Section 2: Mechanics

Sub-topic 1: Vectors

Unit 1: Introduction to vectors and scalars

Learning Outcomes

By the end of the unit, you should be able to:

- state what is meant by scalar and vector quantities, and give examples of each;
- add vectors that are co-linear (in 1-dimension) using a graphical method (head-to-tail) as well as by calculation.

Introduction

In science it is sometimes important to give a direction for some measurements or quantities. Reflect on the following questions, either on your own or with a fellow student:

- Can you think of some measurements or quantities from your everyday life where a direction is important?
- Can you think of some measurements or quantities where a direction is NOT important?

In Physics there are different types of physical quantities that you can measure. A *scalar* is a physical quantity that has a magnitude (size), but not a direction. An example of a scalar quantity is temperature, because it does not have a direction. A *vector* is a physical quantity that has a magnitude (size) and a direction. An example of a vector quantity is force, because it has a clear direction. It is important to know the direction in which a force is being directed to be able to work out its impact.

When you write a vector quantity you need to write it with the magnitude and the direction. In this unit you will also learn about adding vectors in 1 dimension, that is, vectors that are in a straight line. In the following unit you will extend your knowledge to adding vectors in 2 dimensions.

Activity 1: Identify vectors and scalars

Purpose

In this activity, you will try to identify physical quantities as either scalars or vectors.

Suggested time: [10 minutes]

What you will do:

Can you work out which of the following quantities are scalars or vectors? Give a reason for your answer.

a. Temperature
b. Force
c. Mass
d. Density
e. Volume
f. Weight
g. Heat

Guided reflection

The quantities are categorised as scalars or vectors below:

a. Temperature = scalar, since temperature does not need a direction, it is just a measurement with a value.
b. Force = vector, since it is important to know the direction in which a force is being directed to be able to work out its impact
c. Mass = scalar, since mass is a measurement of the quantity of matter in an object, which does not have a direction.
d. Density = scalar, since it is a measure of the mass per unit volume of an object, it does not have a direction.
e. Volume = scalar, since volume is a measurement of the space occupied by an object, it does not have a direction.
f. Weight = vector, since it is a measure of the force of attraction that pulls an object toward the earth (gravitational force), so it has a direction which is towards the earth

(downward). Mass and weight are often confused with one another. You will learn more about the difference between these later in this topic of Mechanics.

g. Heat = scalar, since it is a form of energy, which does not have a direction.

The following website gives some helpful explanations of scalars and vectors if you need more information: https://www.physicsclassroom.com/class/1DKin/Lesson-1/Scalars-and-Vectors

[Wordbox: MAIN IDEA:

- A *scalar* is a physical quantity that has a magnitude but not a direction.
- A *vector* is a physical quantity that has a magnitude and a direction.]

Representing vectors

You represent vectors using arrows to show that they have a direction. The length of the arrow shows the magnitude of the vector. You use a scale to explain what the length means (see the example below). You show the direction of the vector by the direction that the arrow is pointing in. For example, the vector diagram of the force F_1 = 50 N right is shown below:

Scale: 1 cm = 10 N

Figure 1 F_1 = 50 N right

To draw a vector of **50 N right** using the scale, draw a line using your ruler that has a length of 5 cm. Since 1 cm represents 10 N, then a 5 cm line represents 50 N. Now place an arrow head on the right of the line to show the direction of the vector.

A negative vector has the same magnitude as the original vector, but it has the opposite direction. For example, the vector diagram of the force $-F_1$ is shown below.

Scale: 1 cm = 10 N

Figure 2 $-F_1$ = 50 N left

Activity 2: Representing vectors

Purpose

In this activity you will practice representing vectors that have different magnitudes and directions.

Suggested time: [20 minutes]

What you will do:

Draw the following vectors. Don't forget to show the scale that you use for each one.

a) A force F_A of 600 N to the left.

b) A force F_B of 300 N to the right.

c) Force $-F_A$.

d) Force $-2F_B$.

Solutions

The vector diagrams are shown below.

You will use this scale for all of the diagrams: Scale: 1 cm = 100 N

a)

Figure 3 F_A = 600 N left

Draw a line using your ruler that has a length of 6 cm. Since 1 cm represents 100 N, then a 6 cm line represents 600 N. Now place an arrow head on the left of the line to show the direction of the vector.

b)

Figure 4 F_B = 300 N right

For this vector you draw a line that has a length of 3 cm, and place an arrow head on the right of the line.

c) The vector $-F_A$ has the same length as vector F_A but has the opposite direction.

Therefore $-F_A$ = 600 N to the right.

Figure 5 $-F_A$ = 600 N right

d) The vector $-2F_B$ has the double the length of vector F_B but has the opposite direction.

Therefore $-2F_{AB}$ = 600 N to the left.

Figure 6 $-2F_B$ = 600 N left

[WORDBOX] **MAIN IDEA:** A vector can be represented by an *arrow*, where:

- the *length* of the arrow shows the magnitude
- the *direction* of the arrow shows the direction

A *negative* vector has the same magnitude as the original vector, with opposite direction.]

Adding vectors in 1 dimension

When you draw a vector, you call the end that has an arrow the "head" of the vector, and the other end is called the "tail".

Figure 7 A vector with the tail and head labelled

When adding vectors in 1 dimension (in a straight line), you can then find the resultant vector by placing the tail of one of the vectors next to the head of the other vector. This is called the *head-to-tail method*. The resultant vector is the final vector that you draw from the tail of the first vector to the head of the last vector.

Activity 3: Adding vectors in 1-dimension

Purpose

In this activity you will study the steps in an example problem to see how to add vectors in 1-dimension (in a straight line), and then you will do some problems on your own.

Suggested time: [20 minutes]

What you will do:

Look carefully through the steps in the following example problem:

<u>Example:</u>

A truck pulls a trailer with a force of 5 000 N to the right. There is a force of friction of 1 000 N to the left on the trailer. What is the resultant force on the trailer?

<u>Solution</u>:

You can draw a vector diagram of the forces, showing their magnitude and direction.

Scale: let 1 cm = 1 000 N

You draw the vectors by placing the tail of one of the vectors next to the head of the other vector (the head-to-tail method):

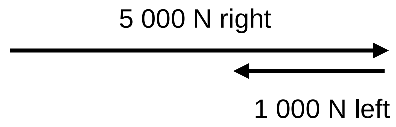


Figure 8 Vectors added head to tail

You can now find the resultant force:

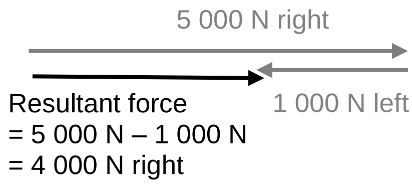


Figure 9 Resultant of vectors added head to tail

The resultant force is therefore 4 000 N to the right.

Now answer these questions about the following force vectors:

F_A = 90 N to the left F_B = 30 N to the right F_C = 60 N to the left

Use vector diagrams to find the resultant vectors:

a. $F_A + F_B$

b. $F_A - 2F_C$

Solutions

The vector diagrams for finding the resultant vectors are shown below.

1.

a) Scale: 1 cm = 10 N

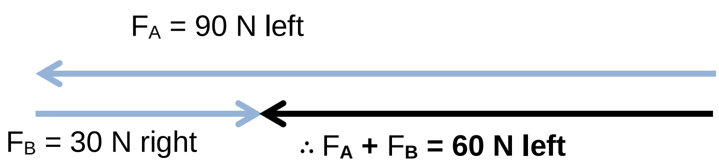


Figure 10 Resultant of vectors $F_A + F_B$

b) Scale: 0,5 cm = 10 N

$-2F_C$ = 120 N right

Figure 11 Resultant of vectors F_A - $2F_C$

This short 11 minute YouTube video explains very clearly how to add vectors in 1 dimension: *Vectors in one dimension*: https://www.youtube.com/watch?v=8Q_w9IICpC8 (Duration: 11.22)

[WORDBOX]: MAIN IDEA: When adding vectors in 1 dimension you use the *head-to-tail method:*

- place the tail (end) of one of the vectors next to the head (start) of the other vector;
- the resultant vector is the final vector that you draw from the tail of the first vector to the head of the last vector.]

Unit 2: Vectors in 2-dimensions

Learning Outcomes

By the end of the unit, students should be able to:

- add two vectors that are at right angles to determine the resultant using a graphical method (head-to-tail or tail-to-tail) as well as by calculation;
- determine the x- and y-components of a vector on the Cartesian plane.

Introduction

In Unit 1 you learnt about scalars and vectors, and you learnt how to add vectors that are in 1 dimension (in a straight line). Reflect on the following questions, either on your own or with a fellow student:

- Why do you say that a force is a vector quantity?
- Is your age a scalar or a vector quantity?

In the previous unit you learnt that a scalar quantity only has a magnitude, whereas a vector quantity has a magnitude and a direction. Your age is a scalar quantity because it only has a magnitude, for example 35 years, and no direction. A force is a vector quantity because it has both a magnitude, for example 100 N, and a direction in which the force is pushing, for example downward.

In the previous unit you also learnt about adding vectors in 1 dimension (in a straight line), by placing the tail of one vector next to the head of another vector, and finding the resultant. In this unit you will extend your knowledge to adding vectors in 2 dimensions (on a flat plane, or at some angle to each other).

Mathematical tools needed for 2 dimensional vectors

In a right-angled triangle, if you know the lengths of any two sides of the triangle, you can find the length of the remaining side using Pythagoras' theorem:

$$\text{hypotenuse}^2 = x^2 + y^2$$

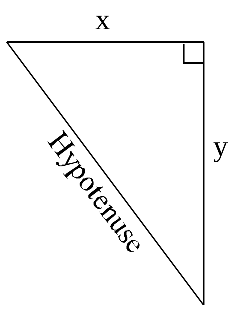


Figure 12 Right-angled triangle

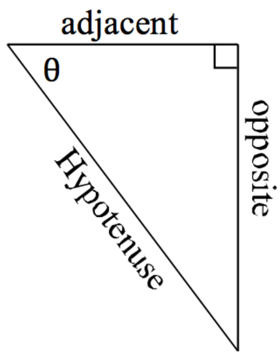


Figure 13 Right-angled triangle with sides labelled

Given a right-angled triangle with one known side and a known angle θ, you can calculate the length of any of the other side using the following trigonometric identities:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

You can also use these identities to calculate an unknown angle if you know the length of any two sides of the triangle.

Frame of reference for vectors in 2 dimensions

When you work with vectors in 2 dimensions, you need to set a frame of reference, which includes a zero point and a positive direction for each of the dimensions that you are looking at. In other words, your frame of reference will look like a small coordinate system, showing the zero point for each axis at the origin, and showing the positive x and y directions. This is called the Cartesian plane. The diagram below shows what the frame of reference would look like in 2 dimensions.

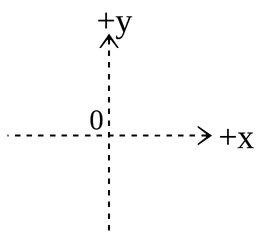


Figure 14 Frame of reference in 2 dimensions using the Cartesian plane

The compass directions of North, East, South and West can be expressed in terms of the Cartesian plane. You can choose to let the +y direction correspond to North, and then the East direction will be the +x direction, as the diagram below shows.

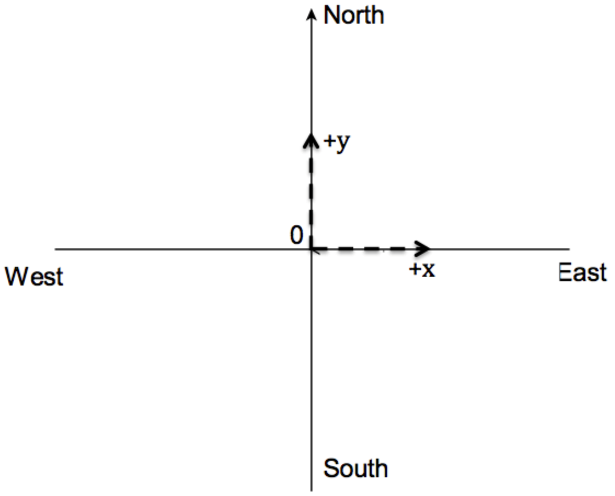


Figure 15 Frame of reference in 2 dimensions using compass directions

Activity 1: Write directions of vectors in 2 dimensions

Purpose

In this activity you will look at an example of how to write the direction for a vector in 2 dimensions, and then you will work out the directions of some other vectors.

Suggested time: [15 minutes]

What you will do:

Study the example below of how to determine the direction of a vector using the Cartesian plane.

Example:

Describe the direction of vector **A** shown on the right on the Cartesian plane.

Solution:

The direction of **A** is 30^o below the +x axis.

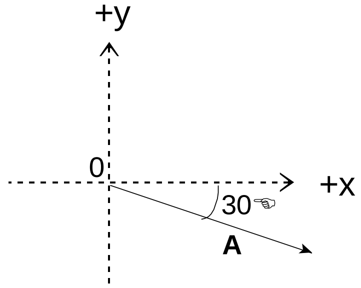


Figure 16 Direction of vector A on the Cartesian plane

(To work this out you think about how an arrow that is fixed at the origin would move from one of the x or y axes to get to the position of the vector. In this example it would need to move downward from the +x axis, to you describe it as being “below the +x axis”.)

Now try to write the directions for the following vectors:

a.

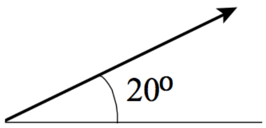


Figure 17 Vector a.

b.

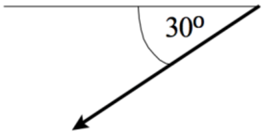


Figure 18 Vector b.

c.

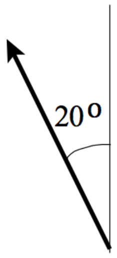


Figure 19 Vector c.

Solutions

a. If you draw the x and y axes onto the vector diagram, it would look like this:

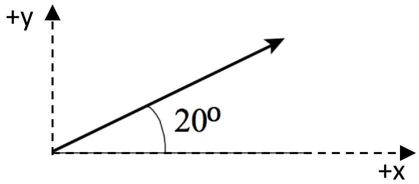


Figure 20 Vector a on the Cartesian plane

Since you would move the arrow upwards from the +x axis you describe this direction as 20^o above the +x axis

b. If you draw the x and y axes onto the vector diagram, it would look like this:

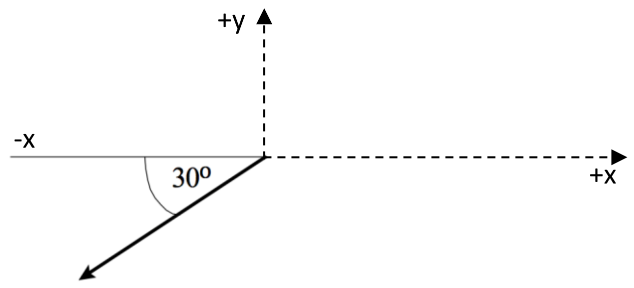


Figure 21 Vector b on the Cartesian plane

Since you would move the arrow downwards from the -x axis you describe this direction as 30^o below the -x axis

c. If you draw the x and y axes onto the vector diagram, it would look like this:

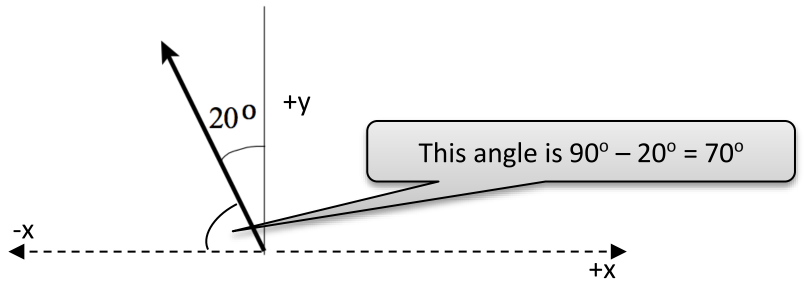


Figure 22 Vector c on the cartesian plane

You could either describe the angle in terms of the -x axis, as 70^o above the -x axis , or you could describe it in terms of the +y axis, as 20^o left of the +y axis.

The resultant of two perpendicular vectors using the head-to-tail method

When you add two vectors that are perpendicular to each other, you can sketch them on the Cartesian plane, and then use Pythagoras' theorem to find the resultant vector. One method that you can use the *head-to-tail* method, where you place the tail (end) of one of the vectors next to the head (start) of the other vector. The resultant vector is the arrow from the tail of the first vector to the head of the second vector.

If you construct the diagram carefully, you can measure the resultant vector from your diagram using a ruler and a protractor. You can also use trigonometry to calculate the magnitude and direction of the resultant vector.

Activity 2: Add vectors in 2 dimensions using the head-to-tail method

Purpose

In this activity you will study an example calculation of finding the resultant of two vectors that are at right angles to one another, and then you will use a construction diagram and a simulation activity to confirm the answer.

Suggested time: [30 minutes]

What you will do:

Look carefully through the following example where two vectors that are at right angles to each other are added to find a resultant vector.

Example:

Renata walked 300 m toward the East, then 400 m toward the North. What was her resultant displacement?

Solution:

Frame of reference: Let the +y direction be North.

You can then draw a head-to-tail vector diagram of Renata's movements R_x and R_y. This is shown on the right. (The frame of reference is shown on the diagram). Her resultant displacement is therefore:

$$R = \sqrt{R_x^2 + R_y^2}$$

$$= \sqrt{300^2 + 400^2}$$

= 500 m

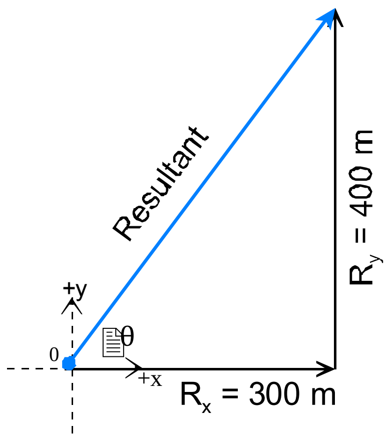


Figure 23 Vector diagram of Renata's movements

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$= \frac{R_y}{R_x}$$

$$= \frac{400\ \text{m}}{300\ \text{m}}$$

Therefore $\theta = 53{,}1^\circ$

Renata's resultant displacement is therefore 500 m in a direction of $53{,}1^\circ$ above the +x-axis.

Now find the resultant for the above problem using an *accurate vector diagram* by following these steps:

1. Do your own construction diagram of this vector sum using a scale of 1 cm = 100 m. Try to be as accurate as possible.
2. Draw in the resultant vector.
3. Measure the magnitude and direction of the resultant vector. In other words, use your ruler to measure the length of the resultant, and use your scale to work out the displacement. Use your protractor to measure the angle θ.
4. How do your values it compare to these calculated values?

You can use an internet simulation to confirm your answer by following these steps:

1. Go to the internet address: https://www.physicsclassroom.com/Physics-Interactives/Vectors-and-Projectiles/Vector-Addition/Vector-Addition-Interactive
2. You will see a screen like the one below with the simulation.

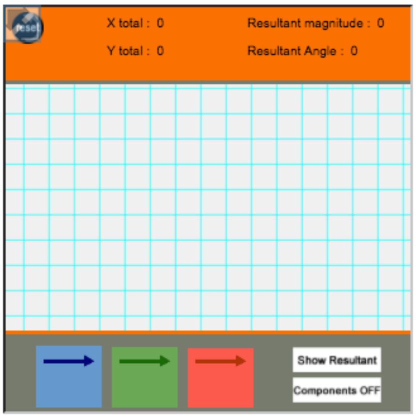


Figure 24 Screen for vector addition simulation

3. At the bottom of the of the screen you can see arrows with three different colours. Use your mouse pointer to click on one of the arrows and move it to the working area in the middle of the screen.
4. You can change the *length and direction* of a vector arrow by clicking on the *head* (front end) of the vector and moving it around before letting go of your mouse button.
5. You can change the *position* of a vector arrow by clicking on the *tail* (back end) of the vector and moving it around before letting go of your mouse button.
6. Place two vectors head-to-tail so that they represent Renata's displacement in the above example. The blocks will guide you about the length of the vectors.
7. What is the scale that is needed so that these vectors represent the displacement from the example above?
8. Click the button that says "Show Resultant" and move the resultant arrow so that it connects the tail of one vector with the head of the other vector.
9. How does the magnitude and direction of this resultant compare with those in the calculation above?

Guided reflection

The magnitude and direction that you found for the resultant vector in your construction diagram should have been similar to the calculated values. You should allow for some differences, since there are always small inaccuracies with construction diagrams.

In this simulation you should find the following:

- The scale that is needed is 1 block = 10 m
- You therefore need a vector in the +y direction that has a length of 4 blocks, and another vector in the +x direction that has a length of 3 blocks.
- The resultant vector is shown to have a length of 5 blocks, at an angle of $53,2^{o}$. Here the direction of the resultant is given from the +x axis.
- If you use the scale, this means that the resultant displacement is 500 m at an angle of $53,2^{o}$ above the +x-axis. (This is $0,1^{o}$ different to your calculation, which is close enough).
- Note that it doesn't matter which vector you draw first, as long as each one has the correct magnitude and direction. *Both* of the following ways of representing the vectors is correct:

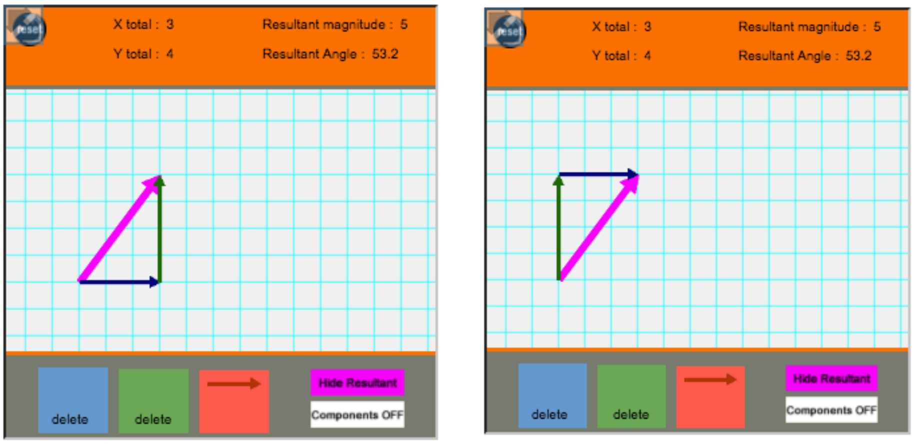


Figure 25 Two ways of representing the same resultant vector

Activity 3: Apply your knowledge of adding vectors using the head-to-tail method

Purpose

In this activity you will apply what you have learnt about adding vectors in 2 dimensions to solve a problem on your own using three different methods.

Suggested time: [30 minutes]

What you will do:

You are given the following problem:

> A jogger travels 6 km toward the South. He then jogs 5 km toward the East.
>
> What is his resultant displacement?

Apply what you have learned to try to solve this problem using the following three methods:

1. An accurate construction diagram using a scale
2. A calculation using trigonometry
3. The online simulation, if you have access to the internet (https://www.physicsclassroom.com/Physics-Interactives/Vectors-and-Projectiles/Vector-Addition/Vector-Addition-Interactive).

Solutions

1. **Construction method:**

Frame of reference: Let the +y direction be North. The best scale to use is: 1 cm = 1 km.

You can then draw a head-to-tail vector diagram of the jogger's movements R_x and R_y.

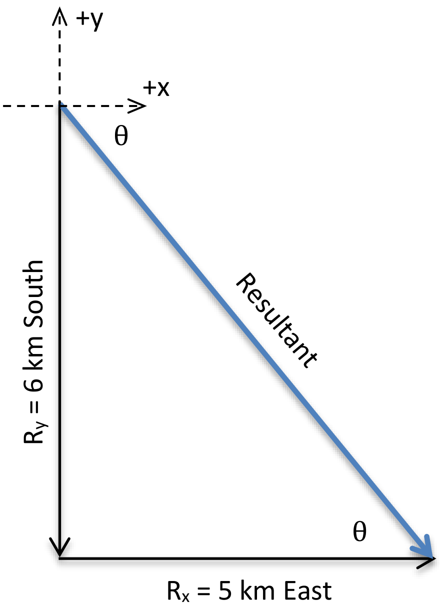


Figure 26 Head-to-tail vector diagram of the jogger's movements

If you measure the length of the resultant on your vector diagram you should find that it has a length of approximately **7,8 cm**. The angle that you measure with your protractor should be approximately **50º**. (Note that you will position your protractor so that the zero line of the protractor lines up with the +x axis, and then you will measure the angle from below this line, as the diagram below shows.)

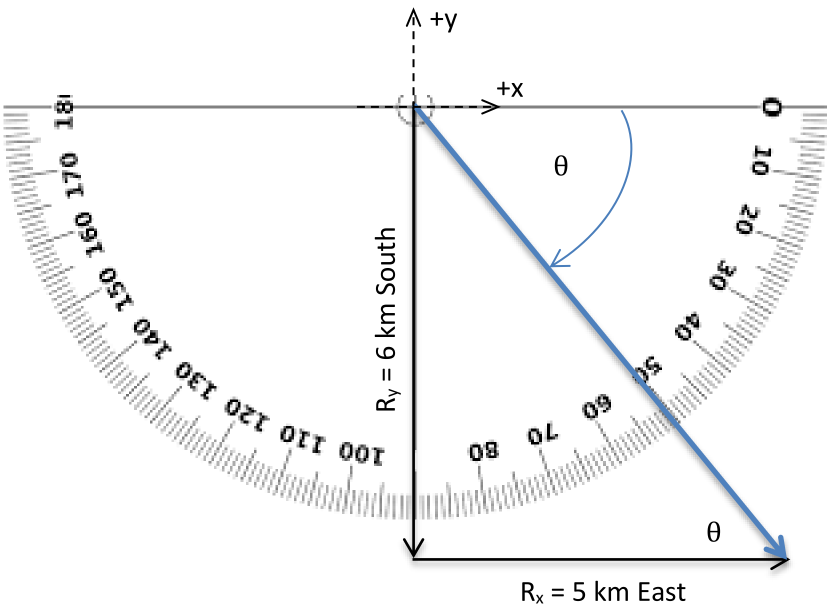


Figure 27 How to measure the angle using a protractor

2. **Calculation method:**

The resultant displacement can be calculated using Pythagoras' theorem:

$$R = \sqrt{R_x^2 + R_y^2}$$

$$= \sqrt{(5\text{ km})^2 + (6\text{ km})^2}$$

= 7,81 km

The angle θ can be calculated using trigonometry:

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$= \frac{R_y}{R_x}$$

$$= \frac{6\text{ km}}{5\text{ km}}$$

Therefore $\theta = 50{,}2^{\circ}$

The jogger's resultant displacement is therefore 7,81 km in a direction of 50,2° below the +x axis (or South of East).

3. **Internet simulation:**

You can construct this vector sum in the following two ways:

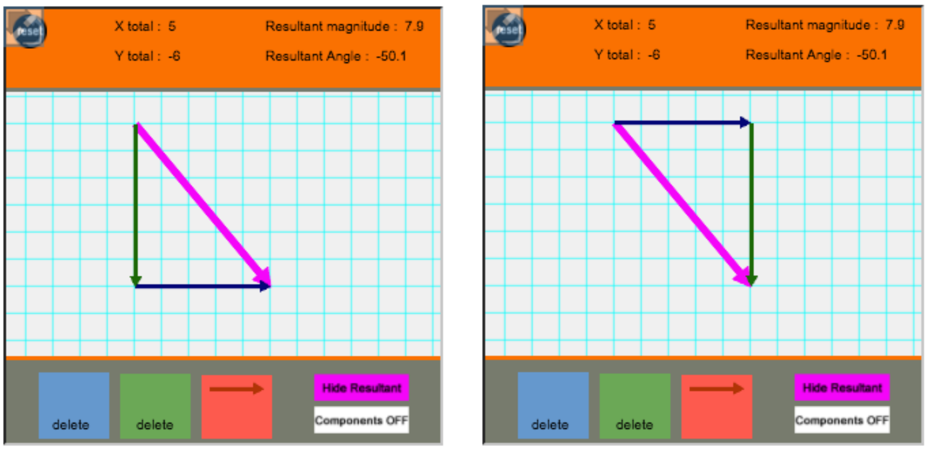


Figure 28 The resultant displacement using the internet simulation

You can see that the answer is similar to the calculated value, with a magnitude of 7,9 km and a direction of 50,1° below the +x axis. (There is a small difference with your calculated answer of 7,81 km in a direction of 50,2° below the +x axis).

The resultant of two perpendicular vectors using the tail-to-tail method

You can also use a *tail-to-tail* method to find the resultant vector. Here you place both of the tails of the vectors together at the origin of the Cartesian plane, and you then *complete a parallelogram* to find the resultant.

Activity 4: Add vectors using the tail-to-tail method

Purpose

In this activity you will be guided through an example of a vector addition using this method.

Suggested time: [20 minutes]

What you will do:

Example:

Two ropes are attached to an object. One rope pulls with a force of 30 N in the -x direction, and the second rope pulls with a force of 50 N in the +y direction. What is the resultant force on the object?

Solution:

The diagram on the right shows the forces on the object, connected tail-to-tail. (The frame of reference is shown on the diagram)

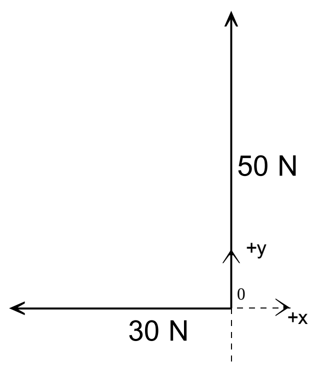


Figure 29 Tail-to-tail vector diagram of forces

In the diagram below the vectors have been redrawn to form a parallelogram. You can use this parallelogram to find the resultant vector.

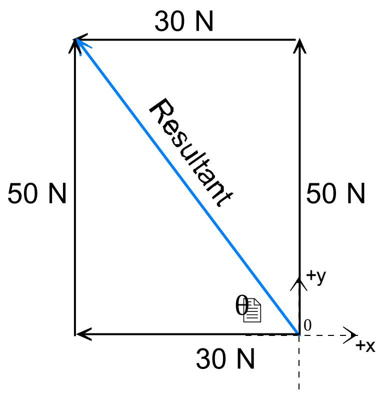


Figure 30 Parallelogram vector diagram of the forces

The resultant is the diagonal of the parallelogram from the position where the two tails meet to the position where the two heads meet. This is shown on the diagram.

Complete the calculation below for this vector diagram:

Use Pythagoras' theorem to calculate the magnitude of the resultant.

Resultant = $\sqrt{\quad + \quad}$

=

Use trigonometry to find the angle θ:

tan θ = $\frac{\quad}{\quad}$

Therefore θ =

The resultant force on the object is therefore _________ at an angle of _______ above the –x axis.

Guided reflection

When you fill in the missing steps in the above calculation you should find:

Magnitude of resultant = $\sqrt{(30N)^2 + (50N)^2}$ = 58,3 N

Using trigonometry:

$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

$= \frac{50\text{ N}}{30\text{ N}}$

Therefore $\theta =$ 59°

The resultant force on the object is therefore 58,3 N at an angle of 59° above the –x axis.

[WORDBOX: MAIN IDEA: To find the resultant of two vectors using the tail-to-tail method:

- The vectors are joined tail-to-tail. A parallelogram is then formed using these vectors.
- The resultant vector is the arrow from the position where the tails join towards the opposite corner of the parallelogram.]

Finding vector components

If you are given the magnitude and direction of a vector relative to the Cartesian plane, this vector has a part in the x-direction, and a part in the y-direction. These are called the x-component and the y-component of the vector. You can use trigonometry to work out the x- and y-components of any vector that is in the Cartesian plane. You will be guided through this process in the following activity.

Activity 5: Work out the components of a vector

Purpose

In this activity you will be guided through a way of working out the x- and y-components of a vector.

Suggested time: [15 minutes]

What you will do:

Look through the example below that shows the method for finding the components of a vector.

Example:

A vector A has a magnitude of 80 N, and a direction of 30° above the -x axis.

Find the x- and y-components of this vector.

Solution:

The diagram on the right shows the vector and its components on the Cartesian plane.

Using trigonometry, you know that

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$ and $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

Therefore $\sin 30° = \frac{A_y}{A}$

So A_y = + A sin 30° = + 80 N sin 30° = + 40 N

Similarly, A_x = - A cos 30° = - 80 N cos 30° = - 69,3 N

(This component has a negative sign since it is in the –x direction.)

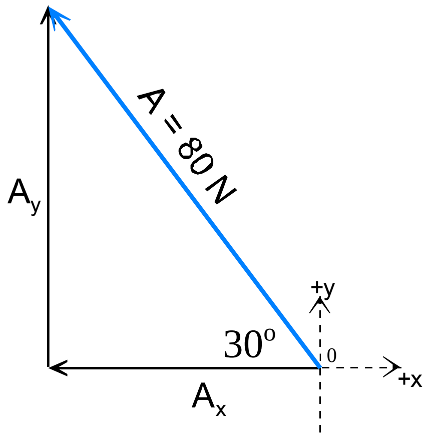


Figure 31 Vector components of vector A

Now try to apply this method to find the components of the following vectors yourself.

a. Vector B has a magnitude of 40 N and a direction of 45° above the +x axis.

b. Vector C has a magnitude of 65 N and a direction of 30° above the –x axis.

Solutions

a. The diagram of vector B is shown on the right.

B_y = + B sin 45° = + 40 N sin 45° = + 28,28 N

B_x = + B cos 45° = + 40 N cos 45° = + 28,28 N

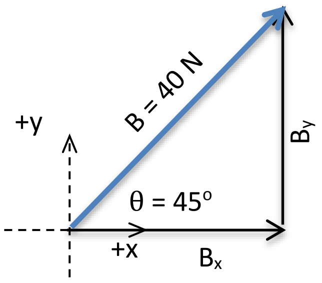


Figure 32 Components of vector B

b. The diagram of vector C is shown on the right.

C_y = + C sin 30° = + 65 N sin 30° = + 32,5 N

C_x = - C cos 30° = - 65 N cos 30° = - 56,29 N

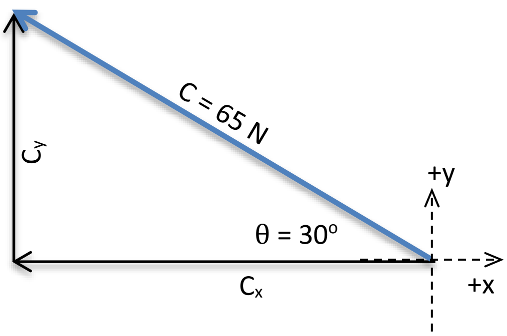


Figure 33 Components of vector C

Activity 6: Add vectors using components

Purpose

Once you have found the components of a set of vectors, you can add the vectors by adding their components. In this activity you will be guided through the steps for adding vectors using components.

Suggested time: [15 minutes]

What you will do:

In Activity 5 you found the x- and y-components of vectors B and C. Find the sum of these two vectors by adding their components in the following way:

1. Add together all of the components that are parallel to the x-direction to find the resultant component in the x-direction (R_x). Write your calculations in the space below:

   R_x =

2. add together all of the components that are parallel to the y-direction to find the resultant component in the y-direction (R_y). Write your calculations in the space below:

   R_y =

3. Find the resultant of R_x and R_y using Pythagoras and trigonometry.

Solutions

1. The sum of all of the components that are parallel to the x-direction are:

$R_x = B_x + C_x$ = 28,28 N – 56,29 N = -31,01 N

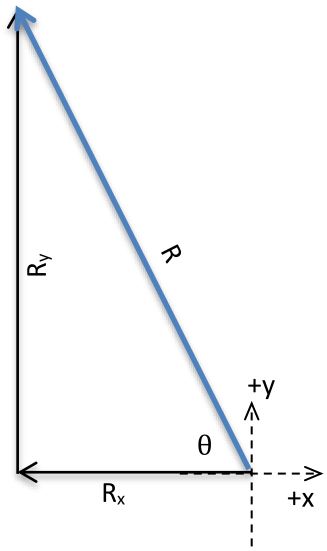


Figure 34 Resultant of all x- and y-components

2. The sum of all of the components that are parallel to the y-direction are:

   $R_y = B_y + C_y$ = 28,28 N + 32,5 N = + 60,78 N

3. Therefore the overall resultant of vectors B + C is:

$$R = \sqrt{R_x^2 + R_y^2}$$

$$= \sqrt{(31{,}01\ \text{N})^2 + (60{,}78\ \text{N})^2}$$

= 68,23 N

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{R_y}{R_x} = \frac{60{,}78\ \text{N}}{31{,}01\ \text{N}}$$

Therefore $\theta = 65{,}6^\circ$

The resultant of vectors B + C is therefore 68,23 N in a direction of 65,6° above the –x axis.

More resources to help you:

For more information on adding vectors in 2 dimensions, visit the website:

https://www.physicsclassroom.com/class/vectors/Lesson-1/Vector-Addition

Activity 7: Consolidate your learning of vectors

Purpose

In this activity you will consolidate your learning of vectors by answering the questions below and then assessing your own understanding using the solutions provided. Give yourself a mark out of the total of 30 marks, which will give you an idea of how well you understand this section of the work.

Suggested time: [60 minutes]

1. What is the difference between a scalar and a vector? (2)

2. List two examples of a scalar, and two examples of a vector quantity. (4)

3. Nonhle walked 300 m in the +x direction, and then 600 m in the +y direction.
   a. Draw an accurate vector diagram using the head-to-tail method to find the magnitude and direction of her displacement. (4)
   b. Find the magnitude and direction of her resultant displacement using calculations and compare your answers to (a). (4)

4. Force A has a magnitude of 48 N and points towards the West. Force B has the same magnitude, but points towards the South. Use the tail-to-tail method to determine the magnitude and direction of A + B. Use a vector diagram to show the direction of the resultant vector, but this does not have to be an accurate construction diagram. (6)

5.
   a. Find the x- and y-components of the following vectors: (4)
      i. Vector A = 30 N at a direction of 15^{o} above the +x axis.
      ii. Vector B = 40 N at a direction of 25^{o} below the –x axis.
   b. Find the resultant of vectors A and B, using a diagram to show how you reach your answer. (6)

Solutions

1. A scalar only has a magnitude, while a vector has a magnitude and a direction. (2)

2. Examples of a scalar: mass, temperature, heat, time (1 mark each for any 2)
   Examples of a vector: displacement, velocity, force, weight (1 mark each for any 2) (4)

3. (4)

a. The head-to-tail vector diagram of Nonhle's movements R_x and R_y is shown on the right.

The best scale to use is 1 cm = 100 m

(1 mark for correct R_x arrow, 1 mark for correct R_y arrow)

The resultant is measured to be about 6,7 cm, which means a displacement of 670 m using the scale. (1 mark)

Measuring the angle from the +x axis, $\theta = 63^{\circ}$ (1 mark)

The displacement is therefore 670 m at an angle of 63° above the +x axis.

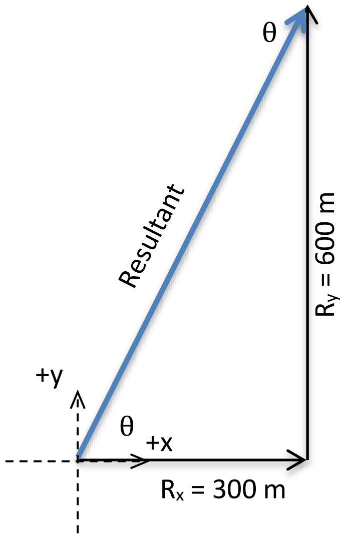


Figure 35 Head-to-tail vector diagram with resultant

b. Using a calculation her resultant displacement is:

$$R = \sqrt{R_x^2 + R_y^2}$$

$$= \sqrt{(300\ \text{m})^2 + (600\ \text{m})^2}$$

= 670,8 m

(1 mark for substitution, 1 mark for correct answer)

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{R_y}{R_x} = \frac{600\ \text{m}}{300\ \text{m}}$$

Therefore $\theta = 63{,}4^{\circ}$ (1 mark for substitution, 1 mark for correct answer)

Nonhle's resultant displacement is therefore 670,8 m in a direction of 63,4° above the +x axis (4)

4. (6)

<u>Frame of reference</u>: Let the +y direction be North.

You can draw a tail-to-tail vector diagram of the forces, and then complete the parallelogram. This is shown on the right. (3 marks for correct diagram)

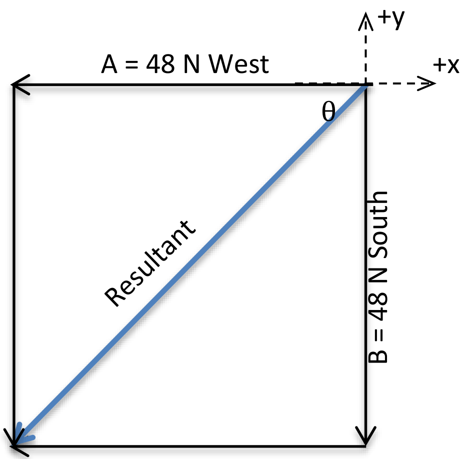


Figure 36 Parallelogram vector diagram with resultant

The resultant force is therefore:

$$R = \sqrt{R_x^2 + R_y^2}$$

$$= \sqrt{(48\ N)^2 + (48\ N)^2}$$

= 67,9 N

(1 mark for substitution, 1 mark for correct answer)

$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{B}{A} = \frac{48\ N}{48\ N}$

Therefore $\theta = 45°$ (1)

The resultant force is therefore 67,9 N in a direction of 45° below the –x axis (or South of West)

5. (4)

a. A diagram representing vector A is shown on the right.

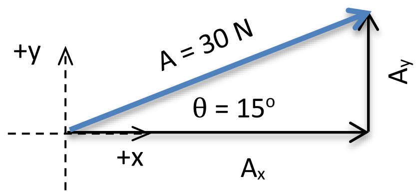


Figure 37 x- and y-components of vector A

A_x = + A cos 15° = + 30 N cos 15° = + 28,98 N

A_y = + A sin 15° = + 30 N sin 15° = + 7,76 N

The diagram of vector B is shown on the right.

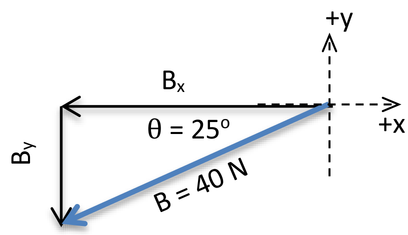


Figure 38 x- and y-components of vector B

B_y = - B sin 25° = - 40 N sin 25° = - 16,91 N (1)

B_x = - B cos 25° = - 40 N cos 25° = - 36,25 N (1)

b. Sum of all of the components that are parallel to the x-direction and y-direction:

$R_x = A_x + B_x$ = 28,98 N – 36,25 N = -7,27 N (1)

$R_y = A_y + B_y$ = 7,76 N – 16,91 N = -9,15 N (1)

Therefore resultant of vectors A + B:

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(7{,}27\ \text{N})^2 + (9{,}15\ \text{N})^2} = 11{,}69\ \text{N}\ \ (1)$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{R_y}{R_x} = \frac{9{,}15\ \text{N}}{7{,}27\ \text{N}}$$

Therefore θ = 51,53° (1)

The resultant of vectors A + B is therefore 11,69 N in a direction of 51,53° below the –x axis. (1) + (1) for the diagram

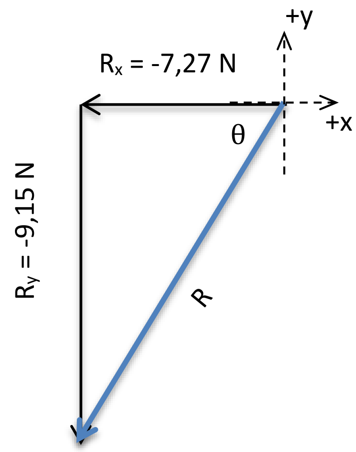


Figure 39 Resultant of all x- and y-components

Sub-topic 2: Motion in 1-dimension

Unit 1: Position, displacement, distance

Learning Outcomes

By the end of the unit, you should be able to:

- define position, displacement and distance;
- define position relative to a frame of reference;
- determine displacement and distance for 1-dimensional motion.

Introduction

When motion is described as being in one dimension, it means that the person or object is moving in a straight line (backward or forwards). In science you use very precise terms to describe quantities of motion. Reflect on the following questions, either on your own or with a fellow student:

- What is the difference between the terms position, displacement and distance?
- Write down sentences that show how you would use each of these words.

In Physics you use these terms very precisely. You will begin with understanding the term *position*. This describes where an object is relative to a frame of reference.

Frame of Reference

A *frame of reference* sets a specific point in space as a zero point (or origin), and then other positions are measured from this zero point. The frame of reference also determines a set of *directions*, called axes. This helps you to describe in which direction an object is moving. When you are describing your direction, you could use terms like East or West, up or down, backwards or forwards. In 1 dimensional motion it is useful to choose one direction as positive, and then the opposite direction is negative.

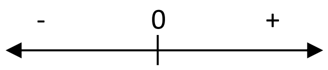


Figure 40 Example of a 1 dimensional frame of reference

Using this frame of reference, any position to the right of the zero point is positive, and any position to the left is negative. Note that the zero point is *not permanently fixed* anywhere (in science you say that it is not absolute), but it is a *chosen point* that allows you to measure positions relative to that point. Once you have chosen a zero point you keep it fixed throughout the problem, and measure positions from this zero point.

Position

Position is a length reading, and so you measure it in units of meters (m). In one dimension, positions are either positive or negative, depending on which side of the zero point they lie.

Activity 1: Find positions

Purpose

In this activity, you will find various positions using a frame of reference.

Suggested time: [20 minutes]

What you will do:

Study the diagram below and answer the questions that follow:

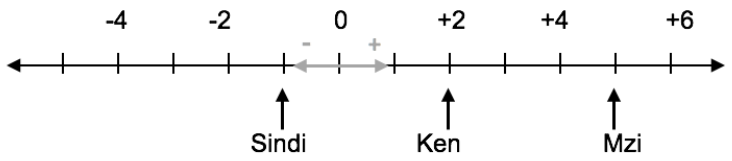


Figure 41 Positions shown on a number line

1. <u>Frame of reference</u>: Positions to the left of the zero point are negative, and positions to the right of the zero point are positive.
   a. What are the positions of Sindi, Ken and Mzi?

2. <u>Frame of reference</u>: Change the frame of reference so that Mzi is the new zero point. Let positions to the left of Mzi be negative, and so positions to the right of Mzi are positive.
   a. What is Sindi's position relative to Mzi?
   b. What is Ken's position relative to Mzi?

3. <u>Frame of reference</u>: Change the frame of reference so that Ken is the new zero point. Positions to the left of Ken are positive, and positions to the right of Ken are negative.
   a. What is Sindi's position relative to Ken?
   b. What is Mzi's position relative to Ken?

Solutions

1. Using the diagram shown with the frame of reference, you write the positions as follows

   a. Sindi: -1 m (meter), since her position is 1 m to the left of the zero point (the position of -1 m is halfway between 0 and -2 m)

   Ken: +2 m, since his position is 2 m to the right of the zero point

   Mzi: +5 m, since his position is 5 m to the left of the zero point (the position of +5 m is halfway between +4 m and +6 m)

2. You can show the new frame of reference on a diagram, while not changing the positions of the people. This helps you to work out the positions.

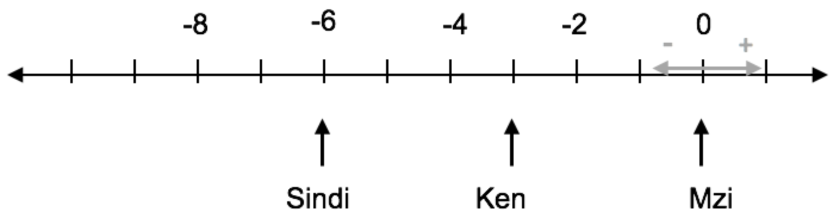


Figure 42 Positions relative to a new frame of reference

   a. Sindi: -6 m, since her position is 6m to the left of Mzi.

   b. Ken: -3 m, since his position is 3m to the left of Mzi.

3. Again you can show the new frame of reference on a diagram:

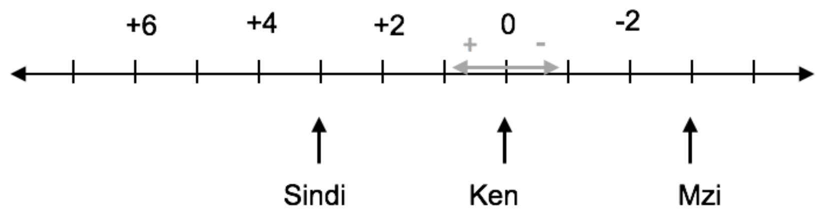


Figure 43 Positions relative to a new frame of reference

   a. Sindi: +3 m, since Sindi is 3 m to the left of Ken

   b. Mzi: -3 m, since Mzi is 3 m to the right of Ken

Activity 2: Simulation of positions

Purpose

In this activity, you will use a simulation to work with positions using a frame of reference, and in the guided reflection below the activity you will develop an understanding of the terms distance and displacement.

Suggested time: [10 minutes]

What you will do:

Go to the following web address: https://connexions.github.io/simulations/moving-man/

You will see a picture of a man, a tree and a house. You will also see a frame of reference and a number line, similar to the picture shown below.

NOTE: If you are not able to use the simulation on the internet you can still answer the questions using the diagram below.

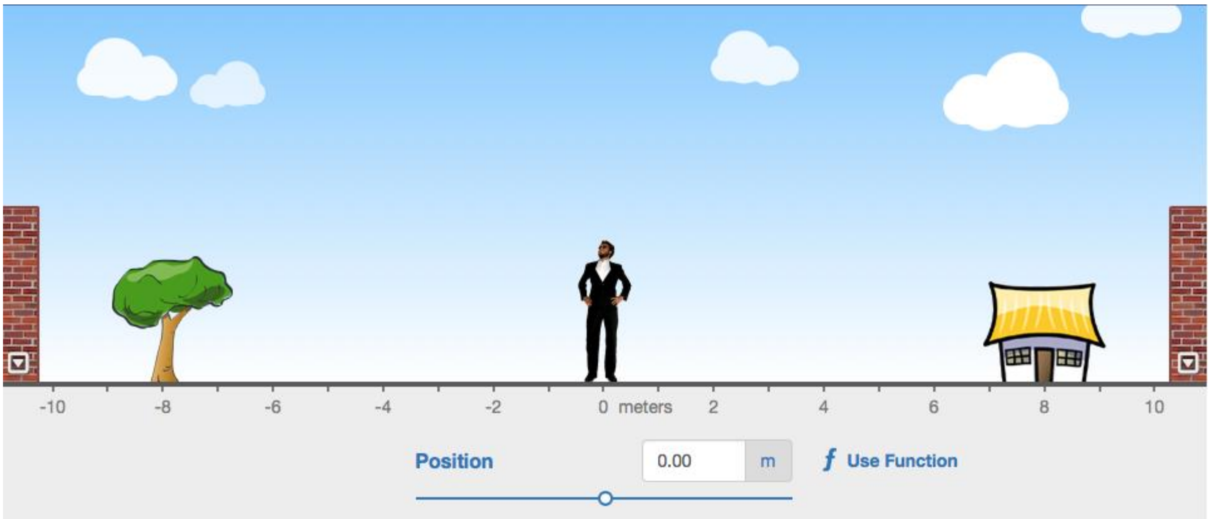


Figure 44 Screen of moving-man simulation

Answer the following questions:

1. Which direction is positive and which direction is negative, relative to the frame of reference shown?
2. What is the initial (starting) position of the man?
3. Move the man to a position of -2 m. Notice that his position is shown in the block below the picture.

4. Move the man to the position where the tree is. What is his position relative to the frame of reference shown?
5. What is his position relative to the position of the tree?
6. Now move the man to the position of the house. What is his position relative to the frame of reference shown?
7. What is his position relative to the position of the tree?

You will use this simulation again in a later activity.

Guided reflection

1. The frame of reference given on the screen shows that the right direction is positive and the left direction is negative.
2. The initial (starting) position of the man is 0 m.
3. When the man is at the same position as the tree, his position relative to the frame of reference shown is -8 m.
4. His position relative to the position of the tree is 0 m.
5. When the man is at the position of the house, his position relative to the frame of reference shown is +8 m.
6. When the man is at the position of the house, he is now 16 m to the right of the tree, so his position relative to the position of the tree is + 16 m.

[Wordbox: MAIN IDEA: *Position* describes where an object is relative to a frame of reference.]

Distance and Displacement

In the above example, when the man walked from the zero point to the tree, and from the tree to the house, he covered a length of path. You need two different words to describe path lengths in Physics, *distance* and *displacement*.

You use the word *distance* for the *total length of the path*. The symbol that you use for distance is D, and it is measured in SI units of meters (m).

In the example of the walking man, the total length of the path that he covered was the 8 m that he walked toward the tree plus the 16 m that he walked toward the house. Therefore the total distance that he covered was 8 m + 16 m = 24 m.

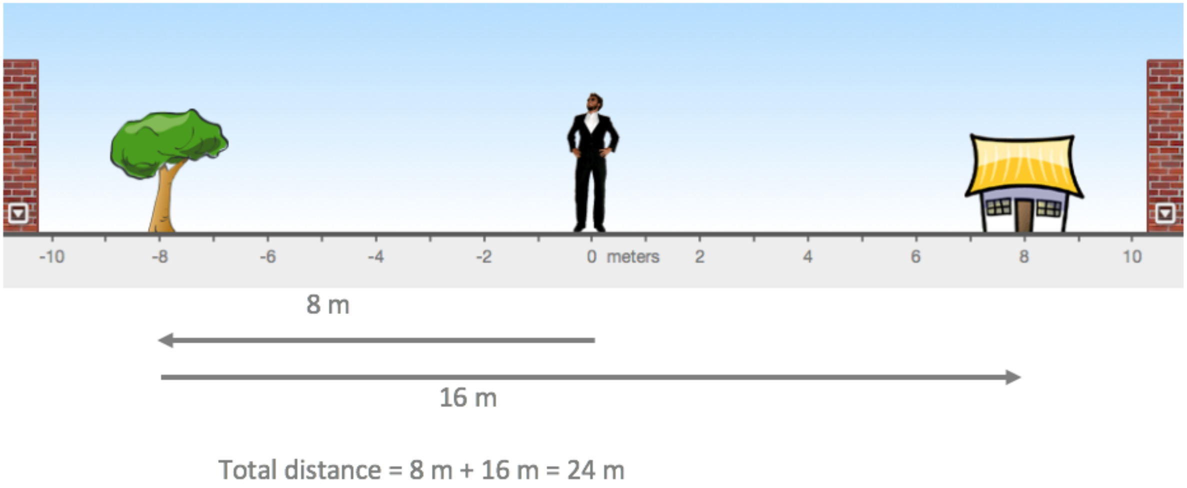


Figure 45 Distance covered by the moving man

You can see that the path of the moving man was in different directions at different times. Distance is a *scalar* quantity because it does not have a specific direction.

You use the word *displacement* to measure the *change in position* from the starting point to the ending point of the movement. You use the symbol Δx for displacement, and it is measured in SI units of m. The displacement is always in a straight line. It is a *vector* quantity because it has a very clear direction. You calculate displacement using the equation: $\Delta x = x_f - x_i$ where x_f is the final position, and x_i is the initial position.

In the example of the walking man, you work out his displacement from his initial position (at the zero point), to his final position (the position of the house) as $\Delta x = x_f - x_i = 8\text{ m} - 0\text{ m} = +8\text{ m}$.

Since displacement is a vector, you can use an arrow to show the displacement, as the diagram below shows:

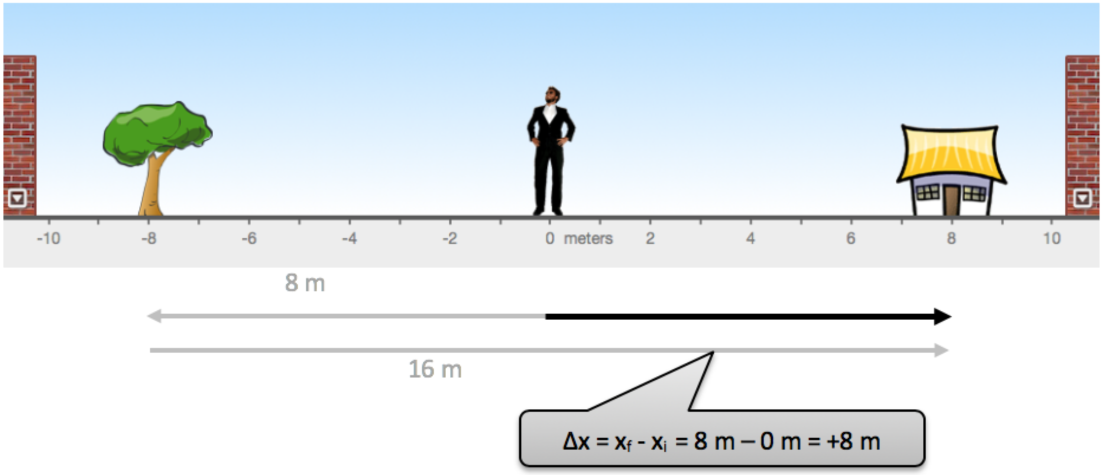


Figure 46 Displacement of the moving man

Note that the vector for the displacement is pointed to the right, which is the positive direction. From this example you can see that the distance covered can often be much greater than the displacement.

Since displacement is defined as the change in position, the symbol that you use for displacement is Δx. It is measured in SI units of meters (m).

Activity 3: Test your understanding of distance and displacement

Purpose

In this activity you will reinforce the learning you have done to now by testing your knowledge of position, distance and displacement.

Suggested time: [20 minutes]

What you will do:

Note that the starting position x_i is not always at the zero point.

Answer the following questions:

1. Keletso started at a position of 0,6 m. Complete the missing information in the calculations given below these questions:
   a) If Keletso walked from her starting position of 0,6 m to a position of -1,2 m, what was her displacement?
   b) If Keletso then walked from her position of -1,2 m to a position of 1 m, what was her displacement for this part of her movement?

c) What was her **total** displacement from her original starting position of 0,6 m?

d) What was the total distance that Keletso covered in (a) and (b)?

Calculations:

a) Given: x_i = 0,6 m and x_f = -1,2 m

Displacement: $\Delta x = x_f - x_i$ = ______________________________

b) Given: x_i = -1,2 m and x_f = 1 m

Displacement: $\Delta x = x_f - x_i$ = ______________________________

c) Given: x_i = _____ m and x_f = ______ m

Displacement: $\Delta x = x_f - x_i$ = ______________________________

d) Total distance D = path length for first movement + path length for second movement

= ______________________________

2. Sindisiwe starts at a position of +2,5 m. She walks to a position of -4 m. What is her displacement? (Remember to show the direction with a + or a – sign)
3. Ashwin walks from a position of -5 m to a position of -3,5 m. What is his displacement?
4. If Celani starts at a position of -4 m and walks in the positive direction for a distance of 2 m, what is her new position?
5. The number line below shows Felix's movement.

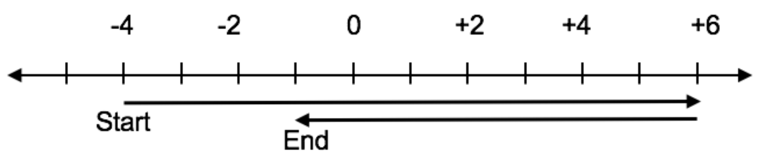


Figure 47 Number line showing Felix's movement

a. What was Felix's starting position?
b. What was the position where Felix changed his direction of movement?
c. What was his ending position?
d. What was the total *distance* covered by Felix?
e. What was Felix's *displacement*?

Solutions

2. The completed steps for the calculation are shown below:

Calculations:

a) Given: x_i = 0,6 m and x_f = -1,2 m

Displacement: $\Delta x = x_f - x_i$ = -1,2 m – 0,6 m = -1,8 m

b) Given: x_i = -1,2 m and x_f = 1 m

Displacement: $\Delta x = x_f - x_i$ = 1 m – (-1,2 m) = +2,2 m

c) Given: x_i = 0,6 m and x_f = 1 m

Displacement: $\Delta x = x_f - x_i$ = 1 m – (0,6 m) = +0,4 m

d) Total distance D = path length for first movement + path length for second movement

= 1,8 m + 2,2 m = 4,0 m

Note that you always show distances as positive, since they don't have a direction.

3. Given: x_i = 2,5 m and x_f = -4 m

Displacement: $\Delta x = x_f - x_i$ = -4 m – 2,5 m = -6,5 m

4. Given: x_i = -5 m and x_f = -3,5 m

Displacement: $\Delta x = x_f - x_i$ = -3,5 m – (-5 m) = +1,5 m

5. Given: x_i = -4 m and Δx = +2 m

From $\Delta x = x_f - x_i$ you get $x_f = x_i + \Delta x$ = -4 m + 2 m = -2 m

6.

a. Felix's starting position x_i = -4 m

b. Felix changed his direction of movement at +6 m

c. Felix's ending position x_f = -1 m

d. The total distance covered by Felix = 10 m + 7 m = 17 m

e. Felix's displacement: $\Delta x = x_f - x_i$ = -1 m – (-4 m) = +3 m

[Wordbox: MAIN IDEAS:
- *Distance* (D) is the total length of the path of movement, measured in units of meters (m).
- *Displacement* (Δx) is the change in position, measured in units of meters (m). $\Delta x = x_f - x_i$]

More resources to help you:

- A web page with some helpful explanations and quizzes on distance and displacement can be found at this address: https://www.physicsclassroom.com/class/1DKin/Lesson-1/Distance-and-Displacement
- If you type "Youtube distance displacement" into the Google search engine, this will help you to find various teaching videos on this section.

Unit 2: Speed, velocity, acceleration

Learning Outcomes

By the end of the unit, you should be able to:

- define speed, instantaneous velocity and average velocity;
- determine speed, instantaneous velocity and average velocity for 1-dimensional motion;
- define acceleration;
- determine acceleration for 1-dimensional motion with uniform acceleration.

Introduction

In Unit 1 you learnt about the two terms that you use to describe the length of a path, namely distance and displacement. In Physics you also need to be able to describe how fast something is moving. Reflect on the following questions, either on your own or with a fellow student:

- What is the difference between the terms speed, velocity and acceleration?
- Write down sentences that show how you would use each of these words.

When you work with motion in Physics, you need to be able to describe how fast something is moving. In the same way that there are two terms to describe path lengths, there are also two terms that you use when you are measuring the fastness of an object: *speed* and *velocity*. You also need to work out how the velocity of the object is changing. This is called *acceleration*. In this unit you will learn about speed, velocity and acceleration.

Speed

If you want to calculate the *speed*, you divide the total distance (D) covered by the time (Δt) that it takes to travel that distance.

$$\text{speed} = \frac{\text{distance covered}}{\text{total time taken}} = \frac{\mathrm{D}}{\Delta \mathrm{t}}$$

Since distance is a scalar quantity, it means that the speed is also a scalar. Scientifically you say that speed is the *rate at which distance is covered*. Because the SI unit for distance is meters (m), and for time is seconds (s), speed is measured in units of meters per second (m/s or $m \cdot s^{-1}$).

When you have units like m/s, what you mean is the number of meters traveled in one second (or the number of meters per second). So the word "*per*" just means "*in one*".

Average Velocity

The *average velocity* is found by dividing the displacement (or change in position) by the time taken:

$$\text{average velocity} = \frac{\text{displacement}}{\text{total time taken}}$$

You will notice that velocity is also measured in m/s or $m \cdot s^{-1}$. Displacement is a vector quantity, in other words, it has a specific direction. This means that the velocity is also a vector quantity, as it has the *same direction* as the displacement.

Mathematically you write the equation for the average velocity as: $v = \frac{\Delta x}{\Delta t}$

Instantaneous Velocity

The *instantaneous velocity* is found by dividing the displacement (or change in position) by an infinitesimal time interval:

$$\text{instantaneous velocity} = \frac{\text{displacement}}{\text{small time interval}}$$

[Wordbox: *instantaneous* – done in an instant (right now)]
[Wordbox: *infinitesimal* - very small]

Instantaneous velocity is also measured in m/s or $m \cdot s^{-1}$, and it is a vector quantity that has the *same direction* as the displacement.

Mathematically you write the equation for the instantaneous velocity as: $v = \frac{dx}{dt}$

where dx is the small displacement, measured in m, and dt is the small time interval, measured in s.

The instantaneous speed of the object is the magnitude of the instantaneous velocity. So the instantaneous speed has a magnitude but does not have a direction, and it is therefore a scalar quantity.

The following examples will give you an idea of how to do calculations of speed and velocity.

Examples: *(Try to solve this problem on your own or with a fellow student while covering the solution, and then check your work using the solution below).*

Zama walked with a constant pace from a position of +2 m to a position of +6 m in 2s.

1. What was her instantaneous velocity at the time 1 second?

Zama then walked with a constant pace to a position of +3m in a time of 3 s.

2. What was Zama's instantaneous velocity after 1 second of this motion?
3. What was her instantaneous speed?
4. What was Zama's average velocity for the total movement in parts 1 and 2?

<u>**Solution**</u>:

1. <u>Given:</u> x_i = +2 m and x_f = +6 m;

   The velocity is constant from 0 s to 2 s, so you use this time interval to find the instantaneous velocity at 1 s.

   Instantaneous velocity at 1 s: $v = \frac{dx}{dt} = \frac{6\ m - 2\ m}{2\ s - 0\ s} = \frac{+4\ m}{2\ s} = +2\ m{\cdot}s^{-1}$

2. <u>Given:</u> x_i = +6 m and x_f = +3 m

   The velocity is constant from for this 3 s, so you use this time interval to find the instantaneous velocity after 1 s.

   Instantaneous velocity after 1 s: $v = \frac{dx}{dt} = \frac{3\ m - 6\ m}{3\ s - 0\ s} = \frac{-3\ m}{3\ s} = -1\ m{\cdot}s^{-1}$

3. Instantaneous speed after 1 s $= \frac{D}{dt} = \frac{3\ m}{3\ s - 0\ s} = 1\ m{\cdot}s^{-1}$

   OR Instantaneous speed =magnitude of Instantaneous velocity =1 $m{\cdot}s^{-1}$

4. <u>Given:</u> x_i = +2 m and x_f = +3 m; Δt = 2 s + 3 s = 5 s

   Average velocity: $v = \frac{\Delta x}{\Delta t} = \frac{3\ m - 2\ m}{5\ s} = \frac{1\ m}{5\ s} = +0{,}2\ m{\cdot}s^{-1}$

Two short videos that explain the difference between speed and velocity are:

- *The difference between Speed & Velocity*: https://www.youtube.com/watch?v=X4Wxd4m-QVc (Duration: 2.28)
- *Speed and Velocity Simple Tutorial*: https://www.youtube.com/watch?v=O22zcaELpaA (Duration: 2.41)

Activity 1: Work with speed and velocity

Purpose

In this activity you will explore speed and velocity, by working with the moving man example and doing some guided calculations.

Suggested time: [20 minutes]

What you will do:

Study the diagram of the moving man below that shows his movement from the zero position to the tree (Path 1) and then to the house (Path 2).

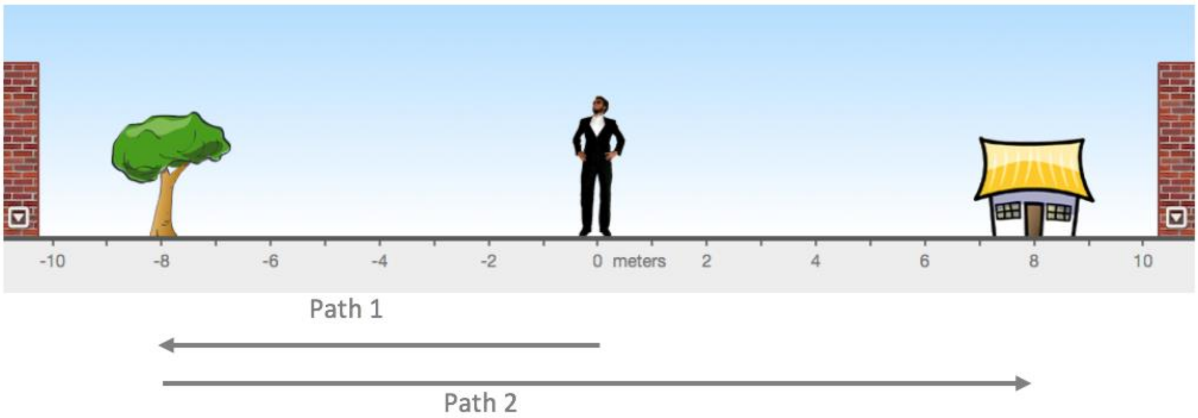


Figure 48 The moving man walks Path 1 and Path 2

Answer the following questions:

1. If the man took 2 seconds to walk Path 1, what was his speed for this movement?
2. What was the man's average velocity for Path 1?
3. If his speed was constant while he walked Path 1, what was his instantaneous velocity at a time of 1 second?
4. The man took 8 seconds to walk from the tree to the house (Path 2). Was he moving faster or slower than his movement for Path 1? Do a calculation to prove your answer.
5. What was the man's average velocity for the total movement from the zero point to the house?
6. What was his average speed for the whole movement?

Guided reflection

The answers to the questions are shown below:

1. If the man took 2 seconds to walk Path 1, then his speed is calculated by dividing the distance covered by the time taken:

$$\text{speed} = \frac{\text{distance covered}}{\text{total time taken}} = \frac{\mathrm{D}}{\Delta \mathrm{t}} = \frac{8\ \mathrm{m}}{2\ \mathrm{s}} = 4\ \mathrm{m/s}$$

Remember that you don't give a direction for distance or speed.

2. The man's average velocity is calculated by dividing the displacement by the time taken:

average velocity: $v = \frac{\Delta x}{\Delta t} = \frac{-8\text{ m} - 0\text{ m}}{2\text{ s}} = \frac{-8\text{ m}}{2\text{ s}}$ = - 4 m/s

The direction of the velocity is negative because he walked to the left (negative direction).

3. If his speed was constant while he walked Path 1, this means that after 1 second (half the time) he would have covered half the distance (4 m). To work out his instantaneous velocity at a time of 1 second you can choose the short time interval from 0 to 1 second:

   Instantaneous velocity after 1 s: $v = \frac{dx}{dt} = \frac{-4\text{ m} - 0\text{ m}}{1\text{ s} - 0\text{ s}} = \frac{-4\text{ m}}{1\text{ s}}$ = - 4 m/s

4. If the man took 8 seconds to walk from the tree to the house (Path 2), then his speed for this path was:

   speed = $\frac{\text{distance covered}}{\text{total time taken}} = \frac{D}{\Delta t} = \frac{16\text{ m}}{8\text{ s}}$ = 2 m/s

   He was therefore moving slower than for Path 1 where his speed was 4 m/s.

5. To find the man's average velocity for the total movement from the zero point to the house, you divide the total displacement by the total time taken.

   average velocity: $v = \frac{\Delta x}{\Delta t} = \frac{8\text{ m} - 0\text{ m}}{2\text{ s} + 8\text{ s}} = \frac{8\text{ m}}{10\text{ s}}$ = + 0,8 m/s

6. To find the man's average speed for the total movement from the zero point to the house, you divide the total distance he covered by the total time taken.

   Average speed = $\frac{\text{distance covered}}{\text{total time taken}} = \frac{D}{\Delta t} = \frac{8\text{ m} + 16\text{ m}}{10\text{ s}}$ = 2,4 m/s

Activity 2: Test your understanding of speed and velocity

Purpose

In this activity you will test your understanding of speed and velocity by solving a few problems and then checking your work using solutions.

Suggested time: [15 minutes]

What you will do:

Answer the following questions:

Isihle walked with a constant velocity from a position of +3 m to a position of -3 m in 6 seconds.

(a) Calculate her instantaneous velocity after 3 seconds of this motion.

Isihle then ran from her position of -3 m with a constant velocity to a position of +6 m in 2 seconds.

(b) What was her instantaneous velocity after 1 second of this motion?

(c) What was Isihle's average velocity for her total movement described above?

(d) What was Isihle's average speed for her total movement?

Isihle then moved from her position of +6 m with a velocity of -3 $m \cdot s^{-1}$ for 6 seconds.

(e) What was Isihle's final position after this movement?

Solutions

(a) <u>Given:</u> x_i = +3 m and x_f = -3 m, Δt = 6 s

Since velocity is constant from 0 s to 6 s, you can use this time interval to find the instantaneous velocity after 3 s.

Instantaneous velocity: $v = \frac{dx}{dt} = \frac{-3\ m - 3\ m}{6\ s - 0\ s} = \frac{-6\ m}{6\ s}$ = -1 $m \cdot s^{-1}$

(b) <u>Given:</u> x_i = -3 m and x_f = +6 m, Δt = 2 s

Since velocity is constant for these 2 seconds, you can use this time interval to find the instantaneous velocity after 1 s of this motion.

Instantaneous velocity: $v = \frac{dx}{dt} = \frac{+6\ m - (-3\ m)}{2\ s - 0\ s} = \frac{+9\ m}{2\ s}$ = +4,5 $m \cdot s^{-1}$

(c) <u>Given:</u> x_i = +3 m and x_f = +6 m and total time = 8 s

Average velocity: $v = \frac{\Delta x}{\Delta t} = \frac{+6\ m - 3\ m}{8\ s} = \frac{+3\ m}{8\ s}$ = +0,375 $m \cdot s^{-1}$

(d) <u>Given:</u> Total distance D = 6 m + 9 m = 15 m and total time = 8 s

Average speed $= \frac{D}{\Delta t} = = \frac{15\ m}{8\ s} =$ 1,875 $m \cdot s^{-1}$

(e) <u>Given:</u> v = -3 $m \cdot s^{-1}$ and Δt = 6 s and x_i = +6 m

From $v = \frac{\Delta x}{\Delta t}$ you get $\Delta x = v \times \Delta t$ = -3 $m \cdot s^{-1}$ × 6 s = -18 m

From $\Delta x = x_f - x_i$ you can find $x_f = \Delta x + x_i$ = -18 m + 6 m = -12 m

Activity 3: Measure position and velocity

Purpose

In this activity you will use simple equipment to measure position and velocity for different kinds of motion.

Suggested time: [40 minutes]

What you need:

- A pin or needle
- A ruler or meter rule
- A plastic packet (not a bottle)
- Water
- A piece of chalk

What you will do:

PART A – Constant motion

1. Using your pin or needle, make a *small* hole in one of the bottom corners of your packet.
2. Fill your packet with water, and make sure that the water comes out of this hole one drop at a time.
3. To make this experiment simpler, assume that the time interval between water drops is 0,5 seconds.
4. Now choose a friend or one of your group members to be the walker. They should hold the water packet.
5. You should hold the piece of chalk, so that you can mark the place where the water drips land using this piece of chalk.
6. Set your frame of reference by choosing some zero point as the position where the walker will start from, and some direction as the positive direction.
7. Make sure that the packet is more than half filled with water. The walker should hold the packet and walk in the positive direction (forward) with a slow but ***steady*** (constant) pace.
8. You should make a mark with chalk at the place where each drop has landed. (If it is a hot day, your drops will dry quickly, and you will lose your data!)
9. Measure the positions of these marks. (NOTE: the positions must be measured relative to the zero point)

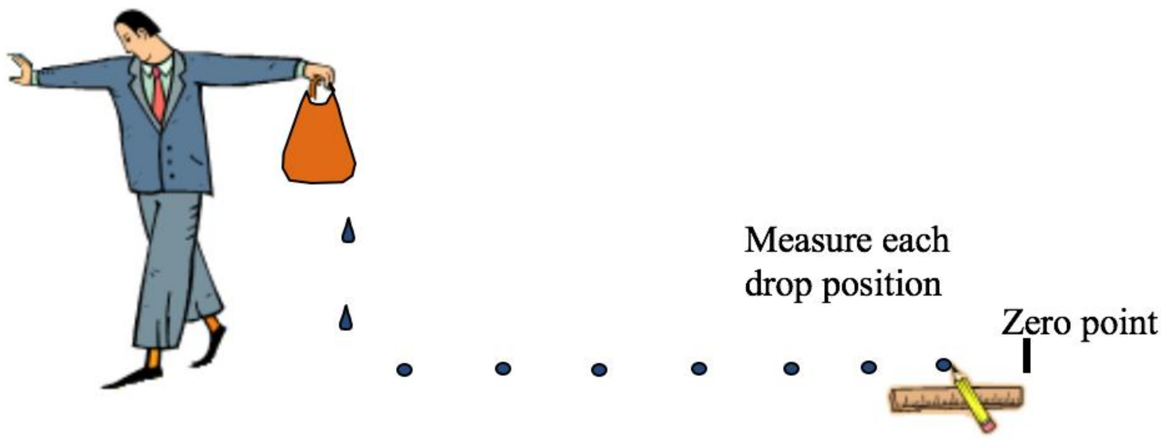


Figure 49 Measure the positions of the drops from the zero points

10. In a table like the one below, record the position and time for each drop.

Drop number	Position (m)	Time (s)
0	0	0
1		0,5
2		1,0
3		1,5
4		2,0
5		2,5
6		3,0
7		3,5
8		4,0

11. Since you are assuming that the time interval between drops is 0,5 seconds, the time values have been filled into the table for you.
12. From your measurements of position and time, calculate the velocity for each of the time intervals shown below:
    a. From 0 s to 1 s: ____
    b. From 1 s to 2 s: ____
    c. From 2 s to 3 s: ____
    d. From 3 s to 4 s: ____
13. How do your velocity values compare with each other?
14. What can you conclude about the velocity of the walker?

PART B – Increasing motion

15. The walker should now return to the starting position and walk in the positive direction (forward), starting slowly and gradually *increasing the pace*.
16. Measure the positions of the marks made from this motion and complete the table.

Drop number	Position (m)	Time (s)
0	0	0
1		0,5
2		1,0
3		1,5
4		2,0
5		2,5
6		3,0
7		3,5
8		4,0

17. From your measurements of position and time, calculate the velocity for each of the time intervals shown below:
    a. From 0 s to 1 s: ____
    b. From 1 s to 2 s: ____
    c. From 2 s to 3 s: ____
    d. From 3 s to 4 s: ____
18. How do your velocity values compare with each other?
19. What can you conclude about the velocity of the walker?
20. Calculate the average velocity value for the whole motion.

Alternative suggestion:

If you have not been able to do the practical part of the activity, some possible values are given in the tables below. You can use these values to complete the calculations and reflections in the above activity (Steps 12 to 14, and steps 17 to 19):

Part 1 – Constant motion

Drop number	Position (m)	Time (s)
0	0	0
1	0,21	0,5
2	0,38	1,0
3	0,62	1,5
4	0,79	2,0
5	1,01	2,5
6	1,18	3,0
7	1,42	3,5
8	1,60	4,0

Part 2 – Increasing motion

Drop number	Position (m)	Time (s)
0	0	0
1	0,09	0,5
2	0,40	1,0
3	0,91	1,5
4	1,59	2,0
5	2,52	2,5
6	3,58	3,0
7	4,91	3,5
8	6,40	4,0

Guided reflection

Part 1 – Constant motion

- From your table you should notice that your values for position increase uniformly (steadily) with time.
- The calculations for velocity using the values form the table above are given below:
  a. From 0 s to 1 s: v = $\frac{\Delta x}{\Delta t} = \frac{0{,}38\ \text{m} - 0\ \text{m}}{1\ \text{s} - 0\ \text{s}} = \frac{0{,}38\ \text{m}}{1\ \text{s}} =$ 0,38 m/s
  b. From 1 s to 2 s: v = $\frac{\Delta x}{\Delta t} = \frac{0{,}79\ \text{m} - 0{,}38\ \text{m}}{2\ \text{s} - 1\ \text{s}} = \frac{0{,}41\ \text{m}}{1\ \text{s}} =$ 0,41 m/s

c. From 2 s to 3 s: $v = \frac{\Delta x}{\Delta t} = \frac{1{,}18\ \text{m} - 0{,}79\ \text{m}}{3\ \text{s} - 2\ \text{s}} = \frac{0{,}39\ \text{m}}{1\ \text{s}} =$ 0,39 m/s

d. From 3 s to 4 s: $v = \frac{\Delta x}{\Delta t} = \frac{1{,}60\ \text{m} - 1{,}18\ \text{m}}{4\ \text{s} - 3\ \text{s}} = \frac{0{,}42\ \text{m}}{1\ \text{s}} =$ 0,42 m/s

(Your own values will be different to this, depending on how fast the walker moved and how quickly your drops came out of your packet).

- The velocity values that you calculate should all be more or less constant (there might be some variation in the values, which you will always find with experimental data).
- From these values you can conclude that the velocity of the walker is constant.
- The average velocity for the whole motion is:

$$v = \frac{\Delta x}{\Delta t} = \frac{1{,}60\ \text{m} - 0\ \text{m}}{4\ \text{s} - 0\ \text{s}} = \frac{1{,}60\ \text{m}}{4\ \text{s}} = 0{,}40\ \text{m/s}$$

PART B – Increasing motion

- When the walker is walking with an increasing pace, you should notice that the position values increase by greater and greater amounts each time.
- The calculations for velocity using the values form the table above are given below:

a. From 0 s to 1 s: $v = \frac{\Delta x}{\Delta t} = \frac{0{,}40\ \text{m} - 0\ \text{m}}{1\ \text{s} - 0\ \text{s}} = \frac{0{,}40\ \text{m}}{1\ \text{s}} =$ 0,40 m/s

b. From 1 s to 2 s: $v = \frac{\Delta x}{\Delta t} = \frac{1{,}59\ \text{m} - 0{,}40\ \text{m}}{2\ \text{s} - 1\ \text{s}} = \frac{1{,}19\ \text{m}}{1\ \text{s}} =$ 1,19 m/s

c. From 2 s to 3 s: $v = \frac{\Delta x}{\Delta t} = \frac{3{,}58\ \text{m} - 1{,}59\ \text{m}}{3\ \text{s} - 2\ \text{s}} = \frac{1{,}99\ \text{m}}{1\ \text{s}} =$ 1,99 m/s

d. From 3 s to 4 s: $v = \frac{\Delta x}{\Delta t} = \frac{6.40\ \text{m} - 3{,}58\ \text{m}}{4\ \text{s} - 3\ \text{s}} = \frac{2{,}82\ \text{m}}{1\ \text{s}} =$ 2,82 m/s

- You should find that the velocity values that you calculate in the table increase with time.
- You can conclude that the velocity of the walker is not constant, but is increasing with time.

[Wordbox: MAIN IDEAS:

- *Speed* is the total distance covered divided by the time taken:

$$\text{speed} = \frac{D}{\Delta t}$$

- Speed is a scalar and is measured in units of meters per second (m/s or $m{\cdot}s^{-1}$).
- *Average velocity* is found by dividing the displacement by the time:

$$v = \frac{\Delta x}{\Delta t}$$

- *Instantaneous velocity* is found by dividing the displacement by an infinitesimal time interval:

$$v = \frac{dx}{dt}$$

- Velocity is a vector and is measured in meters per second (m/s or $m{\cdot}s^{-1}$).]

Acceleration

When you are speeding up or slowing down, this change in your motion is described as acceleration. Scientifically, the definition for acceleration is the *rate of change of velocity*. It is found by dividing the change in velocity by the time it takes for that velocity change:

$$\text{acceleration} = \frac{\text{change in velocity}}{\text{total time taken}}$$

Acceleration is a vector, and its symbol is a. Mathematically you can write the equation as:

$$a = \frac{\Delta v}{\Delta t}$$

Acceleration is measured in meters per second per second (m/s/s), or meters per second squared (m/s^2, or m·s^{-2}).

For example, if somebody starts from rest, and pedals their bike for 4 seconds until they have a velocity of 8 m·s^{-1}, their average acceleration is:

$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t} = \frac{8\ \text{m·s}^{-1} - 0\ \text{m·s}^{-1}}{4\ \text{s}} = +2\ \text{m·s}^{-2}$$

If this person applies the brakes for 3 seconds and slows down so that their speed becomes 2 m·s^{-1}, the average acceleration during this time is:

$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t} = \frac{2\ \text{m·s}^{-1} - 8\ \text{m·s}^{-1}}{3\ \text{s}} = \frac{-6\ \text{m·s}^{-1}}{3\ \text{s}} = -2\ \text{m·s}^{-2}$$

Activity 4: Investigate different kinds of acceleration

Purpose

In this activity you will investigate the acceleration for various different kinds of motion.

Suggested time: [20 minutes]

What you will do:

Calculate the acceleration for each of the following:

1. A car travels in the positive direction with a constant velocity of 4 m·s^{-1} for 2 seconds.
2. A car starts from rest (v = 0 m·s^{-1}), and after 2 seconds it is travelling with a velocity of 4 m·s^{-1}.

3. A car is moving with a velocity of 8 $m \cdot s^{-1}$. After 2 seconds it has slowed down to a velocity of 4 $m \cdot s^{-1}$.
4. A car is travelling with a velocity of -4 $m \cdot s^{-1}$. It slows down to a stop in 2 s. Calculate its acceleration.
5. A car is travelling with a velocity of -4 $m \cdot s^{-1}$. It increases its speed, and reaches a velocity of -8 $m \cdot s^{-1}$ in 2 s.

From your calculations, write a summary of the kind of movement which will give you an acceleration that is:

- Zero
- Positive
- Negative

Guided reflection

The solutions to the calculations are shown below:

1. Given: $v_i = +4\ m \cdot s^{-1}$ and $v_f = +4\ m \cdot s^{-1}$ and $\Delta t = 2\ s$

$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t} = \frac{4\ m.s^{-1} - 4\ m.s^{-1}}{2\ s} = 0\ m.s^{-2}$$

2. Given: $v_i = 0\ m \cdot s^{-1}$ and $v_f = +4\ m \cdot s^{-1}$ and $\Delta t = 2\ s$

$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t} = \frac{4\ m.s^{-1} - 0\ m.s^{-1}}{2\ s} = +2\ m.s^{-2}$$

3. Given: $v_i = +8\ m \cdot s^{-1}$ and $v_f = +4\ m \cdot s^{-1}$ and $\Delta t = 2\ s$

$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t} = \frac{4\ m.s^{-1} - 8\ m.s^{-1}}{2\ s} = -2\ m.s^{-2}$$

4. Given: $v_i = -4\ m \cdot s^{-1}$ and $v_f = 0\ m \cdot s^{-1}$ and $\Delta t = 2\ s$

$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t} = \frac{0\ m.s^{-1} - (-4\ m.s^{-1})}{2\ s} = +2\ m.s^{-2}$$

5. Given: $v_i = -4\ m \cdot s^{-1}$ and $v_f = -8\ m \cdot s^{-1}$ and $\Delta t = 2\ s$

$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t} = \frac{-8\ m.s^{-1} - (-4\ m.s^{-1})}{2\ s} = -2\ m.s^{-2}$$

From your calculations you should notice the following:

- Acceleration is *zero* when there is no motion, or when motion is uniform (constant velocity).
- Acceleration is *positive* when the movement is getting faster in a positive direction, or when the movement is getting slower in a negative direction
- Acceleration is *negative* when the movement is becoming slower in a positive direction, or when the movement is getting faster in a negative direction..

It is important to understand that you cannot use the word "deceleration" to describe negative acceleration. The word deceleration describes the motion of an object that is slowing down, but negative acceleration can mean that an object is speeding up in the negative direction. To avoid making mistakes, it is a good idea to never use the word deceleration in Physics.

You will also notice that the acceleration does not tell you any information about the direction of the motion. It only describes how the velocity is changing.

[Wordbox: MAIN IDEAS: *Acceleration* is the change in velocity divided by the total time, and is a vector quantity:

$$\text{acceleration a} = \frac{\text{change in velocity}}{\text{total time taken}} = \frac{\Delta v}{\Delta t}$$

Acceleration is measured in units of $m{\cdot}s^{-2}$]

More resources to help you:

- A web page with some helpful explanations and quizzes on speed and velocity can be found at this address: https://www.physicsclassroom.com/class/1DKin/Lesson-1/Speed-and-Velocity. This website uses the units of miles distance, and miles per hour for speed or velocity, but the concepts are the same, and are very helpfully explained.
- The same website has a page explaining acceleration which can be found at this address: https://www.physicsclassroom.com/class/1DKin/Lesson-1/Acceleration.
- For a fun and interactive quiz on motion, go to the internet at this address: https://www.physicsclassroom.com/Concept-Builders/Kinematics/Name-That-Motion/Concept-Builder. If you are unsure how to use the interactive activity, you can find instructions at this web address: https://www.physicsclassroom.com/Concept-Builders/Kinematics/Name-That-Motion/Directions
- If you type "Youtube speed velocity acceleration" into the Google search engine you will be able to see many more teaching videos on this section.

Unit 3: Graphs of motion

Learning Outcomes

By the end of the unit, you should be able to:

- plot graphs of position vs time, velocity vs time and acceleration vs time;
- interpret and determine information from graphs of position vs time, velocity vs time and acceleration vs time for 1-dimensional motion with uniform acceleration.

Introduction

In Units 1 and 2 you learnt about the various terms that you use to describe the motion. Refresh your understanding by reflecting on the following questions, either on your own or with a fellow student:

- What is the difference between the terms position, distance and displacement?
- What is the difference between the terms speed, velocity and acceleration?
- Write an equation that links displacement, velocity and time.
- Write an equation that links distance, speed and time.

It is important to be able to represent motion in different ways in Physics. One of these ways is through representing the motion on graphs. There are three different kinds of graphs of motion that describe the movement of an object:

- Graph of position vs time (sometimes called the position-time graph, or x-t graph)
- Graph of velocity vs time (sometimes called the velocity-time graph, or v-t graph)
- Graph of acceleration vs time (sometimes called the acceleration-time graph, or a-t graph)

In this unit you will build on what you have learnt about motion by exploring these graphs.

Graphs of position vs time

A position vs time graph is constructed with position values given on the y-axis, and time values given on the x-axis. The graph below is an example of a position-time graph.

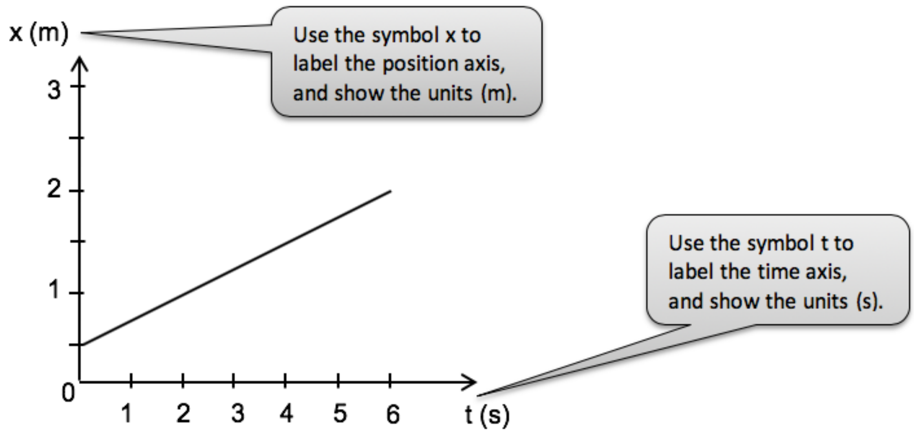


Figure 50 A graph of position vs time

Activity 1: Construct a graph of position vs time

Purpose

A position vs time graph is constructed with position values given on the vertical axis, and time values given on the horizontal axis. In this activity you will construct a graph of position vs time using the values that you measured in your experiment in Activity 3 of Unit 2.

Suggested time: [20 minutes]

What you will do:

PART A – Constant motion

1. Using your data for constant motion from Activity 3 of Unit 2, construct a set of axes which will allow you to draw a graph of position on the vertical axis against time on the horizontal axis. Label your axes carefully. (You can use the data given in the tables in the Alternative Suggestion if you struggled to do the practical activity yourself. This table is repeated below.)

Drop number	Position (m)	Time (s)
0	0	0
1	0,21	0,5
2	0,38	1,0
3	0,62	1,5
4	0,79	2,0

5	1,01	2,5
6	1,18	3,0
7	1,42	3,5
8	1,60	4,0

2. On your set of axes, plot each of the points of position and time from your table.
3. What do you notice about the shape of your position vs time graph for constant motion?

PART B – Increasing motion

1. Using your data for increasing motion from Practical Activity 3 of Unit 2, construct a set of axes which will allow you to draw a graph of position on the vertical axis against time on the horizontal axis. Label your axes carefully. (Again you can use the data given in the tables in the Alternative Suggestion if you struggled to do the practical activity yourself. This table is repeated below for increasing motion.)

Drop number	Position (m)	Time (s)
0	0	0
1	0,09	0,5
2	0,40	1,0
3	0,91	1,5
4	1,59	2,0
5	2,52	2,5
6	3,58	3,0
7	4,91	3,5
8	6,40	4,0

2. On your set of axes, plot each of the points of position and time from your table.
3. What do you notice about the shape of your position vs time graph for increasing motion?

Guided reflection

The graphs for each of these types of motion have been plotted below. These have been done using Excel, which you can use to check your plotted graphs.

PART A – Constant motion

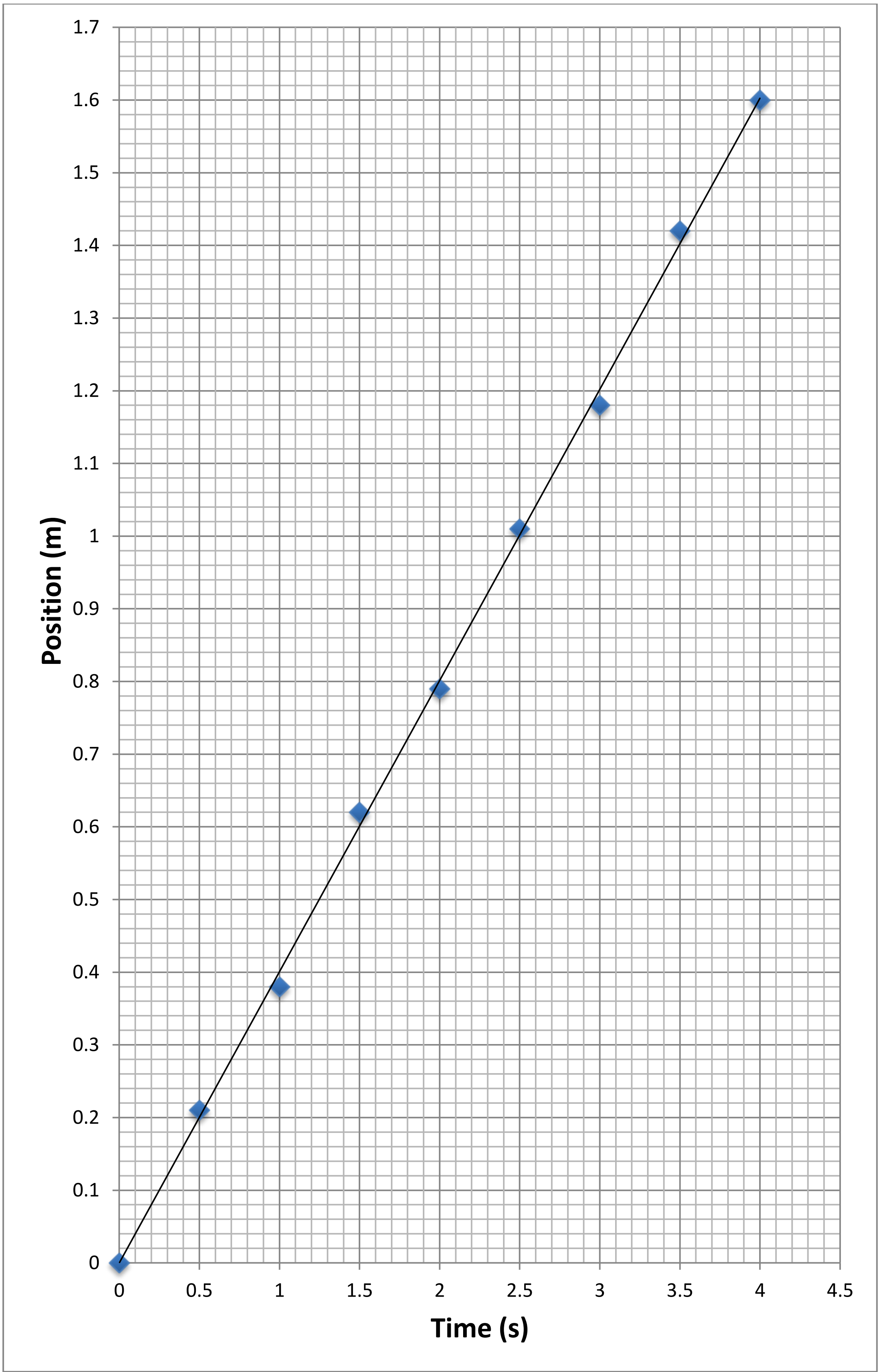


Figure 51 Graph of position vs time for constant motion in a positive direction

Some reminders about drawing scientific graphs:

- When you construct the axes for your graphs, they should choose a scale that fits with your measurements.
- When you number the scale on each axis, the numbers should **not be equal to your data points**. The scale should be even, and should be chosen so that points can be easily plotted.
- The diagrams below show a few examples of incorrect or unhelpful numbering of axes:

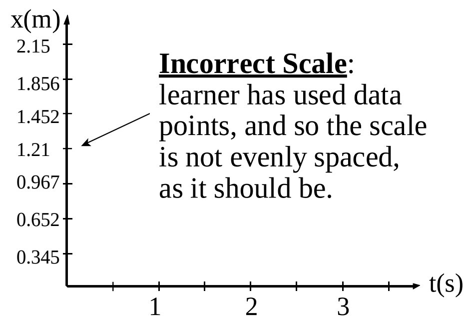


Figure 52 Graph with incorrect scale

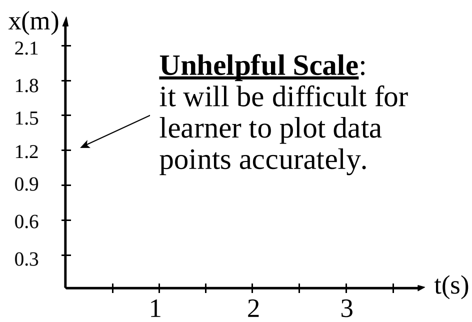


Figure 53 Graph with unhelpful scale

- An example of a useful scale is shown below:

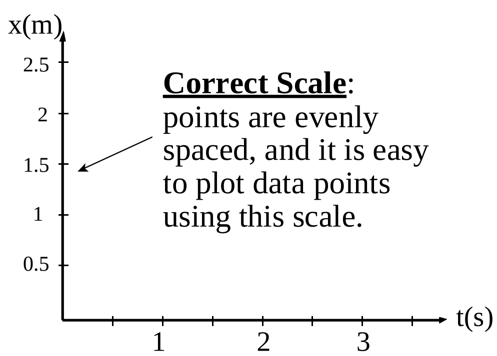


Figure 54 Graph with correct scale

- The shape of your position vs time graph for constant motion should be a straight line graph with a positive gradient, as the example shows.

PART B – Increasing motion

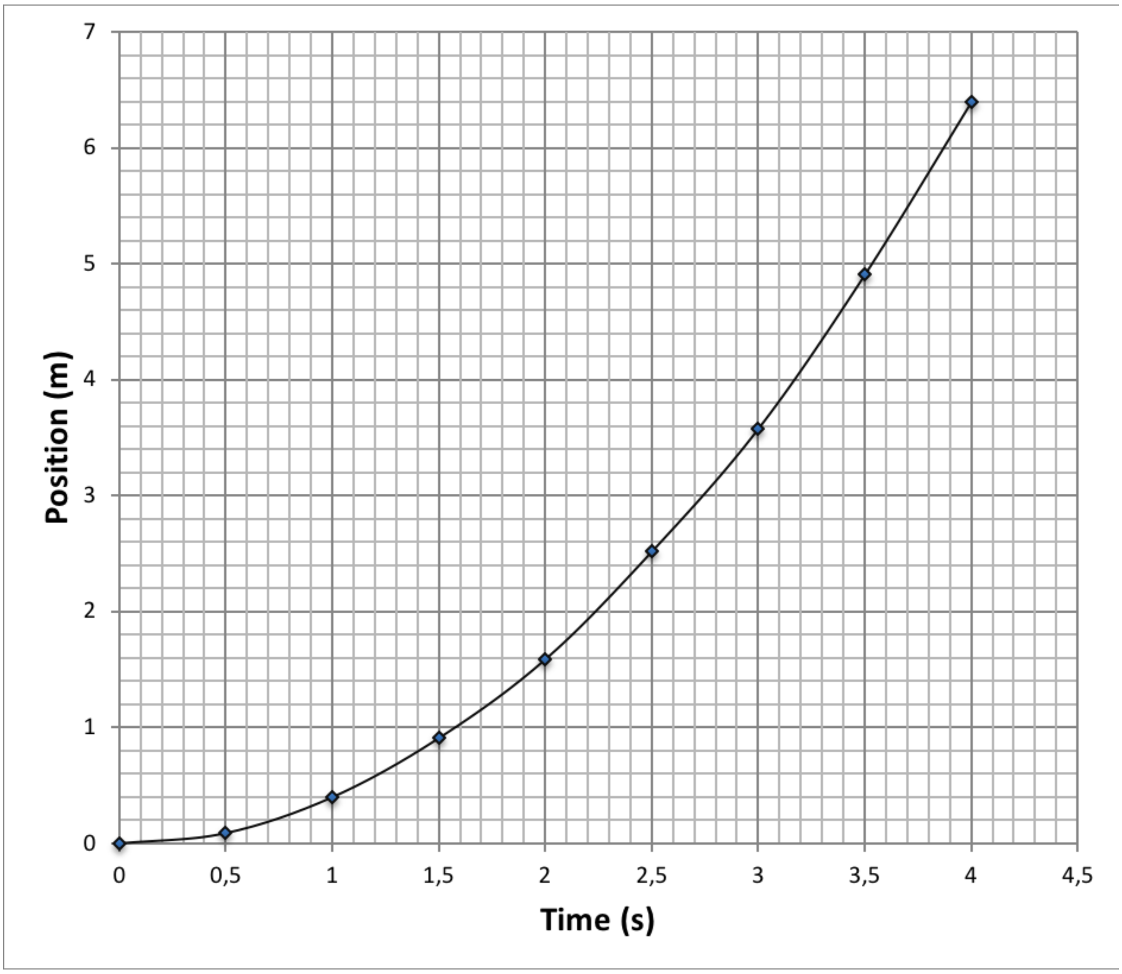


Figure 55 Graph of potion vs time for increasing speed in a positive direction

- The shape of your position vs time graph for increasing motion in a positive direction should be a curved line with an increasing gradient, as the example shows.

You can extend your knowledge from this activity to develop an understanding of the general shapes of graphs for different kinds of motion. These are summarised below:

For constant (uniform) motion:

The position-time graphs for different types of constant motion are shown below:

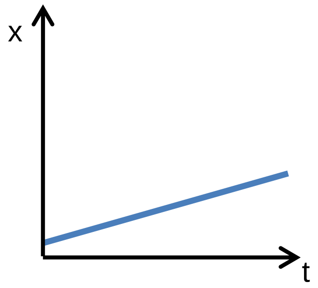


Figure 56 Position-time graph for slow constant motion in a positive direction

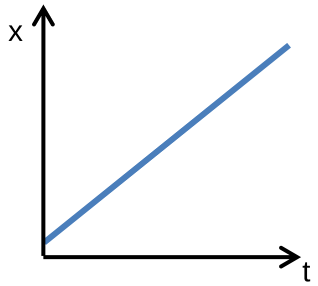


Figure 57 Position-time graph for fast constant motion in a positive direction

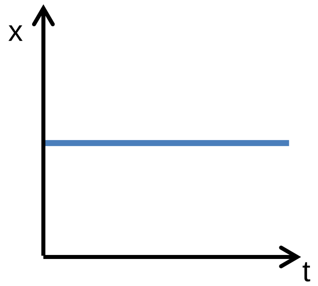


Figure 58 Position-time graph for no motion (velocity is zero)

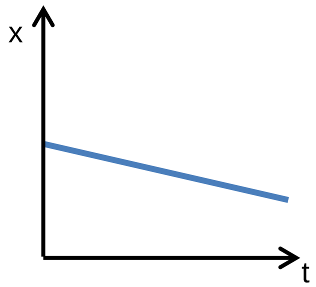


Figure 59 Position-time graph for slow constant motion in a negative direction

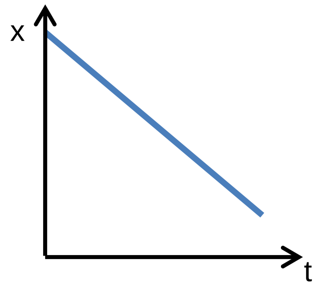


Figure 60 Position-time graph for fast constant motion in a negative direction

The position-time graph is a straight line for constant motion. On a position-time graph, the *value* gives the object's position at a specific time. The *gradient* (slope) of the graph tells you the *velocity* (how fast the movement is, and in what direction). Using an equation you write this as:

$$\text{velocity} = \text{gradient of x-t graph} = \frac{\Delta x}{\Delta t}$$

For changing motion:

Motion does not always have a constant velocity. When an object is accelerated with constant acceleration, you call this *uniformly accelerated motion*. Here the velocity is changing at a constant rate. The position-time graphs for uniformly accelerated motion have a curved shape. Some examples of position-time graphs for uniformly accelerated motion are shown below:

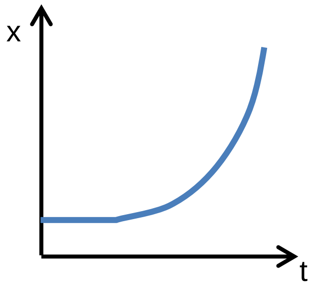


Figure 61 Position-time graph for motion getting faster in a positive direction

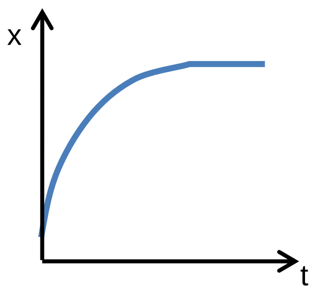


Figure 62 Position-time graph for motion slowing down in a positive direction

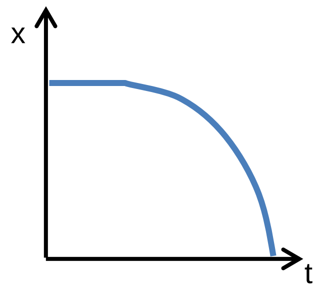


Figure 63 Position-time graph for motion getting faster in a negative direction

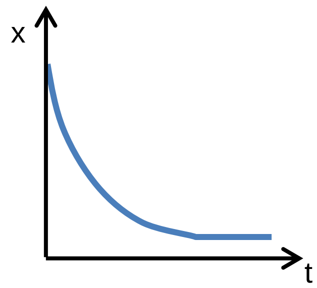


Figure 64 Position-time graph for motion slowing down in a negative direction

Activity 2: Interpret a graph of position vs time

Purpose

It is important to be able to understand the information that a position vs time graph tells you. In this activity you will interpret the motion described by a position vs time graph, and compare your answers with a worked solution.

Suggested time: [20 minutes]

What you will do:

The position vs time graph shown below describes Andile's movement. Study the graph and answer the questions that follow.

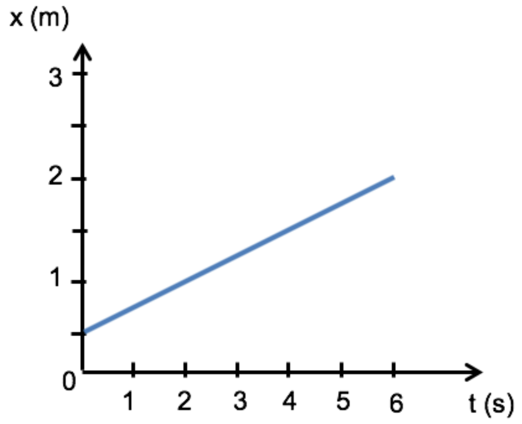


Figure 65 Position-time graph of Andile's movements

1. What was Andile's position at a time of 6 seconds?
2. Describe Andile's motion in words.
3. What was Andile's total displacement?

Sindi started from a position of 2,5 m and walked for 6 seconds with the same speed as Andile, but in the opposite direction to Andile's motion.

4. On the same graph as the one representing Andile's motion, draw a sketch to show what the position-time graph would look like for Sindi's movement.
5. Will Sindi and Andile pass each other? If so, at what time?

Solutions

1. Andile's position at a time of 6 seconds was 2 m. You can work this out from the graph by drawing a line from the 6 second mark on the time axis until it meets the line of the graph, and then from this point draw a line across to the position axis and read the value there.

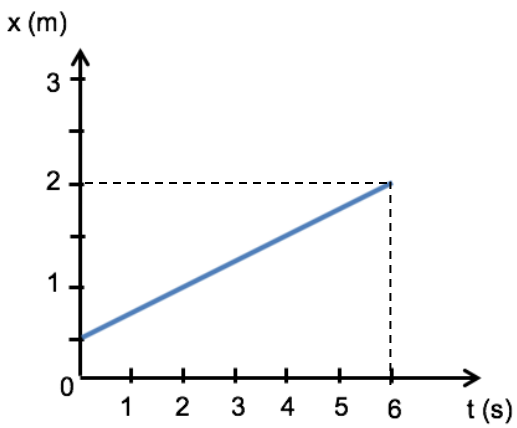


Figure 66 Position-time graph showing Andile's position at a time of 6 seconds

2. Andile moved from a starting point of 0,5 m with a uniform (constant) velocity in the positive direction.
3. His total displacement is his change of position from the start to the end of the movement. Therefore $\Delta x = x_f - x_i = 2\ m - 0,5\ m = 1,5\ m$.
4. The position-time graph for Sindi's motion is shown with the dark line below:

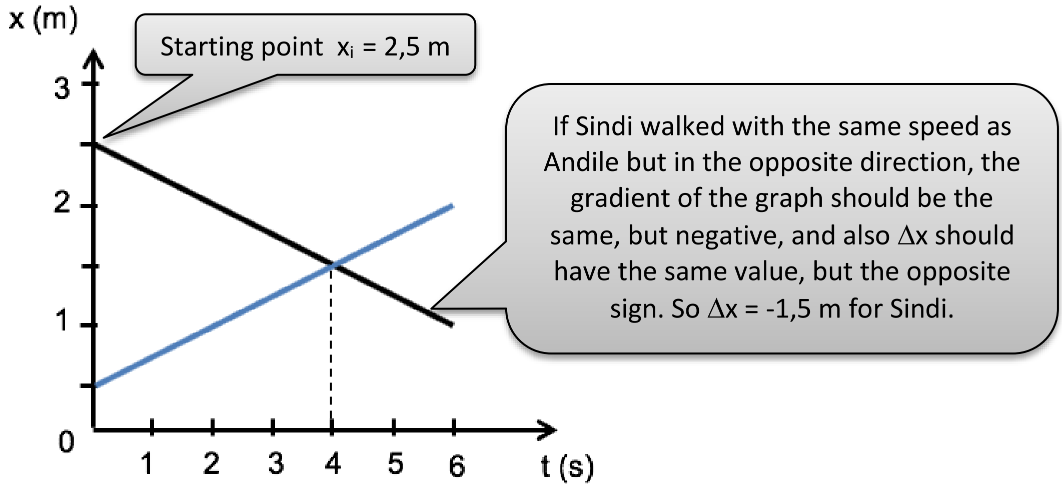


Figure 67 Position-time graph with Andile and Sindi's movements

5. Yes, Sindi and Andile pass each other at a time of 4 seconds (you can see this from where the graphs meet, or intersect, shown by a dotted line above).

Activity 3: Test your understanding of graphs of position vs time

Purpose

In this activity you will reinforce the learning you have done to now by testing your understanding of position vs time graphs.

Suggested time: [20 minutes]

What you will do:

Answer the following questions:

1. The position-time graph below describes the motion of a man on a bicycle.

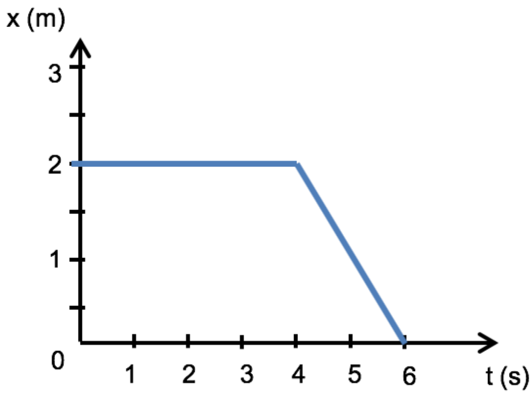


Figure 68 Position-time graph of a man on a bicycle

   a. Calculate the man's velocity between 0 s and 4 s.
   b. Calculate the man's velocity between 4 s and 6 s.

2. Describe in words how you would need to walk to create the following position-time graphs:

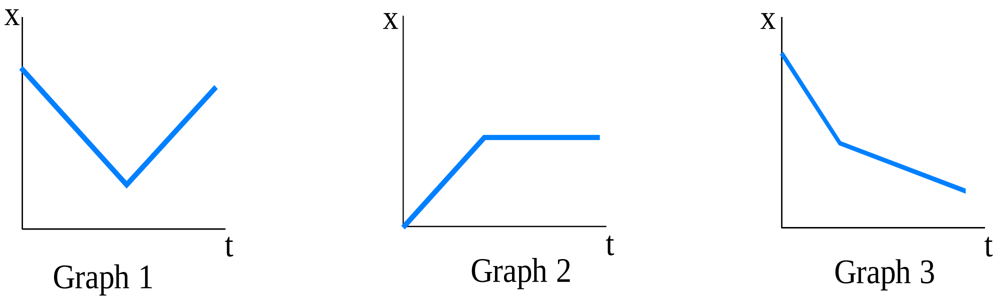


Figure 69 Position-time graphs of three different movements

3. Sketch the shape of the position-time graphs for the following motion:
   a. Start at the zero position, and walk in a positive direction with a constant velocity.
   b. Start at some positive position, and walk in a negative direction with a uniform velocity.
   c. Stand still for some time at a positive position, then walk in a negative direction with uniform velocity.

4. The position-time graph below describes the motion of a woman who is running.

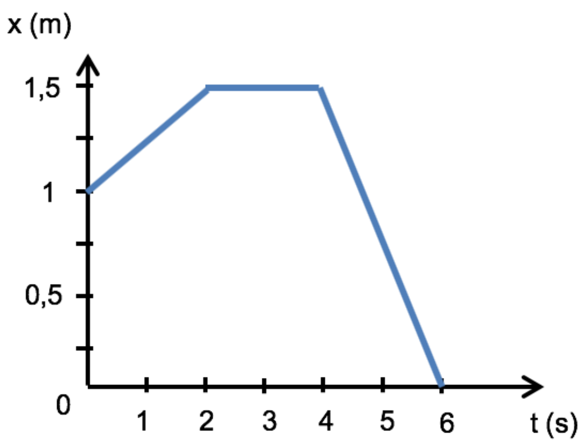


Figure 70 Position-time graph of a woman who is running

a. Describe the woman's movement during the 6 seconds.

b. Calculate the woman's velocity between

   i. 0 s and 2 s

   ii. 2 s and 4 s

   iii. 4 s and 6 s

Solutions

1.

You read these values from the graph, as the diagram below shows

a. Velocity = $\frac{\Delta x}{\Delta t} = \frac{2\text{ m} - 2\text{ m}}{4\text{ s} - 0\text{ s}}$ = 0 m·s^{-1}

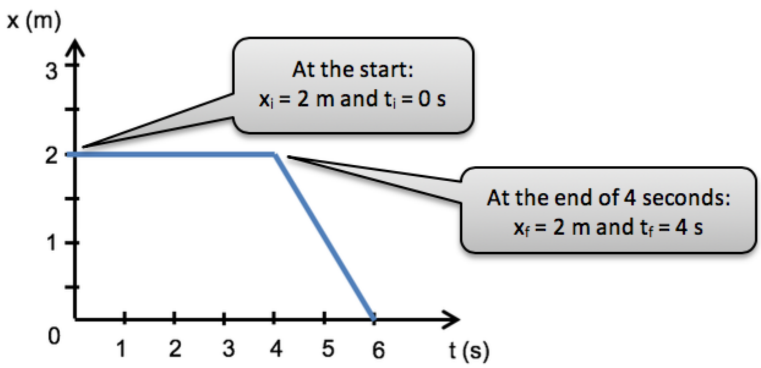


Figure 71 Position-time graph with labels for constant velocity section

b. Velocity = $\frac{\Delta x}{\Delta t} = \frac{0\text{ m} - 2\text{ m}}{6\text{ s} - 4\text{ s}}$ = -1 m·s^{-1}

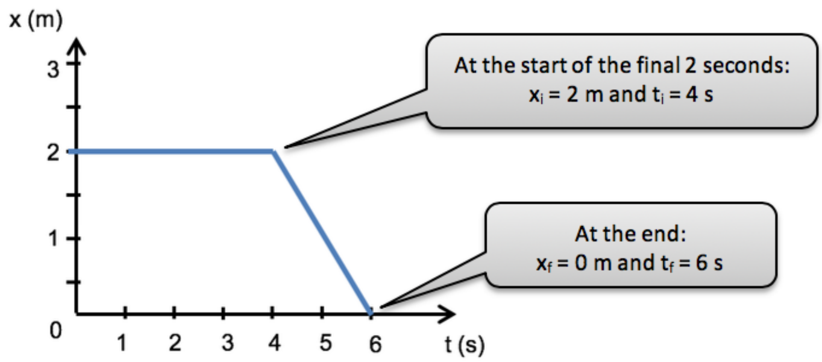


Figure 72 Position-time graph with labels for decreasing velocity section

2. Graph 1 = walk in a negative direction with uniform velocity, then in a positive direction with uniform velocity

   Graph 2 = walk in a positive direction with uniform velocity, then stand still

   Graph 3 = walk in a negative direction with fast uniform velocity, then in a negative direction with a slower uniform velocity

3.

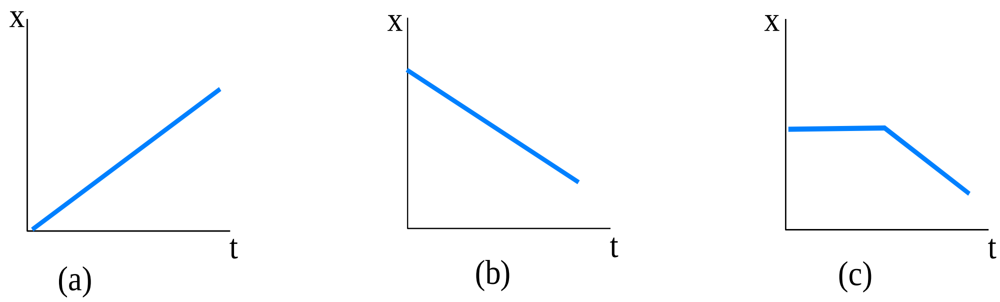


Figure 73 Position-time graphs of three kinds of motion

4.
   a. The woman starts at a position of 1 m and runs with a constant velocity in a positive direction for 2 seconds. She then stops for 2 seconds at a position of 1,5 m, and then she runs in a negative direction with a constant velocity for 2 seconds until she reaches a position of 0 m.

   b.
      i. Velocity between 0 s and 2 s:

$$v = \frac{\Delta x}{\Delta t} = \frac{1{,}5\ \text{m} - 1\ \text{m}}{2\ \text{s} - 0\ \text{s}} = 0{,}25\ \text{m·s}^{-1}$$

      ii. Velocity between 2 s and 4 s:

$$v = \frac{\Delta x}{\Delta t} = \frac{1{,}5\ \text{m} - 1{,}5\ \text{m}}{4\ \text{s} - 2\ \text{s}} = 0\ \text{m·s}^{-1}$$

iii. Velocity between 4 s and 6 s:

$$v = \frac{\Delta x}{\Delta t} = \frac{0\text{ m} - 1{,}5\text{ m}}{6\text{ s} - 4\text{ s}} = -0{,}75\text{ m·s}^{-1}$$

[Wordbox: MAIN IDEA:

- On a position-time graph, the *value* gives the object's position at a certain time
- The *gradient* (slope) of the graph tells us the *velocity*:

$$\text{velocity = gradient of x-t graph} = \frac{\Delta x}{\Delta t}$$]

Graphs of velocity vs time

You can also use graphs of velocity vs time to represent the motion of an object. The velocity vs time graph is constructed with velocity values given by the y-axis, and time values given on the x-axis. The graph below is an example of a velocity-time graph.

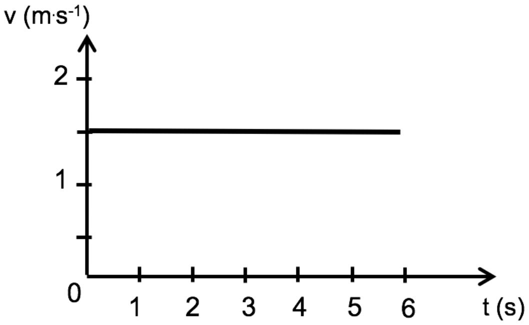


Figure 74 A graph of velocity vs time

Activity 3: Use a simulation to construct graphs of velocity vs time for uniform motion

Purpose

A velocity vs time graph is constructed with velocity values given on the vertical axis, and time values given on the horizontal axis. In this activity you will construct graphs of velocity vs time using the Moving Man simulation for uniform motion (constant velocity).

Suggested time: [30 minutes]

What you will do:

1. Go to the web address to run the simulation called "The Moving Man": https://connexions.github.io/simulations/moving-man/
2. At the top left of the screen click on the tab labelled Charts:

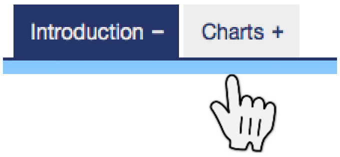


Figure 75 Click on the tab labelled Charts

3. You will see a screen with the man standing at the top, and three charts beneath labelled "Position", "Velocity" and "Acceleration". You are not going to use the acceleration chart in this activity, so you can delete it from the screen by clicking on the small arrow at the top right of this chart, as the following picture shows:

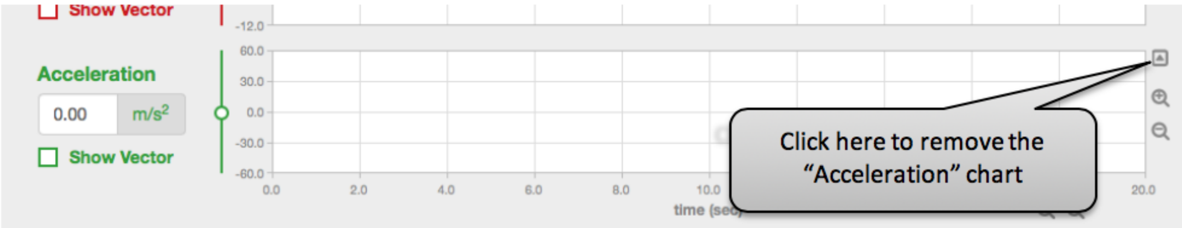


Figure 76 How to remove the Acceleration chart on Moving Man simulation

4. Click on the window just beneath the red "Velocity" label and enter a velocity of 1.00 m/s.

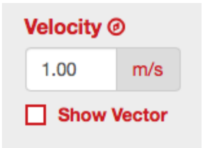


Figure 77 How to change the velocity value

5. *Think about this:* What direction do you expect the man to walk in if he moves with a positive velocity?
6. Click on the "start" button at the bottom of the screen (with the large blue dot in the middle) and the man will start walking with this velocity of 1 m/s. Is this the direction you expected him to move in?
7. If you want to pause the timer you can push the "pause" button at the bottom of the screen (where the "Start" button was). Pause the man's movement when he gets past the house.
8. Notice the shape of the position and velocity graphs that are formed.
9. Sketch the shapes of these graphs on the axes below for this movement.

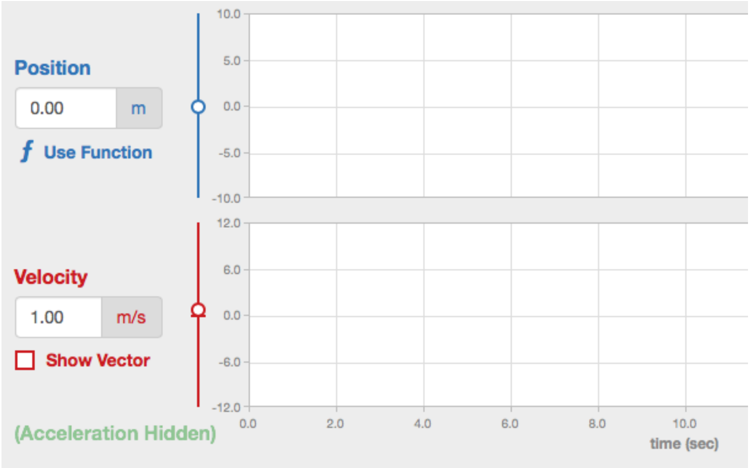


Figure 78 Axes for position and velocity graphs for velocity of 1 m/s

10. Click the button underneath the charts that is labelled "Clear". The graphs should disappear from the chart windows.
11. Enter a velocity of -2.00 m/s in the velocity window.
12. *Think about this:*
    a. What direction do you expect the man to walk in if he moves with a negative velocity?
    b. What do you expect the position and velocity graphs to looks like for a negative velocity?
13. Now click the "start" button again. Notice the graphs that are formed as the man moves. Pause his movement when he reaches the tree.
14. Notice the shape of the position and velocity graphs that are formed.
15. Sketch the shapes of these graphs on the axes below for this movement.

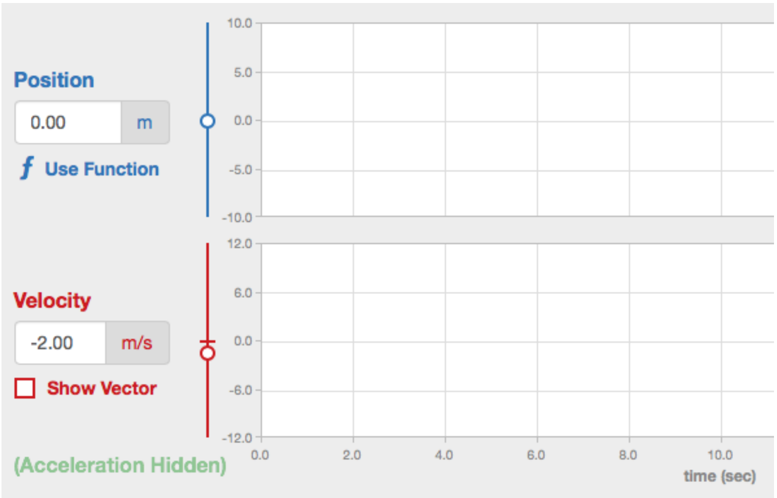


Figure 79 Axes for position and velocity graphs for velocity of -2 m/s

16. Click the button underneath the charts that is labelled “Clear”. In the window for position values enter a position of -8.00 m.
17. Enter a velocity of 0.00 m/s in the velocity window.
18. Now click the “start” button again. Notice the graphs that are formed as the man remains standing.
19. Notice the shape of the position and velocity graphs that are formed.
20. Sketch the shapes if these graphs on the axes below for this movement.

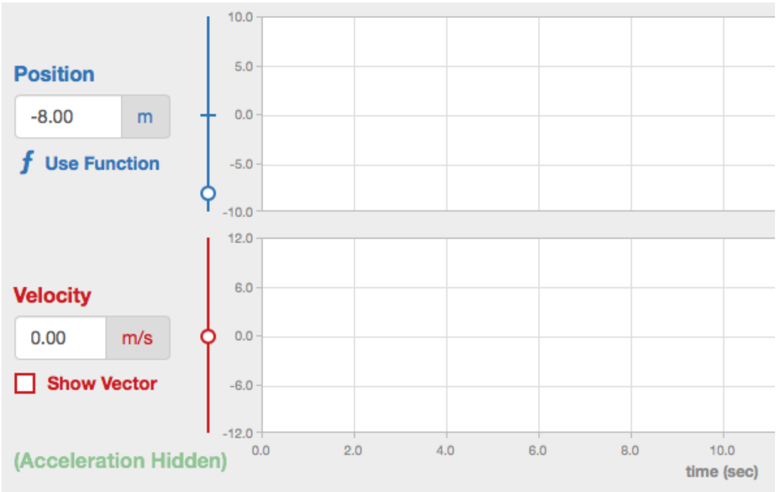


Figure 80 Axes for position and velocity graphs for velocity of 0 m/s

Guided reflection

If you have had difficulty in working out how to use the simulation, a helpful 4-minute video of this can be seen at: https://www.youtube.com/watch?v=xmCvpsQRKyo.

When the man starts at the zero position and moves with a **positive velocity** (v = 1 m/s), the position and velocity graphs look like this:

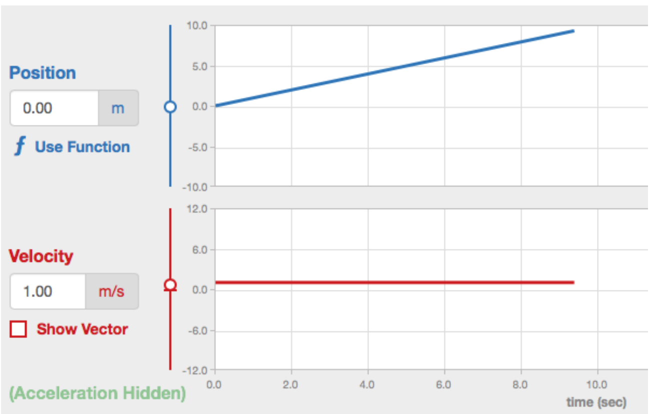


Figure 81 Position and velocity graphs for velocity of 1 m/s

Both of these graphs describe *exactly the same movement*. The position graph plots the values of his position against time, while the velocity graph plots the values of his velocity, and this graph doesn't tell you anything about his exact position.

When the man starts at a positive position and moves with a *negative velocity* (v = -2 m/s), the position and velocity graphs look like this:

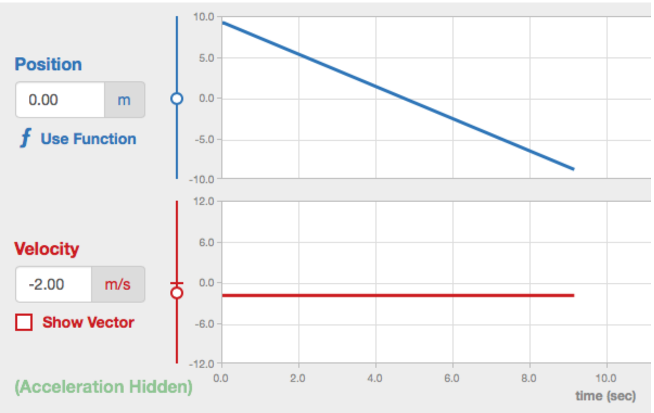


Figure 82 Position and velocity graphs for velocity of -2 m/s

Notice that the gradient of the position graph is negative when he moved with a negative velocity.

When the man starts at a negative position and has a *velocity of zero* (no motion), the position and velocity graphs look like this:

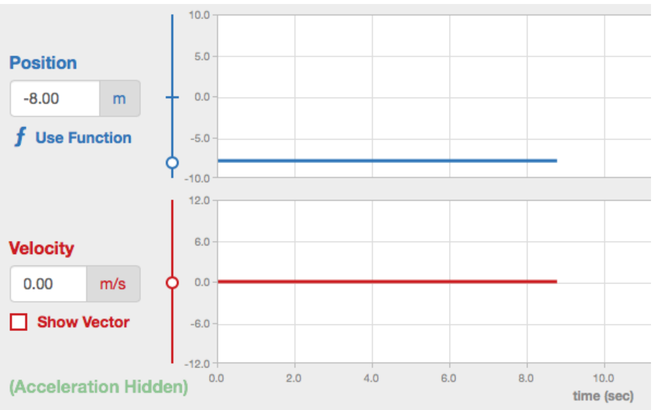


Figure 83 Position and velocity graphs for velocity of 0 m/s

Summary of graphs of velocity vs time

From these graphs you can draw some *general conclusions about velocity-time graphs*:

- The velocity vs time graph is constructed with velocity values given on the vertical axis.
- Velocity values that are above the horizontal time-axis are positive, which means that the direction of the motion is positive.
- Velocity values that are below the time-axis are negative, which means that the direction of the motion is negative.
- When there is *no movement (zero velocity),* the velocity-time graph is on the time-axis.

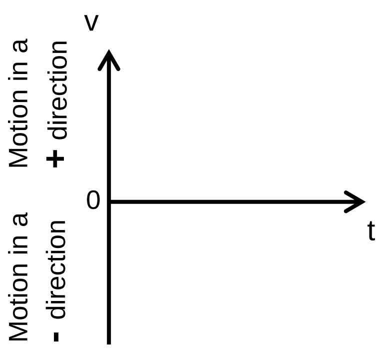


Figure 84 Positive and negative directions on a velocity-time graph

- For ***uniform*** motion (motion with a constant velocity) the velocity-time graph is a straight horizontal line.
- The velocity-time graphs for different types of ***uniform*** motion are shown below:

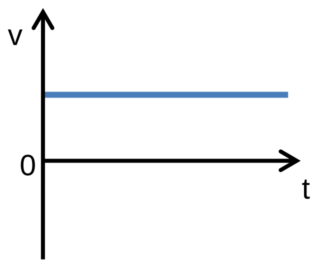


Figure 85 Velocity-time graph showing uniform motion in a positive direction

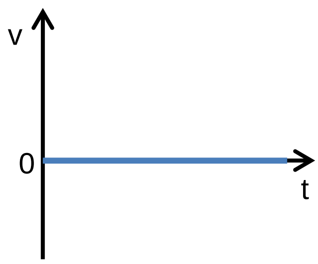


Figure 86 Velocity-time graph showing no motion (v=0)

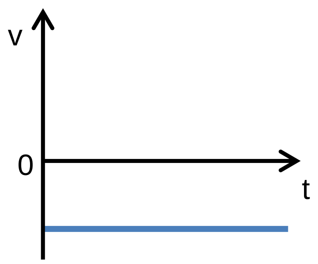


Figure 87 Velocity-time graph showing uniform motion in a negative direction

Activity 4: Use a simulation to construct graphs of velocity vs time and acceleration vs time for accelerated motion

Purpose

Velocity is not always constant, but sometimes changes gradually over time. This is called **accelerated** motion. In this activity you will construct graphs of velocity vs time using the Moving Man simulation for uniformly accelerated motion (where the acceleration is constant).

Suggested time: [30 minutes]

What you will do:

1. Go to the web address to run the simulation called "The Moving Man": https://connexions.github.io/simulations/moving-man/
2. At the top left of the screen click on the tab labelled "Charts".
3. This time you want to keep all three charts, labelled "Position", "Velocity" and "Acceleration".
4. The scale for the acceleration graph is very large, so to see this graph more clearly click the "Zoom in" button (which has a picture of a magnifying glass with a + sign in it) on the right of the acceleration window about 7 times, until the scale has a maximum value of 7.5.
5. In the window for acceleration values enter an acceleration of 1.00 m/s^2.
6. *Think about this:*
    a. How do you expect the position-time graph to look if the man is moving with a positive acceleration?
    b. How do you expect the velocity-time graph and the acceleration-time graph to look if the man is moving with a constant positive acceleration?
7. Click the "Start" button and observe the man's movement. Click the "Pause" button when he gets to the house.
8. Notice the shape of the position, velocity and acceleration graphs that are formed.
9. Sketch the shapes of these graphs on the axes below for this movement.

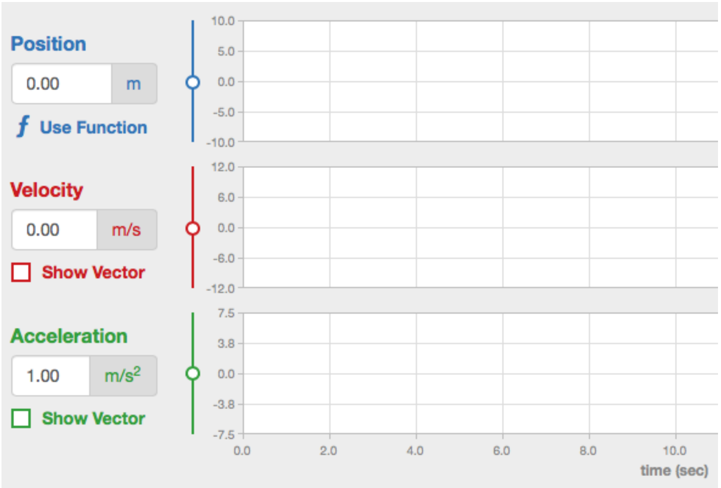


Figure 88 Axes for graphs of motion for acceleration of 1 m/s^2

10. Click the button at the bottom right of the screen labelled "Reset All". This clears everything and places the man at the origin (zero position) again. You will need to click the "Zoom in" button again to adjust the scale for the acceleration graph.
11. In the window for acceleration values enter an acceleration of -1.00 m/s^2.
12. *Think about this:* How do you expect the position, velocity and acceleration graphs to look if the man is moving with a constant negative acceleration?
13. Now click the "start" button again. Notice the graphs that are formed as the man moves. Pause his movement when he reaches the tree.
14. Notice the shape of the position, velocity and acceleration graphs that are formed.
15. Sketch the shapes if these graphs on the axes below for this movement.

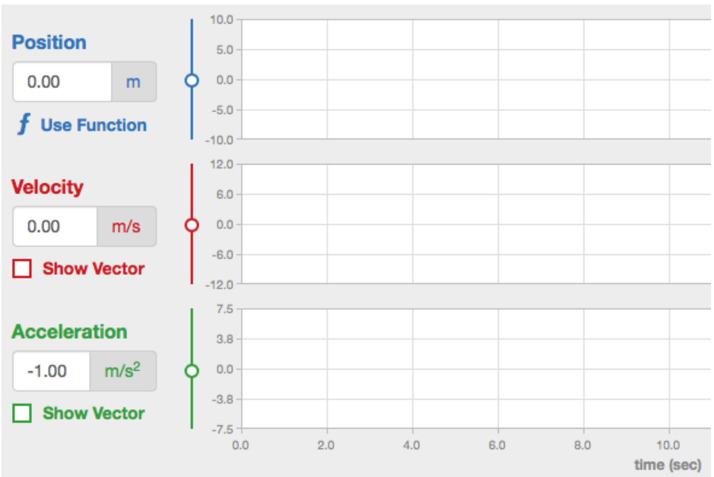


Figure 89 Axes for graphs of motion for acceleration of -1 m/s^2

16. Explore the shapes of the position, velocity and acceleration graphs when you input the following values:

    a. v = 4.00 m/s and a = -2.00 m/s^2

b. v = -4.00 m/s and a = 2.00 m/s²

c. Any other motion that you are curious about

Guided reflection

When the man starts at the zero position and moves with a *positive acceleration* (a = 1 m/s²), the position, velocity and acceleration graphs look like this:

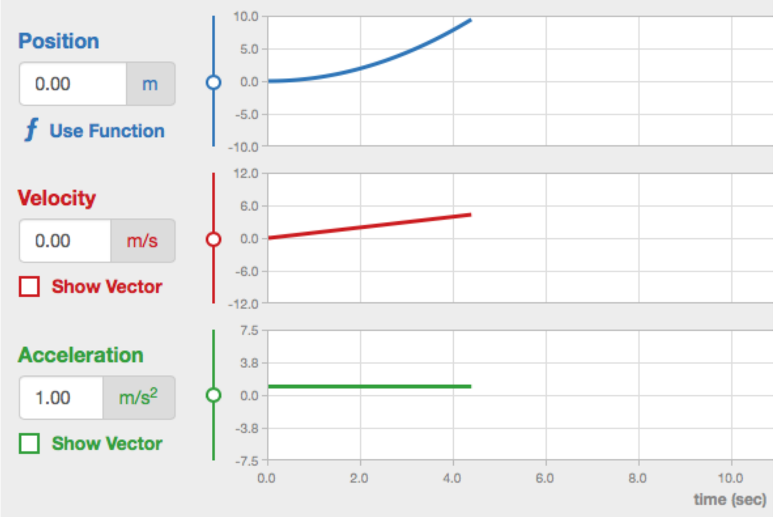


Figure 90 Graphs of motion for acceleration of 1 m/s²

When the man starts at the zero position and moves with a *negative acceleration* (a = -1 m/s²), the position, velocity and acceleration graphs look like this:

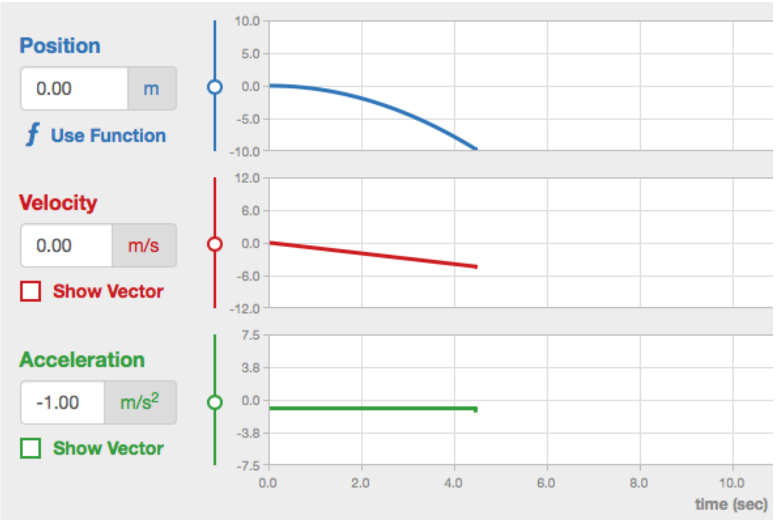


Figure 91 Graphs of motion for acceleration of -1 m/s²

The shapes of the position, velocity and acceleration graphs are shown for the following values:

a.

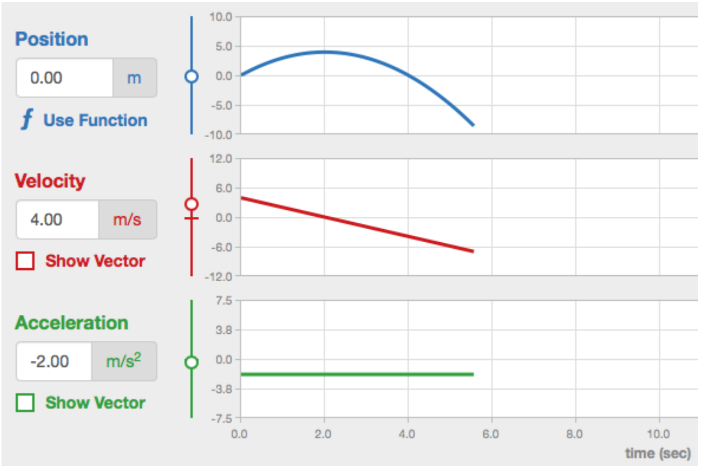


Figure 92 Graphs of motion for v = 4 m/s and a = -2 m/s²

b.

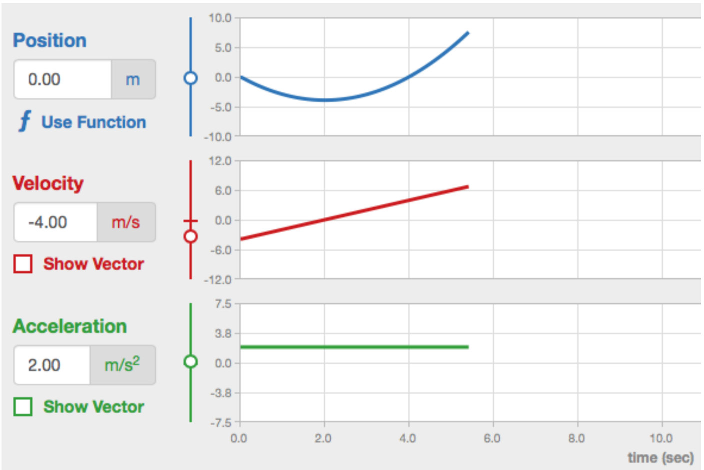


Figure 93 Graphs of motion for v = -4 m/s and a = 2 m/s²

From these investigations the following conclusions can be made:

- For *uniformly accelerated* motion the velocity-time graph is a straight line that is not horizontal.
- The *gradient* of the velocity-time graph tells you the value of the acceleration of the object.
  - A positive gradient on the velocity-time graph shows that the object has a positive acceleration.
  - A negative gradient on the velocity-time graph shows that the object has a negative acceleration.
  - The equation that you use to calculate the acceleration from the velocity-time graph is:

$$a = \text{gradient of v-t graph} = \frac{\Delta v}{\Delta t}$$

The velocity-time graphs for different types of *uniformly accelerated* motion are shown below:

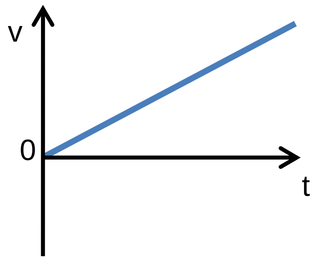


Figure 94 Velocity-time graph for motion in a positive direction, getting faster

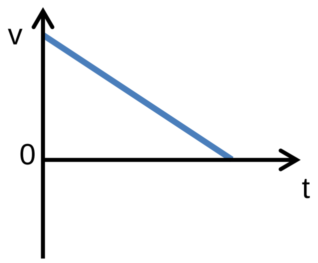


Figure 95 Velocity-time graph for motion in a positive direction, slowing down

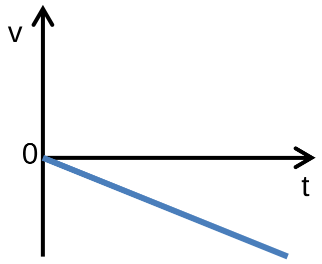


Figure 96 Velocity-time graph for motion in a negative direction, getting faster

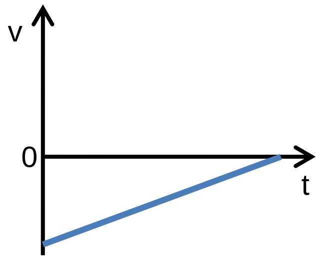


Figure 97 Velocity-time graph for motion in a negative direction, slowing down

[Wordbox: MAIN IDEAS:

- For *uniform motion*, the velocity-time graph is a straight, horizontal line. The line is above the time-axis for movement in the positive direction, and below the time-axis for movement in the negative direction.
- When there is *no motion (zero velocity),* the velocity-time graph is on the time-axis.
- On a velocity-time graph, the **value** gives the object's velocity at a certain time
- The *area underneath* the velocity-time graph gives a value for the *displacement*.
- The *gradient* of the velocity-time graph gives a value for the *acceleration*.

  acceleration = gradient of v-t graph = $\frac{\Delta v}{\Delta t}$]

Graphs of acceleration vs time

An acceleration vs time graph has acceleration values on the y-axis, and time values on the x-axis. Acceleration values that are above the time-axis are positive, and acceleration values that are below the time-axis are negative.

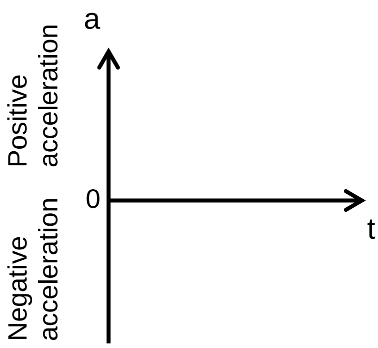


Figure 98 Positive and negative acceleration on an acceleration-time graph

Since you will only look at motion that has uniform acceleration, the acceleration-time graph is always a horizontal line. The sketches of the three acceleration-time graphs for uniformly accelerated motion are shown below:

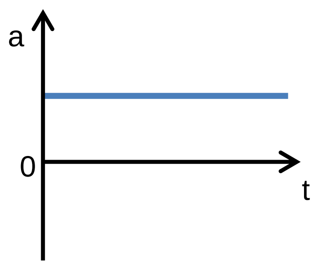


Figure 99 Acceleration-time graph for uniform (constant) positive acceleration

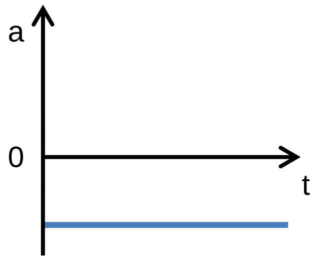


Figure 100 Acceleration-time graph for uniform (constant) negative acceleration

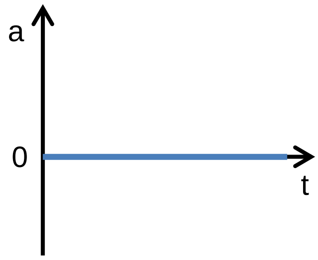


Figure 101 Acceleration-time graph for zero acceleration

Activity 5: Interpret a graph of velocity vs time

Purpose

It is important to be able to understand the information that a velocity vs time graph is telling you. In this activity you will be guided through a process of interpreting the motion described by a velocity vs time graph.

Suggested time: [20 minutes]

What you will do:

The velocity-time graph below describes a the motion of a taxi. Study this graph and answer the questions that follow:

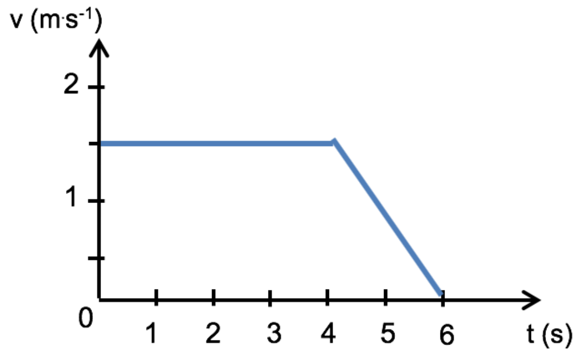


Figure 102 Velocity-time graph of the motion of a taxi

1. Describe the taxi's motion in words.
2. Calculate was the taxi's acceleration between 0 s and 4 s.
3. Calculate was the taxi's acceleration between 4 s and 6 s.
4. Sketch the ***shape*** of the acceleration-time graph for this motion on a set of axes like the ones below (you do not need to show values on your graph):

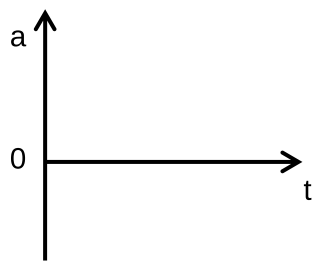


Figure 103 Axes for acceleration-time graph

5. A second taxi is moving with a constant velocity of -1 $m{\cdot}s^{-1}$ for 4 seconds, and then slows to a stop in a time of 2 seconds. Draw its velocity-time graph.
6. Sketch the ***shape*** of the acceleration-time graph for this motion on a set of axes like the ones below (you do not need to show values on your graph):

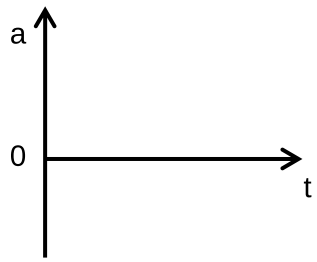


Figure 104 Axes for acceleration-time graph

Solutions

1. The taxi moved with a constant positive velocity of 1,5 m·s^{-1} for the first 4 seconds. It then slowed to a stop in a time of 2 seconds.

2. a = gradient of v-t graph = $\frac{v_f - v_i}{\Delta t} = \frac{1{,}5\ \mathrm{m{\cdot}s^{-1}} - 1{,}5\ \mathrm{m{\cdot}s^{-1}}}{4\ \mathrm{s} - 0\ \mathrm{s}}$ = 0 m·s^{-2}

3. a = $\frac{v_f - v_i}{\Delta t} = \frac{0\ \mathrm{m{\cdot}s^{-1}} - 1{,}5\ \mathrm{m{\cdot}s^{-1}}}{6\ \mathrm{s} - 4\ \mathrm{s}}$ = -0,75 m·s^{-2}

4.

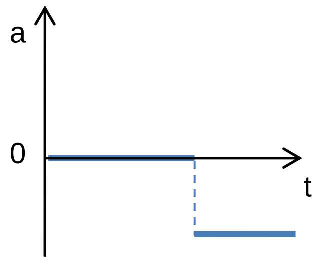


Figure 105 Shape of the acceleration-time graph for the taxi's motion

5.

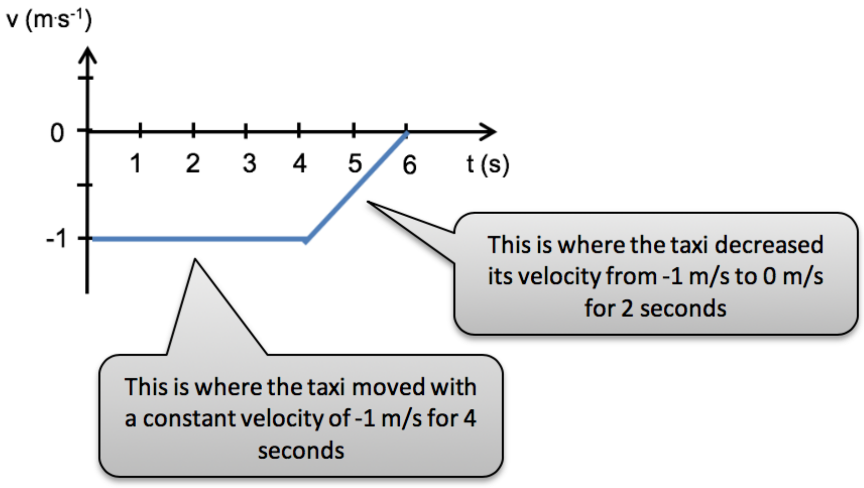


Figure 106 Shape of the velocity-time graph for the second taxi

6.

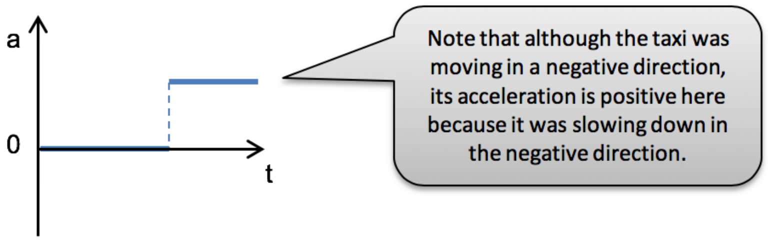


Figure 107 Shape of the acceleration-time graph for the second taxi

Finding displacement from the velocity-time graph

You can use a velocity-time graph to work out the displacement (Δx) of an object by calculating the *area underneath* the v-t graph for that time interval. This does not give you any information about the starting position of the object, it only tells you the change in position of the object.

For example, you can find the total displacement of the motion represented by the velocity-time graph below:

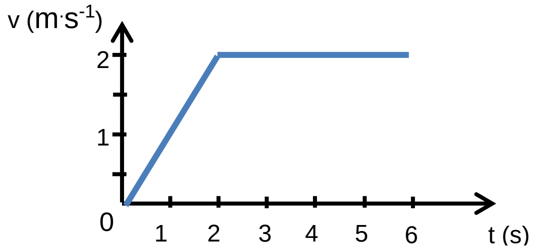


Figure 108 Velocity-time graph for finding displacement

The first thing you need to do is to break up the area underneath the graph into shapes that you know how to find the area of. This is shown in the diagram below:

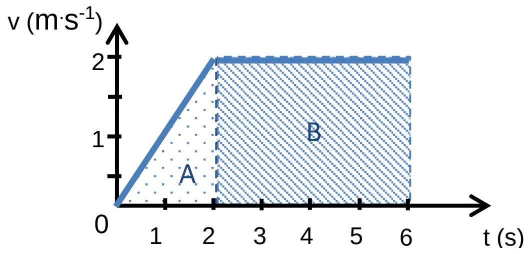


Figure 109 Shapes of the area underneath the velocity-time graph

You can then calculate the displacement from the total area:

Δx = area underneath v-t graph

= area under triangle A + area under square B

= ($\frac{1}{2}$ base of A × height of A) + (base of B × height of B)

= ($\frac{1}{2}$ × 2 s × 2 m·s^{-1}) + (4 s × 2 m·s^{-1})

= 2 m + 8 m

= 10 m

Finding change in velocity from the acceleration-time graph

In a similar way to what you have just seen, the area underneath the acceleration-time graph tells you the change in the object's velocity (Δv) during that time interval. This does not give you any information about the starting velocity of the object, it only tells you the change in velocity of the object.

For example, you can find the change in velocity of the motion represented in the acceleration-time graph given below:

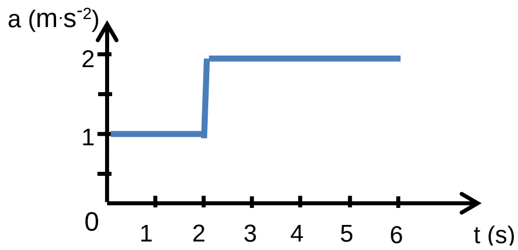


Figure 110 Acceleration-time graph for finding change in velocity

You first break up the area underneath the graph into shapes that you know how to find the area of:

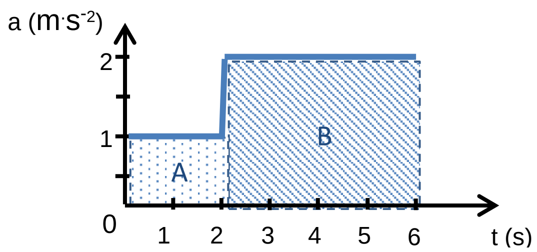


Figure 111 Shapes of the area underneath the acceleration-time graph

You can then calculate the change in velocity from the total area:

Δv = area underneath a-t graph

= area under square A + area under square B

= (base of A × height of A) + (base of B × height of B)

= $(2\text{ s} \times 1\text{ m·s}^{-2}) + (4\text{ s} \times 2\text{ m·s}^{-2})$

= $2\text{ m·s}^{-1} + 8\text{ m·s}^{-1}$ = 10 m·s^{-1}

[Wordbox: MAIN IDEAS:

- The gradient of the velocity-time graph gives the *acceleration* of the object.
- For an object with a *constant positive acceleration*, the acceleration-time graph will be a straight horizontal line above the x-axis.
- For an object with a *constant negative acceleration*, the acceleration-time graph will be a straight horizontal line beneath the x-axis.
- The area underneath the acceleration-time graph gives a value for the amount that the velocity has changed as a result of that acceleration.]

Activity 6: Test your understanding of graphs of motion

Purpose

In this activity you will reinforce the learning you have done to now by testing your understanding of graphs of motion.

Suggested time: [60 minutes]

What you will do:

Answer the following questions, and then check your understanding using the provided solutions:

1. Draw the position-time and velocity-time graphs for the following movements:
    a. Walk in a positive direction with a constant but slow velocity
    b. Walk in a negative direction with a constant but fast velocity
    c. Walk in a negative direction, starting from rest and gradually speeding up
    d. Walk in a positive direction, with a high velocity at first, and gradually slowing to a stop.

2. Cameron rode his bicycle. The velocity-time graph for his motion is shown below:

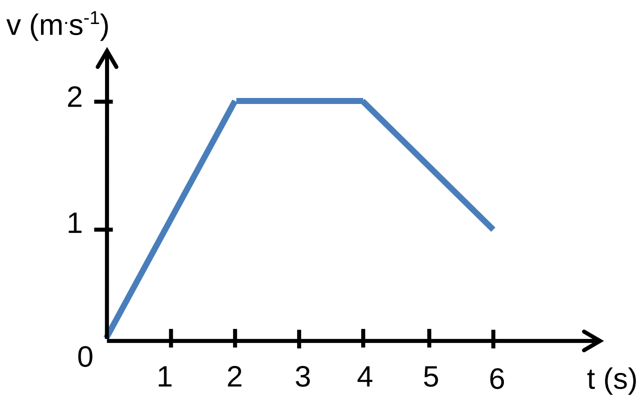


Figure 112 Velocity-time graph for Cameron's motion

a. Describe Cameron's motion in words.

b. Calculate Cameron's acceleration in the following time intervals:

   i. between 0 s and 2 s

   ii. between 2 s and 4 s

   iii. between 4 s and 6 s

c. What was Cameron's total displacement?

d. Sketch the shape of the position-time graph for the first 2 seconds of Cameron's motion.

e. Sketch the shape of the acceleration-time graph for Cameron's motion whole motion.

3. An object started from rest and accelerated uniformly for 5 seconds, as shown in the graph below:

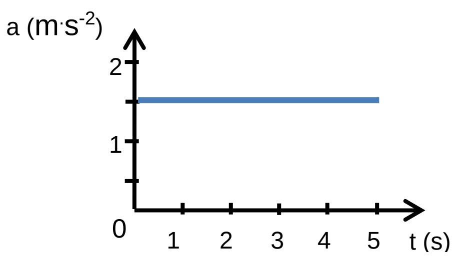


Figure 113 Acceleration-time graph for object's motion

a. What was the object's velocity after 5 seconds?

b. The object now comes to rest after slowing down for 2 seconds. Calculate the object's acceleration

c. Draw graphs of acceleration-time and velocity-time for just *this part* (described in b.) of the object's motion.

Solutions

1. The position-time and velocity-time graphs are shown below for each movement:

   a.

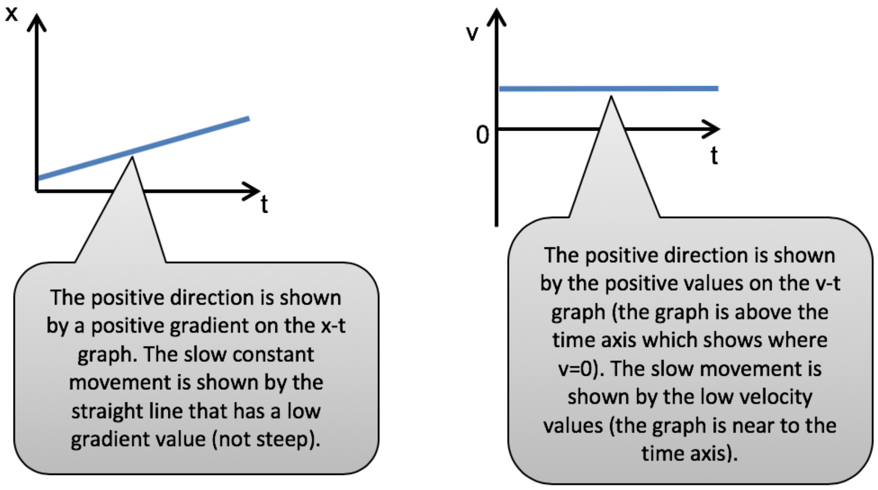


Figure 114 Position-time and velocity-time graphs for motion in a positive direction with a constant but slow velocity

   b.

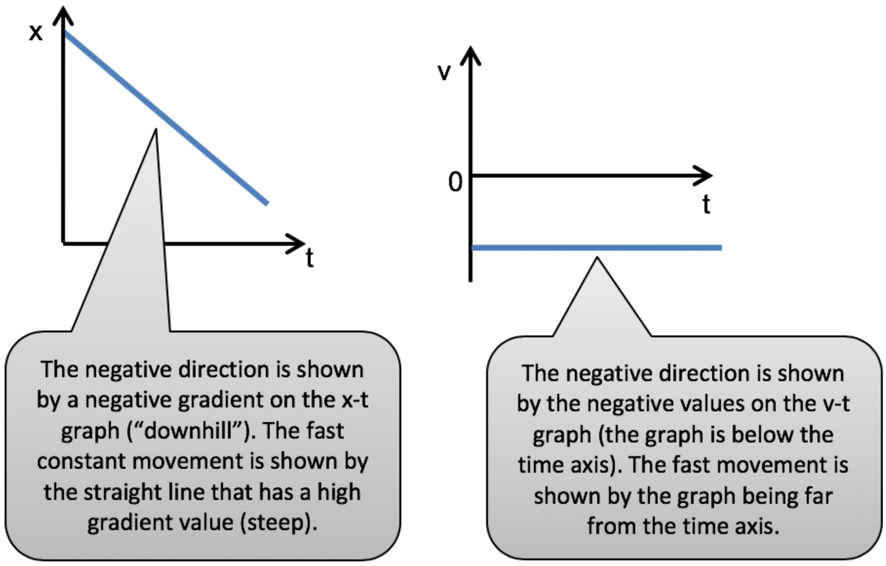


Figure 115 Position-time and velocity-time graphs for motion in a negative direction with a constant but fast velocity

c.

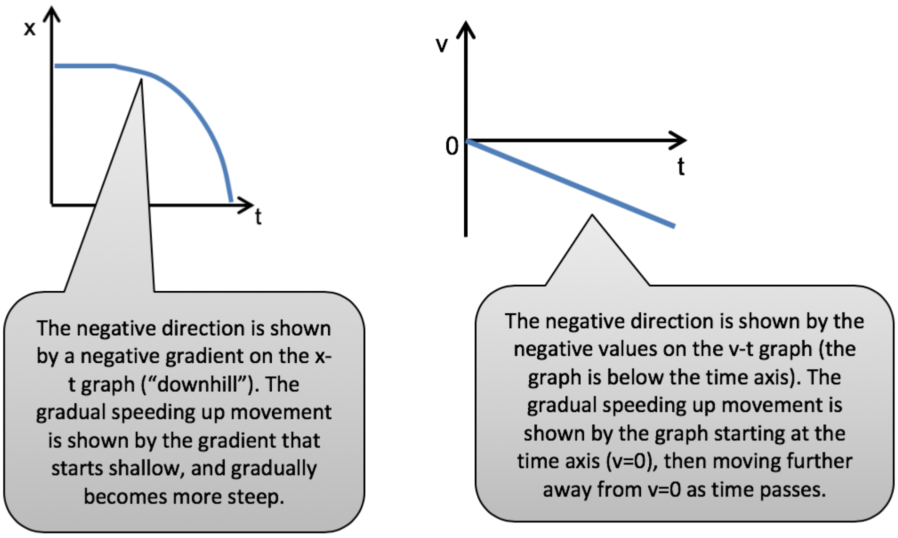


Figure 116 Position-time and velocity-time graphs for motion in a negative direction, starting from rest and gradually speeding up

d.

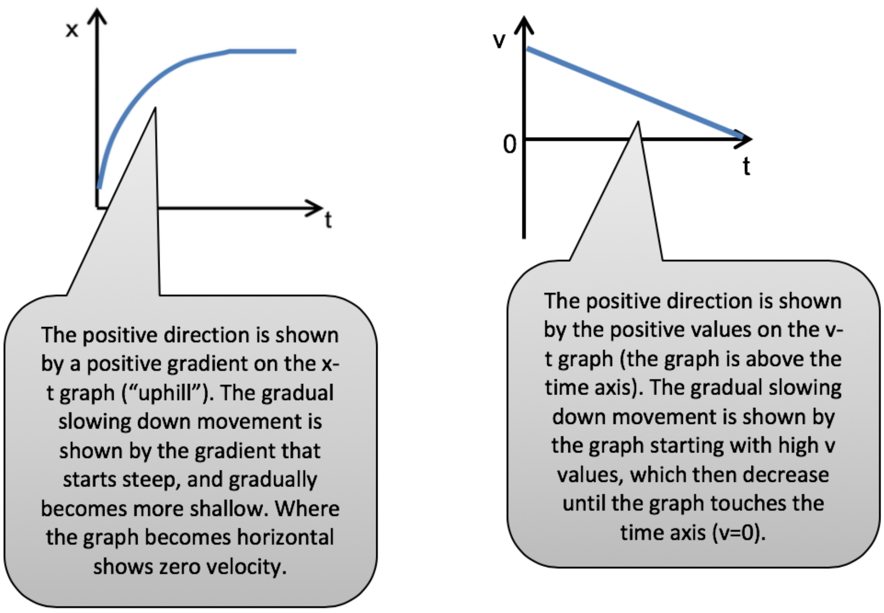


Figure 117 Position-time and velocity-time graphs for motion in a positive direction, with a high velocity at first, and gradually slowing to a stop

2.

a. The graph shows that Cameron rode with increasing velocity in the positive direction for 2 seconds. He then rode with a uniform velocity for another 2 seconds, and then he slowed down while still riding in the positive direction for another 2 seconds.

b.

i. a = gradient of v-t graph = $\frac{v_f - v_i}{\Delta t} = \frac{2\ m\cdot s^{-1} - 0\ m\cdot s^{-1}}{2\ s - 0\ s}$ = 1 m·s^{-2}

ii. a = $\frac{v_f - v_i}{\Delta t} = \frac{2\ m\cdot s^{-1} - 2\ m\cdot s^{-1}}{4\ s - 2\ s}$ = 0 m·s^{-2}

iii. a = $\frac{v_f - v_i}{\Delta t} = \frac{1\ m\cdot s^{-1} - 2\ m\cdot s^{-1}}{6\ s - 4\ s}$ = -0,5 m·s^{-2}

c. Total displacement = area under v-t graph

= area of A + area of B + area of C + area of D

= ($\frac{1}{2}$ base of A × height of A) + (base of B × height of B) + ($\frac{1}{2}$ base of C × height of C)

+ (base of D × height of D)

= ($\frac{1}{2}$ × 2 s × 2 m·s^{-1}) + (2 s × 2 m·s^{-1}) + ($\frac{1}{2}$ × 2 s × 1 m·s^{-1}) + (2 s × 1 m·s^{-1})

= 2 m + 4 m + 1 m + 2 m

= 9 m

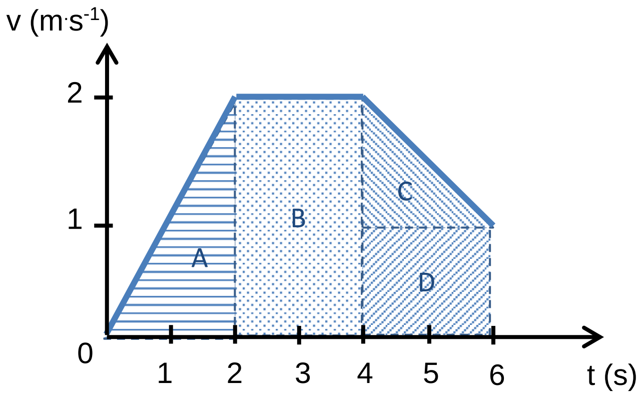


Figure 118 Shapes of the area underneath the velocity-time graph

d.

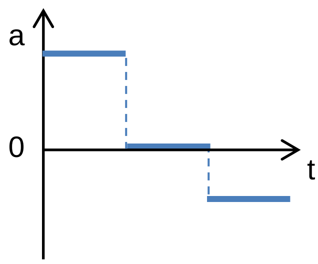


Figure 119 Acceleration-time graph for Cameron's whole motion

3.

a. To find the velocity after 5 seconds you find the area underneath the a-t graph, which is the area of the rectangle A:

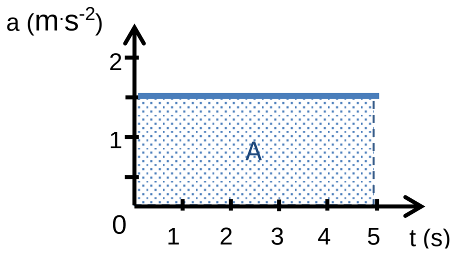


Figure 120 Area A underneath the acceleration-time graph

Δv = area underneath a-t graph

= (base of A × height of A)

= (5 s × 1,5 m·s^{-2})

= 7,5 m·s^{-1}

So the final velocity after the 5 seconds was 0 m·s^{-1} + 7,5 m·s^{-1} = 7,5 m·s^{-1}

b. The acceleration for this part of the motion is:

$$a = \frac{v_f - v_i}{\Delta t} = \frac{0\ \text{m·s}^{-1} - 7{,}5\ \text{m·s}^{-1}}{2\ \text{s} - 0\ \text{s}} = -3{,}75\ \text{m·s}^{-2}$$

c. The velocity-time and acceleration-time graphs for this part of the motion are shown below:

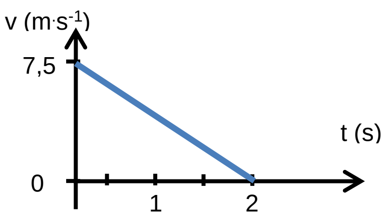


Figure 121 Velocity-time graph for object moving in a positive direction, slowing to a stop

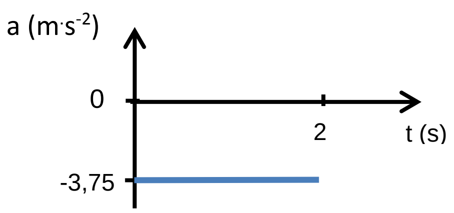


Figure 122 Acceleration-time graph for object moving in a positive direction, slowing to a stop

More resources to help you:

- A web page with some helpful explanations and quizzes on graphs of motion can be found at this address: https://www.physicsclassroom.com/Physics-Tutorial/1-D-Kinematics (Look at the links under the headings "Lesson 3 - Describing Motion with Position vs. Time Graphs" and "Lesson 4 - Describing Motion with Velocity vs. Time Graphs")
- A 7-minute video that explains the graphs of motion using a Moving Man simulation: *Motion graphs explained*: https://www.youtube.com/watch?v=rYbf_-HIJNE (Duration: 1.42)
- Two slightly longer videos that explain graphs of motion very clearly are:
  - For uniform motion: *Graphs of motion-Part 1*: https://www.youtube.com/watch?v=FK6dc9TtEuw (Duration: 14.25)
  - For accelerated motion: *Graphs of motion-Part 2*: https://www.youtube.com/watch?v=3zD8u7DO304 (Duration: 12.30, but just watch the first 10 minutes of this video).
- If you type "Youtube graphs of motion" into the Google search engine you will be able to see many more teaching videos on this section.

Unit 4: Equations of motion

Learning Outcomes

By the end of the unit, you should be able to:

- use the kinematics equations of motion to solve problems for 1-dimensional motion.

Introduction

In Unit 3 you learnt how to represent motion using graphs. Refresh your memory by reflecting on the following questions, either on your own or with a fellow student:

- How do you find the displacement from a graph of velocity vs time?
- How do you find the acceleration from a graph of velocity vs time?

You may recall that the three graphs that you use to represent motion are the position-time graph, the velocity-time graph and the acceleration-time graph. The three graphs are related to each other, and all three of them can be used to describe the same kinds of motion. As you saw in Unit 3, you can use the graphs of motion to work out what will happen with an object's motion at some point in time. In this unit you will learn about the equations of motion, which are tools that help you to solve problems related to motion.

The equations of motion

You can represent motion that has constant acceleration using equations. These equations give you a way of linking displacement, velocity and acceleration. They are very useful, as they give you a way of solving problems for different situations involving motion. (These are sometimes called the "kinematics equations of motion").

The four equations of motion for objects that have constant acceleration are:

$$v_f = v_i + a\Delta t$$

$$\Delta x = v_i\Delta t + \tfrac{1}{2}a\Delta t^2$$

$$v_f^{\,2} = v_i^{\,2} + 2a\Delta x$$

$$\Delta x = \tfrac{1}{2}(v_f + v_i)\,\Delta t$$

In these equations, v_i is the initial velocity, in units of $m{\cdot}s^{-1}$

v_f is the final velocity, in $m{\cdot}s^{-1}$

NOTE ABOUT UNITS:

- Remember that units of $m{\cdot}s^{-1}$ mean the same as m/s, which means "meters per second"
- Units of $m{\cdot}s^{-2}$ mean the same as m/s^2, which means "meters per second squared"

a is the acceleration, in $m \cdot s^{-2}$

Δt is the time interval of the motion, in seconds (s)

Δx is the displacement of the object, in meters (m)

If you are interested in seeing where these equations come from you can see a 10-minute YouTube video explaining this: *Deriving the equations of motion*: https://www.youtube.com/watch?v=vZ1shY8SAU0 (Duration: 9.43) (Note that this video uses the symbol Δd instead of Δx)

Activity 1: Apply the equations of motion

Purpose

In this activity you will work through an example problem where the equations of motion are applied.

Suggested time: [20 minutes]

What you will do:

Look through the example that is given below and fill in the missing information:

<u>Example problem</u>:

A marble was rolled to the right along a rough surface, and came to rest after it rolled for 1 metre. The acceleration of the marble was -2 $m \cdot s^{-2}$.

a. Calculate the marble's initial velocity.
b. Find the time that the marble took to come to a stop.

<u>Solution:</u>

Here you use the steps for problem solving given in Topic 1.

Problem solving step 1: Draw a diagram of the scenario, showing as much information as you can:

Problem solving step 2: **Write a list of the given information**

Given: $a =$ ________

$\Delta x =$ ________

$v_f =$ ________

Problem solving step 3: **Read the question carefully to decide what is being asked**

a. You are asked to find the marble's initial velocity, v_i.
b. You are asked to find the time for the marble to stop, Δt

Problem solving step 4: **Select an appropriate equation or scientific concept for solving this problem**

a. You know the values of a, Δx and v_f and you want to calculate v_i, so the equation that you will use is $v_f^2 = v_i^2 + 2a\Delta x$
b. You know the values of a, v_i and v_f and you want to calculate Δt, so the equation that you will use is $v_f = v_i + a\Delta t$

Problem solving step 5: **Do the calculation carefully**

a. From the equation $v_f^2 = v_i^2 + 2a\Delta x$

You can solve for v_i:

b. From the equation $v_f = v_i + a\Delta t$

You can solve for Δt:

Problem solving step 6: **Reflect on your answer**

Make sure that the values you obtained are sensible, and that you have answered the question identified in Step 3. Check that the units of your answer are correct. Remember to write any vectors with a magnitude as well as a direction.

Fill in the answers below:

a. The marble had an initial velocity of ________________.
b. The time for the marble to stop is ________________.

Solution

Your calculations for the missing steps should look something like this:

Problem solving step 1:

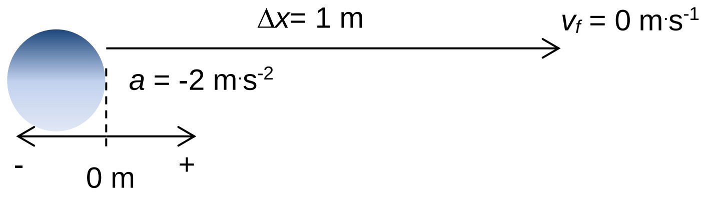


Figure 123 Diagram of the scenario

Problem solving step 2:

<u>Given</u>:

a = -2 m·s^{-2}

Δx = 1 m

v_f = 0 m·s^{-1}

Problem solving step 3:

a. You are asked to find the marble's initial velocity, v_i.

b. You are asked to find the time for the marble to stop, Δt

Problem solving step 4:

a. You know the values of a, Δx and v_f and you want to calculate v_i, so the equation that you will use is $v_f^2 = v_i^2 + 2a\Delta x$

b. You know the values of a, v_i and v_f and you want to calculate Δt, so the equation that you will use is $v_f = v_i + a\Delta t$

Problem solving step 5:

Calculation:

a. From the equation $v_f^2 = v_i^2 + 2a\Delta x$

You can solve for v_i:

$v_i^2 = v_f^2 - 2a\Delta x$

$= 0 - 2 \times (-2 \text{ m·s}^{-2}) \times 1 \text{ m}$

$= 4 \text{ m}^2\text{·s}^{-2}$

Therefore v_i = 2 m/s

To solve for v_i you subtract $2a\Delta x$ from both sides and first solve for v_i^2, and then find the square root to give you v_i

b. From the equation $v_f = v_i + a\Delta t$

You can solve for Δt:

$$\Delta t = \frac{v_f - v_i}{a}$$

$$= \frac{0 - 2 \text{ m/s}}{-2 \text{ m/}s^2}$$

$$= 1 \text{ s}$$

To solve for Δt you subtract v_i from both sides, giving:
$v_f - v_i = a\Delta t$
You can then divide both sides of this equation by a.

Problem solving step 6:

The answers are:

a. The marble had an initial velocity of +2 m/s to the right.
b. The time for the marble to stop is 1 second.

Activity 2: Test your understanding of equations of motion

Purpose

In this activity you will reinforce the learning you have done to now by testing your understanding of graphs of motion.

Suggested time: [60 minutes]

What you will do:

Answer the following questions using the equations of motion, and then check your understanding using the provided solutions:

1. A car is travelling with a velocity of 40 $m \cdot s^{-1}$ and applies its brakes. It comes to a stop after braking for 10 seconds. Calculate the acceleration of the car.
2. A cyclist starts from rest, and accelerates down a hill at 1,6 $m \cdot s^{-2}$ for 5 s.
   a. How far does she travel during the first 5 seconds?
   b. She then continues with a constant velocity for another 5 s. How far does she travel during this time?
   c. After this she cycles up a slight slope and comes to a stop after traveling for 8 m. What is her acceleration as she travels up the slope?

Solution

The solutions below have been done using the 6 steps of problem solving that you used in the previous activity.

1. Problem solving step 1:

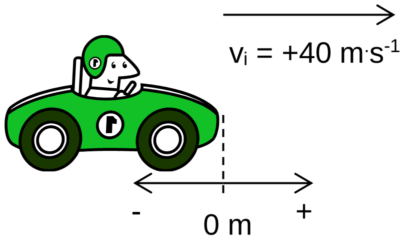


Figure 124 Diagram of the scenario

Problem solving step 2:

Given:

$v_i = +40\ m{\cdot}s^{-1}$

For the car to stop, $v_f = 0\ m{\cdot}s^{-1}$

t = 10 s

Problem solving step 3:

You are asked to calculate the acceleration of the car, so you must solve for a.

Problem solving step 4:

The equation that you will use is $v_f = v_i + a\Delta t$

Problem solving step 5:

Calculation:

From the equation $v_f = v_i + a\Delta t$

You can solve for a:

> To solve for a you subtract v_i from both sides, giving:
> $v_f - v_i = a\Delta t$
> You can then divide both sides of this equation by Δt.

$$a = \frac{v_f - v_i}{\Delta t} = \frac{0-(40\ m{\cdot}s^{-1})}{(10\ s\ )} = -4\ m{\cdot}s^{-2}$$

Problem solving step 6:

The acceleration of the car is $-4\ m{\cdot}s^{-2}$.

2. Problem solving step 1:

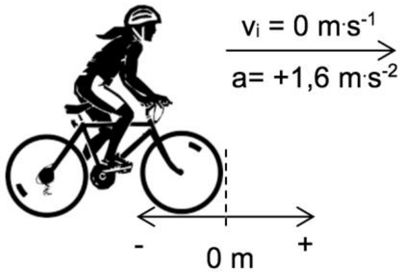


Figure 125 Diagram of the scenario

a. Problem solving step 2: Given: $a = 1{,}6\ m{\cdot}s^{-2}$ and $t = 5\ s$ and $v_i = 0\ m{\cdot}s^{-1}$

Problem solving step 3: You are asked how far she travels during the first 5 seconds.

Problem solving step 4: The equation that you will use is $\Delta x = v_i \Delta t + ½a\Delta t^2$

Problem solving step 5: Calculation:

$\Delta x = v_i \Delta t + ½a\Delta t^2 = 0 + ½ \times 1.6\ m{\cdot}s^{-2} \times (5\ s)^2 = 20\ m$

Problem solving step 6:

She travels 20 m during the first 5 seconds.

b. Problem solving step 2: Given: $a = 0\ m{\cdot}s^{-2}$ and $t = 5\ s$.

You need to calculate the final velocity after the first 5 seconds to give you the initial velocity at the start of the second 5 seconds.

$v_f = v_i + a\Delta t = 0 + (1.6\ m{\cdot}s^{-2}) \times (5\ s) = 8\ m{\cdot}s^{-1}$

So for the second 5 s, $v_i = 8\ m{\cdot}s^{-1}$

Problem solving step 3: You are asked how far she travels during the second 5 seconds.

Problem solving step 4: The equation that you will use is $\Delta x = v_i \Delta t + ½a\Delta t^2$

Problem solving step 5: Calculation:

$\Delta x = v_i \Delta t + ½a\Delta t^2 = (8\ m{\cdot}s^{-1} \times 5\ s) + 0 = 40\ m$

Problem solving step 6:

She travels 40 m during the second 5 seconds.

c. Problem solving step 2: Given: $v_i = 8\ m \cdot s^{-1}$ and $\Delta x = 8$ m and $v_f = 0\ m \cdot s^{-1}$

Problem solving step 3: You are asked to calculate her acceleration, a.

Problem solving step 4: The equation that you will use is $v_f^{\,2} = v_i^{\,2} + 2a\Delta x$

To solve for a you subtract $v_i^{\,2}$ from both sides, giving you:
$v_f^{\,2} - v_i^{\,2} = 2a\Delta x$
You then divide by $2\Delta x$ on both sides, which leaves a on its own.

Problem solving step 5: Calculation:

From the equation $v_f^{\,2} = v_i^{\,2} + 2a\Delta x$

You can solve for a:

$$a = \frac{v_f^2 - v_i^2}{2\Delta x}$$

$$= \frac{0-(8\ \mathrm{m{\cdot}s^{-1}})^2}{2\times 8\ \mathrm{m}}$$

$$= -4\ \mathrm{m{\cdot}s^{-2}}$$

Problem solving step 6:

Her acceleration up the slope is $-4\ m \cdot s^{-2}$.

[Wordbox: MAIN IDEAS: The four equations of motion for objects that have constant acceleration are:

$v_f = v_i + a\Delta t$

$\Delta x = v_i\Delta t + ½a\Delta t^2$

$v_f^{\,2} = v_i^{\,2} + 2a\Delta x$

$\Delta x = ½(v_f + v_i) \Delta t$]

Unit 5: Projectile motion

Learning Outcomes

By the end of the unit, you should be able to:

- verify that the gravitational acceleration of objects in free-fall is constant using experimental data of ball-bearings of different masses in free-fall;
- explain that objects in free-fall (projectiles) accelerate towards the earth with a constant acceleration of 9,8 $m \cdot s^{-2}$;
- apply graphs and kinematics equations of motion to objects in free-fall, in familiar and novel contexts.

Introduction

In Unit 3 and 4 you learnt how to represent motion using graphs and equations. Refresh your memory by reflecting on the following questions, either on your own or with a fellow student:

- What are the three types of graphs that you use to describe motion?
- What are the four equations that you can use to solve problems involving motion?

In the last unit you learnt about equations which you can use to solve problems with motion. In this unit you will extend your knowledge to understand the motion of objects that are falling freely under the influence of gravity. This is called projectile motion.

Activity 1: Find the gravitational acceleration of a falling object

Purpose

In this activity you will calculate the gravitational acceleration of a ball that is dropped in the air. Since it is very difficult to measure time and position values for this kind of motion, you will be given values that you will use for the calculations.

Suggested time: [30 minutes]

What you will do:

Follow the steps outlined below:

Below you will see a picture of a ball that is falling through the air. The ball's velocity and time values are shown at various times along its motion.

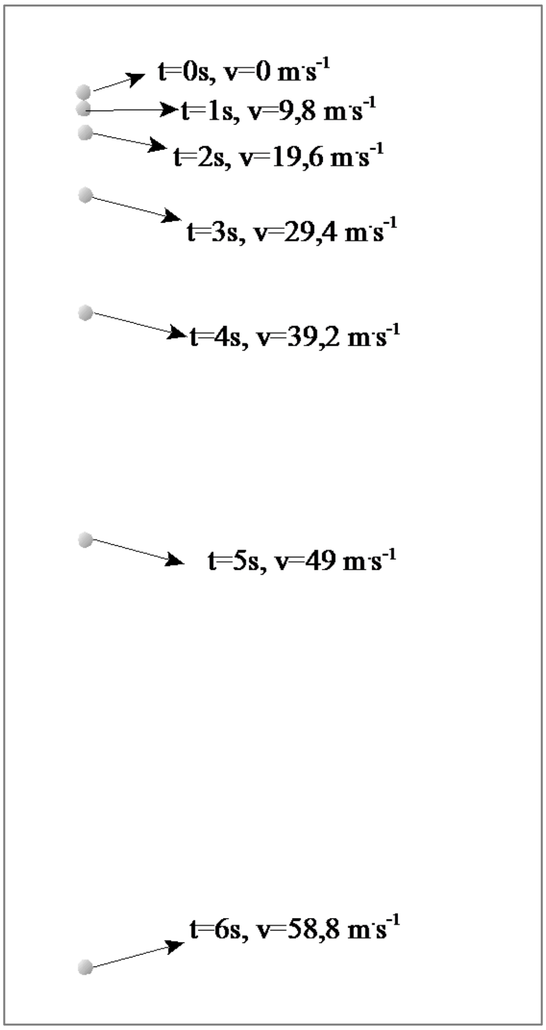


Figure 126 Picture of a ball that is falling through the air

<u>Frame of reference</u>: The downward direction is positive, and the starting position of the ball is zero. Use this information to answer the following questions:

1. Draw a set of axes like the ones below. This will allow you to draw a graph of velocity on the y-axis against time on the x-axis for these values.

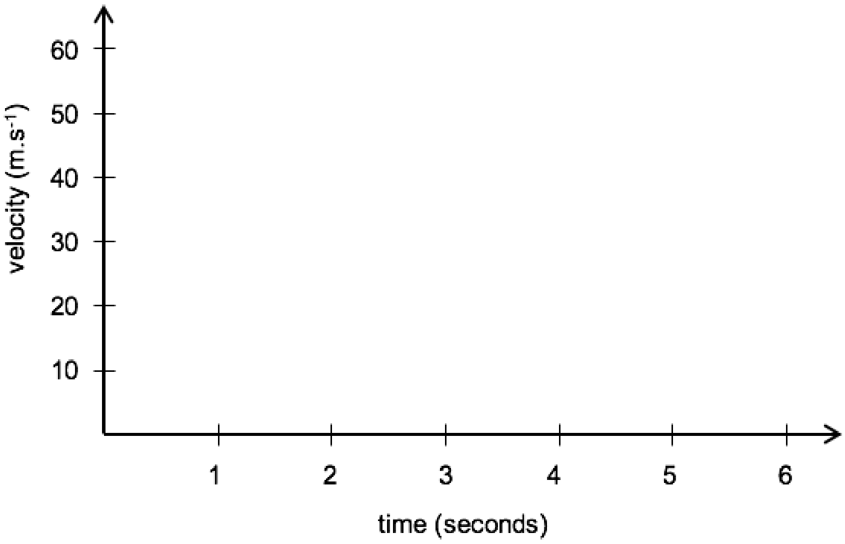


Figure 127 Axes for a graph of velocity vs time

2. On this set of axes, plot each of the points of velocity and time from the diagram of the ball.
3. Draw a best-fit line through your points.
4. Describe the shape of your velocity vs time graph.
5. From the information given in the diagram of the ball falling through the air, calculate the acceleration of the ball between the following times:
    a. between 0s and 1s
    b. between 2s and 3s
    c. between 3s and 6s
6. Use your calculations to complete a table like this one:

Acceleration ($m \cdot s^{-2}$)	**Time (s)**
	1
	2
	3
	4
	5
	6

7. Construct a set of axes that will allow you to draw a graph of acceleration on the y-axis against time on the x-axis. Label your axes carefully.
8. On your set of axes, plot each of the points of acceleration and time from your table.
9. Draw a best-fit line through your points.

10. What do you notice about the shape of your acceleration vs time graph for an object that is falling through the air?

11. The diagram below shows the positions of two balls that are falling together. Ball A and Ball B are dropped at the same time, and hit the ground at exactly the same time. Ball B has three times the mass of Ball A. What can you conclude about the relationship between the acceleration of the balls and their masses?

Figure 128 Two balls with different masses falling in the air

Guided reflection

- If you plot your points carefully and accurately, your velocity-time graphs should look similar to the one given below:

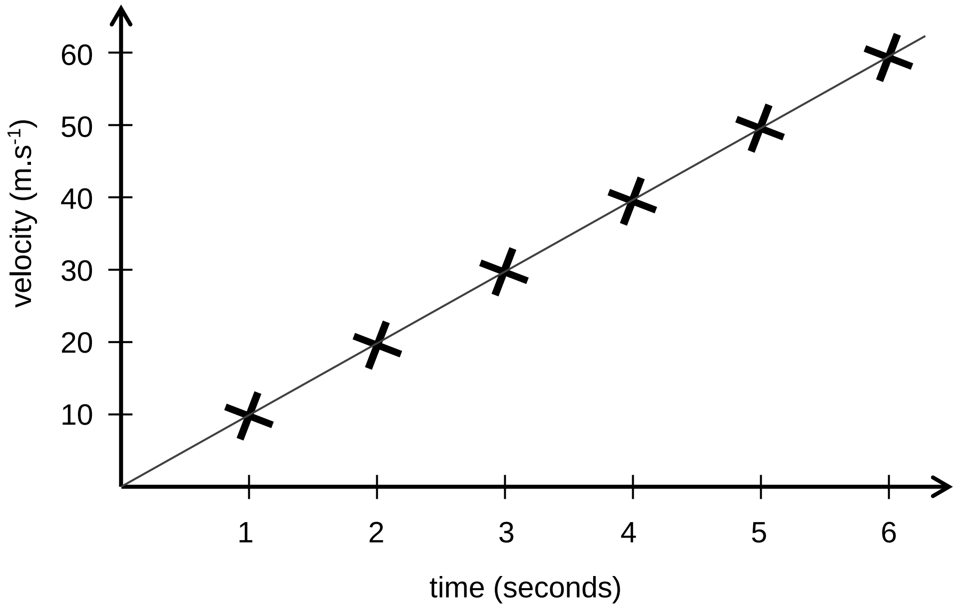


Figure 129 Graph of velocity vs time for the falling ball

- The best-fit line should go through the points as accurately as possible.
- The shape of the graph should be a straight line with a positive gradient. This tells you that the ball is falling with a constant acceleration.
- The acceleration of the ball between the following times is:
    a. between 0s and 1s: a = 9,8 $m \cdot s^{-2}$
    b. between 2s and 3s: a = 9,8 $m \cdot s^{-2}$
    c. between 3s and 6s: a = 9,8 $m \cdot s^{-2}$
-

Acceleration ($m \cdot s^{-2}$)	**Time (s)**
9,8	1
9,8	2
9,8	3
9,8	4
9,8	5
9,8	6

- Your acceleration vs time graph should look like the one below:

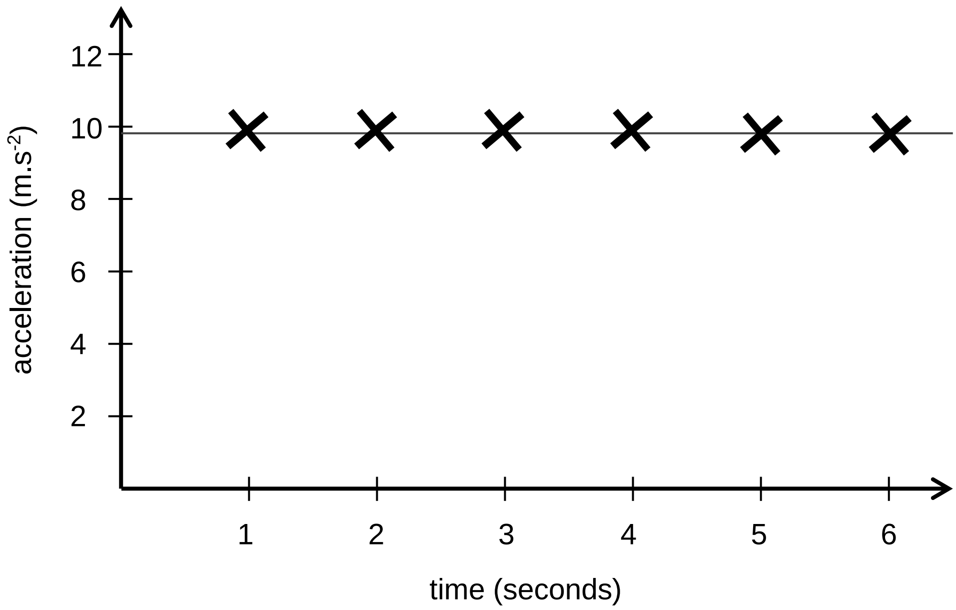


Figure 130 Graph of acceleration vs time for the falling ball

- The shape of the acceleration-time graph for an object falling freely in the air is a straight horizontal line with a value of 9,8 $m \cdot s^{-2}$.
- Since the two balls that are falling are at the same position at each time interval, this tells you that the acceleration of the balls does not depend on their masses, but is a constant downward acceleration (9,8 $m \cdot s^{-2}$), *regardless of the mass of the object*.

This tells you that the acceleration does not depend on the mass of the ball. You call this the *acceleration due to gravity*, which has the symbol *g*. Its direction is always downwards (towards the earth).

Activity 2: Experiment with falling objects

Purpose

In this activity you will explore the rate at which different objects fall to the earth.

Suggested time: [20 minutes]

What you need:

A large ball

A small ball

A feather

What you will do:

Follow the steps below:

1. Hold the large ball in one hand and the small ball in another.
2. Hold these as high as you can above your head, but make sure that they are at the same height.
3. Drop the balls at the same time, and observe when they hit the ground.
4. What do you notice about the rate at which the two balls fell?
5. Repeat steps 1 to 3, but this time hold a feather in one hand and one of the balls in the other.
6. What do you notice about the rate at which the feather fell, compared with the ball?
7. Can you think of a reason for this?
8. Are there any conditions that will allow the feather and the ball to fall at the same rate?

Guided reflection

In Activity 1 you saw that objects that fall through the air accelerate at a constant rate of 9,8 $m \cdot s^{-2}$ towards the earth, no matter what their mass is. Your experiment with the two balls would have confirmed this. But in this activity you saw that a feather and a ball do not fall at the same rate.

To help you with reflecting on this scenario, you can see a very interesting YouTube video of a ball and a feather falling together: *Why do two bodies of different masses fall at the same rate?*: https://www.youtube.com/watch?v=OdBKNnb-tRk (Duration: 2.35)

This video shows the following:

- If a ball and a father are dropped from the same height, the ball falls to the ground more quickly than the feather. This is because of the force of air resistance, which opposes the motion of the objects (you will learn more about this in the next section on forces). Air resistance has more effect on the feather, since the feather has a larger surface area for the air to push against. This causes the feather to fall more slowly in air.
- The video then shows what happens when the air is removed, creating a vacuum in the container that the ball and the feather are in – they fall at exactly the same rate, and hit the

ground at the same time! This shows that as soon as the effect of air resistance is removed, even a feather can fall with the same acceleration as a ball!

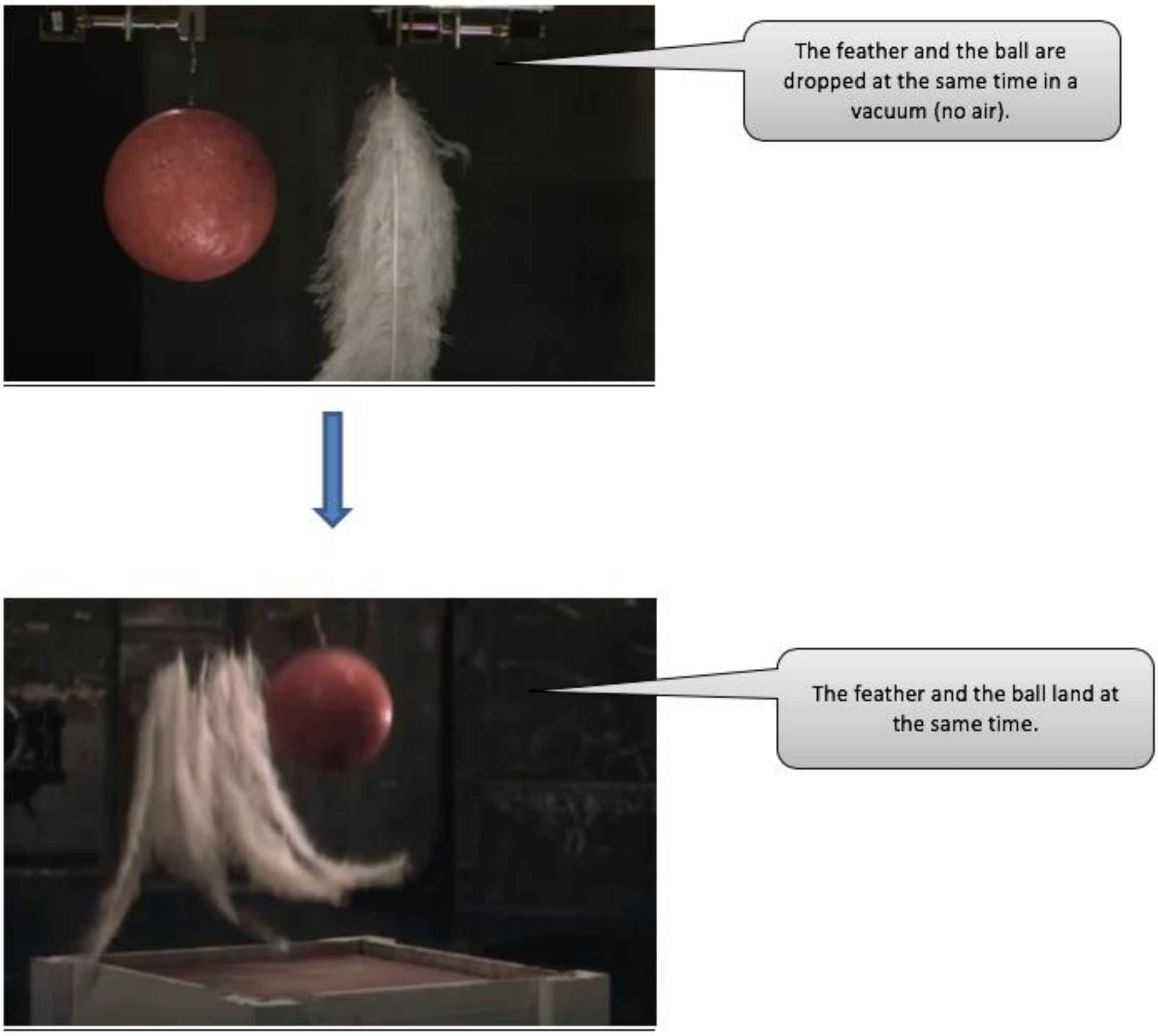


Figure 131 Photographs of a feather and a ball falling in a vacuum

Activity 3: Solve different scenarios for projectile motion

Purpose

In this activity you will explore three scenarios for projectile motion by working through examples.

Suggested time: [30 minutes]

What you will do:

Fill in the missing information in the examples below:

Case 1: An object is dropped from rest

Example:

Ayanda was standing on a ledge, and dropped a ball onto the ground, which was 3 m below the height that he dropped the ball from.

a. Calculate the final velocity of the ball just before it hit the ground.
b. Calculate the amount of time that it would take for the ball to hit the ground.
c. Draw the position, velocity and acceleration graphs of motion for this object's movement.

Problem solving steps 1 and 2: Frame of reference: Choose **down as +**

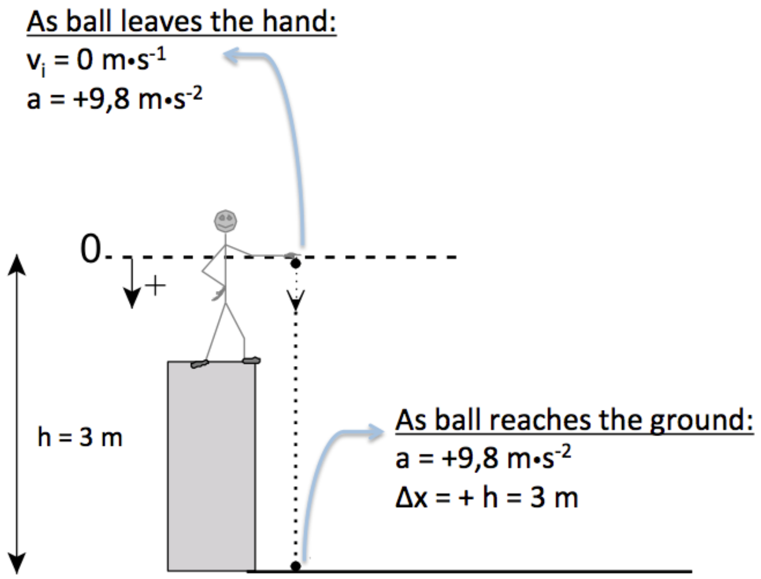


Figure 132 Diagram of the scenario with the given information shown

Problem solving step 3:

a. You are asked to find the ball's final velocity, v_f.
b. You are asked to find the time taken for the ba;; to hit the ground, Δt.

Problem solving step 4:

a. The equation that you will use is $v_f^2 = v_i^2 + 2a\Delta x$
b. The equation that you will use is $v_f = v_i + a\Delta t$

Problem solving step 5: Calculation:

a. First calculate v_f using the equation $v_f^2 = v_i^2 + 2a\Delta x$

b. From the equation $v_f = v_i + a\Delta t$

You can solve for Δt:

Problem solving step 6:

a. The stick had a final velocity of ____________________.

b. The time for the stick to hit the water is ___________________.

c. Sketch the shape of the position, velocity and acceleration graphs of motion for this object's movement on the axes below:

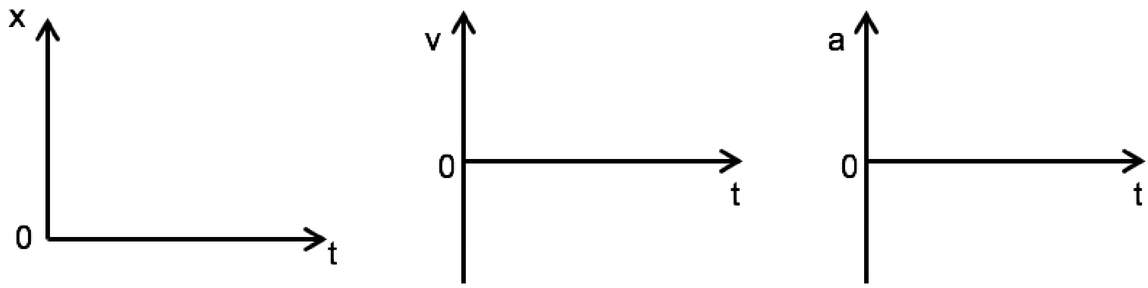


Figure 133 Axes for position, velocity and acceleration graphs

Case 2: An object is thrown upward, and falls back to the same height that it was thrown from

Example:

Kagiso threw a ball upward with an initial velocity of 5 $m \cdot s^{-1}$.

a. What was the maximum height reached by the ball?
b. After how much time did the ball fall back to the position that Kagiso threw it from?
c. Draw the position, velocity and acceleration graphs of motion for this object's movement.

Problem solving step 1 and 2: Frame of reference: Choose up as +

Diagram of the scenario with given information: (fill in the missing information on the diagram)

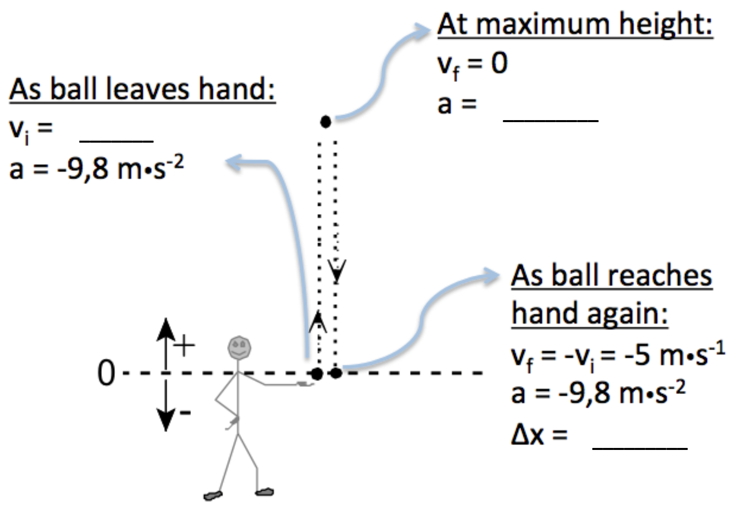


Figure 134 Diagram of the scenario with some missing information for you to fill in

Problem solving step 3:

a. You are asked to find the maximum height, Δx, where $v_f = 0$.

b. You are asked to find the time for the full motion, Δt, where $v_f = - v_i$.

Problem solving step 4:

a. The equation that you will use is $v_f^2 = v_i^2 + 2a\Delta x$

b. The equation that you will use is $v_f = v_i + a\Delta t$

Problem solving step 5: Calculation:

a. From the equation $v_f^2 = v_i^2 + 2a\Delta x$ you solve for Δx:

b. From the equation $v_f = v_i + a\Delta t$ you can solve for Δt:

Problem solving step 6:

a. The maximum height reached is __________ above the starting position.

b. The total time of the motion is ____________.

c. Sketch the shape of the position, velocity and acceleration graphs of motion for this object's movement on the axes below:

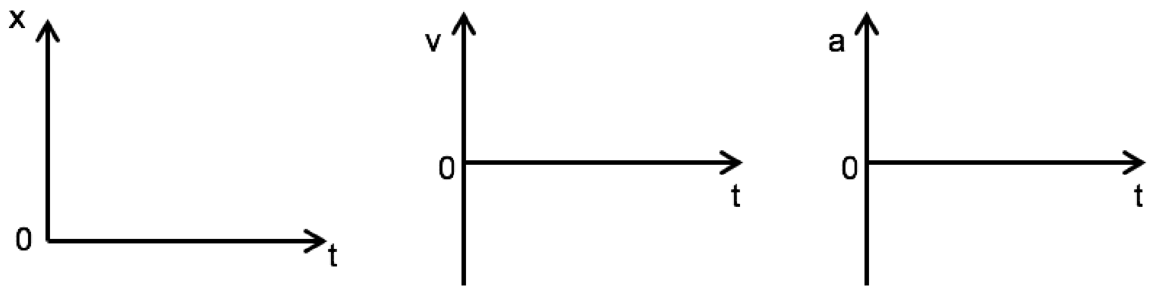


Figure 135 Axes for position, velocity and acceleration graphs

Case 3: An object moves upward and then falls down to a position below its starting position

Example:

A flea jumps from a table to a height of 0,6 m above the table, and falls to the ground below the table 1 second after it jumped.

a. What is the height of the table above the ground?

b. Sketch the shape of the position, velocity and acceleration graphs of motion for the flea's movement.

Problem solving step 1 and 2: Frame of reference: Choose up as +

Diagram of the scenario with given information: (fill in the missing information on the diagram)

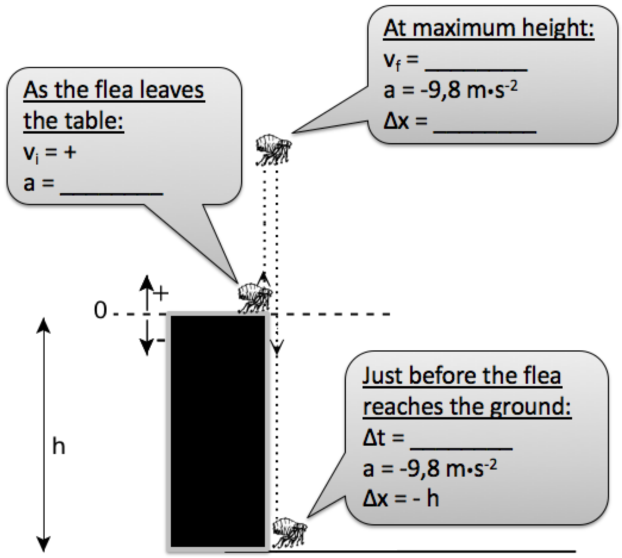


Figure 136 Diagram of the scenario with some missing information for you to fill in

Problem solving step 3:

You are asked to find the height of the table, which is the magnitude of the displacement for the flea's full motion, Δx. To be able to do this, you need to find the initial velocity, v_i.

Problem solving step 4: You will first use the equation $v_f^2 = v_i^2 + 2a\Delta x$ to find v_i

You will then use the equation $\Delta x = v_i\Delta t + ½a\Delta t^2$ to find Δx.

Problem solving step 5:

Calculation:

From the equation $v_f^2 = v_i^2 + 2a\Delta x$ you solve for v_i:

Therefore you can find Δx:

Problem solving step 6:

a. The height of the table is ___________ above the ground.

b. Sketch the shape of the position, velocity and acceleration graphs of motion for the flea's movement on the axes below:

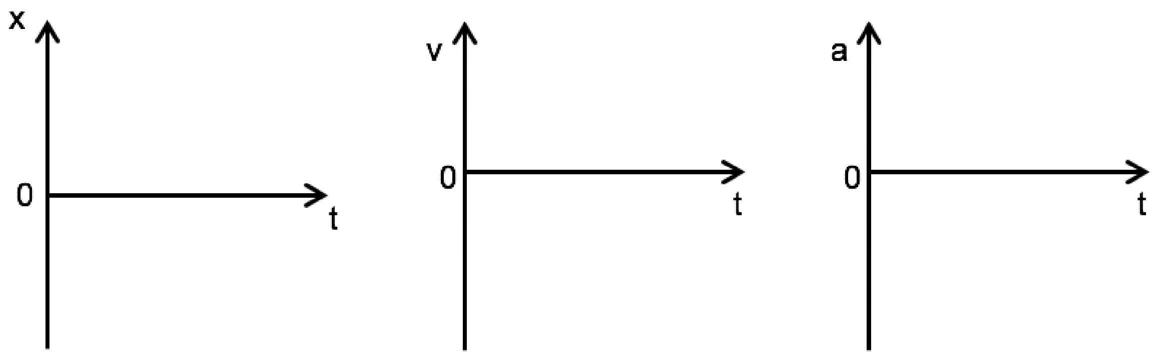


Figure 137 Axes for position, velocity and acceleration graphs

Guided Solution

The missing steps from the above examples are shown below:

Case 1: An object is dropped from rest

Problem solving step 5: Calculation:

a. From the equation

$v_f{}^2 = v_i{}^2 + 2a\Delta x$

$= 0 + 2 \times (9{,}8\ \text{m·s}^{-2}) \times 3\ \text{m}$

$= 58{,}8\ \text{m}^2\text{·s}^{-2}$

Therefore $v_f = \sqrt{58{,}8\ \text{m}^2 \cdot \text{s}^{-2}}$

$= 7{,}67\ \text{m·s}^{-1}$

b. From the equation $v_f = v_i + a\Delta t$ you can solve for Δt:

$$\Delta t = \frac{v_f - v_i}{a}$$

$$= \frac{7{,}67\text{m·s}^{-1} - 0}{9{,}8\text{m·s}^{-2}}$$

$= 0{,}78$ s

Problem solving step 6:

a. The ball had a final velocity of <u>+7,67 m·s^{-1} downwards</u>.

b. The time for the ball to hit the water is <u>0,78 seconds</u>.

c. The graphs of motion for this object that is dropped are shown below:

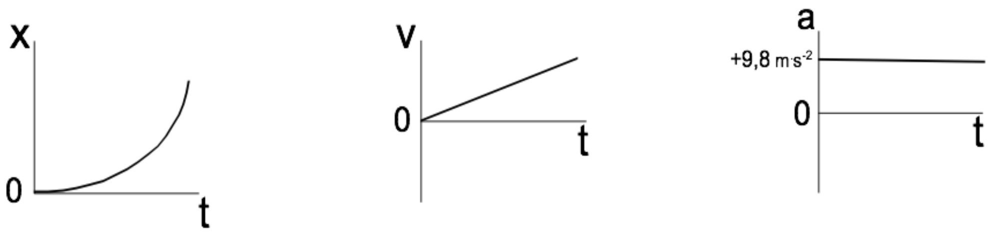


Figure 138 Graphs of motion for an object that is dropped

Case 2: An object is thrown upward, and falls back to the same height that it was thrown from

Problem solving step 1 and 2: The information is shown on the diagram

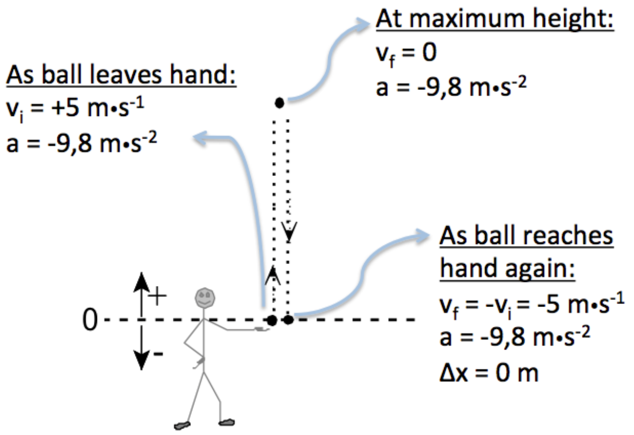


Figure 139 Diagram of the scenario with missing information filled in

Problem solving step 5: Calculation:

a. From the equation $v_f{}^2 = v_i{}^2 + 2a\Delta x$ you solve for Δx:

$$\Delta x = \frac{v_f{}^2 - v_i{}^2}{2a}$$

$$= \frac{0-(5\text{m}\cdot\text{s}^{-1})^2}{2\times(-9{,}8\text{m}\cdot\text{s}^{-2})}$$

$$= +1{,}28 \text{ m}$$

b. From the equation $v_f = v_i + a\Delta t$ you can solve for Δt:

$$\Delta t = \frac{v_f - v_i}{a}$$

$$= \frac{-5\text{m}\cdot\text{s}^{-1} - (+5\text{m}\cdot\text{s}^{-1})}{-9{,}8\text{m}\cdot\text{s}^{-2}}$$

$$= 1{,}02 \text{ s}$$

OR you can find the time to the maximum height:

$$\Delta t = \frac{v_f - v_i}{a}$$

$$= \frac{0 - (+5\text{m}\cdot\text{s}^{-1})}{-9{,}8\text{m}\cdot\text{s}^{-2}}$$

= 0,51 s

Therefore total time = 0,51s × 2 = 1,02 s

Problem solving step 6:

a. The maximum height reached is <u>1,28 m above the starting position</u>.

b. The total time of the motion is <u>1,02 seconds</u>.

c. The graphs of motion for this object that is thrown up in the air are shown below:

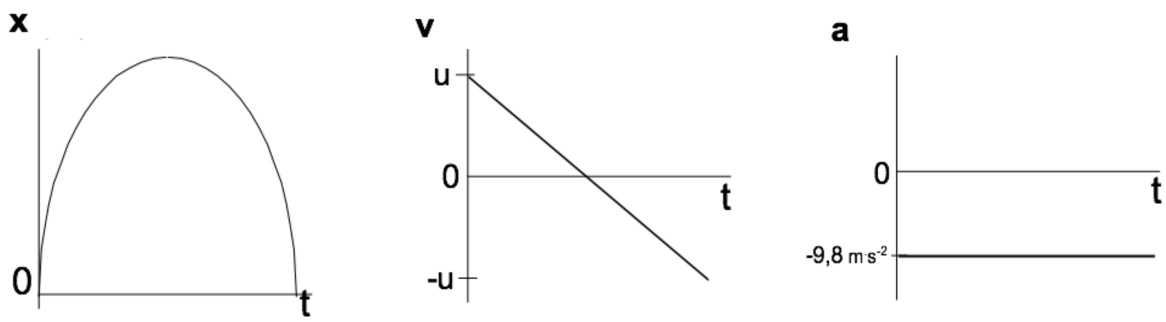


Figure 140 Graphs of motion for an object that is thrown upward, and falls back to the same height that it was thrown from

Case 3: An object moves upward and then falls down to a position below its starting position

Problem solving step 1 and 2:

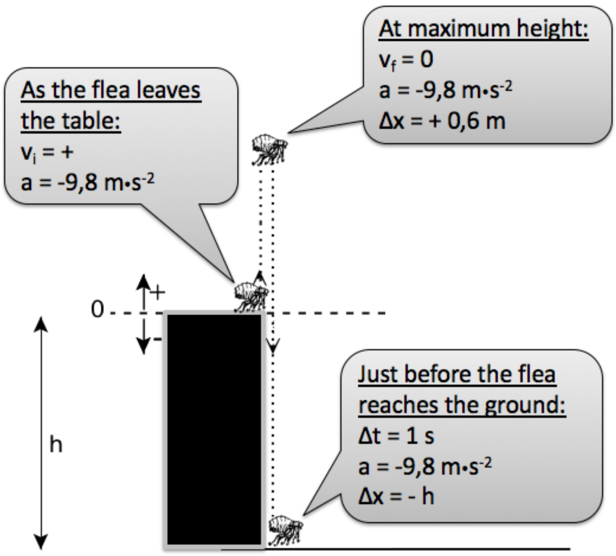


Figure 141 Diagram of the scenario with missing information filled in

Problem solving step 5: Calculation:

From the equation $v_f^2 = v_i^2 + 2a\Delta x$ you solve for v_i:

$v_i^2 = v_f^2 - 2a\Delta x$

$= 0 - 2 \times (-9{,}8\ m{\cdot}s^{-2}) \times 0{,}6\ m$

$= 11{,}76\ m^2{\cdot}s^{-2}$

Therefore $v_i = \sqrt{11{,}76\ m^2 \cdot s^{-2}}$

$= 3{,}43\ m{\cdot}s^{-1}$

Therefore. $\Delta x = v_i\Delta t + ½a\Delta t^2$

$= (3{,}43\ m{\cdot}s^{-1} \times 1\ s) + (½ \times (-9{,}8\ m{\cdot}s^{-2}) \times 1\ s)$

$= -1{,}47\ m$

(This displacement is negative, since you chose up as the positive direction, and the ground is **below** the table.)

Problem solving step 6:

a. The height of the table is 1,47 m above the ground.

b. The graphs of motion for the flea's movement through the air are shown below:

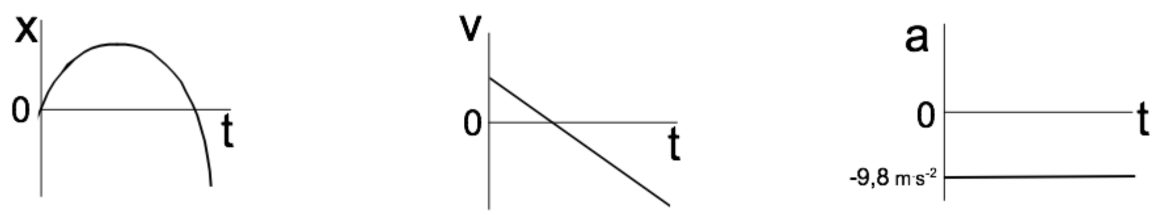


Figure 142 Graphs of motion for a flea jumps up and then falls down to a position below its starting position

Activity 4: Consolidate your learning of motion in 1 dimension

Purpose

In this activity you will consolidate your learning of motion in 1 dimension by answering the questions below and then assessing your own understanding using the solutions provided. Give yourself a mark out of the total of 100 marks, which will give you an idea of how well you understand this section of the work.

Suggested time: [90 minutes]

Answer the following questions:

1. Complete the table below by giving one word for the quantity that is described, showing the units that each of the quantities is measured in, and showing whether each is a scalar or a vector quantity: (15)

Description	**Name of quantity**	**Units**	**Scalar / Vector**
The total length of the path of motion			
The change in position from starting point to ending point			
The rate at which distance is covered			
The rate of change of position			
The rate of change of velocity			

2. Njabulo walked with a constant velocity from a position of -4 m to a position of +5 m in 3,6 seconds.

   a. Calculate his average velocity. Show your working clearly. (2)

   Njabulo then ran with a constant velocity to a position of -6 m in 2 seconds.

   b. What was his velocity for this 2 seconds? (2)

   c. What was his average velocity for the total movement described in a and b? (3)

3. A cyclist, starting from rest, accelerates down a slope at 2 $m \cdot s^{-2}$.

   a. Explain in your own words what 2 $m \cdot s^{-2}$ means. (3)

   b. What is the cyclist's velocity at the end of

      (i) 1 second? (ii) 2 seconds? (iii) 5 seconds? (6)

   c. How far has he travelled at the end of 5 s? (4)

   d. Draw the graphs of the cyclist's position, velocity and acceleration against time. (9)

4. The velocity of a train is 28 $m \cdot s^{-1}$. It applies its brakes, and has an average acceleration of -1,50 $m \cdot s^{-2}$.

   a. How much time is needed for the train to reach a velocity to 10 $m \cdot s^{-1}$? (4)

   b. Draw the graphs of the train's position, velocity and acceleration against time. (9)

5. A stone is dropped from a bridge that is 8 m above a river. What is the stone's velocity just before it hits the river? (4)

6. A ball is thrown straight upward and reaches a maximum height of 6 m.

a. Calculate the time that the ball takes to return to the height that it was thrown from. (5)

b. Draw a graph of the ball's velocity against time. (3)

c. Draw a graph of the ball's acceleration against time. (3)

7. If a ball is thrown vertically upward, and it takes 6 s for it to return to its point of release, calculate its initial velocity. (5)

8. A ball is thrown straight upward with an initial speed of 8 $m \cdot s^{-1}$ from the roof of a building that is 9 m above the ground. The ball falls to the ground below the roof. Let the upward direction be positive.

a. Calculate the final velocity of the ball as it hits the ground. (5)

b. Draw the graph of the ball's velocity against time. (3)

9. The graph below represents the motion of an object in the air.

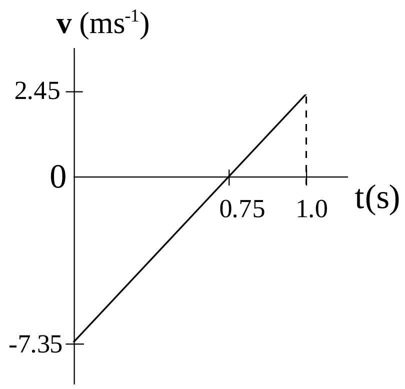


Figure 143 Velocity-time graph of object moving in the air

a. Use this graph to find the object's total displacement. (4)

b. What is the object's velocity at 0,5 s? (3)

c. Sketch a graph of the object's acceleration against time. (3)

d. Describe the motion of this object in words. Include a description of the frame of reference. (5)

Solutions

1. The completed table is shown below (1 mark for each correct entry). (15)

Description	Name of quantity	Units	Scalar / Vector
The total length of the path of motion	***Distance***	***m***	***Scalar***
The change in position from starting point to ending point	***Displacement***	***m***	***Vector***
The rate at which distance is covered	***Speed***	***$m{\cdot}s^{-1}$***	***Scalar***
The rate of change of position	***Velocity***	***$m{\cdot}s^{-1}$***	***Vector***
The rate of change of velocity	***Acceleration***	***$m{\cdot}s^{-2}$***	***Vector***

2.

a. Given: x_i = -4 m and x_f = +5 m and Δt = 3,6 s

Average velocity:

$$v = \frac{\Delta x}{\Delta t}$$

$$= \frac{+5\ \text{m} - (-4\ \text{m})}{3{,}6\ \text{s}}$$

Be careful when you substitute your values, and make sure that the signs are correct.

$$= \frac{+9\ \text{m}}{3{,}6\ \text{s}}$$

= +2,5 $m{\cdot}s^{-1}$ (2)

b. Given: x_i = +5 m and x_f = -6 m and Δt = 2 s

Velocity:

$$v = \frac{\Delta x}{\Delta t}$$

$$= \frac{-6\ \text{m} - 5\ \text{m}}{2\ \text{s}}$$

$$= \frac{-11\ \text{m}}{2\ \text{s}}$$

= -5,5 $m{\cdot}s^{-1}$ (2)

c. Given: x_i = -4 m and x_f = -6 m and Δt = 3,6 s + 2 s = 5,6 s

Average velocity:

$$v = \frac{\Delta x}{\Delta t}$$

$$= \frac{-6\ \text{m} - (-4\ \text{m})}{5{,}6\ \text{s}}$$

$$= \frac{-2\ \text{m}}{5{,}6\ \text{s}}$$

= -0,36 m·s^{-1} (3)

3.

a. 2 m·s^{-2} means that the velocity increases by 2 m·s^{-1} in each second.

OR 2 m·s^{-2} is the acceleration, which is the rate of change of velocity. (3)

b. (i) from the equation $a = \frac{\Delta v}{\Delta t}$ you get:

To solve for Δv you multiply both sides of the equation by Δt

$\Delta v = a\ \Delta t$

$= 2\ \text{m·s}^{-2} \times 1\ \text{s}$

$= 2\ \text{m·s}^{-1}$

(ii) $v = a\ \Delta t$

$= 2\ \text{m·s}^{-2} \times 2\ \text{s}$

$= 4\ \text{m·s}^{-1}$

(iii) $v = a\ \Delta t$

$= 2\ \text{m·s}^{-2} \times 5\ \text{s}$

$= 10\ \text{m·s}^{-1}$ (6)

c. $\Delta x = v_i\Delta t + \frac{1}{2}a\Delta t^2$

$= 0\ \text{m·s}^{-1} \times 5\ \text{s} + (\frac{1}{2} \times (-2\ \text{m·s}^{-2}) \times (5\ \text{s})^2)$

$= -25\ \text{m}$ (4)

d. (3) marks for each correct shape

(9)

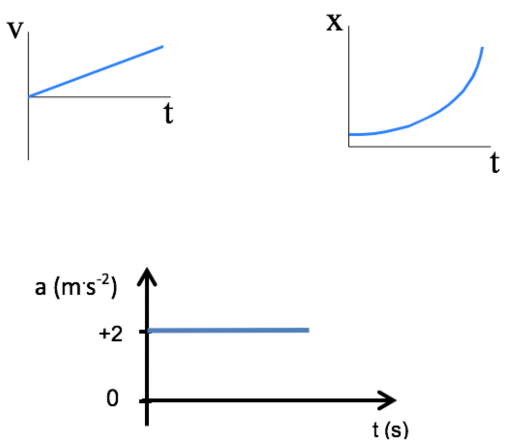


Figure 144 Graphs of motion for object's movement

4.

a. Given: v_i = 28 m·s^{-1} ; a = -1,50 m·s^{-2} ; v_f = 10 m·s^{-1}

Calculation:

From the equation $v_f = v_i + a\Delta t$

you can solve for Δt:

$$\Delta t = \frac{v_f - v_i}{a}$$

$$= \frac{(10\ \text{m·s}^{-1} - (+28\ \text{m·s}^{-1}))}{(-1{,}5\ \text{m·s}^{-2})}$$

= 12 s (4)

To solve for Δt you subtract v_i from both sides, giving:
$v_f - v_i = a\Delta t$
You can then divide both sides of this equation by a.

b. (3) marks for each correct shape

(9)

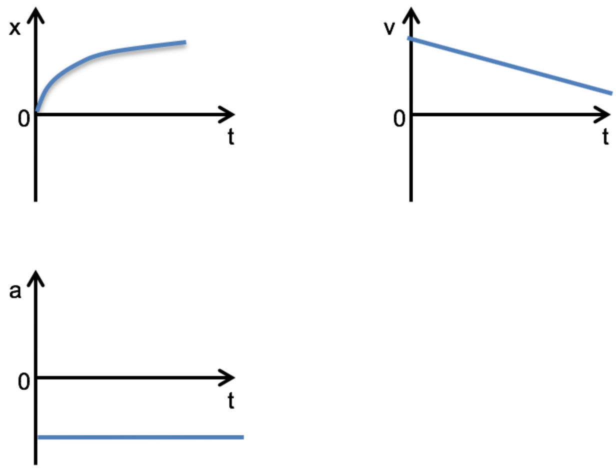


Figure 145 Graphs of motion for object's movement

5. Frame of reference: Take down as +

Given: v_i = 0 m·s^{-1} ; a = +9,8 m·s^{-2} ; Δx = 8 m

Calculation:

$v_f^2 = v_i^2 + 2a\Delta x$

= 0 + 2 × (9,8 m·s^{-2}) × 8 m

= 156,8 m^2·s^{-2}

Therefore $v_f = \sqrt{156{,}8\ \text{m}^2 \cdot \text{s}^{-2}}$ = 12,52 m·s^{-1} (4)

6. Frame of reference: Take up as +

a. <u>Given</u>: At max height $v_f = 0$ m·s^{-1} ; a = -9,8 m·s^{-2} ; Δx = 6 m

<u>Calculation</u>:

From the equation $v_f^2 = v_i^2 + 2a\Delta x$

you solve for v_i:

To solve for v_i you subtract $2a\Delta x$ from both sides and first solve for v_i^2, and then find the square root to give you v_i

$v_i^2 = v_f^2 - 2a\Delta x$

$= 0^2 - (2 \times (-9{,}8\ \text{m·s}^{-2}) \times 6\ \text{m})$

$= 117{,}6\ \text{m}^2\text{·s}^{-2}$

Therefore $v_i = \sqrt{117.6\ \text{m}^2 \cdot \text{s}^{-2}} = +10{,}84\ \text{m·s}^{-1}$

Therefore when the ball return to the height it was thrown from $v_f = -10{,}84$ m·s^{-1}

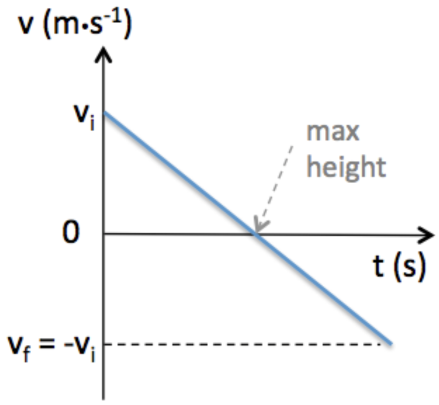
From the equation $v_f = v_i + a\Delta t$ you can solve for Δt:

$$\Delta t = \frac{v_f - v_i}{a}$$

$$= \frac{(-10{,}84\ \text{m·s}^{-1} - (10{,}84\ \text{m·s}^{-1}))}{(-9{,}8\ \text{m·s}^{-2})}$$

= 2,21 s (5)

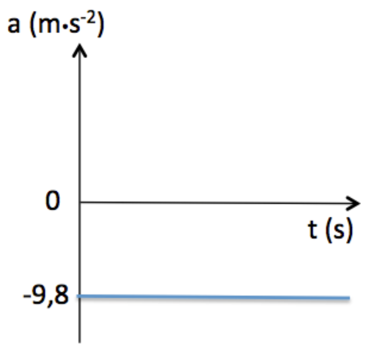
b.



(3)

Figure 146 Velocity-time graph for this motion

c.



(3)

Figure 147 Acceleration-time graph for this motion

7. Frame of reference: Take up as +

   Given: $a = -9{,}8\ m{\cdot}s^{-2}$; $\Delta t = 6$ s ; $v_f = -v_i$

   Calculation:

   From the equation $v_f = v_i + a\Delta t$

   you substitute $-v_i$ in place of v_f:

   $-v_i = v_i + a\Delta t$

   You can then solve for v_i:

   $2v_i = -a\Delta t = -(-9{,}8\ m{\cdot}s^{-2}) \times 6\ s = 58{,}8\ m{\cdot}s^{-1}$

   Therefore $v_i = \frac{58{,}8\ m{\cdot}s^{-1}}{2} = 29{,}4\ m{\cdot}s^{-1}$ (5)

8.

   a. Frame of reference: Take up as positive, and the starting point of the ball as the zero position

   Given: $v_i = +8\ m{\cdot}s^{-1}$; $\Delta x = -9$ m ; $a = -9{,}8\ m{\cdot}s^{-2}$

   To find v_f:

   $v_f^{\,2} = v_i^{\,2} + 2\,a\,\Delta x$

   $= (8ms^{-1})^2 + 2 \times (-9{,}8ms^{-2}) \times -9m$

   $= 240\ m^2s^{-2}$

   $\therefore v_f = \pm\sqrt{240m^2s^{-2}} = -15{,}5\ m{\cdot}s^{-1}$ (You choose the negative sign since final velocity is down) (5)

   b. (3)

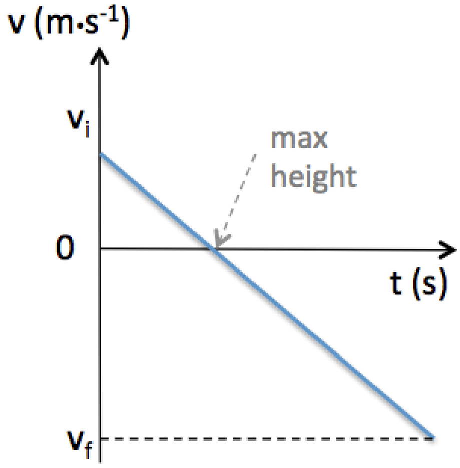


Figure 148 Velocity-time graph for this motion

9.The graph with the areas of two triangles is shown below:

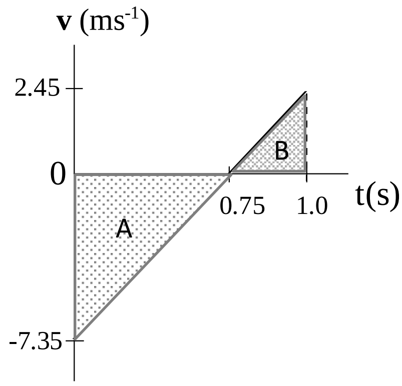


Figure 149 Area underneath the velocity-time graph

c. Δx = area under v-t graph = area of triangle A + area of triangle B

$= (\frac{1}{2}\text{ base} \times \text{height})_A \quad + (\frac{1}{2}\text{ base} \times \text{height})_B$

$= (\frac{1}{2} \times 0{,}75\text{ s} \times 7{,}35\text{ m·s}^{-1}) + (\frac{1}{2} \times 0{,}25\text{ s} \times 2{,}45\text{ m·s}^{-1})$

$= 2{,}76\text{ m} + 0{,}31\text{ m}$

$= 3{,}07\text{ m}$ (4)

d. <u>Given</u>: $v_i = -7{,}35\text{ m·s}^{-1}$; $a = +9{,}8\text{ m·s}^{-2}$; $\Delta t = 0{,}5\text{ s}$

To find v_f:

$v_f = v_i + a\Delta t$

$= -7{,}35\text{ m·s}^{-1} + (+9{,}8\text{ m·s}^{-2})(0{,}5\text{ s})$

$= -2{,}45\text{ m·s}^{-1}$ (3)

e. (3)

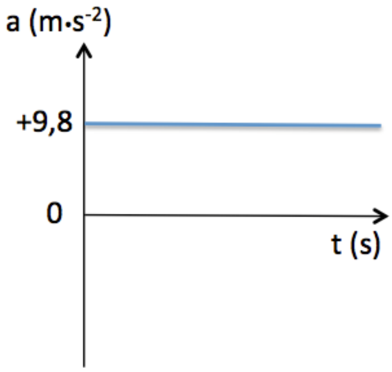


Figure 150 Acceleration-time graph for this motion

f. Frame of reference: Down is positive. ✓

The ball is thrown upward with an initial velocity of 7,35 $m \cdot s^{-1}$ ✓ and after it has reached its maximum height ✓ it falls back down to a height that is above its starting position ✓. Its final velocity just before it lands is 2,45 $m \cdot s^{-1}$. ✓ (5)

Sub-topic 3: Force

Unit 1: Forces

Learning Outcomes

By the end of the unit, you should be able to:

- differentiate between mass and weight;
- calculate the weight of an object on earth;
- identify all of the forces acting on an object, including weight, normal force, applied force, frictional force and tension force;
- draw free body diagram(s) to represent the forces acting on an object.

Introduction

Force is a very important concept in Physics. Reflect on the following questions, either on your own or with a fellow student:

- What do you understand by the term "force"?
- If an object is moving, does this always mean that a force is being applied to it?

You use the term "force" in many ways in your everyday life. Sometimes people talk about a "forceful personality", or a "force to be reckoned with". These are not what are meant by the term when you use it in Physics. It is also often assumed that when an object is moving there must be a force that is keeping it in motion. This is not always the case, as you will learn in this section.

In Physics you have to use terms very precisely to avoid misconceptions. In this unit you will learn the precise scientific meaning of the term "force", and you will learn about some of the different types of force.

What is a force?

A force is a push or a pull exerted upon an object. [**Wordbox**: "*exerted*" means that the force is applied to, or acting on an object]. A force can also be a twist exerted on an object. Force is measured in units called newtons (N). Since all forces have a clear magnitude and direction, forces are *vector quantities*. You can therefore represent a force that acts on an object using an arrow.

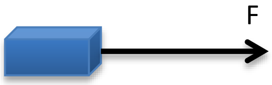


Figure 151 A force labelled “F” acts to the right on a block

Some of the forces that you will look at in this unit are the net force, the gravitational force, the normal force, resistance forces (such as friction and air resistance), and tension forces. These are explained below.

The net force

The resultant or net force on an object (F_{net}) is the combined effect of all of the forces acting on that object. This means that F_{net} is the vector sum (resultant) of all of the forces acting on an object. You write this as: $F_{net} = F_1 + F_2 + \ldots$

(Look back in Unit 1 of the Mechanics chapter to remind yourself of what a resultant vector is).

If forces acting on an object are *balanced* there is no net force ($F_{net} = 0$), and so the forces will not have an effect on the motion of the object. To have an effect, the forces must be *unbalanced*, so that there is a *net force* acting on an object.

Activity 1: Balanced and unbalanced forces

Purpose

In this activity, you will explore the concept of balanced and unbalanced forces.

Suggested time: [10 minutes]

What you need:

A length of rope or string

What you will do:

You need a friend or fellow student to help you with this activity.

1. Ask your friend to hold one end of the rope, and you should hold the other.

2. Stand in such a way that you can pull on the rope towards your left. Ask your friend to stand next to you on your right to make sure that the rope does not move.
3. What is the direction of the force that your friend has to exert to keep the rope from moving?
4. What is the net force that is acting on the rope?
5. What would your friend have to do to create an unbalanced net force on the rope?

Guided reflection

In this activity you would have noticed the following:

- To keep the rope from moving your friend would have to exert a force toward the right that is equal and opposite to the force that you are exerting to the left.
- While these forces are balanced, the net force on the rope is zero. You write this as:

  $F_{net} = 0$

- For there to be an unbalanced net force on the rope, your friend will have to exert a force that is greater or less than the force that you are exerting on the rope.

[WORDBOX: MAIN IDEAS:

- The *net force*, F_{net}, is the sum of all of the forces acting on an object. It is also called the resultant force. $F_{net} = F_1 + F_2 + \ldots$
- If forces are balanced there is no net force: $F_{net} = 0$.]

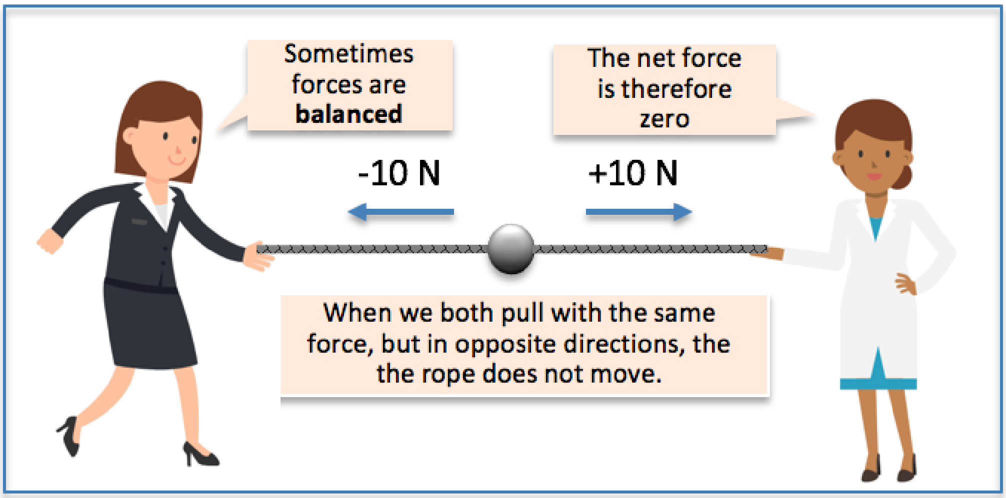


Figure 152 Diagram of balanced forces on a ball

Applied force

When an external force is exerted on an object, for example a push by a person's hand or the push from a car's engine, this is called the *applied force* (F_A). To correctly identify an applied force, you need to be able to identify the person or object that is exerting the force. For example, when a netball is being thrown upward, the applied force is exerted by the hand on the ball.

You only include the applied force in your calculations *while it is acting* on the object. For example, if a person kicks a soccer ball along the floor, the applied force on the ball only has an effect on the ball while the ball is in contact with the foot. After the ball has left the foot, there is no applied force on the ball. The only force on the ball is the gravitational force that attracts it toward the earth (if you ignore the effects of air resistance).

Figure 153 The applied force on the ball only has an effect on the ball while the ball is in contact with the foot

[WORDBOX]: MAIN IDEAS:

- The *applied force,* F_A, is an external force that is exerted on an object (push or pull).
- You only include the applied force in your calculations *while it is acting* on the object.

Gravitational force

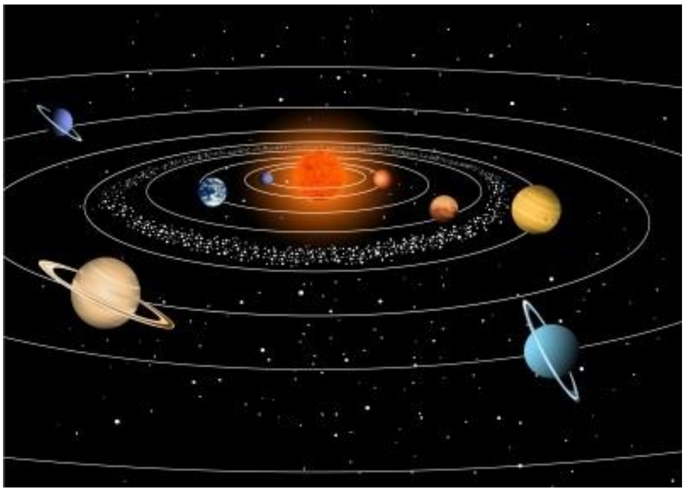


Figure 154 The sun and the planets in the Solar System

The gravitational force is the force of attraction (pull) that objects/bodies have on one another due to their masses.

For example:

- the attraction of the Sun and planets,
- the attraction between the Earth and the Moon,
- the attraction between the Earth and objects on the surface of the Earth.

Objects with greater mass have more gravitational pull on each other. The gravitational force decreases as the distance between the objects increases.

You use the symbol F_g for the gravitational force, and it is measured in newtons (N).

Activity 2: Observe the gravitational force

Purpose

In this activity you will observe the effect of the gravitational force on an object.

Suggested time: [10 minutes]

What you need:

A piece of scrap paper, rolled up into a ball

What you will do:

Throw your ball of paper up in the air and observe what happens. Answer the following questions:

1. Describe the ball's motion.
2. Can you explain what is happening to the ball?
3. Why does the ball not continue moving upward forever?

Now hold the ball and let it drop towards the Earth.

4. Describe the ball's motion.

5. Can you explain what is happening to the ball?

Guided reflection

NOTE: This is an investigation activity, so it is not important that you get these answers correct. The most important thing is for you to think about the movement of an object in the air.

1. As the ball rises in the air, it slows down, until it stops at the top. It then turns to fall down again, speeding up as it falls.

2. There is a gravitational force acting on the ball pulling it towards the earth. This is why it slows down on its way up, and speeds up on its way down again.

3. The ball does not continue moving upward forever because there is a downward force acting on the ball (gravity).

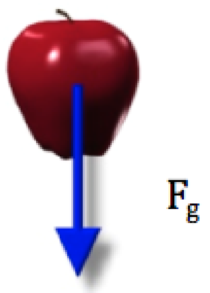


Figure 155 Gravitational force or weight, F_g

As the ball is dropped towards the Earth you should observe the following:

1. The ball falls down towards the ground, speeding up as it falls.

2. There is a gravitational force acting on the ball pulling it towards the earth. This is why it speeds up as it falls.

<u>**Weight and mass**</u>

The weight of an object is the gravitational force exerted on it by the Earth (or the Moon, or another planet). Since weight is a force, it is also measured in newtons (N).

The *mass* of the object is measured in kilograms (kg). An object's mass stays the same no matter where it is measured. However, the *weight* of an object will change when it is weighed in different places with different gravitational force. For example, an object on the moon has a lower weight than on Earth.

In everyday life you use the word "weight" to describe your mass in kilograms. This is not the scientifically correct meaning of the word "weight". This short YouTube video explains the difference between mass and weight: *Mass and Weight*: https://www.youtube.com/watch?v=_bRWtqCkXUA (Duration: 1.53)

To calculate an object's weight on Earth you use the equation:

weight = mass in kg $\times$ 9,8 $m{\cdot}s^{-2}$

You can write this as a mathematical formula:

$F_g = m\,g$

If you work out the units in the formula, the units of force are newtons (N) and the units of $m{\cdot}g$ are $kg{\cdot}m{\cdot}s^{-2}$. This tells us that 1 N is equivalent to 1 $kg{\cdot}m{\cdot}s^{-2}$

Activity 3: Calculate mass and weight

Purpose

In this activity you will study an example problem, and then apply this to do calculations where you convert between weight and mass.

Suggested time: [20 minutes]

What you will do:

Look through the steps in the following example problem:

Example:

Nkosi has a mass of 60 kg. What is his weight on Earth?

Solution:

$F_g = m\,g = 60\text{ kg} \times 9{,}8\text{ m}{\cdot}\text{s}^{-2} = 588\text{ N}$

Nkosi's weight is 588 N downward.

Now complete the following table by filling in the missing information:

Mass (g)	Mass (kg)	Weight (N)
500 g	500 g ÷ 1000 = 0,5 kg	0,5 × 9,8 = 4,9 N
100 g		
	5 kg	
		98 N
	20 kg	
0,5 g		
	0,03 kg	
		196 N
	3 500 kg	
2 500 g		
		882 N

Solutions

To calculate the weight if you are given the mass you use to formula: $F_g = m\ g$

To calculate the mass if you are given the weight you make m the subject fo the formula by dividing both sides by g:

$$m = \frac{F_g}{g}$$

The missing information is shown in the table below:

Mass (g)	Mass (kg)	Weight (N)
500 g	***500 g ÷ 1000 = 0,5 kg***	***0,5 kg × 9,8 $m{\cdot}s^{-2}$ = 4,9 N***
100 g	***100 g ÷ 1000 = 0,1 kg***	***0,1 kg × 9,8 $m{\cdot}s^{-2}$ = 0,98 N***
5 kg × 1000 = 5000 g	5 kg	***5 kg × 9,8 $m{\cdot}s^{-2}$ = 49 N***

10 kg × 1000 ***= 10 000 g or 1 × 10^4 g***	***98 N ÷ 9,8 m·s^{-2} = 10 kg***	98 N
20 kg × 1000 ***= 20 000 g or 2 × 10^4 g***	20 kg	***20 kg × 9,8 m·s^{-2}*** ***= 196 N***
0,5 g	***0,5 g ÷ 1000 = 5 × 10^{-4} kg***	***5 × 10^{-4} kg × 9,8 m·s^{-2}*** ***= 196 N***
0,03 kg × 1000 = 30 g	0,03 kg	***0,03 kg × 9,8 m·s^{-2}*** ***= 0,294 N***
3500 kg × 1000 ***= 3,5 × 10^6 g***	3 500 kg	***3500 kg × 9,8 m·s^{-2}*** ***= 3,43 × 10^4 N***
2 500 g	***2500 g ÷ 1000 = 2,5 kg***	***2,5 kg × 9,8 m·s^{-2}*** ***= 24,5 N***
90 kg × 1000 ***= 9 × 10^4 g***	***882 N ÷ 9,8 m·s^{-2} = 90 kg***	882 N

[WORDBOX: MAIN IDEAS:

- The *gravitational force,* F_g, is the downward force exerted by the Earth on an object. It is also called the weight of the object.
- You calculate weight using the equation: $F_g = m\ g$.]

The normal force

When a book is on a table, the gravitational force acts downward on the book, but the book is not moving. This means that the downward force on the book must be balanced by an upward force, to keep the two forces balanced. Therefore the upward force must be equal in magnitude, but opposite in direction, to the weight. This upward force is exerted on the book by the table surface, and is called the *normal force*, F_N. As a result of the downward weight being balanced by the upward normal force, there is no net force on the book.

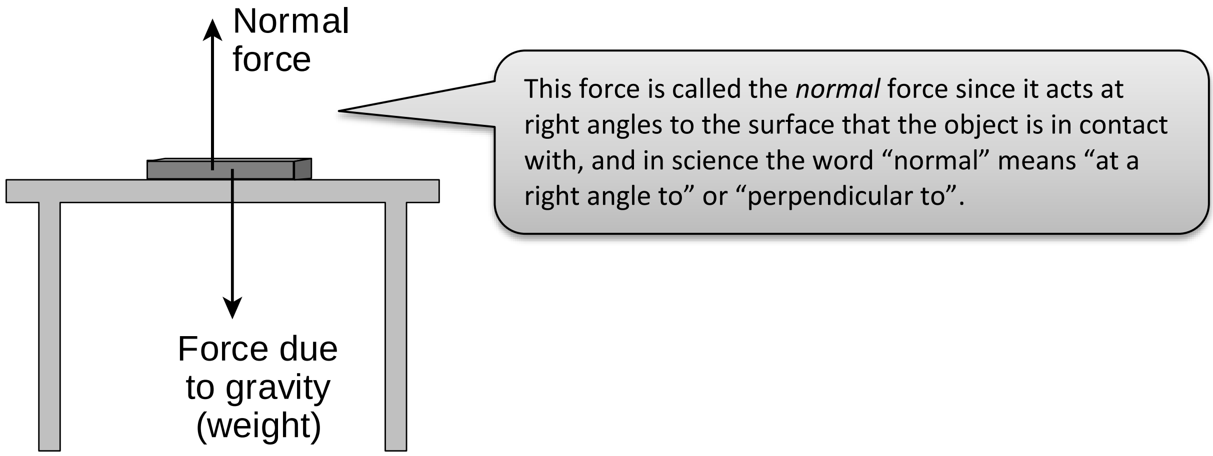


Figure 156 The weight and normal force on an object on a flat surface

When an object is on a surface that is not horizontal, the direction of the normal force on this object is again perpendicular to the surface. In this case you can see that the normal force is not equal and opposite to the weight. Again you can see that it is the force that the surface exerts on the object, and is at right angles to the surface.

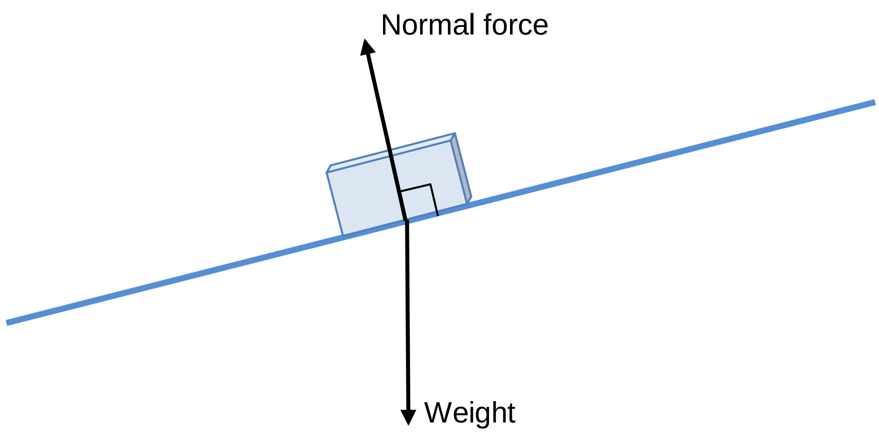


Figure 157 The weight and normal force on an object on a gradient (slope)

[WORDBOX: MAIN IDEAS:

- The *normal force,* F_N, is the force exerted by a surface on an object that is in contact with it.
- The normal force is always *perpendicular* to the surface.]

Resistance forces

When an object moves in a certain direction, there are forces that oppose the motion of the object, in other words, they act in the opposite direction to the motion. For example, when you kick a ball it does not move along the ground forever, because there is a force of *friction* that acts in the opposite direction to its movement. This force is caused by the interaction of the two surfaces. This frictional force is parallel to the surface, and opposes the motion of the object.

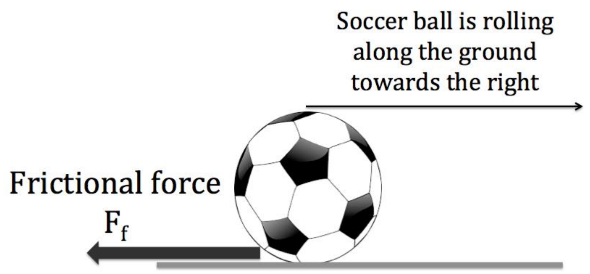


Figure 158 The frictional force is opposite to the direction of motion

Another example of a resistance force is *air resistance*. When an object is moving through the air, its motion is opposed by the air resistance force (F_a).

Figure 159 The motion of a falling object is opposed by the air resistance force

Activity 4: The effect of air resistance on a falling object

Purpose

In this activity you will investigate the air resistance force on an object that is falling.

Suggested time: [10 minutes]

What you need:

A feather

A ball or stone

What you will do:

1. Hold the feather and the ball at the same height above the Earth.
2. Drop the feather and the ball at the same time. Which one hits the ground first?
3. Can you explain your observations? (Hint: identify all of the forces acting on each object)
4. If the feather and the ball were dropped at the same time on the moon, where there is no air, what would you observe?

Guided reflection

In this activity you should have observed that when the feather and the ball are dropped at the same time, the ball hits the ground first. The reason for this is that the air resistance force has a greater effect on the feather, since it has a larger surface area for the air resistance to have effect. There is a smaller air resistance force acting on the ball, so the effect of the weight is much greater.

If the feather and the ball were dropped at the same time on the moon, where there is no air, there would be no air resistance force, so they would both have the same acceleration (9,8 $m \cdot s^{-2}$ downward). Therefore they would hit the ground at the same time.

In Activity 2 of the section on motion you should have watched a video showing the movement of a ball and a feather that are dropped in a container that has no air in it. You may find it helpful to watch this video again: *Why do two bodies of different masses fall at the same rate*?: https://www.youtube.com/watch?v=OdBKNnb-tRk (Duration: 2.35)

This video shows the following:

- If a ball and a father are dropped from the same height, the ball falls to the ground more quickly than the feather, as you observed in Activity 4.
- The video then shows what happens when the air is removed, creating a vacuum in the container that the ball and the feather are in – they fall at exactly the same rate, and hit the ground at the same time. This shows that as soon as the effect of air resistance is removed, even a feather can fall with the same acceleration as a ball.

[WORDBOX: MAIN IDEAS:

- The *frictional force*, F_f, is the force that opposes the motion of an object that is moving on a surface. The frictional force acts parallel to the surface that the object is in contact with.
- The *air resistance force*, F_a, is the force that opposes the motion of an object that is moving through the air.]

Tension forces

When an object that is hanging from a rope, and is not moving, the object's weight causes a downward force on the object, pulling it towards the earth. But since the object is not moving, there must be an upward force on it from the rope. This force is equal and opposite to the weight. You call this force the *tension* (due to the rope).

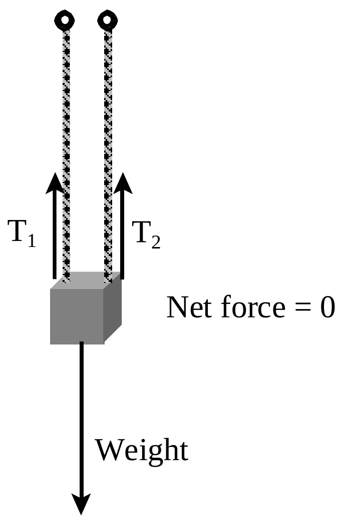


Figure 160 The tension force acts in a rope, string or cable

You can have more than two forces that are in equilibrium. The diagram on the left shows an object that is hung from two ropes. If the ropes have tension T_1 and T_2, then, since the net force must equal zero, the weight must be equal to $T_1 + T_2$.

When a rope is attached to two objects at each end of the rope, the tension pulls equally on the objects on the opposite ends of the rope.

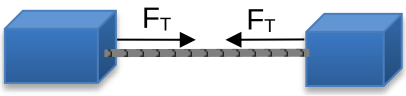


Figure 161 The tension force pulls equally on the objects on the opposite ends of the rope

[WORDBOX: MAIN IDEAS:

- The *tension force*, F_T, is the force exerted on an object by a rope or string that is attached to the object.
- The tension force pulls equally on the objects on the opposite ends of the rope.]

Activity 5: Identify forces

Purpose

In this activity you will identify all of the forces acting in various scenarios.

Suggested time: [15 minutes]

What you will do:

Answer the questions about the forces in each of these pictures

1. Draw and label the forces acting on the swing

Figure 162 Child on a swing

2. Draw and label the forces acting on the tennis ball

Figure 163 A tennis racquet hits a ball

3. Draw and label the forces acting on the elastic band

Figure 164 An elastic band is pulled on both ends

4. Draw and label the forces acting on the ball

Figure 165 A man pushes a ball

5. Can you see the effect of the force that the man is exerting on the wall in the picture below? Does this mean that there are no forces acting? Explain your answer.

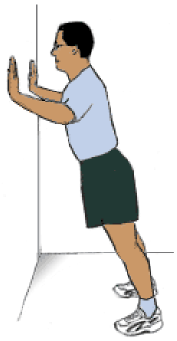


Figure 166 A man pushes on a wall

Solutions

1.

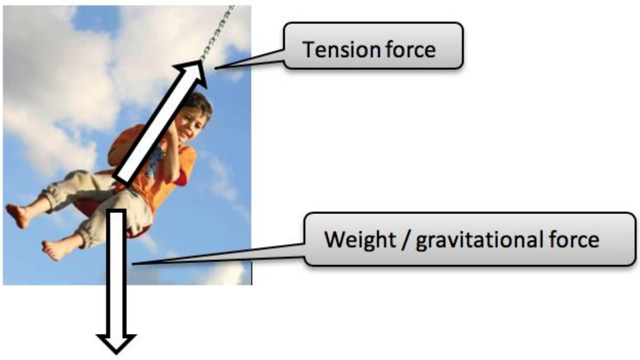


Figure 167 Forces on the child on a swing

2.

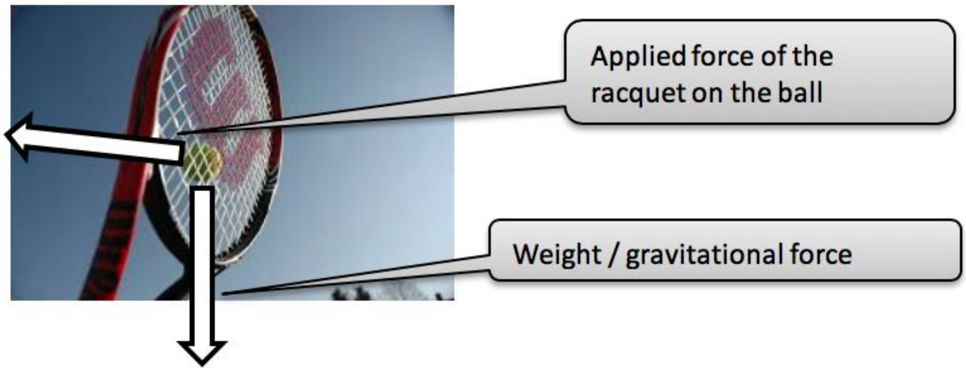


Figure 168 Forces on the tennis ball

3.

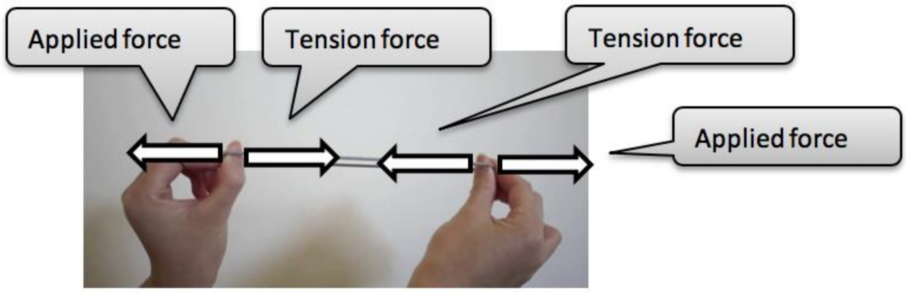


Figure 169 Forces on the elastic band

4.

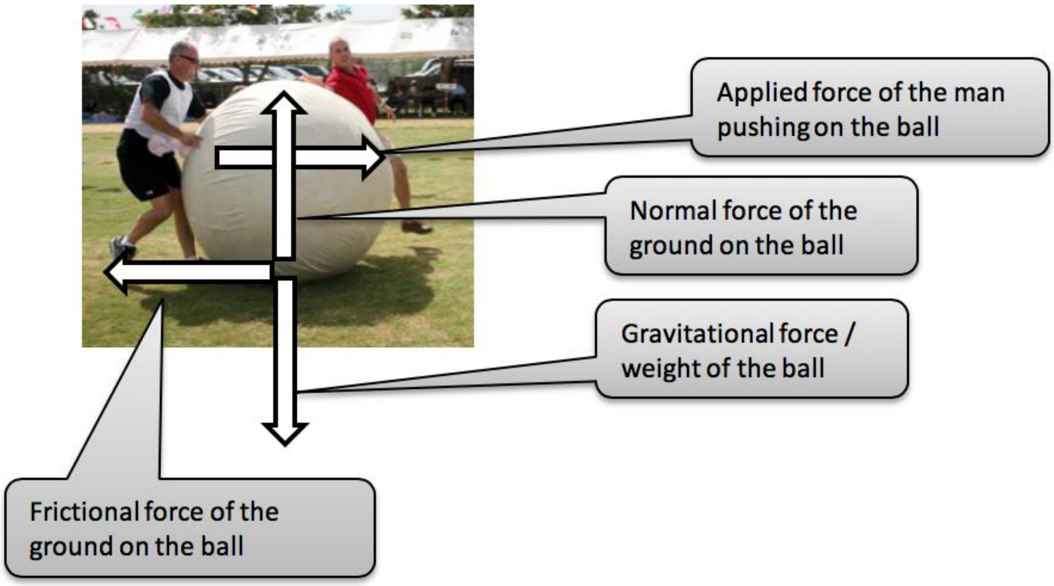


Figure 170 Forces on the ball

5. No, you cannot see the effect of the force that the man is exerting on the wall. This does not mean that there are no forces acting, it just means that all of the force on the wall are balanced, and there is no net force on the wall.

Force diagrams and free body diagrams

You can represent the forces acting on an object using a *force diagram*, which is a picture of the object with all of the forces acting on it drawn in as arrows. You can also represent these forces in a *free body diagram*. Here you represent the object with a dot, and all the forces acting on the object are shown as arrows pointing outward from this dot. The diagrams below show the difference between a force diagram and a free body diagram.

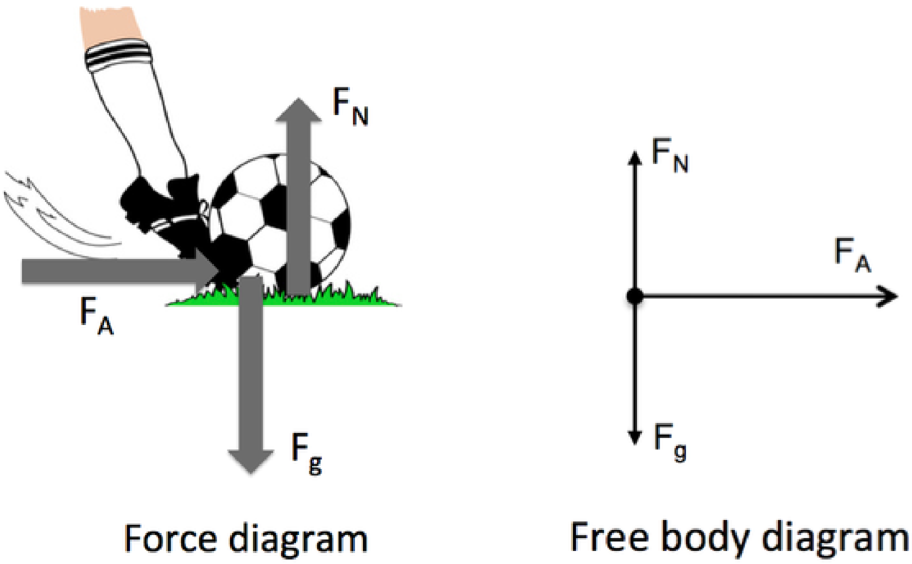


Figure 171 A force diagram and a free body diagram

Activity 6: Free body diagrams

Purpose

In this activity you will practice the skill of drawing free body diagrams.

Suggested time: [20 minutes]

What you will do:

Answer the following questions:

Bernard threw an apple up in the air, and watched it move up to its maximum height and fall back down again. Draw a free body diagram of the forces on the apple for the following times. (Include air resistance forces in your diagrams):

a. While the apple is still in contact with the hand, which is pushing up on the apple
b. When the apple is rising in the air
c. When the apple is at its maximum height
d. When the apple is falling back down to the ground.

Solutions

a. While the apple is still in contact with the hand, which is pushing up on the apple, the free body diagram will be:

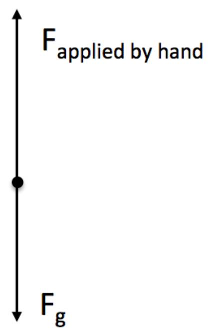


Figure 172 Forces on the apple while it is in contact with the hand

b. When the apple is rising in the air, the free body diagram will be:

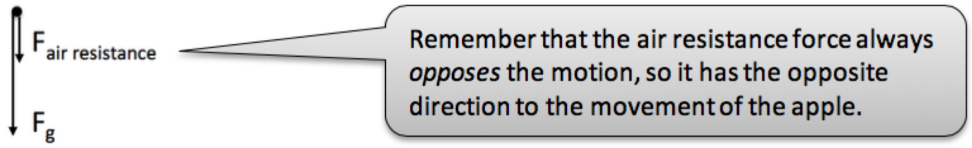


Figure 173 Forces on the apple as it is rising in the air

c. When the apple is at its maximum height, the free body diagram will be:

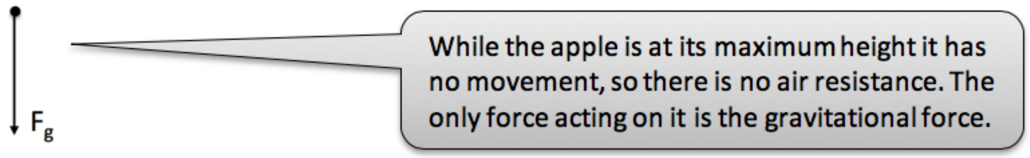


Figure 174 Force on the apple at its maximum height

d. When the apple is falling back down to the ground, the free body diagram will be:

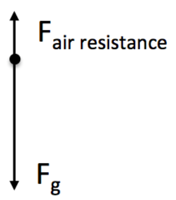


Figure 175 Force on the apple as it falls down

[WORDBOX: MAIN IDEAS:

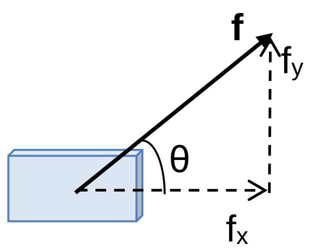
- A *force diagram* shows a picture of an object with all of the forces acting on it drawn in as arrows.
- A *free body diagram* shows the object as a dot, and all the forces acting on the object are shown as arrows pointing outward from this dot.]

Forces in 2 dimensions

When forces act at an angle to each other, rather than in a straight line, you need to use 2 dimensions to describe their direction. In other words, you need to describe the direction of the forces using their components in the x- and y-direction.

Finding the components of a force

Not all forces act parallel or perpendicular to a surface. Some forces are at an angle to a surface. When this is the case, it is useful to find the *components* of the force. The diagram below shows the x- and y-components of a force **f** that makes an angle θ with the x-axis.



$f_x = f \cos \theta$

$f_y = f \sin \theta$

Figure 176 x- and y- components of force f

Here f_x is the component of the force in the x direction (parallel to the surface), and f_y is the component of the force in the y direction (perpendicular to the surface). Once you have found the components of a force, you can replace the force itself with these components to help us in solving problems.

Activity 7: Find the components of a force

Purpose

In this activity you will practice finding the components of a force.

Suggested time: [10 minutes]

What you will do:

Answer the following question:

Dave is pulling with a rope on a box with a force of 100 N at an angle of 30° with the x-axis, as the diagram below illustrates. Calculate the x- and y-components of the force that Dave exerts on the box.

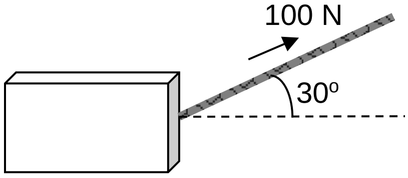
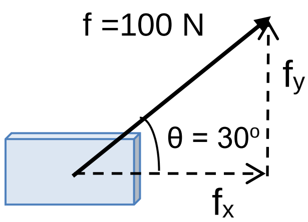


Figure 177 A force of 100 N at an angle of 30° with the x-axis

Solutions

You can show the components of the force on a diagram:



$F_x = F \cos\theta = 100\text{ N} \times \cos 30^\circ = 86{,}6\text{ N}$

$F_y = F \sin\theta = 100\text{ N} \times \sin 30^\circ = 50\text{ N}$

Figure 178 Components of the 100 N force

[WORDBOX: MAIN IDEAS: The x- and y-components of a force **f** that makes an angle θ with the x axis are:

- $f_x = f \cos\theta$
- $f_y = f \sin\theta$]

Finding the net force in 2 dimensions

When you are dealing with forces that are in two dimensions, then you need to find the net force in the x direction, and the net force in the y direction. The net force in the x direction $F_{net\ x}$ is a vector sum of all the components in the x direction. The net force in the y direction $F_{net\ y}$ is a vector sum of all the components in the y direction. (Here you will apply what you learnt in the Vectors section of this chapter).

Activity 8: Find the net force

Purpose

In this activity you will work through an example to find the net force of a number of forces in 2 dimensions.

Suggested time: [20 minutes]

What you will do:

Work through the following following example, and fill in the missing steps:

Example:

A 50 N weight that is attached to a rope is being pulled upward with a force of 120 N. An applied force is pushing on the weight with a force of 20 N to the left. What is the net force on the weight?

Calculation:

The free body diagram of the forces is shown on the right.

You use the Cartesian plane (the x and y axis) as your frame of reference.

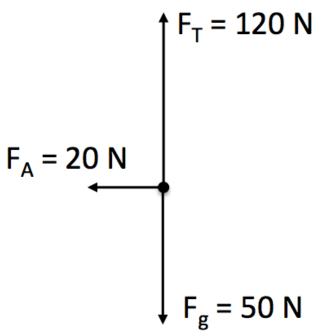


Figure 179 Free body diagram

Forces in the x direction:

There is only one force in the x direction, so

$F_{net\ x} = F_A = -20$ N

This is negative since the +x direction is right, so a force to the left is in the –x direction

Forces in the y direction are:

Tension force F_T = + 120 N

The +y direction is up, and the –y direction is down.

and weight F_g = – 50 N

Therefore $F_{net\ y}$ = + 120 N – 50 N

= ___________

You use Pythagoras' theorem to find the magnitude of the *net force*.

$F_{net}^2 = F_{net\ x}^2 + F_{net\ y}^2$

Therefore $F_{net} = \sqrt{(\_\_\_\_)^2 + (\_\_\_\_)^2}$ =

You find the *direction* using trigonometry:

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{\_\_\_\_\_\ \text{N}}{\_\_\_\_\_\ \text{N}}$$

Therefore $\theta =$ _______

The net force on the weight is therefore ________ at an angle of ________ above the -x axis.

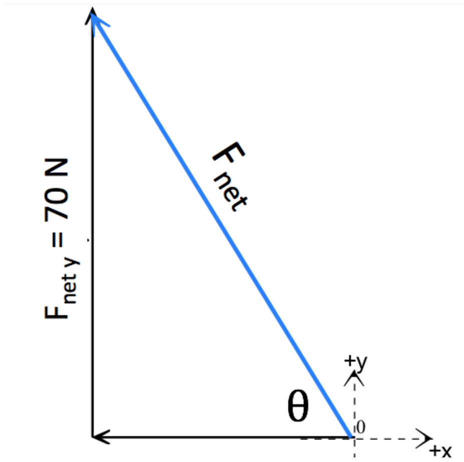


Figure 180 Vector diagram for net force

Solution:

The missing parts of the calculation are shown below:

$F_{net\ x} = F_A$ = -20 N

Therefore $F_{net\ y}$ = + 120 N – 50 N

= + 70 N

$F_{net}^2 = F_{net\ x}^2 + F_{net\ y}^2$

Therefore F_{net} = = $\sqrt{(20\ \text{N})^2 + (70\ \text{N})^2}$ = 72,8 N

Using trigonometry, $\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{70\ \text{N}}{20\ \text{N}}$

Therefore $\theta =$ 74,1°

The net force on the weight is therefore 72,8 N at an angle of 74,1° above the -x axis.

Unit 2: Newton's Laws of Motion

Learning Outcomes

By the end of the unit, students should be able to:

- state Newton's first, second and third laws of motion;
- apply Newton's laws of motion to various scenarios involving forces in equilibrium and non-equilibrium (include multiple coupled objects, but *exclude* object on an inclined plane), in familiar and novel contexts**.**

Introduction

In Unit 1 you learnt about different kinds of force, and how to apply what you learnt about vectors to add forces together. To refresh your memory, reflect on the following questions, either on your own or with a fellow student:

- What is the difference between mass and weight?
- What is the difference between a force diagram and a free body diagram?
- What is the normal force?

In this unit you will learn about Newton's laws of motion. Isaac Newton was a scientist who lived more than 350 years ago. When he was only 23 years old he developed three laws of motion which were so simple but important that they formed the basis of scientific thinking for the next 300 years, and are still used in many scientific areas today.

Newton's First Law

Activity 1: Reflect on forces and motion

Purpose

In this activity you will reflect on the motion of a ball as an introduction to Newton's first law of motion.

Suggested time: [15 minutes]

What you will do:

Discuss these questions with a fellow student, or reflect on them yourself. *It is not important that you get the correct answers for these questions, but that you reflect on them for yourself*:

- Imagine that you flick a ball and let it roll along a horizontal surface. Describe the motion of the ball.
- What are the forces acting on the ball after your hand has stopped being in contact with it?
- Why does the ball not continue moving in a straight line forever?
- What would need to happen to keep the ball moving in a straight line with a constant velocity?

Guided reflection

- If you flick a ball and let it move along a horizontal surface, it will gradually slow down and eventually come to a stop.
- The forces acting on it are the gravitational force, the normal force, and the frictional force.
- The ball does not continue moving in a straight line forever because the frictional force acting on it opposes its motion and eventually causing it to stop.
- To keep the ball moving in a straight line with a constant velocity you would either have to find a way of removing the frictional force so that there is no force resisting its motion, or you would need to balance the frictional force with an applied force that has the same magnitude as the frictional force.

This activity is an introduction to the idea behind Newton's first law of motion. This law states:
An object continues in a state of rest or uniform (moving with constant) velocity unless it is acted upon by an unbalanced (net or resultant) force.

In other words:

- If *no net force* acts on an object, the object will either remain *at rest*, or if it is moving, it will continue moving with a *constant velocity* in a straight line.
- If there *is a net force* acting on an object, it will cause a change in that object's velocity.
- If an object is *at rest*, or travelling in a straight line with *constant velocity*, this means that there is no net force acting on the object.

What Newton's first law implies is that if there is a net force acting on an object, this will cause a change in that object's velocity, in other words it will cause the object to accelerate. This relationship between the force and the acceleration is described in Newton's second law.

[Word box: **MAIN IDEA:** Newton's first law states: "*An object continues in a state of rest or uniform velocity unless it is acted upon by an external unbalanced force.*"]

You can see a short 2 ½ minute YouTube video that explains Newton's first law of motion at this website: Physics – Newton's First Law of Motion: https://www.youtube.com/watch?v=5oi5j11FkQg (Duration: 2.31)

Activity 2: Simulation with Newton's first law

Purpose

In this activity you will work with a computer simulation to explore how Newton's first law relates to the movement of an object.

Suggested time: [30 minutes]

What you will do:

Go to the following web address: https://phet.colorado.edu/sims/html/forces-and-motion-basics/latest/forces-and-motion-basics_en.html

You will see a simulation called "Forces and Motion: Basics"

- Click on the first of the four windows, labelled "Net Force". You will see a cart with a rope attached to each side of it, and blue and red figures underneath.
- Move one of the smallest blue figures up to the rope, and notice that there is now an applied force to the left. Click on the button labelled "Go!" and see what happens as a result of this force.

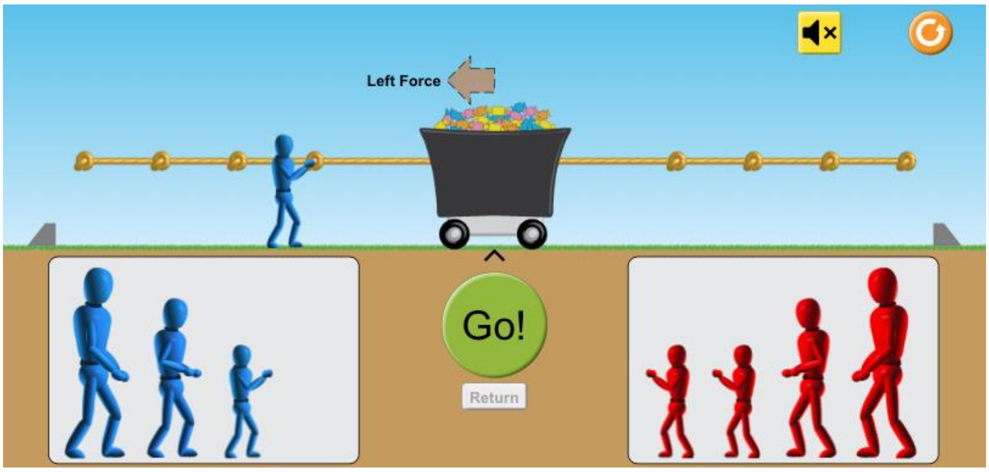


Figure 181 Unbalanced force pulling toward the left

- To start everything again click the "reset" button at the top right hand side .
- Move one of the smallest blue figures up to the rope again, and this time add one of the smallest red figures to the other side of the rope.
    - What do you notice about the forces on the cart?
    - What happens when you press the "Go!" button?

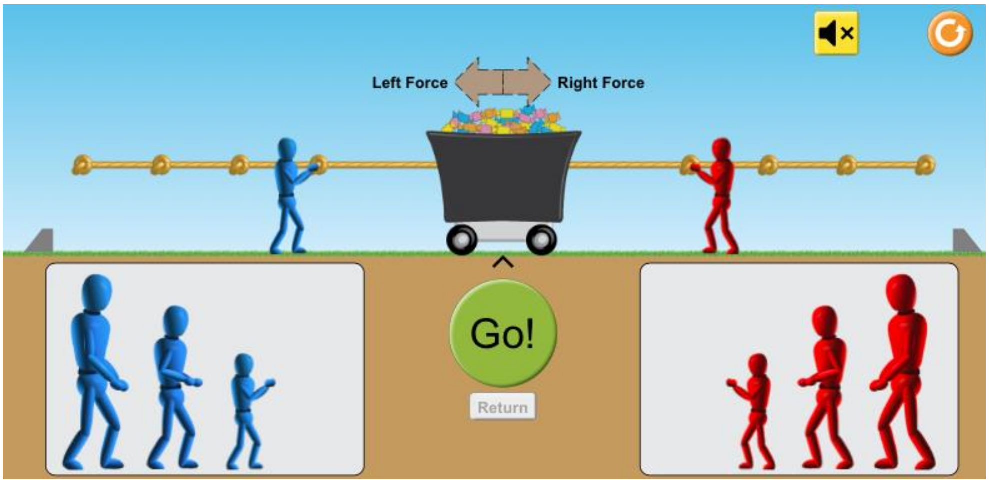


Figure 182 Balanced forces

- Now add another of the smallest blue figures to the rope, and see what happens to the forces.
    - What is the direction of the net force? (You can see the net force by clicking in the box labelled "Sum of forces", as shown on the right).

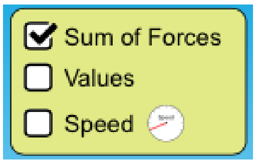


Figure 183 Click the "Sum of Forces" option

- Once the cart is moving, add the other smallest red figure to the other side of the rope so that there is **no net force**.

- What happens to the movement of the cart when there is no net force on it while it is moving?
- What do you need to do to stop the cart from moving? (Experiment with adding figures to the rope to see how to stop the cart).

- Use your observations to complete these sentences:
  - When the cart is standing still, and the left and right forces on the cart are balanced, the cart will ______________________________.
  - When the cart is moving to the left, and the left and right forces on the cart are balanced, the cart will ______________________________.
  - What you need in order to change the movement of the cart is ______________________.

Guided reflection

In this simulation you would have observed the following:

- When one of the blue figures was pulling on the rope, there was only one applied force to the left, which meant that there was a net (unbalanced) force on the cart. When the button labelled "Go!" was clicked, the cart started moving towards the left as a result of this net force.
- When there was a blue figure and a red figure pulling on opposite sides of the rope, the force to the left was balanced by the force to the right, and as a result there was no net force on the cart. When you press the "Go!" button there is no movement, since there is no net force to cause a change in the cart's movement.
- If another of the small blue figures is added to the rope, the forces are no longer balanced. There is a net force acting on the cart to the left. This will cause the cart to move to the left.
- Once the cart is moving, if one of the smallest red figures is added to the other side of the rope, the forces are balanced and there is no net force.
  - As a result the cart will keep moving to the left with constant speed. (You can check this by clicking on the box labelled "Speed" at the top right of the screen).

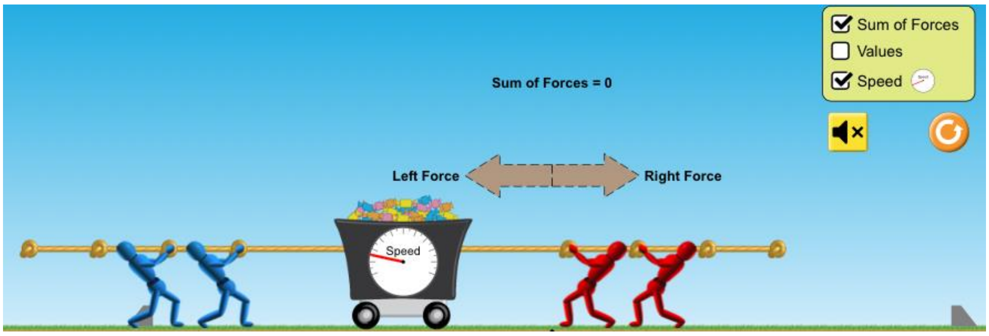


Figure 184 Balanced forces result in constant velocity

- What you need to do to stop the cart from moving is to add another red figure so that there is a net force to the right. This net force will cause the cart to slow down until it stops. If you don't pause the motion once it has stopped the net force to the right will then cause the cart to accelerate towards the right.

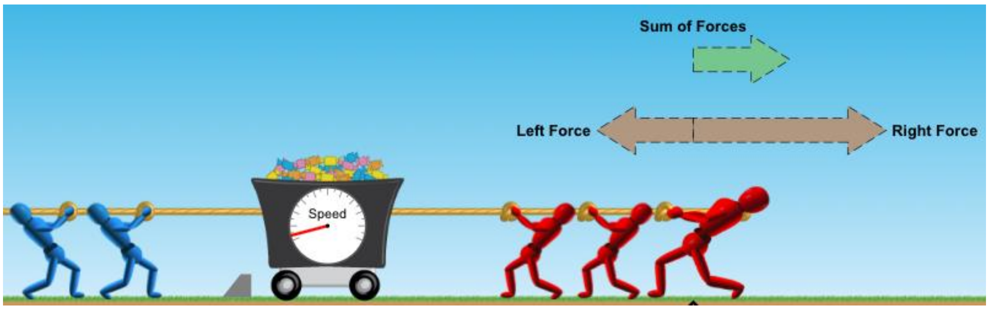


Figure 185 A net force to the right will cause a cart moving to the left to slow down

- The completed sentences are shown below:
    - When the cart is standing still, and the left and right forces on the cart are balanced, the cart will ***<u>continue standing still</u>***.
    - When the cart is moving to the left, and the left and right forces on the cart are balanced, the cart will ***<u>continue moving to the left with a constant speed</u>***.
    - What you need in order to change the movement of the cart is ***<u>a net (or unbalanced) force</u>***.
- These sentences are a description of Newton's first law. (Have a look at it again above, and compare it with these sentences).

Newton's second law

Newton's second law of motion follows on from Newton's first law, since it tells us how you can calculate the change of motion of an object as a result of a net force.

Newton's second law states: *"When a net force F_{net}, is applied to an object of mass m, the object accelerates in the direction of the net force. The acceleration a is directly proportional to the net force and inversely proportional to the mass."*

You can write Newton's second law as a mathematical equation:

$$F_{net} = ma$$

where F_{net} is the net force on an object, measured in units of newton (N)
m is the mass, measured in kg
a is the acceleration of the object, measured in m/s^2

Force is always measured in units of newton (N). 1 N of force is the amount of force that will cause an object with a mass of 1 kg to accelerate at 1 m/s^2. So 1 N = 1 $kg \cdot m/s^2$, which you can also write as 1 $kg \cdot m \cdot s^{-2}$.

Because acceleration is a vector quantity, force is also a vector quantity. The force always has the *same direction* as the acceleration.

Checklist for solving problems with Newton's second law:

The following points should be kept in mind when you apply Newton's second law:

1. When you apply this equation, you must apply it to *one object* at a time. Then you need to identify all of the forces acting on that object.
2. It is helpful to draw *free body diagrams* showing all of the forces acting on the object.
3. If all of the forces on an object are in *equilibrium*, they balance each other out and the net force is zero. This means that the object will remain at rest, or will travel with a constant velocity.
4. If the forces on the object do not balance each other out (*non-equilibrium*), there is a net force on the object. This means that the object will have a non-zero acceleration.
5. The components of the forces in the *vertical (y) direction* should be looked at separately

to find $F_{net\ y}$. This can then be used to find the acceleration in the y direction: $F_{net\ y} = ma_y$

6. Similarly, the forces in the *horizontal (x) direction* should be looked at separately to find $F_{net\ x}$. This can then be used to find the acceleration in the x direction: $F_{net\ x} = ma_x$

You can use this check-list whenever you are solving problems using Newton's second law to make sure that you are using the right problem solving approach. The example given in Activity 3 will show you how these steps are applied.

Activity 3: Apply Newton's second law

Purpose

In this activity you will first work through an example problem where Newton's second law has been applied, and then you will solve various types of problems using this law.

Suggested time: [45 minutes]

What you will do:

Work carefully through the example below, which will give you an idea of how to set out a problem where Newton's second law is applied. Once you have done this, try to solve the problems that follow. Don't forget to use the checklist given before this example to make sure that you use the right problem solving approach. Once you have completed the exercises, you can check your work against the solutions that are provided.

Example:

A box with a mass of 5 kg is being pushed along the ground in the +x direction with a force of 30 N. There is a frictional force of 10 N between the box and the ground. Calculate the acceleration of the box.

Solution:

You will only look at the forces acting on the box (Checklist step 1), since you want to calculate the acceleration of the box.

You first draw a free body diagram of all of the forces on the box (Checklist step 2).

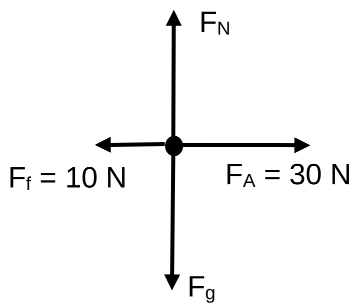


Figure 186 Free body diagram

The forces perpendicular to the surface are in equilibrium, so $F_{net\ y} = 0$. (Checklist step 3 and 5)

The net force parallel to the surface is not balanced (Checklist step 4), so you find the net force which allows us to calculate the acceleration (Checklist step 6):

$F_{net\ x} = F_A + F_f$

$= 30\ N – 10\ N$

$= +20\ N.$

Note that you always express F_{net} as the *sum of the forces* (regardless of their direction). You then show the direction using negative values when you substitute values into the

So $a_x = \frac{F_{net\ x}}{m}$

$= \frac{20\ N}{5\ kg}$

$= 4\ m{\cdot}s^{-2}$

The acceleration is 4 m·s^{-2} in the +x direction.

<u>Now try to solve these problems yourself</u>:

<u>Problem 1</u>:

A car with a mass of 800 kg was being towed using a tow rope. The tension in the tow rope was 1 500 N, and the acceleration of the car was 1,25 m/s^2. There is a frictional force between the car and the road surface. (Ignore the mass of the tow-rope).

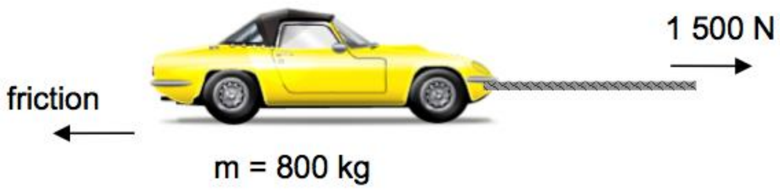


Figure 187 A car being towed using a tow rope

a. Draw a free body diagram of all of the forces on the car.

b. Calculate the frictional force between the car and the road.

Problem 2:

A car with a mass of 1 000 kg is being lifted vertically upward at a constant speed of 5 $m \cdot s^{-1}$ by a strong cable. The tension in the cable that is lifting the car is 10 000 N. There is an air resistance force acting on the car, and there are no horizontal forces acting on the car.

a. Draw a free body diagram of the forces acting on the car.

b. Calculate the magnitude of the air resistance force. (HINT: choose up as positive, and then be careful with the signs when substituting into the equation for the net force.)

Solution:

Problem 1:

a.

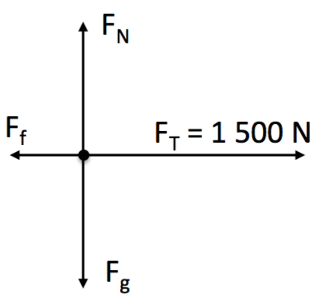


Figure 188 The free body diagram shows the forces acting on the car

b. You first need to calculate the net force using the acceleration:

F_{net} = m a = 800 kg × 1,25 m/s^2 = 1 000 N

The forces in the y direction are in equilibrium, so you don't need to use these to find the frictional force.

In the x direction: $F_{net\,x} = F_T + F_f$

You rearrange this to make the frictional force the subject of the formula:

$F_f = F_{net\ x} - F_T$

$= 1\ 000\ N – 1\ 500\ N$

$= - 500\ N$

Therefore frictional force is 500 N in the opposite direction to the tension force.

Problem 2:

a. The picture below shows the free body diagram of the forces acting on the car. (The direction of the air resistance force is down, since it opposes the upward motion of the car.)

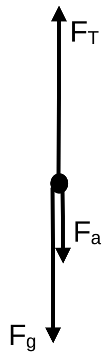


Figure 189 Free body diagram of the forces on the car

b. $F_{net\ x} = 0$ N since there are no horizontal forces.

The direction of motion is upward, so you choose up as the positive direction.

The vertical speed is constant, so $a_y = 0\ m{\cdot}s^{-2}$

Therefore $F_{net\ y} = m\ a_y = 0$ N

But $F_{net\ y} = F_T + F_g + F_a$

Therefore $0 = F_T + F_g + F_a$

You rearrange this to make the air resistance force the subject of the formula:

$F_a = – F_T – F_g = – 10\ 000\ N – (1\ 000\ kg \times (–9{,}8\ m{\cdot}s^{-2}))$

$= – 200\ N$

Therefore the air resistance force is 200 N in a downward direction.

[Word box: MAIN IDEAS:

- Newton’s second law states: “*When a net force F_{net}, is applied to an object of mass m, the object accelerates in the direction of the net force. The acceleration a is directly proportional to the net force and inversely proportional to the mass.”*
- You can write this as an equation: $F_{net} = ma$]

You can see a short YouTube video that explains Newton's second law of motion: *Physics – Newton's Second Law of Motion:* https://www.youtube.com/watch?v=8YhYqN9BwB4 (Duration: 3.56)

Activity 4: Test your understanding of Newton's first and second laws

Purpose

In this activity you will test your understanding of Newton's first and second laws by answering various questions.

Suggested time: [40 minutes]

What you will do:

Answer the questions below:

1. An object with a mass of 150 kg has an acceleration of 2 $m{\cdot}s^{-2}$. Calculate the magnitude of the net force on the object.
2. The net force on a 25 kg object is 500 N. Find the acceleration of the object.
3. Thomas gave a 50 g ball a flick so that it rolled along a horizontal surface, and the ball eventually slowed down to a stop.
   a. Why did the ball not continue to move with a constant velocity after it had been flicked by Thomas' hand? Use Newton's first law in your explanation.
   b. If the frictional force between the ball and the surface is 0,2 N, calculate the acceleration of the ball.
4. A car with a mass of 1000 kg is pulled along a horizontal road with a force of 5000 N to the right. There is a frictional force opposing the car's movement. As a result the net force on the car is 4000 N to the right.
   a. Draw a free body diagram of all of the forces acting on the car.
   b. Calculate the acceleration of the car.
   c. What is the magnitude and direction of the frictional force?

5. A car with a mass of $1,4 \times 10^3$ kg is being towed by a tow-truck. The car is accelerating at a rate of $0,5\ m{\cdot}s^{-2}$. There is a frictional force of 200 N between the car's wheels and the road. What is the magnitude of the tension in the tow-rope?

6. An object with a mass of 1,2 kg is falling in air with an acceleration of $9\ m{\cdot}s^{-2}$. What is the magnitude and direction of the force due to air resistance?

Solutions:

1. Given: m = 150 kg

   a = $2\ m{\cdot}s^{-2}$

   F_{net} = ma

   = 150 kg × $2\ m{\cdot}s^{-2}$

   = 300 N

2. Given: F_{net} = 500 N

   m = 25 kg

   From the equation F_{net} = ma you solve for a:

   $$a = \frac{F_{net}}{m}$$

   $$= \frac{500\ N}{25\ kg}$$

   = $20\ m{\cdot}s^{-2}$

3.

   a. According to Newton's first law, there should be no net force on the ball for it to move with a constant velocity in a straight line. The ball did not continue to move with a constant velocity after it had been pushed by Thomas' hand because there was a frictional force that opposed the motion of the ball, resulting in a net force on the ball in the opposite direction to its motion.

b. Given: $F_{friction}$ = -0,2 N

$F_{friction}$ is negative since friction opposes the motion.

m = 50 g ÷ 1000 = 0,05 kg

Since there are no other unbalanced forces on the ball,

$F_{net} = F_{friction}$ = -0,2 N.

From the equation F_{net} = ma you solve for a:

$$a = \frac{F_{net}}{m}$$

$$= \frac{-0{,}2\ \text{N}}{0{,}05\ \text{kg}}$$

$= -4\ m{\cdot}s^{-2}$

4.

a.

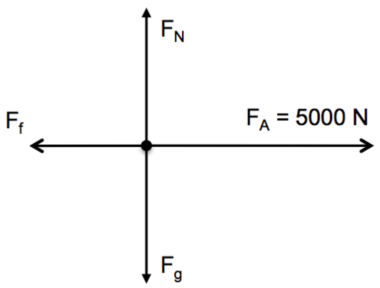


Figure 190 Free body diagram of the forces on the car

b. From the equation F_{net} = ma you solve for a:

$$a = \frac{F_{net}}{m}$$

$$= \frac{4000\ \text{N}}{1000\ \text{kg}}$$

$= 4\ m{\cdot}s^{-2}$

c. Choose right as +

You write F_{net} as the sum of the forces:

$F_{net} = F_A + F_f$

You rearrange this to solve for F_f

$F_f = -F_A + F_{net}$

= -5000 N + 4000 N

= -1000 N

The frictional force is 1000 N to the left.

5.

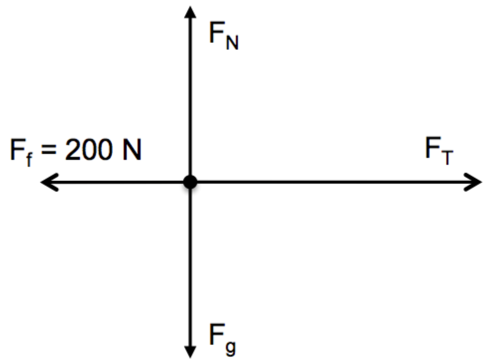


Figure 191 Free body diagram of the forces

Given: $m = 1{,}4\times10^3$ kg

$a = 0{,}5\ m{\cdot}s^{-2}$

Choose right as +

Calculation:

$F_{net} = ma$

$= 1{,}4\times10^3\ kg \times 0{,}5\ m{\cdot}s^{-2}$

$= 700$ N

You can also express F_{net} as the sum of the forces:

$F_{net} = F_T + F_f$

You rearrange this to solve for F_T

$F_T = -F_f + F_{net}$

$= -200\ N + 700\ N$

$= +500$ N

The tension force is 500 N in the direction of the acceleration of the car.

6.

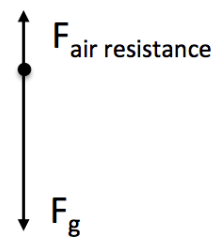


Figure 192 Free body diagram of the forces

Given: F_g = 11,76 N

a = 9 $m{\cdot}s^{-2}$

Choose down as +

You usually choose the direction of motion as the positive direction in your frame of reference

Calculation:

F_{net} = ma

= 1,2 kg × 9 $m{\cdot}s^{-2}$

= 10,8 N

But you can express F_{net} as the sum of the forces:

$F_{net} = F_g + F_a$

You rearrange this to solve for F_a

$F_a = -F_g + F_{net}$

= -11,76 N + 10,8 N

= -0,96 N

The air resistance force is 0,96 N upward.

Newton’s Third Law

Newton’s third law of motion states:

When object A exerts a force on object B, object B simultaneously exerts an oppositely directed force of equal magnitude on object A.

Another way of stating this law is: “For every action there is an equal and opposite reaction.”

Forces always take place in pairs, which are known as "action-reaction force pairs." If you look at the example of a book that is placed on a table, the action force is the weight of the book acting downward on the table. The reaction force is the normal force that the surface of the table is exerting on the book.

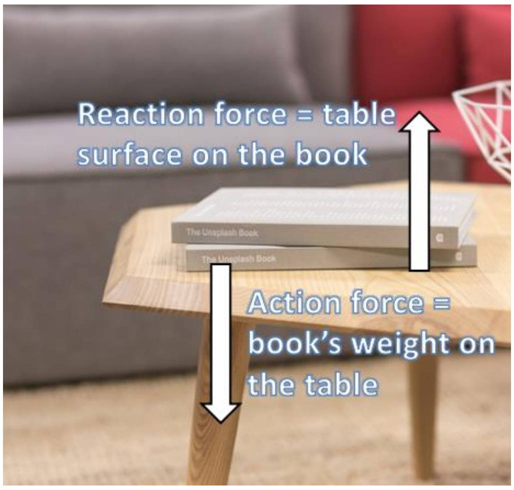


Figure 193 Action-reaction force pair for a book on a table

So the properties of action-reaction force pairs are:

- the reaction force is *equal in magnitude* to the action force
- the reaction force is *opposite in direction* to the action force
- the two forces take place simultaneously (at the same time)
- the pair of forces act on *different objects*.

Newton's third law is applied in many examples of motion. In a rowing boat, the action force is the force of the oar pushing backward on the water. The reaction force is the forward movement of the boat.

Figure 194 Action-reaction force pair for a rowing boat moving in water

Activity 5: Apply Newton's third law

Purpose

In this activity you will apply Newton's third law to a range of everyday contexts.

Suggested time: [20 minutes]

What you will do:

Identify the action and reaction forces in each of the pictures shown below:

1.

Figure 195 A swimmer moving through water

2.

Figure 196 A rocket blasts off from the ground

3.

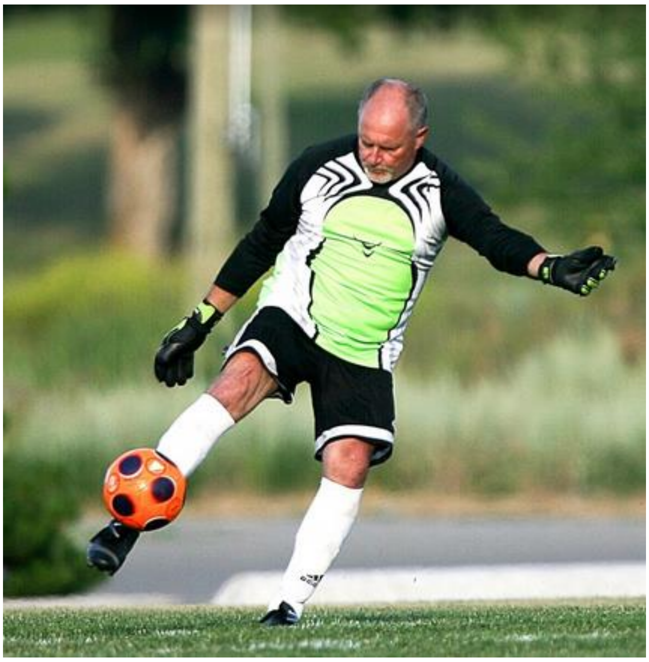


Figure 197 A man kicks a soccer ball

Solutions:

1.

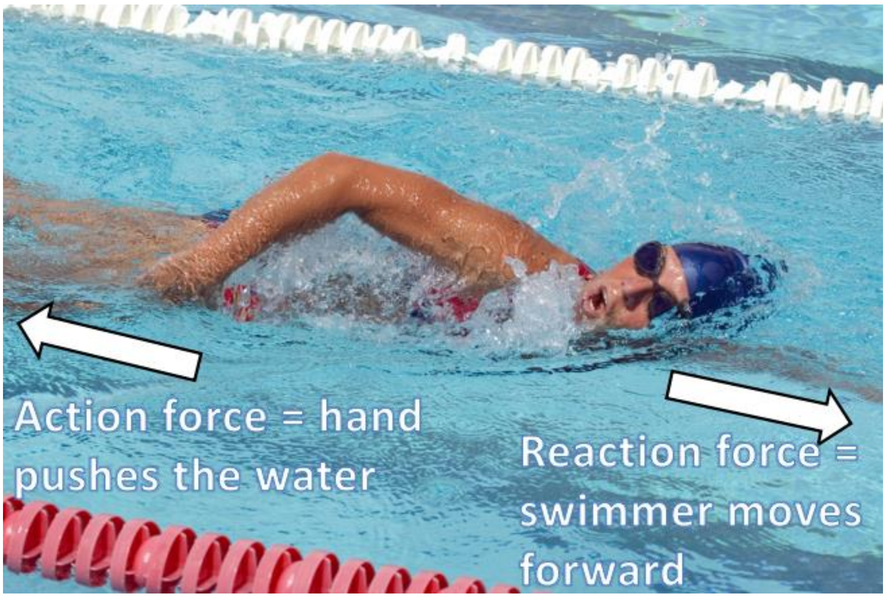


Figure 198 Action-reaction force pair for a swimmer moving in the water

2.

Figure 199 Action-reaction force pair for a rocket

3.

Figure 200 Action-reaction force pair for a man kicking a soccer ball

[Word box: MAIN IDEAS:

- Newton's third law states: "*When object A exerts a force on object B, object B simultaneously exerts an oppositely directed force of equal magnitude on object A.*"
- Action-reaction force pairs are equal in magnitude but opposite in direction to each other.]

You can see a short 4 minute video that explains Newton's third law of motion at this website: *Physics – Newton's Third Law of Motion*: https://www.youtube.com/watch?v=TVAxASr0iUY (Duration: 4.08)

Newton's laws with joined (coupled) objects

You can apply Newton's laws to objects that are joined and moving together. Some important notes when solving problems for joined objects:

- The *tension is the same everywhere* in the rope that is joining the objects. You represent this force as F_T in the equation for Newton's second law for both objects.
- Since the objects are joined, they *accelerate with the same acceleration*, which you represent by the symbol a for both objects.
- Since F_T and a are common for both objects, you can write an expression for Newton's second law for each object, which gives us 2 equations with 2 unknowns. You can then solve these using the simultaneous equation method.

The example given in the following activity will show you how these problem solving strategies are applied.

Activity 6: Apply Newton's laws to joined objects

Purpose

In this activity you will be guided through an example problem where Newton's second law is applied to the movement of two objects that are joined together.

Suggested time: [20 minutes]

What you will do:

Work through the example that is given, and fill in the missing information:

Example:

A box with a mass of 500 g is sitting on a horizontal surface, and is connected by a rope that is threaded over a frictionless pulley to an object that has a mass of 300 g, which is hanging from the end of the rope. There is a frictional force of 1,2 N between the surface and the box. Find the tension in the rope.

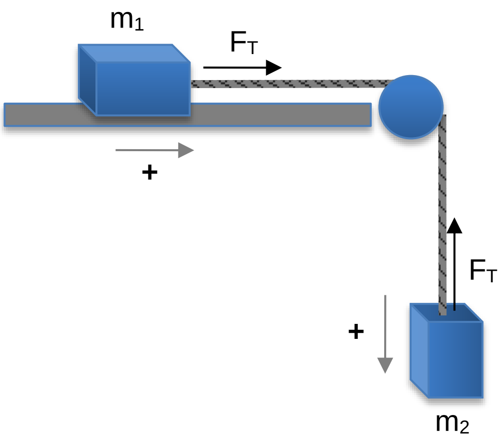


Figure 201 Coupled objects connected by a rope hanging over a pulley

<u>Calculation</u>:

Let the box on the horizontal surface be m_1 and the mass hanging from the rope be m_2.

Given: m_1 = 500 g = _______ kg

m_2 = 300 g = _______ kg

F_f = -1,2 N

F_f is negative since friction always opposes the motion, and you choose the direction of motion as positive.

Frame of reference: Since m_1 will move to the right, you choose *right* as + for this object, and since m_2 will move down, you choose *down* as + for this object.

The tension is the same everywhere in the rope that is joining the objects. This tension pulls to the right on m_1, so you will represent this force as $+F_T$. The tension force pulls upward on m_2, so you will represent this force as $-F_T$.

Since the masses are joined, they will both accelerate with the same acceleration, which you represent by the symbol a.

Draw the free body diagram for m_1:

Draw the free body diagram for m_2:

Since F_T and a are common for both objects, you can write an expression for Newton's second law for each object, which gives us 2 equations with 2 unknowns. You can then solve these using the simultaneous equation method:

Expressing Newton's second law for object m_1:

The net force on object m_1 can be expressed as $F_{net\ 1} = m_1 a =$ _____ $\times$ a

But you can also express the net force as the sum of the forces:

$F_{net\ 1} = F_T + F_f$

You can substitute the expressions for $F_{net\ 1}$ and F_f into this equation

$\therefore$ ________ $\times$ a = F_T + (________) (*Equation 1*)

Expressing Newton's second law for object m_2:

The weight of object m_2 is: $F_{g2} = m_2 g =$ ______ $\times$ 9,8 = ________

The net force on object m_2 can be expressed as $F_{net\ 2} = m_2 a =$ _____ $\times$ a

But you can also express the net force as: $F_{net\ 2} = F_{g2} + F_T$

$\therefore$ _____ $\times$ a = _______ + ($-F_T$) (*Equation* 2)

The magnitude of F_T is negative because its direction is opposite to the motion.

You can rearrange equation (2):

F_T = _________________________

Insert the above expression for F_T here.

Substitute this into equation (1):

________ $\times$ a = _____________ + (________)

Now solve for a:

$\therefore$ a = ________________

Substitute this back into equation (2) and solve for F_T:

F_T = ____________________

Therefore the tension in the rope is _________ N.

Solution:

The missing information from the steps in the above calculation is shown below.

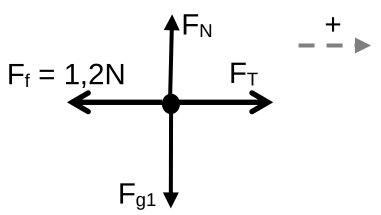


Figure 202 Free body diagram for m_1

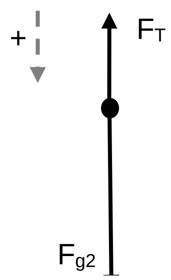


Figure 203 Free body diagram for m_2

The net force on object m_1 can be expressed as $F_{net\ 1} = m_1a$ = **0,5**×a

But you can also express the net force as: $F_{net\ 1} = F_T + F_f$

∴ **0,5**×a = F_T + **(-1,2)** (*Equation 1*)

The weight of object m_2 is: $F_{g2} = m_2g$ = **0,3** × 9,8 = **2,94N**

The net force on object m_2 can be expressed as $F_{net\ 2} = m_2a$ = **0,3** × a

But you can also express the net force as: $F_{net\ 2} = F_{g2} + F_T$

∴ **0,3** × a = **2,94** + ($-F_T$) (*Equation* 2)

From equation (2): F_T = **2,94 – 0,3a**

Substitute this into equation (1): **0,5** × a = **(2,94 – 0,3a)** + (**-1,2**)

∴ 0,8a = 1,74

∴ a = **2,175 m·s^{-2}**

Substitute this back into equation (2): F_T = 2,94 – 0,3a = **2,29 N**

The tension in the rope is **2,29 N**.

More resources to help you:

For more information on Newton's laws of motion, visit the website:

https://www.physicsclassroom.com/Physics-Tutorial/Newton-s-Laws

Activity 7: Consolidate your learning of forces

Purpose

In this activity you will consolidate your learning of forces by answering the questions below and then assessing your own understanding using the solutions provided. Give yourself a mark out of the total of 70 marks, which will give you an idea of how well you understand this section of the work.

Suggested time: [80 minutes]

1. State Newton's first law of motion. (2)

2. Thabo has attached a rope to a box, and is pulling on the rope, but the box does not move at all.
   a. What is the net force on the box? (1)
   b. Explain what is keeping the box from moving. (2)
   c. Draw a free body diagram of the box, and label all the forces acting on it. (4)

3. Mandy is riding her bicycle, and is pushing on the peddles with a force of 50 N, and as a result she is moving with a constant velocity. What is the net force on the bicycle? Explain your answer. (3)

4. A 50 g ball is sitting on the ground.
   a. What is the size of the force that the earth exerts on the ball? (3)
   b. What is the name of this force? (1)
   c. What is the size of the force that the ball exerts on the earth? Give a reason for your answer. (2)

d. A force of 10 N is applied to this object, and there is a frictional force of 2 N acting on the object. Find the acceleration of the object. (6)

5. An object with a mass of 10 kg is being pushed, and as a result it has an acceleration of 2 $m \cdot s^{-2}$.
   a. Calculate the magnitude of the net force on the object. (3)
   b. If there is a frictional force of 5 N that is opposing the movement of the object, what is the magnitude of the applied force on the object? (4)

6. Two people are pulling a box that has a mass of 15 kg. Grace is pulling on the box with a force of 20 N in the +x direction. Sam is pulling on the box with a force of 12 N in the –x direction. There is a frictional force of 3 N opposing the box's movement.
   a. Draw a free body diagram that shows all the forces on the box. (5)
   b. What is the acceleration of the box? (6)

7. A child pushes his 50g toy car with a force of 0,06N at the angle of 60^{o} to the ground. As a result, the car moves forward with a constant acceleration of 0,4 $m \cdot s^{-2}$. Calculate the frictional force on the car. (6)

8. A 1 200 kg car is towing a trailer with a mass of 600 kg. The engine of the car applies a total pushing force of 2 000 N.
   a. What is the acceleration of the car and trailer if you ignore friction? (5)
   b. Calculate the force that is exerted on trailer. (3)
   c. If there is a frictional force of 10 N on the trailer, and 15 N on the car, find the net force on the car. (6)

9. A book with a mass of 2 kg is sitting on a horizontal surface, and is connected by a rope that is threaded over a frictionless pulley to an object that has a mass of 1 kg, which is hanging from the end of the rope. There is a frictional force of 3 N between the surface and the book. Find the acceleration of the book. (8)

Solutions

1. Newton's first law of motion: *An object continues in a state of rest or uniform velocity (1) unless it is acted upon by an external unbalanced force*. (1) (2)

2.
   a. Net force on the box is 0 N since the box is not moving. (1)
   b. There is a frictional force (1) that opposes (1) the applied force (or acts in the opposite direction). (2)
   c. Free body diagram: (1 mark for each correctly labelled force) (4)

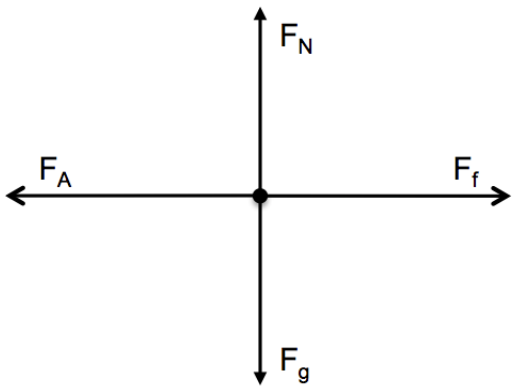


Figure 204 Free body diagram of the forces

3. The net force on the bicycle is 0 N (1). According to Newton's first law (1), there is no net force on an object for it to move with a constant velocity in a straight line. (1)

   (3)

4.
   a. $F_g = 0{,}05 \text{ kg} \times 9{,}8 \text{ m·s}^{-2} = 0{,}49 \text{ N}$ (3)
   b. Gravitational force, or weight (1)
   c. The ball exerts a force of 0,49 N on the earth (Newton's third law). (2)
   d. $F_{net} = F_A + F_f$

      $= 10 \text{ N} – 2 \text{ N}$

      $= 8 \text{ N}$.

      From the equation $F_{net} = ma$ you solve for a:

      $a = \frac{F_{net}}{m}$

      $= \frac{8 \text{ N}}{0{,}05 \text{ kg}}$

$= 160\ m{\cdot}s^{-2}$ in the direction of the applied force (6)

5. Given: $m = 10\ kg$; $a = 2\ m{\cdot}s^{-2}$

   a. $F_{net} = ma = 10\ kg \times 2\ m{\cdot}s^{-2} = 20\ N$ (3)

   b. Let the direction of the acceleration be positive.

   $F_{net} = F_A + F_f$

   Therefore $F_A = -F_f + F_{net} = -(-5\ N) + 20\ N = 25\ N$

   The applied force is 25 N in the direction of the acceleration of the object. (4)

6.

   a. Free body diagram: (1 mark for each correctly labelled force) (5)

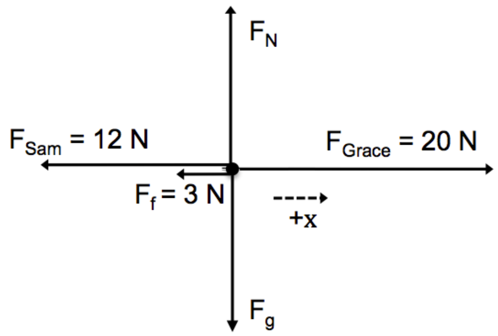


Figure 205 Free body diagram of the forces

   b. $F_{net} = F_{Grace} + F_{Sam} + F_f = 20\ N - 12\ N - 3\ N = 5\ N$

   From the equation $F_{net} = ma$ you solve for a:

   $a = \frac{F_{net}}{m}$

   $= \frac{5\ N}{15\ kg}$

   $= 0{,}33\ m{\cdot}s^{-2}$ in the direction of the force applied by Grace (6)

7. Given: $m = 50\ g \div 1000 = 0{,}05\ kg$

   $a = 0{,}4\ m{\cdot}s^{-2}$

   $F_{Ax} = F_A \cos\theta$

   $= 0{,}06 \times \cos 60°$

   $= 0{,}03\ N$

   $F_{Net} = m \times a$

   $= 0{,}05\ kg \times 0{,}4\ m{\cdot}s^{-2}$

$= 0{,}02N$

But $F_{Net} = F_f + F_{Ax}$

Therefore $F_f = F_{Net} - F_{Ax} = 0{,}02N – 0{,}03N = -0{,}01N$

The frictional force is 0,01 N in the opposite direction to the motion of the car. (6)

8. Given: $m_1 = 1\ 200$ kg

   $m_2 = 400$ kg

   $F_A = 2\ 000$ N.

   a. If you ignore friction, you can treat the car and trailer as one combined mass. The only force on the system is the applied force.

      Therefore $F_{net} = F_A = 2\ 000$ N

      From the equation $F_{net} = ma$ you can solve for a:

      $$a = \frac{F_{net}}{m_1 + m_2}$$

      $$= \frac{2000\ N}{1200\ kg + 400\ kg}$$

      $= 1{,}25\ m{\cdot}s^{-2}$ in the direction of the applied force (5)

   b. For the trailer:

      $F_{net} = ma$

      $= 400\ kg \times 1{,}25\ m{\cdot}s^{-2}$

      $= 500$ N (3)

   c. Given: $m_1 = 1\ 200$ kg

      $m_2 = 400$ kg

      $F_A = 2\ 000$ N

      Total frictional force on the system:

      $F_f = -100\ N - 150\ N = -250\ N$

      Remember that F_f is negative since friction always opposes the motion.

      Therefore $F_{net} = F_A + F_f$

      $= 2000\ N - 250\ N$

      $= 1750$ N

      $$a = \frac{F_{net}}{m_1 + m_2}$$

      $$= \frac{1750\ N}{1200\ kg + 400\ kg}$$

      $= 1{,}09\ m{\cdot}s^{-2}$ in the direction of the applied force.

      For the car: $F_{net} = ma$

      $= 1200\ kg \times 1{,}09\ m{\cdot}s^{-2}$

      $= 1308$ N (6)

9. **Frame of reference:** Since the book (m_1) will move to the right, you choose *right* as + for this object, and since the weight (m_2) will move down, you choose *down* as + for this object.

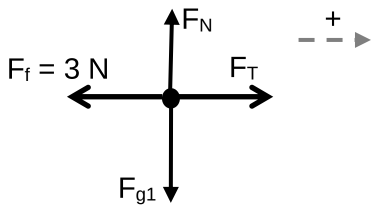


Figure 207 Free body diagram for m_1

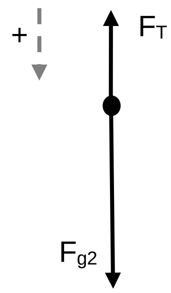


Figure 206 Free body diagram for m_2

Calculation:

The net force on object m_1 can be expressed as $F_{net\,1} = m_1a = 2a$

But you can also express the net force as: $F_{net\,1} = F_T + F_f$

$\therefore 2a = F_T + (-3)$ (*Equation 1*)

The weight of object m_2 is: $F_{g2} = m_2g = 1 \times 9,8 = 9,8$ N

The net force on object m_2 can be expressed as $F_{net\,2} = m_2a = 1a$

But you can also express the net force as: $F_{net\,2} = F_{g2} + F_T$

$\therefore 1a = 9,8 + (-F_T)$ (*Equation 2*)

From (*Equation 2*): $F_T = 9,8 - a$

Substitute this into (*Equation 1*): $2a = (9,8 - a) - 3$

$\therefore 3a = 6,8$

$\therefore a = 2,27\ m{\cdot}s^{-2}$ to the right (8)

Sub-topic 4: Momentum and impulse

Unit 1: Linear momentum and impulse

Learning Outcomes

By the end of the unit, you should be able to:

- define linear momentum and impulse;
- calculate the momentum of a moving object;
- calculate the change in momentum of an accelerating object;
- define force as the rate of change of momentum.

Introduction

Imagine rolling a ball along a frictionless surface. Discuss your ideas with another student, or write down your ideas.

- What will you observe about the ball's velocity?
- What is the net force on the ball after it has left your hand?
- What is it that keeps the ball moving forward after it has left your hand?

In the example of a ball rolling after it has left your hand, you will know from the previous section that there is no force acting on the ball in the direction of its motion once it has left your hand. The only force acting on it is the frictional force that slows it down. There is a property that keeps it moving forward that is not a force. This property is called *momentum*.

What is momentum?

Momentum is defined as the quantity of motion of a moving body (or the property that keeps an object moving in a certain direction).

Activity 1: Explore momentum

Purpose

In this activity, you will explore the concept of momentum by rolling two balls of different mass and seeing how far they move a cardboard object.

Suggested time: [20 minutes]

What you need:

- Two balls of different masses
- A piece of cardboard
- A thin piece of wood or stiff cardboard

What you will do:

6. Using your piece of wood or stiff cardboard, lift one end and place it on a book so that it forms a ramp for your balls to roll down.
7. Cut your other piece of cardboard into a rectangular shape, and fold this in half so that it forms an upside down V shape. Place this about 10 cm from the bottom of your ramp, and make a mark to show where you placed it (see the picture to guide you).

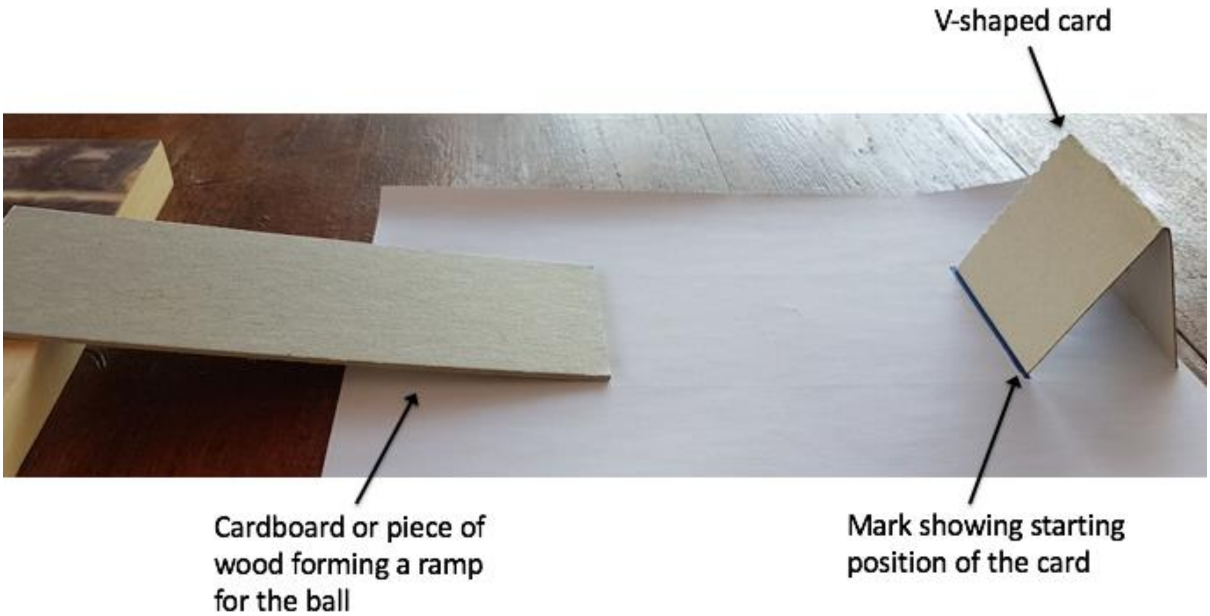


Figure 208 Photograph of the experimental setup

8. Place your smaller ball at the top of the ramp, and roll it down the ramp. When it strikes the V-shaped card, notice how far the card moves. Measure this distance.
9. Now return the card to its starting position, and roll the larger ball down the ramp. What difference do you see in the distance that the card moves?
10. Now place the raised end of your ramp on two books, so that you raise the angle of your ramp. This will increase the velocity of the ball. Roll your smaller ball down the ramp, and notice how far it causes the V-shaped card to move. How does this compare with the distance moved when the velocity was less?

Guided reflection:

- In this experiment, you should have noticed that the V-shaped card was moved a greater distance when it was struck by the larger ball than by the smaller ball.
- Since the distance moved by the card tells us about the momentum of the ball that strikes it, you can conclude that the larger the mass of an object, the greater its momentum.
- You should also have noticed that when you raised the slope using two books instead of one, the V-shaped card was moved a greater distance than when you had a lower slope.
- You can therefore conclude that the larger the velocity of an object, the greater its momentum.

What you have seen in this experiment is that the momentum of a moving object is *proportional* to its *mass*, and to its *velocity*.

You can write this mathematically as:

Momentum = mass × velocity

The equation for this is:

$p = m\,v$

where p is the momentum of the object, measured in $kg \cdot m \cdot s^{-1}$,

m is the mass of the object, measured in kg, and

v is the velocity of the object, measured in $m \cdot s^{-1}$.

Momentum is a *vector* quantity. The momentum will always have the *same direction as the velocity.*

Activity 2: Test your understanding of momentum

Purpose

In this activity you will test your understanding of momentum by solving various problems.

[30 minutes]

What you will do:

Answer the following questions.

2) A car that has a mass of $1{,}2 \times 10^3$ kg is travelling at a speed of 72 $km \cdot hr^{-1}$. Calculate the momentum of the car.

3) A 20 kg child is riding on his go-cart, which has a mass of 7,5 kg, with a speed of 8 $m \cdot s^{-1}$. What is the momentum of the child and the go-cart together?

4) The child applies the brakes of the go-cart so that it is now moving with a speed of 6 $m \cdot s^{-1}$. By how much has its momentum changed?

5) Which has more momentum: a 900 kg car moving at 100 $km \cdot hr^{-1}$ or a 1800 kg vehicle moving at 50 $km \cdot hr^{-1}$?

Solution:

1. Given: $m = 1,2 \times 10^3$ kg

   $v = 72\ km \cdot hr^{-1} \times \frac{1000\ m/km}{3600\ s/hr}$

   $= 20\ m \cdot s^{-1}$

   $p = mv$

   $= 1,2 \times 10^3\ kg \times 20\ m \cdot s^{-1}$

   $= 2,4 \times 10^4\ kg \cdot m \cdot s^{-1}$

2. Given: $m = 20\ kg + 7,5\ kg = 27,5\ kg$

   $v = 8\ m \cdot s^{-1}$.

   $p = mv$

   $= 27,5\ kg \times 8\ m \cdot s^{-1}$

   $= 220\ kg \cdot m \cdot s^{-1}$

3. Given: $m = 20\ kg + 7,5\ kg = 27,5\ kg$

   $v_i = 8\ m \cdot s^{-1}$

   $v_f = 6\ m \cdot s^{-1}$

   Change in momentum :

   $\Delta p = m \Delta v$

   $= m\ (v_f - v_i)$

   $= 27,5\ kg \times (8\ m \cdot s^{-1} - 6\ m \cdot s^{-1})$

   $= 55\ kg \cdot m \cdot s^{-1}$

4. $p_1 = m_1 v$

   $= 900\ kg \times 100\ km \cdot hr^{-1} \times \frac{1000\ m/km}{3600\ s/hr}$

   $= 25\ 000\ kg \cdot m \cdot s^{-1}$

   $p_2 = m_2 v$

= 1800 kg × 20 m·s^{-1}

= 36 000 kg·m·s^{-1}

The 1800 kg vehicle moving at 20 m·s^{-1} has the higher momentum.

[WORDBOX: MAIN IDEAS:

- *Momentum* (symbol p) is defined as the quantity of motion of a moving body. It is the property that keeps a body in motion:

  $p = mv$

- Momentum is measured in units of kg·m·s^{-1}]

Impulse

Newton's first law can also be written in this way: In order to change the momentum of an object, a net force must be applied to that object.

You can write this mathematically as:

Change in momentum = force × time interval

The equation for this is:

$\Delta p = F\,\Delta t$

where Δp is the change in the momentum of the object, measured in kg·m·s^{-1},

F is the force exerted on the object, measured in newtons (N), and

Δt is the time for which the force acts, measured in seconds (s).

The name that is given to the force multiplied by the time interval is the *impulse*.

You can therefore write: Impulse = $F\,\Delta t$

But since Δp means the change in momentum, you can write it as $\Delta p = m\,\Delta v$

Therefore: Impulse = $F\,\Delta t = m\,\Delta v$

You can see a YouTube video that explains impulse: *impulse examples///Homemade Science with Bruce Yeany*: https://www.youtube.com/watch?v=oXW6RqEwVWA (Duration: 7.34)

Activity 3: Explore impulse

Purpose

In this activity you will explore impulse using an egg and a sheet.

Suggested time: [15 minutes]

What you need:

- A large bed sheet
- An egg

What you will do:

1. Ask two of your fellow students or friends to hold up a bed sheet or large piece of fabric. They just need to hold the top two corners, and pull these fairly firmly so that the top of the sheets is held firmly between them, and the rest of the sheet hangs loosely below.
2. Throw an egg at the middle of the sheet, as hard as you can.
3. What do you observe? Can you explain your observations?
4. What would happen if you threw the same egg against a wall? (You don't have to actually do this, I am sure you know what will happen!)
5. Can you explain the difference between throwing an egg against a sheet and a wall?
6. Can you think of some everyday applications of this principle?

Guided reflection:

- What you throw an egg against a sheet, it should not break, because the collision between the egg and the sheet takes place over an extended period of time since the sheet is free to move when the egg strikes it.
- By increasing the time of the collision, the force of the impact is decreased.
- In comparison, when you throw an egg against a wall, the time over which the collision takes place is very small, since the wall can not move at all.
- By decreasing the time of the collision, the force of the impact is increased.
- Some everyday applications of this are:
    - **Air bags and seat belts** are used in motor vehicles to reduce the effect of the force that a person experiences during an accident, since each of these devices extends the time needed to stop the momentum of the person in the car.

- **Arrestor beds** are used to helps trucks to slow down or to make an emergency stop. An arrestor bed is a thick sand patch that increases the time interval over which the momentum of a truck is changed. This decreases the force on the truck while it is slowing down.
- **Crumple zones** in cars reduce the injury to people during a collision. When a car crumples on impact, rather than bouncing backwards, there is a smaller change in momentum and therefore a smaller impulse. The smaller impulse means that the occupants of the cars experience a smaller force. When the car crumples, the change in the car's momentum happens over a longer time, resulting in a smaller force on the people in the car.

Activity 4: Test your understanding of impulse and momentum

Purpose

In this activity you will test your understanding of impulse and momentum by solving problems.

Suggested time: [30 minutes]

What you will do:

Answer the following questions.

1. A 5 kg ball falls into a sand pit with an initial velocity of 4 $m \cdot s^{-1}$, and comes to rest in a tenth of a second. What is the force that the ball exerted on the sand?

2. A 0,02 kg ball is travelling towards a wall with a velocity of 20 $m \cdot s^{-1}$. It bounces off the wall, and returns in the opposite direction with a velocity of 18 $m \cdot s^{-1}$. The ball is in contact with the wall for 0,2 seconds.

a. Calculate the net force exerted on the ball.

b. Calculate the acceleration of the ball.

3. A 50 g bullet hits a block of wood with an initial velocity of 500 $m \cdot s^{-1}$, and comes to rest in 0,025 seconds.

a. What is the force that the block exerted on the bullet?

b. What is the force that the bullet exerted on the block?

4. During a bungee jump, a person jumps off a high bridge with a very strong elastic rope tied to their ankles. Explain, in terms of impulse and momentum, why a bungee rope can not be made out of normal non-elastic rope.

5. A 50 g ball is falling towards floor with a velocity of 10 $m{\cdot}s^{-1}$. It bounces off the floor with a velocity of 8 $m{\cdot}s^{-1}$. If the ball was in contact with the floor for 0,2 seconds, what was the force exerted on the ball by the floor?

Solutions:

1. Given: m = 5 kg

   $v_i = 4\ m{\cdot}s^{-1}$

   $v_f = 0\ m{\cdot}s^{-1}$

   $\Delta t = 1/10\ s = 0,1\ s$

   From the equation $F\ \Delta t = m\ \Delta v$ you can rewrite this to make F the subject of the formula:

   $$F = \frac{m\ \Delta v}{\Delta t}$$

   $$= \frac{5kg \times (4\ m{\cdot}s^{-1} - 0\ m{\cdot}s^{-1})}{0,1\ s}$$

   = 200 N

   The force that the ball exerted on the sand was 200 N downward.

2. Frame of reference: Let the direction of the initial velocity of the ball be positive

   Given: m = 0,02 kg

   $v_i = 20\ m{\cdot}s^{-1}$

   $v_f = -18\ m{\cdot}s^{-1}$

   $\Delta t = 0,2\ s$

   $p_i = 0,02 \times 20 = 0,4\ kg{\cdot}m{\cdot}s^{-1}$

   $p_f = 0,02 \times (-18) = -0,36\ kg{\cdot}m{\cdot}s^{-1}$

   Calculation:

   a. $F_{net} = \frac{\Delta p}{\Delta t}$

   $$= \frac{-0,76\ kg{\cdot}m{\cdot}s^{-1}}{0,2\ s}$$

   = - 3,8 N

The net force exerted on the ball is 3,8 N in the opposite direction to its initial velocity.

b. $a = \frac{\Delta v}{\Delta t}$

$= \frac{-18 \text{ m}\cdot s^{-1} - 20 \text{ m}\cdot s^{-1}}{0{,}2 \text{ s}}$

$= -190 \text{ m}\cdot\text{s}^{-2}$

OR from Newton's second law

$a = \frac{F}{m}$

$= \frac{-3{,}8 \text{ N}}{0{,}02 \text{ kg}}$

$= -190 \text{ m}\cdot\text{s}^{-2}$

The acceleration of the ball is 190 $\text{m}\cdot\text{s}^{-2}$ in the opposite direction to its initial velocity.

3. Given: $m = 50 \text{ g} \div 1000 = 0{,}05 \text{ kg}$

Remember that there are 1000 g in 1 kg. Therefore to convert from g to kg you divide by 1000.

$v_i = 500 \text{ m}\cdot\text{s}^{-1}$

$v_f = 0 \text{ m}\cdot\text{s}^{-1}$

$\Delta t = 0{,}025 \text{ s}$

a. $F = \frac{m\,\Delta v}{\Delta t}$

$= \frac{0{,}05\text{kg} \times (0 \text{ m}\cdot s^{-1} - 500 \text{ m}\cdot s^{-1})}{0{,}025 \text{ s}}$

$= -1000 \text{ N}$

The block exerted a force of 1000 N on the bullet in the opposite direction to the bullet's motion.

b. The bullet exerted a force of 1000 N on the block in the same direction as the bullet's motion.

4. The bungee rope is made of elastic, therefore this means that it takes a longer time for the person to come to a stop than if the rope was made of non-elastic material. As a result, the force on the person is much lower with a bungee rope than with a normal non-elastic rope.

5. Frame of reference: Let down be +

Given: $m = 0{,}1 \text{ kg}$

$v_i = +10 \text{ m}\cdot\text{s}^{-1}$

$v_f = -8\ m{\cdot}s^{-1}$

$\Delta t = 0,2\ s$

$F = \frac{m\,\Delta v}{\Delta t}$

$= \frac{0{,}1\text{kg} \times (-8\ \text{m}{\cdot}s^{-1} - 10\ \text{m}{\cdot}s^{-1})}{0{,}2\ \text{s}}$

$= -9\ N$

The floor exerted a force of 9 N on the ball in the upward direction.

[BOX]: MAIN IDEAS:

- **Impulse** is the change in the momentum of an object, which is equal to the force on an object multiplied by the time interval: Impulse = $F\,\Delta t = m\,\Delta v$

More resources to help you:

You can find more information on momentum and impulse at these web pages:

https://www.physicsclassroom.com/Class/momentum/u4l1a.cfm

https://www.physicsclassroom.com/class/momentum/Lesson-1/Momentum-and-Impulse-Connection

Unit 2: Momentum in collisions

Learning Outcomes

By the end of the unit, you should be able to:

- state the principle of conservation of momentum;
- apply the principle of conservation of momentum to solve problems involving collisions between two bodies in 1-dimension, in familiar and novel contexts.

Introduction

In the previous unit you learnt about the concept of momentum. Refresh your understanding of this concept by reflecting on the following questions:

- What would be the impact on the momentum of a moving object if:
  - its mass was doubled?
  - its velocity was halved?

You will recall from the previous unit that momentum is the property that keeps an object moving in a certain direction. Momentum is proportional to the mass and the velocity of the moving object. In this unit you will learn what happens when two moving objects collide (come into contact with each other).

The momentum of colliding objects

Activity 1: Explore the momentum of colliding objects

Purpose

When two objects come into contact with one another, they are involved in a collision. In this activity you will explore the combined momentum of objects before and after they collide with one another.

Suggested time**:** [30 minutes]

What you need:

- 2 marbles or balls that have the same size and mass
- A piece of wood or desktop that has a groove cut into it
- 2 stop-watches (you could use the stop-watch feature on 2 cellphones)
- A mass measuring scale
- A ruler or measuring tape

What you will do:

1. Measure the mass of each marble (or ball), and record the mass under m_1 and m_2 in the table below. (If you do not have a mass measuring scale, assume the mass of each marble or ball to be 10 g).

m_1 (kg)	Δt_1 (s)	v_1 ($m{\cdot}s^{-1}$)	m_1v_1 ($kg{\cdot}m{\cdot}s^{-1}$)	m_2 (kg)	Δt_2 (s)	v_2 ($m{\cdot}s^{-1}$)	m_2v_2 ($kg{\cdot}m{\cdot}s^{-1}$)

2. Place one marble (m_1) at one end of the groove, and the other marble (m_2) half way along the groove.
3. Measure the distance from marble m_1 to marble m_2, and from marble m_2 to the end of the groove.
4. Start one of your stop-watches as you flick (not too hard!) marble m_1 towards marble m_2, and stop the stop-watch as it collides with marble m_2.
5. Start your second stop-watch as marble m_1 strikes marble m_2, and stop the stop-watch as marble m_2 reaches the end of the groove.
6. Use your values for the distances and times to calculate the velocity and momentum of marble m_1 before the collision, and the velocity and momentum of marble m_2 after the collision. Fill these values into the table above. (Let the direction of marble m_1 be positive).
7. Repeat this experiment 3 times to verify your findings.
8. Answer the following questions:
   a. What was the combined momentum of the marbles before the collision?
   b. What was the combined momentum of the marbles after the collision?

c. Do your answers agree with the law of conservation of momentum? If not, can you explain any experimental errors in your results?

Guided reflection:

1. The table below contains possible values for this experiment (your values will be different to these, so these are just a guide).
2. To calculate the velocity, the distance travelled by the marble should be divided by the time taken for that distance. (In this example the distance travelled by both marbles was 50 cm = 0,5 m.)

m_1 (kg)	Δt_1 (s)	v_1 ($m \cdot s^{-1}$)	m_1v_1 ($kg \cdot m \cdot s^{-1}$)	m_2 (kg)	Δt_2 (s)	v_2 ($m \cdot s^{-1}$)	m_2v_2 ($kg \cdot m \cdot s^{-1}$)
0,01	1,18	0,424	0,00424	0,01	1,20	0,417	0,00417
0,01	1,41	0,355	0,00355	0,01	1,42	0,352	0,00352
0,01	1,03	0,485	0,00485	0,01	1,05	0,476	0,00476

3. Answers to the questions:
   a. The combined momentum of the marbles before the collision were very similar to the combined momentum of the marbles after the collision in each case.
   b. These values to verify the law of conservation of momentum, although a small amount of momentum was lost in the collision.

This activity shows that the combined momentum of the objects before the collision is equal to the combined momentum of the objects after the collision. The total amount of momentum in this system is therefore conserved during the collision. This is called the *Principle of Conservation of Momentum,* which states that *in an isolated system the total linear momentum remains constant, in magnitude and direction*. (An isolated system is one where there are no external forces acting on the objects.)

In other words, the total momentum before the collision is equal to the total momentum after the collision. You can write this mathematically as:

$$p_{before} = p_{after}$$

Therefore $m_1v_{i1} + m_2v_{i2} = m_1v_{f1} + m_2v_{f2}$

where v_i refers to the initial velocity and v_f the final velocity.

NOTE: When solving problems with momentum it is very important to choose a frame of reference to make sure that the directions are correct!

Activity 2: Apply the principle of conservation of momentum

Purpose

In this activity you will study the steps in an example problem to see how to solve problems involving conservation of momentum, and then you will do some problems on your own.

Suggested time: [30 minutes]

What you will do:

Look carefully through the steps in the following example problem:

Example:

A 40 g green block is moving with a velocity of 5 m·s^{-1} towards a yellow 50 g block that is standing still. After hitting the yellow block, the green block bounces backwards along the path it has travelled with a velocity of 1 m·s^{-1}. Calculate the velocity of the yellow block after the collision.

Solution:

Frame of Reference: Let the direction of the green block's initial velocity be positive.

Given: Let the mass of the green block be m_1 and of the yellow block be m_2.

Then m_1 = 40 g = 0,04 kg

m_2 = 50 g = 0,05 kg

Before the collision v_{i1} = 5 m·s^{-1}; v_{i2} = 0 m·s^{-1};

After the collision v_{f1}=-1 m·s^{-1}; v_{f2}=?

When you substitute into this formula it is easier to leave out the units to avoid confusion with a very long expression

Calculation:

$m_1v_{i1} + m_2v_{i2} = m_1v_{f1} + m_2v_{f2}$

Therefore $(0,04 \times 5) + (0,05 \times 0) = (0,04 \times (-1)) + (0,05 \times \mathbf{v}_{f2})$

$$v_{f2} = \frac{0,2+0-(-0,04)}{0,05} = 4,8 \text{ m·s}^{-1}$$

Don't forget to insert units for your final answer!

The final velocity of the yellow block is 4,8 m·s^{-1} in the direction that the green block was initially travelling in.

Now try to answer the following problems yourself:

1. Thandi, who has a mass of 50 kg, is standing still on her roller skates, and throws a 1,5 kg ball to her friend. If the ball leaves her hands with a speed of 8 $m{\cdot}s^{-1}$, what was Thandi's velocity as a result of the throw?
2. Karl is running along the passage with a speed of 3 $m{\cdot}s^{-1}$, and runs straight into his father, who is walking in the opposite direction to Karl with a speed of 2 $m{\cdot}s^{-1}$. As a result of the collision, his father comes to a stop. If Karl has a mass of 65 kg, and his father has a mass of 130 kg, what was Karl's final velocity?

Solutions:

1. Frame of Reference: Let the direction of the ball be positive.

   Given: Let the mass of Thandi be m_1 and of the ball be m_2.

   $m_1 = 50$ kg

   $m_2 = 1,5$ kg

   Before the throw $v_{i1} = 0\ m{\cdot}s^{-1}$; $v_{i2} = 0\ m{\cdot}s^{-1}$

   After the throw $v_{f2} = 8\ m{\cdot}s^{-1}$; $v_{f1} = ?$

   Calculation:

   You use the equation for conservation of momentum:

   $m_1v_{i1} + m_2v_{i2} = m_1v_{f1} + m_2v_{f2}$

   $$\therefore v_{f1} = \frac{(m_1v_{i1} + m_2v_{i2}) - (m_2v_{f2})}{m_1}$$

   $$\therefore v_{f1} = \frac{(50 \times 0) + (1,5 \times 0) - (1,5 \times 8)}{50}$$

   $= -0,24\ m{\cdot}s^{-1}$

   Thandi's final velocity is 0,24 $m{\cdot}s^{-1}$ in the opposite direction to the ball's movement.

2. Frame of Reference: Let Karl's initial direction be positive.

   Given: Let Karl's mass be m_1 and his father's mass be m_2.

   $m_1 = 65$ kg

   $m_2 = 130$ kg

Before the collision $v_{i1} = 3\ m{\cdot}s^{-1}$; $v_{i2} = -2\ m{\cdot}s^{-1}$

After the collision $v_{f2}=0\ m{\cdot}s^{-1}$; $v_{f1}=?$

<u>Calculation:</u>

You use the equation for conservation of momentum:

$m_1v_{i1} + m_2v_{i2} = m_1v_{f1} + m_2v_{f2}$

$$\therefore v_{f1} = \frac{(m_1v_{i1} + m_2v_{i2}) - (m_2v_{f2})}{m_1}$$

$$\therefore v_{f1} = \frac{(65 \times 3) + (130 \times -2) - (130 \times 0)}{65} = -1\ m{\cdot}s^{-1}$$

Karl's final velocity is 1 $m{\cdot}s^{-1}$ in the opposite direction to his initial movement.

[WORDBOX: MAIN IDEAS:

- The Principle of Conservation of Momentum states that *in an isolated system the total linear momentum remains constant, in magnitude and direction*.
- You can write this mathematically as: $p_{before} = p_{after}$
  Therefore $m_1v_{i1} + m_2v_{i2} = m_1v_{f1} + m_2v_{f2}$.]

More resources to help you:

You can find more information on conservation of momentum at the following web address:

https://www.physicsclassroom.com/class/momentum/Lesson-2/Momentum-Conservation-Principle

Activity 3: Consolidate your learning of momentum and impulse

Purpose

In this activity you will consolidate your learning of momentum and impulse by answering the questions below and then assessing your own understanding using the solutions provided. Give yourself a mark out of the total of 35 marks, which will give you an idea of how well you understand this section of the work.

Suggested time**:** [40 minutes]

1. A ball is thrown upward in the air, and after it has reached its maximum height it falls back down to its starting point. Complete the following table for the different parts of the ball's motion by writing only **UP / ZERO / DOWN** in each block: (9)

Movement of the ball	**Net force on the ball (UP / ZERO / DOWN)**	**Momentum of the ball (UP / ZERO / DOWN)**	**Acceleration of the ball (UP / ZERO / DOWN)**
Ball is moving upward			
Ball is at its maximum height			
Ball is moving downward			

2. Which has more momentum, a 150 g cricket ball with a velocity of 100 $km \cdot hr^{-1}$, or a 10 kg child walking at 0,5 $m \cdot s^{-1}$? Show your calculations clearly. (5)

3. What do you mean by the term *impulse*? (2)

4. A 4 g dart is thrown at a dartboard that is hanging on a wall, and it strikes the board with an initial velocity of 50 $m \cdot s^{-1}$. If comes to rest in 0,05 seconds, what is the force that the block exerted on the bullet? (4)

5. State the principle of conservation of momentum. (2)

6. Kopano throws a 20 g ball at a wall with a velocity of 15 m·s^{-1}. It bounces off the wall, and returns back to him with a velocity of 14 m·s^{-1}. The ball is in contact with the wall for 0,1 seconds.

   a. Calculate the force exerted on the ball by the wall. (4)
   b. What is the net force exerted on the wall by the ball? (1)
   c. Calculate the acceleration of the ball. (3)

7. A 5 g bullet is fired from a gun that has a mass of 400 g. The bullet has a velocity of 360 m·s^{-1} as it leaves the gun barrel. What is the recoil velocity of the gun? (5)

Solutions

1. The completed table is shown below (1 for each correct block): (9)

Movement of the ball	**Net force on the ball (UP / ZERO / DOWN)**	**Momentum of the ball (UP / ZERO / DOWN)**	**Acceleration of the ball (UP / ZERO / DOWN)**
Ball is moving upward	*DOWN*	*UP*	*DOWN*
Ball is at its maximum height	*DOWN*	*ZERO*	*DOWN*
Ball is moving downward	*DOWN*	*DOWN*	*DOWN*

2. Given: m_1 = 150 g = 0,15 kg

Remember that there are 1000 g in 1 kg. Therefore to convert from g to kg you divide by 1000.

v_1 = 100 km·hr^{-1} × 1000 m/km ÷ 3600 s/hr = 27,78 m·s^{-1}

There are 1000 m in 1 km and 3600 seconds in 1 hour. Therefore to convert from km·hr^{-1} to m·s^{-1} you multiply by 1000 and divide by 3600.

m_2 = 10 kg

v_2 = 0,5 m·s^{-1}

Calculation:

$p_1 = m_1 v_1$

$= 0,15 \text{ kg} \times 27,78 \text{ m·s}^{-1}$

$= 4,17 \text{ kg·m·s}^{-1}$

$p_2 = m_2 v_1$

$= 10 \text{ kg} \times 0,5 \text{ m·s}^{-1}$

$= 5 \text{ kg·m·s}^{-1}$

The child has the higher momentum. (5)

3. I*mpulse* means the change in an object's momentum, or the force multiplied by the time for which the force acts. (2)

4. <u>Frame of reference:</u> Let the dart's initial direction be +

<u>Given</u>: $m = 4 \text{ g} \div 1000 = 0,04 \text{ kg}$

$v_i = +50 \text{ m·s}^{-1}$

$v_f = 0 \text{ m·s}^{-1}$

$\Delta t = 0,05 \text{ s}$

<u>Calculation</u>:

$$F = \frac{m\,\Delta v}{\Delta t}$$

$$= \frac{0,04\text{kg} \times (0 \text{ m·}s^{-1} - 50 \text{ m·}s^{-1})}{0,05 \text{ s}}$$

$= -40 \text{ N}$

The dartboard exerted a force of 40 N on the dart in the opposite direction to the dart's initial velocity. (4)

5. The principle of conservation of momentum states that in an isolated system the total linear momentum remains constant, in magnitude and direction. (2)

6. <u>Frame of reference:</u> Let the direction of the initial velocity of the ball be positive

<u>Given</u>: $m = 20 \text{ g} \div 1000 = 0,02 \text{ kg}$

$v_i = 15\ m{\cdot}s^{-1}$

$v_f = -14\ m{\cdot}s^{-1}$

$\Delta t = 0{,}1\ s$

$p_i = 0{,}02 \times 15 = 0{,}3\ kg{\cdot}m{\cdot}s^{-1}$

$p_f = 0{,}02 \times (-14) = -0{,}28\ kg{\cdot}m{\cdot}s^{-1}$

Calculation:

a. $F_{net} = \frac{\Delta p}{\Delta t}$

$= \frac{-0{,}28-0{,}3}{0{,}1}$

$= -\ 5{,}8\ N$

The net force exerted on the ball by the wall is 5,8 N in the opposite direction to its initial velocity. (4)

b. The net force exerted on the wall by the ball is 5,8 N in the same direction as the ball's initial velocity (these are action-reaction force pairs, from Newton's third law). (1)

c. $a = \frac{\Delta v}{\Delta t}$

$= \frac{-14\ m{\cdot}s^{-1} - 15\ m{\cdot}s^{-1}}{0{,}1\ s}$

$= -290\ m{\cdot}s^{-2}$

OR from Newton's second law

$a = \frac{F}{m}$

$= \frac{-5{,}8\ N}{0{,}02\ kg}$

$= -290\ m{\cdot}s^{-2}$

The acceleration of the ball is $290\ m{\cdot}s^{-2}$ in the opposite direction to its initial velocity. (3)

7. Let the bullet be m_1, and let the gun be m_2. Choose the bullet's direction as positive.

Given: $m_1 = 5\ g \div 1000 = 0{,}005\ kg$

$v_{i1} = 0 m{\cdot}s^{-1}$

$m_2 = 400\ g \div 1000 = 0{,}400\ kg$

$v_{i2} = 0\ m{\cdot}s^{-1}$

$v_{f1} = 360\ m{\cdot}s^{-1}$

Calculation:

You use the equation for conservation of momentum:

$m_1 v_{i1} + m_2 v_{i2} = m_1 v_{f1} + m_2 v_{f2}$

But since both v_{i1} and v_{i2} are 0, then the initial momentum is zero.

The equation becomes: $0 = m_1 v_{f1} + m_2 v_{f2}$

$$\therefore v_{f2} = \frac{-m_1 v_{f1}}{m_2} = \frac{-(0{,}005\text{kg} \times 360\text{m}\cdot\text{s}^{-1})}{0{,}4\text{kg}}$$

$= -4{,}5\ \text{m}\cdot\text{s}^{-1}$

Therefore the gun's recoil velocity is $4{,}5\ \text{m}\cdot\text{s}^{-1}$ in the opposite direction to that of the bullet.

(5)

Sub-topic 5: Work, power and energy

Unit 1: Energy conversion and conservation

Learning Outcomes

By the end of the unit, you should be able to:

- list examples of different forms of energy, including kinetic energy, potential energy (gravitational, chemical, elastic), electrical energy, light energy, thermal energy and nuclear energy;
- define kinetic energy and gravitational potential energy in words and using mathematical expressions: $E_k = ½ mv^2$ and $E_p = mgh$;
- define mechanical energy;
- state the principle of the conservation of mechanical energy;
- apply the principle of conservation of mechanical energy to various contexts, including objects that are dropped or thrown vertically upwards, and the motion of a swing or pendulum.

Introduction

Energy is a very important concept in science. Everything you do in your daily lives requires some kind of energy. In this unit you will learn the scientific meaning of energy.

Reflect on what you know about energy by thinking about the following questions:

- What do you understand by the term “energy”?
- Name some of the ways in which you use energy in your daily life.
- What do you understand by the terms “energy”, “power” and “work”? When you use these terms in your everyday language, do they all mean the same thing?

All forms of energy can be categorised into two main types of energy – *potential energy* and *kinetic energy*.

- *Potential energy* is energy that is *stored* in some way, e.g. in an elastic band or spring.
- *Kinetic energy* is the energy of *movement*.

In this unit you will begin by looking at gravitational potential energy, as this helps with understanding the concept of potential energy.

Measuring Energy

A range of different units are used for measuring energy. One that you might be familiar with is the unit of calories (Cal). This unit is often used to describe the amount of energy in food products. The SI unit for energy is the joule (J). Calories are related to joules by the equation: 1 cal = 4,186 J. In this unit you will use the unit of joules for energy.

Gravitational Potential Energy

When you lift an object, the effort, or work, that you put into lifting it is stored in this object as energy. This stored energy is called *gravitational potential energy*. Since gravitational potential energy is a form of energy, it is measured in the SI units of energy, namely joules (J). In the following activity you will explore the factors that affect the gravitational potential energy of an object.

Activity 1: Explore gravitational potential energy

Purpose

In this activity, you will explore the concept of gravitational potential energy by lifting masses to different heights above the floor.

Suggested time**:** [20 minutes]

What you need:

- 2 objects, one that has a larger mass than the other (e.g. a small book and a large book)
- A scale for measuring mass
- A tape measure or ruler
- A chair and a desk or table

What you will do:

1. Measure the length from the floor to the seat of the chair. Record this length as h_1. Now measure the length from the floor to the desk or table top. Record this length as h_2.

   Length from floor to chair: h_1 = _____________

Length from floor to desk: h_2 = _____________

2. Measure the mass of each of your objects, and record these as m_1 for the lighter object and m_2 for the heavier object.

   Lighter object: m_1 = _____________

   Heavier object: m_2 = _____________

   (If you don't have a way of measuring their masses, assume that m_1 = 0,5 kg and m_2 = 1 kg)
3. Lift object m_1 to the height of the chair. Now lift object m_2 to the height of the chair. Which do you think needed more effort (work) to lift it to this height?
4. The work done in lifting an object to a height above the ground is stored in the object as gravitational potential energy. Which object has the greater gravitational potential energy, m_1 or m_2?
5. Return your objects to the floor. Now lift object m_2 to the height of the desk (h_2). Did this require more work to be done than lifting object m_2 to the height of the chair (h_1)?
6. Which object has the greater gravitational potential energy, the one lifted to h_1 or to h_2?
7. From your investigations, complete the following sentences by choosing the correct word from the underlined words:

   - The greater the mass of an object that is lifted, the HIGHER / LOWER the gravitational potential energy of the object.
   - The greater the height that an object is lifted to, the HIGHER / LOWER the gravitational potential energy of the object.

Guided reflection:

1. When you lift the lighter object m_1 to the height of the chair, it needs less effort than lifting the heavier object m_2.
2. The heavier object m_2 therefore has the greater gravitational potential energy.
3. When you lift an object to the greater height of the desk (h_2), this requires more work to be done than lifting the object to the lower height of the chair (h_1).
4. The object that is lifted to a greater height therefore has the greater gravitational potential energy.
5. The correctly completed sentences are:

   - The greater the mass of an object that is lifted, the HIGHER the gravitational potential energy of the object.

- The greater the height that an object is lifted to, the HIGHER the gravitational potential energy of the object.

This activity shows that when an object is lifted upward from a reference point, for example the floor, the greater the mass of the object being lifted, the more effort needs to be done in lifting it, and therefore the greater the amount of stored energy. In other words, the gravitational potential energy is proportional to the mass of an object.

When an object is lifted to a greater height above the reference point, the effort needed to lift the object is greater than if the object is lifted a small height from the reference point. So the gravitational potential energy is proportional to the height of an object.

You can calculate the gravitational potential energy of an object using the following equation:

$E_p = mgh$

where E_p is the gravitational potential energy, measured in joules (J)

m is the mass of the object, in kg

g is the gravitational acceleration (on earth g = 9,8 $m \cdot s^{-2}$)

h is the height of the object above the zero point, in metres (m)

If you study the units in this equation, you can see that 1 J = 1 $kg \cdot m^2 \cdot s^{-2}$.

Activity 2: Apply your understanding of potential energy

Purpose

In this activity you will apply your understanding of potential energy by solving various problems.

Suggested time: [30 minutes]

What you will do:

Answer the following questions.

1. A professor lifts a pile of books that has a mass of 6 kg from the floor to the top of her bookshelf, which is at a height of 1,5 m above the floor. By how much has the gravitational potential energy of the pile of books increased?
2. Nonhlanhla lifts her child straight up onto a platform. Cindi pushes her child in a pram up a ramp onto the same platform so that her child is at the same height as Nonhlanhla's child.

The two children have the same mass. How does the potential energy of these two children compare with each other?

3. A taxi has a mass of 1 200 kg. It drives up a hill so that its gravitational potential energy has increased by 235,2 kJ. Calculate the increase in the taxi's height above its starting point.
4. When a mass *m* is lifted to a height *h* above its starting point, its gravitational potential energy increases by 1 000 J. If a mass of 2*m* is lifted to a height of 3*h* above this same starting point, by how much will its gravitational potential energy increase?

Solutions:

1. Given: m = 6 kg

   h = 1,5 m

   g = 9,8 $m.s^{-2}$

   Calculation:

   You use the equation for potential energy:

   $E_p = mgh$

   = 6 kg × 9,8 $m.s^{-2}$ × 1,6 m

   = 88,2 $kg{\cdot}m^2{\cdot}s^{-2}$

   = 88,2 J

   Note that the units of $kg{\cdot}m^2{\cdot}s^{-2}$ are equivalent to the units of joules (J).

2. The potential energy of these two children is equal, since they have the same mass, and have been lifted to the same height.

   Remember that there are 1000 J in 1 kJ, so you multiply kJ by 1000 to convert to J.

3. Given: m = 1 200 kg

   E_p = 235,2 kJ × 1000 = 235 200 J

   g = 9,8 $m.s^{-2}$

   Calculation:

   You use the equation for potential energy:

   $E_p = mgh$

   To find the increase in height you solve for h:

   $$h = \frac{E_p}{mg} = \frac{235\ 200\ \mathrm{J}}{1200\ \mathrm{kg} \times 9{,}8.\ \mathrm{m{\cdot}s^{-1}}}$$

   = 20 m

4. The equation $E_p = mgh$ shows that E_p is proportional to m and h.

Therefore if you halve the mass, and the height is increased by a factor of 4, the potential energy will increase by a factor of ½ × 4 = 2.

Therefore $E_p = 2 \times 1\,500$ J

$= 3$ kJ.

[Wordbox: MAIN IDEAS:

- *Gravitational potential energy* is the stored energy of an object because of its position above some reference point.
- The gravitational potential energy of an object can be calculated by the equation:

  $E_p = mgh$.]

Kinetic Energy

Kinetic energy is the energy of movement. The greater the mass of a moving object, the higher the value of the kinetic energy when it moves. Similarly, the faster the object is moving, the greater its kinetic energy.

The kinetic energy can be calculated using the following equation:

$$E_k = \frac{1}{2} mv^2$$

where E_k is the kinetic energy of the object, measured in joules (J), m is the mass of the object, in kg, and v is the velocity of the object, in $m \cdot s^{-1}$.

Activity 3: Apply your understanding of kinetic energy

Purpose

In this activity you will apply your understanding of kinetic energy by solving various problems.

Suggested time: [30 minutes]

What you will do:

Answer the following questions.

1. A woman is running with a netball that has a mass of 500 g. The woman's mass is 82 kg and she is running with a velocity of 6 $m \cdot s^{-1}$. What is the total kinetic energy of the woman and the ball together?

2. You are standing by the roadside waiting for a taxi, while holding a heavy suitcase. Does this involve kinetic energy?

3. The taxi that you get into has a mass of 900 kg (including your mass). When it is traveling at a speed of 100 $km{\cdot}hr^{-1}$, what is its kinetic energy in kJ?

4. If this taxi reduces its speed to 50 $km{\cdot}hr^{-1}$, by what *factor* has its kinetic energy decreased?

Solutions:

1. <u>Given</u>: m = 82 kg + 0,5 kg = 82,5 kg

   v = 6 $m.s^{-1}$

   <u>Calculation:</u>

   You use the equation for kinetic energy:

   $E_k = \frac{1}{2} mv^2$

   $= \frac{1}{2} \times 82{,}5\ kg \times (6\ m{\cdot}s^{-1})^2$

   $= 1485\ kg{\cdot}m^2{\cdot}s^{-2}$

   $= 1485\ J$

   Again here you will see that the units of $kg{\cdot}m^2{\cdot}s^{-2}$ are equivalent to the units of joules (J).

2. No, you are not moving while you stand waiting for a taxi, so no kinetic energy is involved.

   Remember that to convert from $km{\cdot}hr^{-1}$ to $m{\cdot}s^{-1}$ you multiply by 1000 and divide by 3600.

3. <u>Given</u>: m = 900 kg

   $v = 100\ km{\cdot}hr^{-1} \times 1000\ m/km \div 3600\ s/hr$

   $= 27{,}78\ m{\cdot}s^{-1}$

   <u>Calculation:</u>

   You use the equation for kinetic energy:

   $E_k = \frac{1}{2} mv^2$

   $= \frac{1}{2} \times 900\ kg \times (27{,}78\ m{\cdot}s^{-1})^2$

   $= 347\ 278\ J$

   $= 347{,}278\ kJ$

4. If this taxi reduces its speed to 50 $km{\cdot}hr^{-1}$, its speed has halved, so since kinetic energy is proportional to the square of the speed, the kinetic energy is reduced by a factor of 4. (In other words, the new kinetic energy is 4 times less than the previous kinetic energy.)

Activity 4: Explore potential and kinetic energy using a simulation

Purpose

In this activity you will use a computer simulation to explore the relationship between kinetic and potential energy.

Suggested time**:** [20 minutes]

What you will do:

Go to the following web address: https://phet.colorado.edu/sims/html/energy-skate-park-basics/latest/energy-skate-park-basics_en.html. This will open the simulation called *Energy Skate Park: Basics*

1. At the bottom of the screen you will see three windows. Choose the one in the middle, labelled “Friction”.

Figure 209 Choose the window labelled "Friction" at the bottom of the screen

2. You will see a screen with a person on a skate board next to a skating track.
3. Click the “Pause” button underneath this 
4. Move the slider labelled “Friction” to the very left so that there is no friction to begin with.

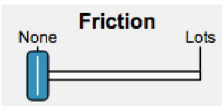


Figure 210 Move the slider labelled “Friction” to the left

5. Click the button on the right labelled “Bar Graph”. This allows you to see a graph on the left of the screen with the potential, kinetic, thermal and total energy. (Thermal energy is the energy that is lost as a result of friction, in the form of heat energy).
6. Move the skater to the top of the track and notice what happens to the potential, kinetic and total energy on the Bar Graph. Be careful to keep the skater on the track.
7. Click the “Play” button 
8. Notice what happens to the potential, kinetic and total energy on the Bar Graph as the skater moves.
9. From your observations, answer the following questions:

a. When is the potential energy at its maximum value?

b. When is the kinetic energy at its maximum value?

c. What happens to the total energy as the skater moves?

d. Why do you think the skater keeps moving back and forth without losing any height?

10. When the skater is at a maximum height, click the "Pause" button. Now move the slider labelled "Friction" to the middle, so that there is some friction acting on the skater.

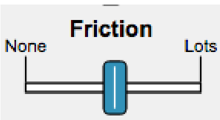


Figure 211 Move the slider labelled "Friction" to the middle

11. Click the "Play" button, and notice what happens to the potential, kinetic, thermal and total energy on the Bar Graph as the skater moves.
12. Can you explain your observations?

Guided reflection:

- In this simulation, you will notice that the skater has maximum potential energy and minimum kinetic energy at the top of the track, and at the bottom of the track the potential energy is minimum and the kinetic energy is maximum, as the diagrams below illustrate.

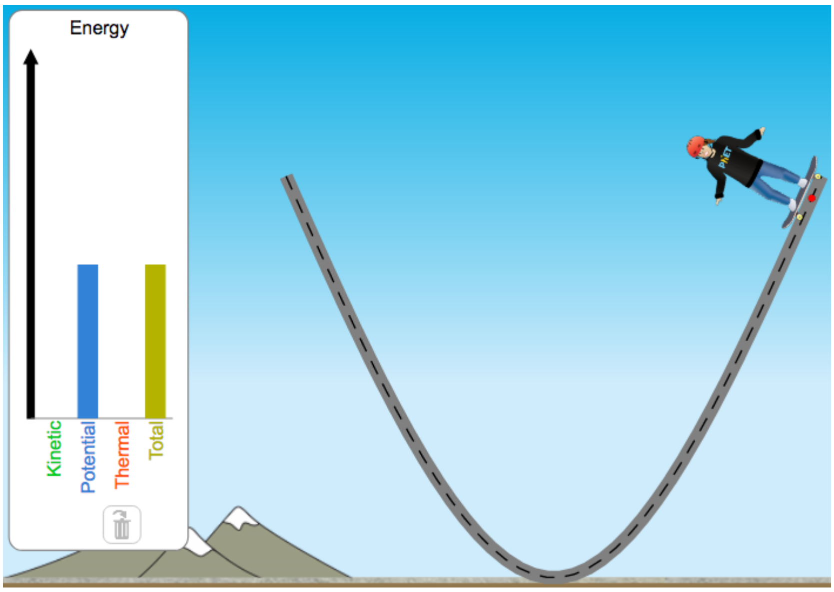


Figure 212 Screenshot with the skater at the top of the track

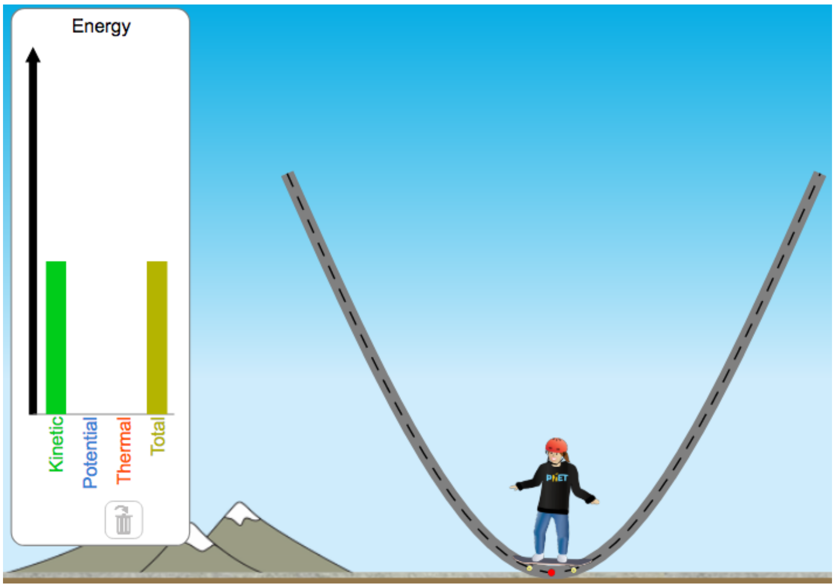


Figure 213 Screenshot with the skater at the bottom of the track

- You will also notice that the total energy stays constant.
- If you look at the height of the bar graphs, you will see that at any stage in the movement, the sum of the potential and kinetic energy gives you the total energy.
- The reason that the skater can keep moving back and forth without losing height each time is that there is no frictional force.
- When there is a frictional force, you will notice that the thermal energy gradually increases, and the maximum potential and kinetic energy reached decreases as the skater continues to move. This shows that friction removes (or dissipates) the sum of the potential and kinetic energy, and converts this into thermal or heat energy.

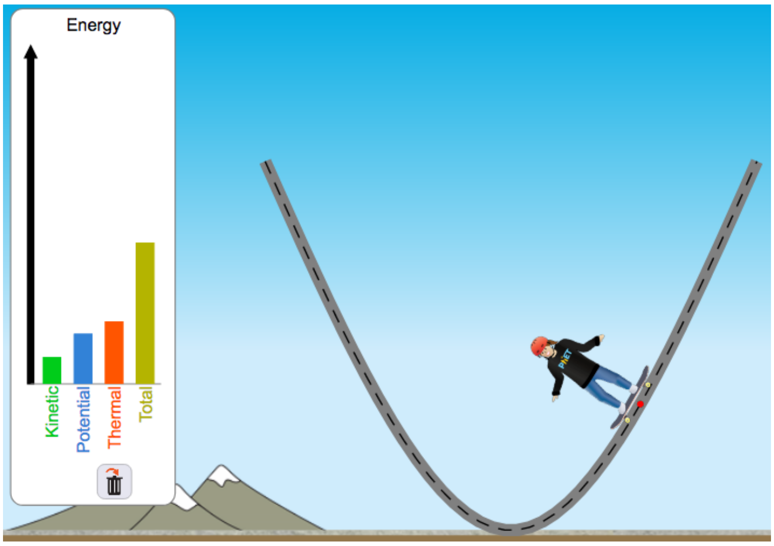


Figure 214 Screenshot with friction included in the skater's movement

Mechanical Energy

The sum of the potential energy and the kinetic energy of an object is called the mechanical energy. You can write this as an equation:

$$E_m = E_p + E_k$$

where E_m is the total mechanical energy, E_p is the potential energy, and E_k is the kinetic energy of the object, all measured in units of joules (J).

In the example of the skater from the simulation in Activity 4, when the skater was at the top of the track, the gravitational potential energy was maximum, since the height above the ground was maximum, but since the skater was not moving (v=0) the kinetic energy was zero. At the bottom of the track the skater had maximum velocity, and minimum height, so the kinetic energy was maximum and the potential energy was minimum. When there was no friction acting on the skater, the mechanical energy, which is the sum of the potential and kinetic energy, was conserved. The gravitational potential energy and kinetic energy were converted from one form to another, but the total remained constant.

You can use an equation to show the conservation of mechanical energy.
Since the mechanical energy before some change is equal to the mechanical energy afterwards, you can write this as:

$$E_{m1} = E_{m2}$$

where E_{m1} is the total mechanical energy before the change, and E_{m2} is the total mechanical energy after the change.

You can write this equation using symbols for the kinetic and potential energy:

$$E_{p1} + E_{k1} = E_{p2} + E_{k2}$$

where E_{p1} is the initial potential energy, E_{k1} is the initial kinetic energy, E_{p2} is the final potential energy after the change, and E_{k2} is the final kinetic energy.

In reality, an object cannot keep moving forever, because there are always frictional forces acting which cause the object to lose some of its total mechanical energy. You could see this in the simulation when you added friction to the skater. This caused some of the energy to be converted to thermal energy, reducing the total mechanical energy of the movement. In this way, energy is *dissipated* (lost) from the system, and as a result, the skater reaches a slightly lower height each time, until the motion eventually stops.

Another example that helps to illustrate this is the example of a ball that is swinging from a rope. The diagram below shows how energy is converted between potential energy and kinetic energy as the ball swings.

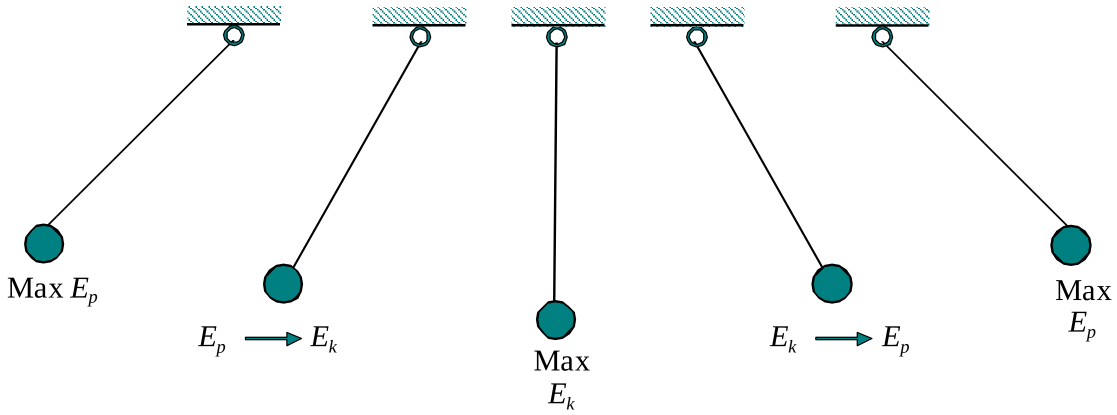


Figure 215 Conversion between potential and kinetic energy for a swinging ball

Air resistance and frictional forces cause the ball to lose some of its total mechanical energy, and so energy is dissipated from the system, causing the ball to swing to a slightly lower height each time, until it eventually stops.

The principle of conservation of mechanical energy states that *the total mechanical energy in a system is conserved when there is no loss of energy due to dissipative forces*.

You can see a YouTube video that shows a fun example: *Conservation of Mechanical Energy*: https://www.youtube.com/watch?v=d4K6ATZSJwk (Duration: 2.06)

Activity 5: Apply your understanding of conservation of energy

Purpose

In this activity you will apply your understanding of the conservation of mechanical energy to an example of motion.

Suggested time: [20 minutes]

What you will do:

Answer the following questions, or discuss them with a fellow student.

A 500 g ball is lifted to a height of 1,5 m above the ground. Answer the following questions:

a. By how much has the ball's gravitational potential energy increased?

b. If the ball is dropped from this height, what is its kinetic energy just before it hits the ground?

c. Find *two ways* of calculating the ball's velocity just before it reaches the ground.

Solutions:

<u>Given</u>: m = 500 g = 0,5 kg

h = 1,5 m

g = 9,8 m.s^{-2}

a. <u>Calculation:</u>

You use the equation for potential energy:

$E_p = mgh$

$= 0{,}5 \text{ kg} \times 9{,}8 \text{ m.s}^{-2} \times 1{,}5 \text{ m}$

$= 7{,}35 \text{ J}$

b. Since the mechanical energy is conserved, this potential energy is converted to kinetic energy, therefore kinetic energy just before it hits the ground = 7,35 J.

c. $E_k = \frac{1}{2} mv^2 = 7{,}35 \text{ J}$

$$\therefore \mathrm{v} = \sqrt{\frac{2\mathrm{E_k}}{\mathrm{m}}}$$

To make v the subject of the formula you multiply both sides by 2 and divide both sides by *m*, and then take the square root of that.

$$\therefore \mathrm{v} = \sqrt{\frac{2 \times 7{,}35 \text{ J}}{0{,}5 \text{ kg}}} = 5{,}42 \text{ m·s}^{-1}$$

[Wordbox: MAIN IDEAS:

- Mechanical energy = gravitational potential energy + kinetic energy: $E_m = E_p + E_k$
- The total mechanical energy in a system is conserved when there is no loss of energy due to dissipative forces: $E_{p1} + E_{k1} = E_{p2} + E_{k2}$.]

More resources to help you:

You can find more information on conservation of mechanical energy at this web page:
https://www.physicsclassroom.com/class/energy/Lesson-1/Mechanical-Energy

The following website has a Mindset Learn teaching video: *Mechanical Energy:*
http://learn.mindset.co.za/resources/physical-sciences/grade-10/energy/learn-xtra-lessons/mechanical-energy (Duration: 1.12.49)
In this video, that the first 27 minutes gives a helpful explanation of potential energy, kinetic energy and mechanical energy, and then the video goes on to use a similar simulation that you used in Activity 4 to explore these concepts in more depth.

Unit 2: Work and power

Learning Outcomes

By the end of the unit, you should be able to:

- define work done as the force multiplied by the distance moved in the direction of the force $W = F_x \Delta x$;
- apply the relationship for work done to various related problems, in familiar and novel contexts;
- define power as the work done divided by the time taken to do the work;
- apply the relationship for power to various related problems, in familiar and novel contexts.

Introduction

In this unit you will learn about the scientific concepts of work and power. Reflect on your understanding of these terms by discussing the following questions with a fellow learner, or reflecting on them yourself:

- Think of some everyday examples where you use the words “work” and “power”.
- Have you learned about the scientific meaning of these concepts before? If so, what do you recall?
- What is the difference between “work” and “power”?

Work

In science the word “work” has a very specific meaning. Work is done on an object if a force is applied to the object, and if this force results in motion that involves a displacement of the object in the direction of the force.

- When a force acts at some angle θ to the direction of motion of an object, you only consider the component of the force that acts parallel to the direction of the motion of the object. You can therefore calculate the work done by the force on the object using the equation:

  $$W = F_x \Delta x$$

  where Δx is the magnitude of the displacement.

- To calculate the x-component of the force F, you use the equation: $F_x = F \cos \theta$
- Work is a form of energy, and so it is measured in units of joules (J).

Activity 1: Apply your understanding of work

Purpose

In this activity you will apply your understanding of work to everyday life situations by reflecting on various scenarios.

Suggested time: [20 minutes]

What you will do:

Answer the following questions.

1. Read each of the scenarios below, and answer the question for each one.

a) Naledi spent all afternoon sitting at her desk writing an essay for homework. Did Naledi do work according to the scientific definition?

b) Tebogo pushed her younger brother up a hill in his go-cart. Did Tebogo do work? Did her brother do work?

c) Moses and Lydia had a tug-of-war by holding onto the opposite ends of a long piece of rope and pulling. They were evenly matched, so they stayed in one place, even though they pulled as hard as they could. Did either Moses or Lydia do work?

d) A weight-lifter lifted a heavy weight and then held it steadily above his head.

i) Did the weight lifter do work by lifting the weight upward?

ii) Did the weight lifter do work by holding the weight above his head?

2) Ryan pushes a mower with a force of 50 N, at an angle of $\theta = 30^o$ to the horizontal, as the diagram shows.

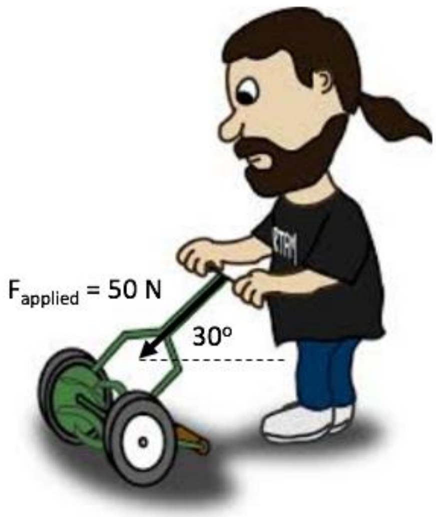


Figure 216 Force applied by Ryan on the mower

a) Find the component of the force parallel to the movement of the mower.

b) What is the work done on the mower if he pushes it for 10 m?

Solutions:

1.

a) No work was done by Naledi while sitting at her desk writing an essay for homework, since there was no motion. According to the scientific definition there must be displacement in the direction of the force for work to be done.

b) Tebogo did work while she pushed her younger brother up a hill in his go-cart, since she applied a force in the direction of his displacement. Her brother did not do any work since he was not applying a force in the direction of displacement.

c) Neither Moses nor Lydia did any work while they pulled the rope, since there was no motion.

d)

i) Yes, the weight lifter did do work by lifting the weight upward, since he exerted a force in the direction of motion of the weight.

ii) No work was done in holding the weight above his head, since there was no motion.

2.

a) To calculate the x-component of the force F, you use the equation:

$F_x = F \cos \theta$

$= 50 \text{ N} \times \sin 30^o$

$= 25 \text{ N}$

b) The work done on the mower if he pushes it for 10 m is:

$W = F_x \Delta x$

$= 25 \text{ N} \times 10 \text{ m}$

$= 250 \text{ J}$

Power

Power is defined as the amount of work done per unit time, or the rate at which work is done. In other words, you can calculate power by dividing the amount of work done by the time taken to do that work. In equation form, you write:

$$P = \frac{W}{\Delta t}$$

where P is the power, measured in watts (W), W is the work done, measured in joules (J), and Δt is the amount of time taken to do the work, in seconds (s).

What this equation means is that power is directly proportional to the work done, and inversely proportional to the time it takes to do the work.

Activity 2: Apply your understanding of power

Purpose

In this activity you will apply your understanding of power by answering various questions.

Suggested time: [20 minutes]

What you will do:

Answer the following questions.

1. Mandla and Ravin are lifting weights in the gym. Mandla lifts a 50kg weight above his head 10 times in half a minute. Ravin lifts a 50kg weight above his head 10 times in one minute.

   a. Who did the most work? Explain your answer.

   b. Who delivered more power? Explain your answer.

2. Blessing pulls a box with mass 25 kg on a horizontal surface for a distance of 10 m by applying a force of 12 N to a rope that is attached to the box, at an angle of 30^{o} to the horizontal. A frictional force of 2 N opposes the motion of the box.

   a. Calculate the work done on the box by Blessing.

   b. If Blessing pulled the box for 30 seconds, what was the power that he exerted in this time?

   c. Calculate the work done by the frictional force.

   d. What is the work done by the gravitational force?

Solutions:

1.

   a. Mandla and Ravin do the same amount of work, since they apply the same force to lift the same 50kg weight the same distance above their heads for the same number of times.

   b. Mandla delivered the most power, since he did the same work in half the time. Power is inversely proportional to time.

2. The free body diagram of all of the forces on the box is shown on the right.

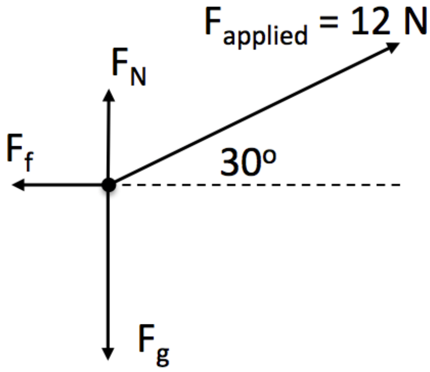


Figure 217 Free body diagram of the forces

a. First find the x-component of the applied force:

$F_{applied\,x} = F_{applied} \cos \theta$

$= 12 \text{ N} \times \cos 30^{o}$

$= 10{,}4 \text{ N}$

Therefore work done on the box by Blessing is:

$W = F_x \, \Delta x$

$= 10{,}4 \text{ N} \times 10 \text{ m}$

$= 104 \text{ J}$

b. To find the power, use the equation:

$$P = \frac{W}{\Delta t}$$

$$= \frac{104 \text{ J}}{30 \text{ s}}$$

$= 3{,}47 \text{ W}$

c. First find the x-component of the frictional force:

$F_{f\,x} = F_f \cos \theta$

$= 2 \text{ N} \times \cos 180^{o}$

$= -2 \text{ N}$

Therefore work done on the box by the frictional force is:

$W_f = F_f \, \Delta x$

$= -2 \text{ N} \times 10 \text{ m}$

$= -20 \text{ J}$

If a force is opposed to the direction of the motion then it does negative work.

d. The work done by the gravitational force is 0 J, since this force is perpendicular to the displacement of the object, so it has no component in the direction. You can show this using a calculation:

$F_{g\,x} = F_g \cos \theta$

$= (25 \text{ kg} \times 9{,}8 \text{ m·s}^{-2}) \times \cos 90^{o}$

$= 0 \text{ N}$

[Wordbox: MAIN IDEAS:

- *Work* is done on an object if a force is applied, and causes displacement of the object in the direction of the force: $W = F_x \Delta x$
- The x-component of the force F is: $F_x = F \cos \theta$
- Work is measured in units of joules (J).
- *Power* is defined as the amount of work done per unit time, or the rate at which work is done: P = \frac{W}{\Delta t}

  Power is measured in watts (W).]

More resources to help you:

You can find more information on power at the following web address:

https://www.physicsclassroom.com/class/energy/Lesson-1/Power

You can see a helpful Youtube teaching video on this chapter: *Work, Energy, and Power: Crash Course Physics:* https://www.youtube.com/watch?v=w4QFJb9a8vo (Duration: 9.54)

Activity 3: Consolidate your learning of work, energy and power

Purpose

In this activity you will consolidate your learning of work, energy and power by answering the questions below and then assessing your own understanding using the solutions provided. Give yourself a mark out of the total of 60 marks, which will give you an idea of how well you understand this section of the work.

Suggested time**:** [60 minutes]

What you will do**:**

Answer the following questions:

Multiple choice questions:

10. Mandla drops a stone into a pile of soft sand from two different heights:

    Case 1: He drops the stone from a height h.

Case 2: He drops the same stone from a height 2 h.

Which of the following is true for the mechanical energy and work in both the cases? (3)

	Mechanical energy	**Work done**
A.	Case 2 < Case 1	Case 2 > Case 1
B.	Case 2 < Case 1	Case 2 < Case 1
C.	Case 2 > Case 1	Case 2 > Case 1
D.	Case 2 > Case 1	Case 2 < Case 1

11. For the two cases described in Question 1, which of the following is true for the acceleration and final velocity of the stone (just *before* hitting the sand) in both the cases? (3)

	Acceleration	**Final velocity**
A.	Case 2 = Case 1	Case 2 > Case 1
B.	Case 2 = Case 1	Case 2 = Case 1
C.	Case 2 > Case 1	Case 2 > Case 1
D.	Case 2 > Case 1	Case 2 = Case 1

Written response questions:

1. Complete the table below by giving one word for the quantity that is described, showing the units that each of the quantities is measured in, and writing the equation that you use to calculate each one. (12)

Description	**Name of quantity**	**Units**	**Equation**
Energy of movement			
Energy that an object has because of its height above the earth			
The sum of the gravitational potential energy and the kinetic energy of the object			
The rate at which work is done			

2. A 1,5 kg book is lifted from the ground to a height of 300 cm above the ground.

   a. By how much has the book's gravitational potential energy increased? (3)

b. If the book is dropped from this height, what is its kinetic energy just before it hits the ground? (1)

c. Calculate the book's velocity just before it strikes the ground. (3)

d. Calculate the time taken for the book to fall to the ground. (3)

3. A 2 000 kg car is parked at the top of a hill, as the diagram shows. The car is allowed to roll down the hill. If no energy is lost through friction, what is the car's velocity at the bottom of the hill? (6)

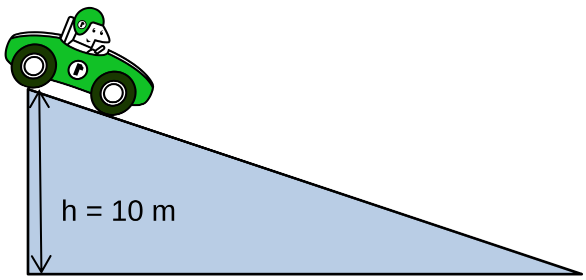


Figure 218 Car at the top of the hill

4. A ball is attached to a rope that hangs from a hook. Kealeboga pulls back the ball to a height that gives it an increase in gravitational potential energy of $1{,}176 \times 10^{-2}$ J, and lets the ball go. The mass of the ball is 30 g. Ignore the effects of friction.

a. Find the height that Kealeboga raised the ball to (above its rest position) (3)

b. Calculate the kinetic energy of the ball at its lowest position (1)

c. Calculate the speed of the ball at the lowest position (3)

d. What is the total mechanical energy of the system? (1)

e. Explain why the ball does not keep swinging to the same height forever. (3)

5. A 50 N force is applied to a 15 kg weight at an angle of 40^{o} to the horizontal. A frictional force of 10 N opposes the motion of the box, and it moves for a distance of 8 m along a horizontal surface in a time of 1 minute.

a. Draw a free body diagram of all of the forces on the box. (4)

b. Calculate the work done on the box by the applied force. (4)

c. Calculate the work done by the frictional force. (4)

d. What was the power that was exerted by the applied force in this time? (3)

Solutions:

Multiple choice questions:

1. C. *Reason:* In Case 2, the height is double the height in Case 1. Therefore the maximum potential energy is greater in Case 2 than in Case 1 (since $E_p = mgh$), and therefore the mechanical energy is greater in Case 2 than in Case 1. The displacement in Case 2 is double the displacement in Case 1, but the force is the same in both cases (since it is just the weight), therefore the work done in Case 2 is greater than the work done in Case 1 (since $W = F_x \Delta x$) (3)

2. A. *Reason:* In both cases the acceleration is 9,8 $m{\cdot}s^{-2}$ (the acceleration due to gravity). The maximum potential energy is greater in Case 2 than in Case 1 (since $E_p = mgh$), and therefore the kinetic energy is greater just before it lands in Case 2 than in Case 1. As a result the velocity is greater just before it lands in Case 2 than in Case 1 (since $E_k = ½ mv^2$). (3)

Written response questions:

1. The correct answers are shown below. (1) mark for each correct answer. (12)

Description	Name of quantity	Units	Equation
Energy of movement	*Kinetic energy*	*J*	$E_k = ½ mv^2$
Energy that an object has because of its height above the earth	*Gravitational potential energy* ***OR*** *Potential energy*	*J*	$E_p = mgh$
The sum of the gravitational potential energy and the kinetic energy of the object	*Mechanical energy*	*J*	$E_m = E_p + E_k$
The rate at which work is done	*Power*	*W*	$P = \frac{W}{\Delta t}$

2. <u>Given</u>: m = 1,5 kg

h = 300 cm ÷ 100 = 3 m

Remember that there are 100 cm in 1 m, so to convert cm to m you divide by 100.

g = 9,8 m.s^{-2}

a. <u>Calculation:</u>

$E_p = mgh$

$= 1,5 \text{ kg} \times 9,8 \text{ m.s}^{-2} \times 3 \text{ m}$

$= 44,1 \text{ J}$ (3)

b. All of the potential energy that the book has at the maximum height is converted into kinetic energy as it falls, therefore just before the book strikes the ground $E_k = 44,1$ J (1)

To make v the subject of the formula you multiply both sides by 2 and divide both sides by *m*, and then take the square root of that.

c. $E_k = \frac{1}{2} mv^2 = 44,1$ J

$$\therefore \text{v} = \sqrt{\frac{2\text{E}_\text{k}}{\text{m}}}$$

$$\therefore \text{v} = \sqrt{\frac{2 \times 44,1 \text{ J}}{1,5 \text{ kg}}} = 7,67 \text{m·s}^{-1}$$ (3)

d. <u>Frame of reference</u>: choose down as +

<u>Given</u>: $v_f = 7,67 \text{ m·s}^{-1}$

$a = 9.8 \text{ m·s}^{-2}$

From the equation $v_f = v_i + a\Delta t$

You can solve for Δt:

To make Δt the subject of the formula you subtract v_i from both sides, and then divide both sides by *a*.

$$\Delta \text{t} = \frac{\text{v}_\text{f} - \text{v}_\text{i}}{\text{a}}$$

$$\therefore \Delta \text{t} = \frac{(7,67 \text{ m·s}^{-1} - 0)}{9,8 \text{ m·s}^{-2}}$$

$= 0,78$ s (3)

3. <u>Given</u>: $m = 2 \times 10^3$ kg

$h_1 = 10$ m

$h_2 = 0$ m

$v_1 = 0 \text{ m·s}^{-1}$

The total initial mechanical energy at the top of the hill (E_{m1}) is equal to the total mechanical energy at the bottom (E_{m2}). You first find the initial mechanical energy:

$E_{m1} = E_{p1} + E_{k1}$

$= mgh_1 + ½\, m(v_1)^2$

$= (2 \times 10^3 \text{ kg} \times 9{,}8 \text{ m·s}^{-2} \times 10 \text{ m}) + 0$

$= 196\,000$ J

Therefore since $E_{m1} = E_{m2}$

You get $E_{m2} = 196\,000 \text{ J} = E_{p2} + E_{k2}$

$= mgh_2 + ½\, m(v_2)^2$

$= 0 + (0{,}5 \times 2 \times 10^3 \text{ kg} \times (v_2)^2)$

$$\therefore v_2{}^2 = \frac{(196000 \text{ J} - 0)}{0{,}5 \times 2000 \text{ kg}} = 196 \text{ m}^2\text{·s}^{-2}$$

$\therefore v_2 = \sqrt{196 \text{ m}^2\text{·s}^{-2}} = 14 \text{ m·s}^{-1}$ (6)

4.

a. **Given**: m = 0,03 kg

$E_p = 1{,}176 \times 10^{-2}$ J

g = 9,8 m.s^{-2}

To make *h* the subject of the formula you divide both sides by *mg*.

From the equation $E_p = mgh$ you can find the height:

$$h = \frac{E_p}{mg}$$

$$\therefore h = \frac{1{,}176 \times 10^{-2} \text{ J}}{0{,}03 \text{ kg} \times 9{,}8 \text{ m·s}^{-2}}$$

= 0,04 m (3)

b. The kinetic energy of the ball at its lowest position is equal to the potential energy at the highest position = $1{,}176 \times 10^{-2}$ J

(1)

c. $E_k = ½\, mv^2 = 1{,}176 \times 10^{-2}$ J

$$\therefore v = \sqrt{\frac{2E_k}{m}}$$

$$\therefore v = \sqrt{\frac{2 \times 1{,}176 \times 10^{-2} \text{J}}{0{,}03 \text{kg}}}$$

$= \quad 0,885\ m.s^{-1}$ (3)

d. Total mechanical energy of the system = E_p at maximum height = $1,176 \times 10^{-2}$ J (1)

e. The ball does not keep swinging to the same height forever because in a real life situation there are frictional forces, causing energy to be dissipated (or lost), until eventually all of the energy is used up and the ball stops moving. (3)

5.

a. Free body diagram: (1 mark for each force correctly labeled) (4)

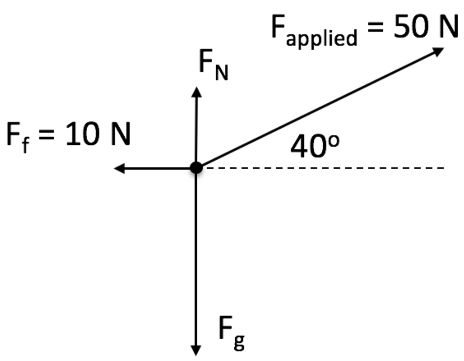


Figure 219 Free body diagram of the forces

b. $F_{applied\ x} = F_{applied} \cos\theta = 50\ N \times \cos 40^{o} = 38,3\ N$

Therefore work done on the box by the applied force is:

$W_{applied} = F_{applied\ x}\ \Delta x$

$= 38,3\ N \times 8\ m$

$= 306,4\ J$ (4)

c. To find the work done on the box by the frictional force, first find the x-component of the frictional force:

$F_{f\ x} = F_f \cos\theta$

$= 10\ N \times \cos 180^{o}$

$= -10\ N$

Therefore work done on the box by the frictional force is:

$W_f = F_f\ \Delta x$

$= -10\ N \times 8\ m$

$= -80\ J$ (3)

The frictional force is opposed to the direction of the motion, so it does negative work.

d. $P = \frac{W}{\Delta t}$

$= \frac{306,4\ J}{60\ s}$

$= 5,11 \text{ W}$ (3)