusion:** E,F, G and H are concyclic points

( i.e a circle can be drawn and pass through E,F, G and H)

**Reason:** line segment subtends equal angles on the same side

We may now write conjecture 6, which is converse of Conjecture 5 as follows:

**Conjecture 11:** If a line segment joining two points subtends equal angles at two other points on the same sides of that line segment, then the four points are concyclic (i.e. they lie on a circle).

**Activity 3:**

**Purpose:** To discover that the opposite angles of a cyclic quadrilateral are supplementary

**Resources:** A pen or pencil, eraser, ruler, compass, protractor and paper

**Suggested time:** 15 minutes

**Task 1:**

In this activity you will investigate some properties of a quadrilateral inscribed in a circle – in other words, a quadrilateral whose vertices lie on a circle. Such a quadrilateral is called a cyclic quadrilateral.



1. Use your protractor to measure each of the following

angles in the accompanying figure: ; and .

1. Calculate the sum of each pair of opposite angles:

and = \_\_\_\_\_\_\_\_

= \_\_\_\_\_\_\_\_\_

1. What can you say about each pair of opposite

angles of a cyclic quadrilateral.

1. Draw your own cyclic quadrilateral CDEF. Repeat step 2,
2. And then write down, what you observe about each pair of opposite

angles of a cyclic quadrilateral.

1. State your findings as your next conjecture.

**Conjecture 12**: The \_\_\_\_\_ angles of a cyclic quadrilateral are \_\_\_\_\_\_\_.

#### Guided reflection on Activity 3

|  |
| --- |
| 1. John says he found that the opposite angles of a cyclic quadrilateral are 150? State whether you agree/disagree with John. If you disagree give a reason.   Disagree.  Reason: The opposite angles in any cyclic quadrilateral always add up to 180 |

**Activity 4 Investigation: Exterior angle of a cyclic quadrilateral**

**Purpose:**

To discover the relationship between the exterior angle of a cyclic quadrilateral and its interior opposite angle.

**Resources:** A pen or pencil, eraser, ruler, compass, protractor and paper

**Suggested time:** 15 minutes

**Task 2**

In this task you will investigate the relationship between the exterior angle of a cyclic quadrilateral and its interior opposite angle.



In the figure alongside, ABCD is a cyclic quadrilateral.

If we extend BC to E, then *DE* is called

an exterior angle of cyclic quad ABCD.

Copy this chart. Complete this chart as you answer Questions 1-6.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Measure of | Measure of | Measure of | Measure of | Measure of |
| Diagram 1 |  |  |  |  |  |
| Diagram 2 |  |  |  |  |  |
| Diagram 3 |  |  |  |  |  |

1. Use a protractor to measure , , , , and in diagram 1

Record your readings in the above table.

1. Draw another cyclic quadrilateral ABCD in your workbook. Extend BC to E. Then is called an exterior angle of cyclic quad.ABCD. You will consider this drawing as diagram 2.
2. Use a protractor to measure , , , , and i in diagram 2

Record your readings in the above table.

1. Draw another cyclic quadrilateral ABCD in your workbook. Extend BC to E. Then is called an exterior angle of cyclic quad.ABCD. You will consider this drawing as diagram 3.
2. Use a protractor to measure , , , , and i in diagram 3

Record your readings in the above table.

1. Study the chart and make a conjecture about the exterior angle of a cyclic quadrilateral.

**Conjecture 13**: The exterior angle of a cyclic quadrilateral is ……………………………………………..

1. What kind of reasoning did you use?

#### Guided reflection on Activity 4

|  |
| --- |
| 1. Is the exterior angle of a cyclic quad chords always/sometimes equal to the interior opposite angle   Always. |

**Answers to Task 2**

6. **Conjecture 13**: The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle

7. Inductive reasoning

**Activity 5 Application of cyclic quadrilateral conjectures to do simple geometry calculations**

**Purpose:**

* To apply the cyclical quadrilateral conjectures to do basic geometry calculations.
* To develop the skill of setting out a geometry calculation using geometrical statements supported by reasons
* To encourage students to use deductive reasoning when doing geometrical calculations.

**Resources:** A pen or pencil, eraser, ruler, compass, protractor and paper

**Suggested time:** 15 minutes

Two worked examples which follow show you how to set your reasons

**Examples**

**Worked example 1**



O is the centre of the circle, *BC* = 116˚ and *AD = DC*.

Calculate *DE.*

**Solution**

|  |  |  |
| --- | --- | --- |
| **RTC** | *DE* |  |
| **Given** | ⨀O,  *BC* = 116˚ , *AD = DC* |  |
|  | **Calculation** | **Reason** |
| **Step 1** | *BC* = 2*B*C | ∠at centre = 2 ×∠ at circumference |
| **Step 2** | But  *BC* = 116˚ | given |
| **Step 3** | 2*B*C = 116°  *B*C = 58° |  |
| **Step 4** | *AC* = 90˚ | s in semi circle |
| **Step 5** | *DC* = 45˚ | AD = DC |
| **Step 6** | *DE* = *D*  But *D* = 58˚+ 45˚  *DE =* | ext. ∠ of cyclic quad ABCD |

Why is *DC* = 45˚.

In ,

*DC + DA + AC =*

*DC + DA +* 90˚ =

*DC + DA=*

*But DC = DA why?*

*DC = DA= ½ ()=*

**Worked example 2**



In the diagram on the right, ABCD is a cyclic quadrilateral.

*AD* = 32˚ and *DC* = 56˚

Calculate: (a) *AD*

(b) *AC*

|  |  |  |
| --- | --- | --- |
| **RTC** | (a) *AD*  (b) *AC* |  |
| **Given** | ⨀P,  *ABCD is a cyclic quad,*  *AD* = 32˚ and *DC* = 56˚ |  |
|  | **Calculation** | **Reason** |
| **Step 1** | AD =  *AD* = 32˚ | ∠ in the same segment |
| **Step 2** | *AC* =  *AD + DC*  *=* 32˚ +56˚ = 88˚ |  |
| **Step 3** | *AC* = 180˚ - *AC*  = 180˚ - 88˚  = 92˚ | opp s of cyclic quad |

the opposite angles of a cyclic quadrilateral are supplementary

**Task 3**



1. In © O on the right,  *=* 96˚ and BC//DE. Calculate

(a) (b) (c) *AB*

2. In © O on the right, O is joined to M and P and OM= 48˚. Calculate



(a) 1 (b) c)

3. In the figure on the right, MNPT is a cyclic quadrilateral.



O is the centre of the circle. OM bisects TN and = 34˚. (a) Name two other angles equal to 34˚.

(b) Determine the size of

(c) Calculate the size of 1

#### Guided reflection on Activity 3

|  |
| --- |
| 1(a) To calculate , which property parallel lines will you use?  Co-interior angles are supplementary  1 (b) Which conjecture will you use to calculate ?  **Conjecture 12**: The opposite angles of a cyclic quadrilateral are supplementary.  1(c) Which conjecture will you use to calculate *AB*?  **Conjecture 13**: The exterior angle of a cyclic quadrilateral is equal is equal to the interior opposite angle.  2. (a) Complete the following statements:  (i) The sum of the angles of triangle are equal to \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.  (ii) The angles opposite equal sides of a triangle are \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.  2(b) Why is 1 =  ∠at centre = 2 ×∠ at circumference  2(c) Why is .  opp s of cyclic quad are supplementary  3(a) OM bisects *TN in the given figure in Question 3. What can be deduces from this statement?*  2 **=** ON  (b) Give a reasons for : opp s of cyclic quad are supplementary  (c) Give reason for *N =* 1 *+* 1 *)*  sum of s in ∆ |

**Answers to Task 3**

1. (a)

When a pair of lines are parallel and cut by a transversal , each pair of co-interior angles are supplementary

|  |  |  |
| --- | --- | --- |
| **RTC** | (a) |  |
| **Given** | © O , = 96˚ and BC//DE |  |
|  | **Calculation** | **Reason** |
| **Step 1** |  | co-int s; BC ||DE |
| **Step 2** |  | Given = 96˚ |
| **Step 3** |  |  |

1. (b)

is opposite in cyclic quad ABCD.

The opposite angles of a cyclic quad are always supplementary

|  |  |  |
| --- | --- | --- |
| **RTC** | (b) |  |
| **Given** | © O , = 96˚ and BC//DE |  |
|  | **Calculation** | **Reason** |
| **Step 1** |  | opp s of cyclic quad |
| **Step 2** |  | From 1(a)… |
| **Step 3** |  |  |

1 (c)

|  |  |  |
| --- | --- | --- |
| **RTC** | *AB* |  |
| **Given** | © O , = 96˚ and BC//DE |  |
|  | **Calculation** | **Reason** |
| **Step 1** | *AB =*  = 96˚ | ext  of cyclic quad |

is opposite in cyclic quad ABCD.

The opposite angles of a cyclic quad are always supplementary

|  |  |  |
| --- | --- | --- |
| **RTC** | (a) 1 (b)  c) |  |
| **Given** | © O and OM= |  |
|  | **Calculation** | **Reason** |
| **Step 1** | 1. 1 = 180- (1 + *OM*) | sum of s in ∆ |
|  | But 1 = *OM =* | OM = OP, radii  s opp equal sides |
|  | 1 = 180- ( + ) |  |
|  | 1 = |  |
|  |  |  |
|  | 1. 1 = | ∠at centre = 2 ×∠ at circumference |
|  | 1 = | From 2(a) |
|  | ∴ |  |
|  | ∴ |  |
|  |  |  |
|  |  | opp s of cyclic quad |
|  |  | From 2(b)… ∴ |
|  |  |  |

**3(a)** 2 **=**  = 34˚ (OM bisects *TN)*

1 **=**  = 34˚ (OM = ON, radii; s opp equal sides)

Can you see how we used our calculations from 3(a)

**3(b)**

|  |  |  |
| --- | --- | --- |
| **RTC** | (b) |  |
| **Given** | © O, OM bisects *TN* and = 34˚ |  |
|  | **Calculation** | **Reason** |
| **Step 1** | *(b) TN =* 2 *+* 1  = 34˚ + 34˚ = 68 | 2 **=**  = 34 from 3(a) |
| **Step 2** |  | opp s of cyclic quad |
| **Step 3** |  |  |
| **Step 4** |  |  |

**3(c)**

|  |  |  |
| --- | --- | --- |
| **RTC** | (b) 1 |  |
| **Given** | © O, OM bisects *TN* and ON = 34˚ |  |
|  | **Calculation** | **Reason** |
| **Step 1** | *N =* 1 *+* 1 *)* | sum of s in ∆ |
| **Step 2** | *N =* 34˚ *+* 34˚ *)*  *=* | From 3(b) |
| **Step 3** | But *N = 2* 1 | ∠at centre = 2 ×∠ at circumference |
| **Step 4** | *2* 1= |  |
| **Step 5** | 1= |  |

We had to calculate *N so that we can use it is this step*

**Activity 6 Proof of cyclical quadrilateral conjectures are always true**

**Purpose:**

* To develop the skill of using deductive reasoning to prove that each of the cyclical quadrilateral conjectures is always true.

**Resources:** A pen or pencil, eraser, ruler, compass, protractor and paper

**Suggested time:** 10 minutes

**Task 4**

In Activity 3, you established the following conjecture:



The opposite angles of a cyclic quadrilateral are supplementary

We will now use the **algebraic approach**, to explain

and establish the validity of the above conjecture.

We will use the diagram on the right.

1. Write down the given information.
2. Write down what you are required to prove.
3. Describe the constructions needed in order to facilitate

an explanation of the required proof.

1. Let’s start by letting
2. Express in in terms of Give a reason for your statement.
3. Express in terms of. Give a reason for your statement.
4. What is the relationship between and . Why?
5. Now express in terms of .
6. What can you conclude about the sum of and .
7. What can you conclude about the sum of *AC* and *AC*? Why

***We now look over questions 1-10 to write a two-column proof of your conjecture.***

Given: A, B, C and D are 4 points on a circle with centre O.

Construction: Draw OB and OD

Required to prove: + = and +

Proof: Let

= 2 = 2 (∠at centre = 2 ×∠ at circumference)

= 360 (revolution)

(∠at centre = 2 ×∠ at circumference)

=

+ =

=

*AC* + *AC* = (sum of s in quad)

= 180

Now that we proved that the conjecture is always true, we will from now onwards refer to it as a theorem.

Now that we proved that the conjecture is always true, we will from now onwards refer to it as a theorem:

**Theorem 6**: The opposite angles of a cyclic quadrilateral are supplementary.

**Abbreviated reason**: opp s of cyclic quad

The **converse of theorem 6** can be stated as follows:

If the opposite angles of a quadrilateral are supplementary then the quadrilateral is cyclic.

**Abbreviated reason:** opp s quad supp **OR** converse opp s of cyclic quad.

We could also through the use of deductive reasoning prove that the exterior angle of a cyclic quadrilateral is always equal to the interior opposite angle. We are not going prove this conjecture in this unit but will accept it as a theorem for now:

**Theorem 7**: The exterior angle of a cyclic quadrilateral is always equal to the interior opposite angl

**Abbreviated reason:** ext  of cyclic quad

The **converse of theorem 7** can be stated as follows:

If the exterior angle of a quadrilateral is equal to the interior opposite angle of the quadrilateral, then the quadrilateral is cyclic.

**Abbreviated reason:** ext  = int opp  **OR** converse ext  of cyclic quad

#### Guided reflection on Activity 6

|  |
| --- |
| 1. In this task you used the algebraic approach to prove that the opposite angles of a cyclic quadrilateral are always supplementary.   How do you feel about your ability to use the algebraic approach to develop a proof? |

**Activity 7 Proving that a quadrilateral is cyclic**

**There are three ways to prove that a quadrilateral is cyclic**

1. If a line subtends equal angles at two different points on the same side of the line, then the four points are cyclic.



If *m= n*, then EFGH is a cyclic quadrilateral.

1. If one pair of opposite angles of a quadrilateral is supplementary, then the quadrilateral is cyclic.



If =180, then ABCD is cyclic quadrilateral.

1. If the exterior of a quadrilateral equals the interior opposite angles, then the quadrilateral is cyclic.



If x = y, then ABCD is a cyclic quadrilateral

**Worked example**

****

In the diagram above, ABCD is a quadrilateral, with diagonals AC and DB intersecting at E, DA = DE, and. Prove that ABCD is a cyclic quadrilateral.

**Proof**

|  |  |  |
| --- | --- | --- |
| **RTP** | ABCD is a cyclic quadrilateral | **Reason** |
| **Given** | DA = DE, and |  |
| **Proof** |  |  |
| **Step 1** |  | s on a str line |
| **Step 2** |  | s opp equal sides |
| **Step 3** |  | ext  of  |
| **Step 4** |  |  |
| **Step 5** | But |  |
| **Step 6** |  |  |
| **Step 7** | is a cyclic quad | DC subtends =s |

This is one of the ways to prove that any quadrilateral is a cyclic quadrilateral

**Task 6**

1. Prove that A, B, C and D are concyclic 

1. Prove that ABCD is a cyclic quadrilateral



1. Prove that ABCD is a cyclic quadrilateral





1. In the figure on the right, AB is a diameter of

circle O. Chord CD is perpendicular to AB and

cuts AB in G. Chord DE cuts AB in F. CE,CB,DB,

OC and AC are drawn.

Prove that CEFO is a cyclic quadrilateral**.**

#### Guided reflection on Activity 7 (Task 6)

|  |
| --- |
| 1. Which angles does BC subtends?   and  Do you think if you can show that if =, then you can conclude that ABCD is a cyclic quadrilateral? If yes, give a reason.  Yes, because the line segment joining B and C subtends equal angles at two points on the same side of it.   1. Complete the following statement: The sum of the angles of a quadrilateral are equal to …   .   1. If you prove that , what will then be your reason to conclude that ABCD is a cyclic quadrilateral.   ext  = int opp  **OR** converse ext  of cyclic quad   1. How can you prove that .   By showing that case SS is satisfied. That is show that 2 sides and an included angle in are correspondingly equal 2 sides and an included angle .  Why is ?  Why is ?  They are angles subtended by the same chord CD, and when this happens the subtended  angles are equal  State the convers of the theorem: The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.  If the exterior angle of a quadrilateral is equal to the interior opposite angle of the quadrilateral, then the quadrilateral is cyclic. |

**Answers to Task 6**

When a quadrilateral ABCD it means that circle can pass through points A, B, C and D.

|  |  |  |
| --- | --- | --- |
| **RTP** | A, B, C and D are concyclic | **Reason** |
| **Given** | , and |  |
| **Proof** |  |  |
| **Step 1** |  | sum of s in ∆ |
| **Step 2** | but | given |
| **Step 3** |  |  |
| **Step 4** | ABCD is a cyclic quad | BC subtends equal s |
|  | A, B, C and D are concyclic |  |

**2.**

|  |  |  |
| --- | --- | --- |
| **RTP** | ABCD is a cyclic quadrilateral | **Reason** |
| **Given** | , , |  |
| **Proof** |  |  |
| **Step 1** |  | sum of s in quad |
| **Step 2** |  |  |
| **Step 3** |  |  |
| **Step 4** |  |  |
| **Step 5** | = |  |
| **Step 6** | ABCD is a cyclic quad | opp s quad supp **OR**  converse opp s of cyclic quad |

We have shown that a pair of opposite angles of quad ABCD are supplementary

**3.**

|  |  |  |
| --- | --- | --- |
| **RTP** | ABCD is a cyclic quadrilateral | **Reason** |
| **Given** | and |  |
| **Proof** |  |  |
| **Step 1** |  | s on a str line |
| **Step 2** | But | given |
| **Step 3** |  | We have shown that an exterior angle of quad ABCD is equal to its interior opposite angle |
| **Step 4** | ABCD is a cyclic quad | ext  = int opp  **OR**  converse ext  of cyclic quad |

**4.**

|  |  |  |
| --- | --- | --- |
| **RTP** | CEFO is a cyclic quadrilateral | **Reason** |
| **Given** | AB dimeter of ⨀O and  CD⊥AB |  |
| **Proof** |  |  |
| **Step 1** | In and |  |
|  | Given CD ⊥ AB |
| 1. BG is common |  |
| 1. CG = DG | perp. From centre to chord |
|  | SS  We showed that 2 sides and an included angle in are correspondingly equal 2 sides and an included angle . |
| **Step 2** |  |  |
| **Step 3** | =  = 2 | s in the same seg  proved in step 2 |
| **Step 4** |  | ∠at centre = 2 ×∠ at circumference |
| = | proved in step 3 |
| **Step 5** | CEFO is a cyclic quad | ext  = int opp  OR  converse ext  of cyclic quad |

We have shown that an exterior angle of quad CEFO is equal to its interior opposite angle

#### Guided reflection on Summary Assessment

|  |
| --- |
| 1. Use the answers given for the summary assessment to check if your answers were correct. 2. Did you realize why you got some your answers incorrect? 3. Have you been able to correct errors you made? |

### Unit 4: Tangents

#### Learning Outcomes

By the end of the unit, you should be able to:

* Make the following conjectures through investigation:
* The angle between tangent and radius is (i.e a right angle) (Conjecture 13)
* Two tangents drawn to a circle from the same point outside the circle are equal in length ( Conjecture 14)
* The angle between a tangent to a circle and a chord drawn from the point of contact is equal to an angle in the alternate segment. ( Conjecture 15)
* Apply tangent conjectures to solve simple geometry problems
* Prove that the angle between a tangent to a circle and a chord drawn from the point of contact is equal to an angle in the alternate segment.
* Prove that a line to a circle is a tangent

### Sub-Topic: Circle Geometry



**Figure 1: Tangent to circle O**

Key Words:

**Tangent** – a straight line which touches the circle at one point only but does not pass through it.

In Figure 1, straight line is a tangent to circle O at point B.

Point B is the point of contact of the tangent ABC to circle 0. Therefore B is called a point of **tangency**.

OB is the **radius** of circle O

is the angle between tangent ABC and the radius OB of circle O.

**Activity 1: Investigation - Angle between tangent and radius**

**Purpose:**

To discover the measure of the angle between a tangent and a radius of a given circle

**Resources:** A pen or pencil, eraser, ruler, compass, protractor and paper

**Suggested time:** 15 minutes

**Task 2**

In this investigation, you will discover something about the measure of the angle formed by a

tangent and the radius drawn to the point of tangency.

1. (a)Construct a circle with a radius of 4cm. Label the center 0.
2. Using your ruler, draw a line that appears to touch the circle at only one point.
3. Label the point T. Point T in this case is called the point of **tangency**.
4. Construct OT.
5. Use your protractor to measure the angles at T.
6. (a) Construct another circle with a radius 5cm.
7. Using your ruler, draw a line that appears to touch the circle at only one point.
8. Label the point T. Point T in this case is called the point of **tangency**.
9. Construct OT.
10. Use your protractor to measure the angles at T.

What do you observe about the size of the angle between the tangent and radius? \_\_\_\_\_\_

Complete the following conjecture statement from what you observed:

**Conjecture 13**: A tangent to a circle is \_\_\_\_\_to a radius drawn to the point of

Tangency (Tangent- Radius Conjecture)

Note: For the purposes of geometry we will refer to this conjecture as an axiom.

**Axiom**: A tangent to a circle is perpendicular to the radius at point of contact.

Abbreviation: (tan  radius).

The corollary to the axiom can be stated as follows:

**Corollary**: A line drawn perpendicular to a radius of a circle, at the point where the radius meets the circle, is a tangent

#### Guided reflection on Activity 1

|  |
| --- |
| What is an axiom?  An *axiom i*s a statement we accept as true with no proof.  What is a theorem?  A theorem a mathematical statement that is proved using mathematical deductive reasoning.  What is a corollary?  A result in which the (usually short) proof relies heavily on a given theorem (we often say that “this is a corollary of Theorem A”).  Are you convinced that the angle between the tangent and radius is always ?  Yes. |

**Activity 2 Application of tangent-radius axiom**

**Purpose:**

**Resources:** A pen or pencil, eraser, ruler, compass, protractor and paper

**Suggested time:** 15 minutes



*Given:* Circle O with ABC tangent at B and OB the radius.

*Conclusion: ABC is perpendicular to OB ( i.e. ABC*

*Reason:*  tangent perpendicular to radius (tan  radius)

**Worked Example**



In the given diagram, ABC is a tangent to the circle O and OB is the radius of circle O.

. Determine the size of and .

OB is the radius and AC is a tangent to ⨀O at B

|  |  |  |
| --- | --- | --- |
| **RTC** | and |  |
| **Given** | ⨀O,  *ABC tangent, OR radius,*  *\* |  |
|  | **Calculation** | **Reason** |
| **Step 1** |  | tan  radius |
|  |  | tan  radius |
|  |  | (given) |
|  |  |  |

**Task 2**

**1.**



In the given diagram, ABC is a tangent to the circle O and BEO is the diameter of the circle O.

. Determine the values of and .

2.



In the given diagram, ABC is a tangent to the circle O and . Determine the sizes of and

#### Guided reflection on Activity 2

|  |
| --- |
| 1. Check step 1 for solution 1, and identify the diameter which subtends the right angle.   BE   1. In step 3 for solution 1, why did we say that ?   Tangent is always perpendicular to radius (diameter).   1. Reflect on the solution provided to question 2, and write down the sequence of steps that was used to calculate .   Calculate the size of using the angle at centre theorem.  Calculate the size of using the following geometrical results: the sum of sum of  s in ∆, OD = OB, radii of circle are equal, and ∠’s opposite equal sides  Calculate using the tan-radius axiom. |

**Answers to Task 2**

1.

|  |  |  |
| --- | --- | --- |
| **RTC** | and . |  |
| **Given** | ⨀O,  *ABC tangent, BOE is diameter,* |  |
|  | **Calculation** | **Reason** |
| **Step 1** |  | diameter subtends right angle  or  in semi-circle |
| **Step 2** |  | sum of s in ∆ |
| **Step 3** |  | tan  diameter or tan  radius |

EB is a diameter of ⨀O

2.

|  |  |  |
| --- | --- | --- |
| **RTC** | and |  |
| **Given** | ⨀O,  *ABC tangent,* |  |
|  | **Calculation** | **Reason** |
| **Step 1** |  | ∠at centre = 2 ×∠ at circumference |
| **Step 2** | +  +  But | sum of s in ∆  OD = OB, radii of circle. ∠’s opposite equal sides |
| **Step 3** |  | tan  radius |

is the angle at centre of ⨀O

# Activity 3 Investigation - two tangents from a common point

**Purpose:** To discover the relationship between lengths of the tangents to a circle drawn from a common point outside a given circle

**Resources:** A pen or pencil, eraser, ruler, compass, protractor and paper

**Suggested time:** 15 minutes

**Task 3**

In this task, you’ll learn how to construct two tangents to a circle from the same point outside the circle. Then you will compare the lengths of two tangents from the same point to the points of tangency.



1. Draw circle M.
2. Select a point outside circle M. Label this point N.
3. Draw MN.
4. Construct GH, the perpendicular bisector of MN.
5. Let O be the point of intersection of GH and MN.
6. Using OM as the radius, draw circle 0.
7. Let E and F be the points of intersection of circle O

and circle M.

1. Draw NE and NF. Then NE and NF are tangents to

circle M from point N.

1. Points E and F are points of tangency of tangents NE

and NF respectively to circle M.

1. Measure NE and NF [ or use your compass to

compare tangents NE and NF].

1. Write down a conjecture about tangents drawn from the same point outside the circle to respective the points of tangency.

**Conjecture 14**: Two tangents drawn to a circle from the same point outside the

circle are \_\_\_\_\_\_\_\_.

**Discussion**



We can use deductive reasoning to prove that two tangents drawn to a circle from the same point outside the circle are equal. However, we will not do this in unit, but will accept it as a theorem from now onwards:

**Theorem:** Two tangents drawn to a circle from the same point outside the circle are equal in length



*Given:* Tangents NA and NB drawn from point N to circle M.

*Conclusion:* NA =NB

*Reason***:** Tangents drawn from a common point (*Tans from common pt* ***OR*** *Tans from same pt*)

#### Guided reflection on Activity 3

|  |
| --- |
| 1. What did you discover when tangents to a circle are drawn from a common point?   Each of the tangents from the common point to their respective points of contact with the circle are equal. |

**Activity 4 Application of tangents drawn from common point to solve geometry problems**

* **Purpose:** To apply the tangent drawn from a common point theorem to do basic geometry calculations.
* To develop the skill of setting out a geometry calculation using geometrical statements supported by reasons
* To encourage students to use deductive reasoning when doing geometrical calculations.

**Resources:** A pen or pencil, eraser, ruler, compass, protractor and paper

**Suggested time:** 20 minutes

**Worked Example**



In the given diagram, ABC and ADE are tangents to the circle O and . Determine the sizes of and

|  |  |  |
| --- | --- | --- |
| **RTC** | and |  |
| **Given** | ⨀O,  *ABC and ADE tangents,* |  |
|  | **Calculation** | **Reason** |
| **Step 1** | AB = AD | Tangents from common point |
| **Step 2** |  | ∠’s opposite equal sides |
| **Step 2** | +  +  But | sum of s in ∆  proved in step 2 |
| **Step 3** |  | tan  radius |

OD is the radius and EDA is a tangent

**Task 4**

1.



In the diagram, BA and BC are tangents to circle E and BC and BD are tangents to circle F. Circles E and F touch at point C. If BA = 13cm, What is the length of BD

2.



In the diagram, , , *QR = 14cm* and . PQ, QR and PR are tangents to circle O at S, U and T respectively. Calculate the value of

#### Guided reflection on Activity 4

|  |
| --- |
| 1. When you look at the solution to question 1, what can you comment about the solution?   We applied deductive reasoning, and the theorem which states: Two tangents drawn to a  circle from the same point outside the circle are equal in length.   1. Reflect on the solution provided for Question 2, and write down which step you do not understand? |

**Answers to Task 4**

|  |  |  |
| --- | --- | --- |
| **RTC** |  |  |
| **Given** | BA and BC are tangents ⨀E,  BC and BD are tangents to ⨀F. BA = 13cm |  |
|  | **Calculation** | **Reason** |
| **Step 1** | BC = BA = 13cm | Tangents from common point |
| **Step 2** | But BD = BC | Tangents from common point |
| Step 3 | BD = 13 cm |  |



|  |  |  |
| --- | --- | --- |
| **RTC** |  |  |
| **Given** | PQ, QR and PR are tangents to circle O*.* , ,  *QR = 14cm* and |  |
|  | **Calculation** | **Reason** |
| **Step 1** |  | *QR = 14cm* and |
| **Step 2** |  | Tangents from common point |
| Step 3 |  |  |
| Step 4 | But PS = PT | Tangents from common point |
| Step 5 | RT = RU = | Tangents from common point |
| Step 6 | PT + TR = PR |  |

Do you know how to build this linear equation

**Activity 5 Investigation of the angle between tangent to a circle and the**

**chord**

**Purpose:** To discover the relationship between the angle between the tangent to a circle and the chord drawn from the point of contact.

**Resources:** A pen or pencil, eraser, ruler, compass, protractor and paper

**Suggested time:** 10 minutes



In the given Figure, MNP is a tangent to circle O at N and RN is a chord of circle O.

Chord RN of circle O divides the circle into a minor segment and major segment [Artist must shade major segment in Blue and minor segment in red].

When looking at chord RN we say:

1. is the angle between tangent MNP and chord RN, and
2. is the angle in the alternate segment with respect to

Now, you look at chord QN in circle O and tangent MNP to circle and write down:

1. The name of the angle between tangent MNP and chord QN.
2. The name of the angle in the alternate segment with respect to

**Task 5**

In this task, you will investigate the relationship that exists for an angle between the tangent to a circle and the chord and the angle subtended by the same chord in the alternate segment.



Use the given diagram, and do the following:

1. (a) Measure and write down the size of .
2. Measure , which is the angle in the alternate segment with respect to .
3. What do you observe about the size of and size of
4. (a) Measure and write down the size of
5. Measure , which is the angle in the alternate segment with respect to .
6. What do you observe about the size of and size of .
7. From your observations in 1 (c) and 2(c) complete the following conjecture statement:

Conjecture 15: The angle between the tangent to a circle and the chord drawn from the point of contact is \_\_\_\_\_to the angle in the alternate segment.

To develop the tan-chord conjecture you used inductive reasoning. We will now use deductive reasoning to show that the tan-chord conjecture is always true.

We will use the given diagram to help build the proof via deductive reasoning.



Before we finalize our three -column proof, let us explore what information we have and what it means to us.

**(a)Can you write down what information is given?**

Do you see we are given: Q, R, N are points on the circle with centre O. MNP is a tangent to t the circle at N.

(b) Write down what you are required to prove?

(c) Can you suggest a construction that was necessary?

Join O to N and O to R.

(d) Why is ON = OR?

They are radii of the same circle

(e) If you let , what do you think will be the size of . Give a reason for your answer.

.  at centre = 2 × at circumference.

1. How can you now calculate in terms of .
2. Consider . What do you know about the sum of the angles of .
3. Can you now explain with reasons why

But (ON =OR, radii; s opp equal sides)

But

1. What do you know about the angle between a radius and tangent to a circle?

It is a right angle (i.e.

Now let us see how we can set up our three-column proof using the information we gathered thus far:



|  |  |  |
| --- | --- | --- |
| **Given** | Q, R, N are points on the circle with centre O. MNP is a tangent to t the circle at N. | **Reason** |
| **Required**  **to prove** |  |  |
| **Construction** | Join O to N and O to R |  |
| **Proof** |  |  |
| **Step 1** | Let |  |
| **Step 2** |  |  at centre = 2 × at circumference. |
| **Step 3** |  | sum of s in ∆  ON = OR, radii of circle. ∠’s opposite equal sides |
| **Step 4** |  | tan  radius  from step 3 |
| **Step 5** | = Q |  |

#### Guided reflection on Activity 5

|  |
| --- |
| Look at the three column proof which has been developed, and write down the sequence of steps that was used to develop the proof for the tan-chord conjecture.  Why do you think we can now call the tan-chord conjecture a theorem?  We have proved that is always true using deductive reasoning. |

**Activity 6 Application of tangent-chord theorem to solve geometry problems**

**Purpose:**

* To apply the tangent drawn from a common point theorem to do basic geometry calculations.
* To develop the skill of setting out a geometry calculation using geometrical statements supported by reasons
* To encourage students to use deductive reasoning when doing geometrical calculations.

**Resources:** A pen or pencil, eraser, ruler, and paper

**Suggested time:** 30minutes

## Conjecture 12: *The angle between a tangent to a circle and a chord drawn from the point of contact is equal to an angle in the alternate segment* *of the circle* .(Tangent-chord conjecture).



**Given**: Circle with a tangent at and a chord subtending

**Conclusion**:

**Reason**: tan chord theorem

**Worked example:**



In the diagram, ABC is a tangent to the circle O at B., , and .

Calculate the sizes of , , and .

Can you identify the angle between the chord and the tangent and the angle in the alternate segment

|  |  |  |
| --- | --- | --- |
| **RTC** | , , and |  |
| **Given** | ABC is a tangent to the circle O at B., , and . |  |
|  | **Calculation** | **Reason** |
| Step 1 |  | tan chord theorem |
| Step 2 |  | sum of s in ∆ |
| Step 3 |  | tan chord theorem |
| Step 4 |  | opp s of cyclic quad |

**Task 6**



In the diagram, MNP is a tangent to the circle O at N. and

Calculate the sizes of and .



In the diagram, MNP is a tangent to the circle O at N. and

Calculate the sizes of and .



In the diagram EFG is a tangent to circle O. and. In .

Calculate the sizes of , ,, , and

#### Guided reflection on Activity 6

|  |
| --- |
| 1. Describe in your own words how you will identify the angle in the alternate segment when given an angle between a tangent and chord?   I will take my 2 fingers and run them from the endpoints of the chord until they subtend an angle in the circumference away from the given angle.   1. Refer to the solution provided to Question 3, and write down all the geometrical theorems we applied.   tan chord theorem; opp s of cyclic quad and sum of s in ∆ |

**Answers to Task 6**

|  |  |  |
| --- | --- | --- |
| **RTC** | and |  |
| **Given** | MNP is a tangent to the circle O at N. |  |
|  | **Calculation** | **Reason** |
| Step 1 |  | tan chord theorem |
| Step 2 |  | tan chord theorem |

|  |  |  |
| --- | --- | --- |
| **RTC** | and . |  |
| **Given** | MNP is a tangent to the circle O at N. and |  |
|  | **Calculation** | **Reason** |
| Step 1 |  | tan chord theorem |
| Step 2 |  | tan chord theorem |

|  |  |  |
| --- | --- | --- |
| **RTC** | , ,, , |  |
| **Given** | EFG is a tangent to circle O. and. In |  |
|  | **Calculation** | **Reason** |
| Step 1 |  | tan chord theorem |
| Step 2 |  | tan chord theorem |
| Step 3 |  | opp s of cyclic quad |
| Step 4 |  | sum of s in ∆;  s opp equal sides (JI=JF) |
| Step 4 |  | tan chord theorem |

Note: The above example illustrates the use of deductive logic, which is also referred to as deductive reasoning.

**Activity 7 Proving that a line is a tangent to a circle**

**Purpose:**

* To develop the skill of setting out a geometry proofs using geometrical statements supported by reasons, when proving that a line is a tangent to a circle
* To encourage students to use deductive reasoning when doing geometrical proofs.

**Resources:** A pen or pencil, eraser, ruler, and paper

**Suggested time:** 30minutes

*We can summarise our earlier results as ways to prove that a line is a tangent to a circle.*

**Ways of proving that a line to a circle is a tangent**

**1.**

****

If line ABC through a point (like B) on the circumference is perpendicular to the radius (like OB), then the line ABC is a tangent to the circle O.

**Abbreviated reason**: line radius

****

If the angle (like between the line (like MNP) and a chord (in this instance NR) is equal to the angle (like ) in the alternate segment, then the line (MNP) is a tangent to the circle at N.

**Abbreviated reason**: converse tan chord theorem OR  between line and chord

**Worked example**

**1.**

****

In the diagram, QR bisects and . Prove that MNP is a tangent to circle O.

**Proof**

QR bisects What does mean?

⇒

|  |  |  |
| --- | --- | --- |
| **RTP** | MNP is a tangent to circle O. | **Reasons** |
| **Given** | QR bisects and |  |
| **Proof** |  |  |
| Step 1 |  | QR bisects |
| Step 2 | But | given |
| Step 3 |  |  |
| Step 4 | MNP is a tangent | converse tan chord theorem **OR**  between line and chord |

**Task 7 Proving Lines are tangents**

****

In the given diagram, S,T and R lie on the circumference of circle O. SR = ST, and

. Prove that QRP is a tangent to circle O at T.



In the given diagram, R,Q and N lie on the circumference of circle O. MNP is a straight line.

, , and . Prove that MNP is a tangent to circle O at N.





In the diagram, O is the centre of the circle, TS//UR. Prove that RS is a tangent to the circle passing through points Q, U and R.

#### Guided reflection on Activity 7

|  |
| --- |
| 1. Reflect on the solution to question 1, and write down the key steps that was followed in proving that QRP is a tangent. 2. Reflect on the solution to question 3, and write down the key steps that was followed in proving that RS is a tangent. |

**Answers to Task 7**

**TASK – Proving Lines are tangents**

|  |  |  |
| --- | --- | --- |
| **RTP** | QRP is a tangent to circle O. | **Reasons** |
| **Given** | ⨀O, SR = ST, and |  |
| **Proof** |  |  |
| Step 1 | SR = RT | given |
| *Step 2* |  | s opp equal sides |
| Step 3 |  | sum of s in ∆ |
| Step 4 | but | Given |
|  |  |  |
|  | QRP is a tangent | converse tan chord theorem **OR**  between line and chord |

Do you know why we must show that

|  |  |  |
| --- | --- | --- |
| **RTP** | MNP is a tangent to circle O. | **Reasons** |
| **Given** | ⨀O, SR = ST, and |  |
| **Proof** |  |  |
| Step 1 |  | s on a str line |
| *Step 2* |  |  |
| Step 3 |  | Given  from step 2 |
| Step 4 | but | Given |
|  |  |  |
|  | MNP is a tangent | converse tan chord theorem **OR**  between line and chord |

3.

|  |  |  |
| --- | --- | --- |
| **RTP** | RS is a tangent to circle QUR | **Reasons** |
| **Given** | ⨀O, S TS//UR |  |
| **Proof** |  |  |
| Step 1 | Let |  |
| *Step 2* |  | opp s of cyclic quad |
| Step 3 |  | Co-int.s, TS//UR |
| Step 4 |  |  |
|  | RS is a tangent is a tangent | converse tan chord theorem **OR**  between line and chord |

#### Guided reflection on Summary Assessment

|  |
| --- |
| 1. Use the answers given for the summary assessment to check if your answers were correct. 2. Did you realize why you got some your answers incorrect? 3. Have you been able to correct errors you made? |