NASCA WORKBOOK

QUANTITATIVE LITERACY

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Introduction

Quantitative Literacy is a fairly new subject offered in South Africa and can be described as the ability to identify, understand and use quantitative arguments in every day real-life contexts. A necessary element is the ability to modify a quantitative argument from a familiar context to an unfamiliar context. Although a quantitatively literate person requires one to know some mathematics, the subject is not defined by the mathematics known. Quantitative literacy demands understanding, which must be flexible enough to enable one to apply quantitative ideas in new as well as in familiar contexts.

Quantitative Literacy provides students with an awareness and understanding of the role that basic quantitative skills, namely, numerical, statistical and spatial, play in the real world. This subject is driven by real-life contexts which underpin these basic quantitative skills.

The inclusion of Quantitative Literacy as a fundamental subject in the NASCA curriculum will empower future citizens to become highly numerate and statistically informed. In the teaching and learning of Quantitative Literacy, students will be provided with opportunities to engage with real-life problems in a variety of contexts to enable them to consolidate and extend basic quantitative skills. The subject will enhance one's ability to understand relevant terminology; make sense of quantitative information communicated in tables, graphs, diagrams, maps, plans and texts and develop the use of a variety of basic quantitative skills to critically analyse situations and creatively solve everyday problems.

In everyday life students are faced with quantitative situations which require them to be confident self-managing persons. These situations frequently relate to financial issues involving hire-purchase agreements, mortgage bonds and investments. Other everyday situations include the ability to read maps, interpret timetables, estimate and calculate areas and volumes, and understand building and seating plans. Everyday situations such as food preparation and the use of medications, requiring the efficient use of ratio and proportion also embrace quantitative situations.

The workplace constantly requires the use of basic quantitative skills to strengthen a workers resolve in order to efficiently and effectively meet work-related demands making him a contributing worker. To benefit from specialised training for the workplace, a flexible understanding of basic quantitative skills is often necessary. These skills must enable the person to, for example, deal with work-related formulae, read statistical charts, deal with schedules and understand instructions involving quantitative components.

To be a participating citizen in a developing democracy, it is essential that the student has acquired a critical stance with regard to quantitative situations presented in the media and other social platforms. The concerned citizen needs to be aware that statistical data can often be used to support opposing arguments, for example, for or against the use of an ecologically sensitive stretch of land for mining purposes. In this information and technological age, the power of numbers and quantitative ways of thinking often shape policies.

Moreover, the intention of the Quantitative Literacy curriculum is geared to equip and empower students to become self-managing persons and acquire the competence to confidently cope with life, employment and further studies. The over-arching design principle of this curriculum is to foreground authentic quantitative contexts in which adults find themselves and enable them to use mathematical skills and concepts to solve real-life problems.

Quantitative Literacy Curriculum

Dear NASCA Quantitative Literacy learner, this workbook is intended to equip you with the knowledge and skills you need in dealing with real-life problems. The skills and knowledge you will master are drawn from the pure Mathematical knowledge you acquired in previous grades. Real-life problems you may encounter are organised into FOUR main content topics, namely: finance; measurement, maps, plans and other representations of the physical world; data handling with probability. Over and above the four content topics, you will be required to master the foundational skills, namely basic essential skills that you will need which will be embedded in the FOUR main content area topics.

Structure of the main topics in the Curriculum

The content in the Quantitative Literacy curriculum is divided into two sections: Basic Essential Mathematical Skills and **FOUR** Content Area Topics.

Much of the content in the Basic Essential Mathematical Skills Topics comprise elementary mathematical content and skills that students are expected to have already been exposed to in their formal schooling, namely, Grade 9 (*e.g.* number formats and conventions, percentages, drawing graphs from tables of values, and so on). The inclusion of this content provides lecturers/tutors with the opportunity to revise these important skills and thus provide students with the opportunity to explore these skills in contexts. It is envisaged that a well-founded grasp of the concepts in the Basic Essential Mathematical Skills Topics is necessary in order to make sense of the content and contexts outlined in the Content Area Topics which follow.

There are FOUR content area (CA) topics which will constitute the Quantitative Literacy curriculum. Each content area topic will be accorded equalweighting and therefore the same allocation of time should be allocated to each content area topic.The following curriculum area topics are important and should receive equal emphasis:

- **Numbers and Operations in Finance (CA 1) –** Estimate, calculate, investigate and monitor the financial aspects of personal, business and national life and to investigate and solve problems in other contexts.

- **Measurement (CA 2) –** Use appropriate measuring instruments to estimate, calculate physical quantities and to describe and represent properties of and relationships, between TWO and THREE dimensional objects. Calculate area, perimeter, volume, surface area, mass, temperature, time and complete conversions within Imperial and Metric systems.
- **Maps, Plans and Models (CA 3) –**Use maps(distance charts and strip charts), time schedules, seating plans, design plans, elevation plans, layout plans and models to interpret, plan trips and analyse spatial relationships.
- **Data Handling and Probability (CA 4) –** Collect, summarise, display and analyse data and apply knowledge of statistics and basic probability to communicate, justify, predict and critically analyse findings and draw conclusions.

Structure of Formal Assessments

There will be **TWO** final examination papers each of 3 hours duration, consisting of 150 marks each. These question papers will be nationally examined and moderated at the end of the academic year. The structure and format of each paper will be comprised as follows:

Examination Paper 1 (3 Hours, 150 marks)

- A basic knowledge and routine applications paper.
- May consist of five questions.
- The first question will focus on integrated short cognitive level 1 topics while Q2 will focus on Finance; Q3 will focus on Measurement; Q4 will focus on Maps and Plans and Q5 will be made up of of Data Handling topic.
- Each question will be context-based, noting that there can be more than one context within a question.
- Each question will focus on a different context.
- Contexts will represent real-life situations and will as far as possible be realistic.
- All questions will include sub-questions assessing the first three cognitive levels (Cognitive Level 1, Cognitive Level 2 and Cognitive Level 3).

Examination Paper 2 (3 Hours, 150 marks)

- An application, reasoning and reflecting paper.
- May consist of FOUR or FIVE questions.
- Questions will require more interpretation and application of the information provided.
- Questions may integrate more than one content area topic.
- Each question will focus on a different context, noting there could be more than

one context within a question.

- Contexts will represent real-life situations and will as far as possible be realistic.
- All questions should integrate Assessment Criteria from more than one Content Area topic.
- All questions will include sub-questions assessing the last three cognitive levels (Cognitive Level 2, Cognitive Level 3 and Cognitive Level 4).

Weighting of Content Area Topics in the 2 Examination Papers

Content Area Topic(CA)	Numbers & Operations in Finance	Measurement	Maps, plans and Models	Data Handling and Probability	TOTAL
	±25%	±20%	±20%	±35%	100%
Paper 1	38 (±5)	30 (±5)	30 (±5)	52 (±5)	150
Paper 2	38 (±5)	30 (±5)	30 (±5)	52 (±5)	150

Note: Basic Essential Mathematical Skills will be integrated with the 4 content area topics and not assessed separately.

Weighting of Cognitive Levels as percentages in the 2 Examination Papers

Cognitive Levels	LEVEL 1 Knowing	LEVEL 2 Routine Procedures in Familiar Contexts	LEVEL 3 Multi-Step Procedures in a Variety of Contexts	LEVEL 4 Reasoning and Reflecting
Paper 1	60% (±5%)	20% (±5%)	20% (±5%)	0%
Paper 2	0%	20% (±5%)	40% (±5%)	40% (±5%)
Total	30% (±5%)	20% (±5%)	30% (±5%)	20% (±5%)

Cognitive Levels in the Quantitative Literacy assessment taxonomy

Cognitive Level 1: Knowing

Knowing questions involve basic recall, knowledge or definitions which serve two functions:

- To familiarise students with the context in which problems are posed by examining questions about the context and
- To assess student's ability to interpret contextualised information and use familiar techniques to perform basic calculations and to explain common terms.

Cognitive Level 2: Applying routine procedures in familiar contexts

- Questions posed at this level of the Quantitative Literacy taxonomy require students to perform well-known procedures and answer questions in familiar contexts.
- Students know which procedure is required from the way the problem is posed and all the necessary information to solve the problem is immediately available to the student.
- Routine procedure questions commonly involve single-step calculations, repeating the same calculation several times, or the completion of questions with which students are familiar (e.g. constructing an income-and-expenditure statement to reflect an individual's finances).

Cognitive Level 3: Applying multi-step procedures in a variety of contexts

- Questions at this level of the Quantitative Literacy taxonomy require students to solve problems or complete answers to questions using well-known procedures and methods, but where the procedure or method is not immediately obvious from the way the problem is posed.
- Students may have to decide on the most appropriate procedure or method to find the solution to the question, and they may have to perform one or more preliminary calculations or complete one or more preliminary tasks before determining a solution.
- Situations, in which a variety of mathematical and non-mathematical content, skills and/or considerations should be utilised from different topics in the curriculum in order to make sense of a problem, are also at the multi-step procedures level of the taxonomy.
- Questions at the multi-step procedures level contain far less direction or guidance than questions at the routine procedures level and require that students make decisions regarding the appropriate content, methods and non-mathematical considerations needed to solve problems.

Cognitive Level 4: Reasoning and reflecting

Questions at this level of the Quantitative Literacy taxonomy can be divided into two groups of questions:

- Firstly questions that require a decision, opinion or prediction about a particular scenario based on calculations in a previous question or on given information.

Examples of these types of reasoning and reflecting questions include:

- comparing given data on the performance of two groups of learners in an examination and explaining which group performed better based on the available data;
- offering an opinion on how a particular government minister might react to a particular set of statistics;
- analysing a completed income-and-expenditure statement for a household and making suggestions on how the members of the household could change their expenditure to improve their financial position.

- Secondly questions which require students to pose and answer questions about which mathematics they require to solve a problem, select and use that mathematical content, recognise the limitations of using mathematics to solve the problem, and consider other non-mathematical techniques and factors that may define or determine a solution to the problem.

  Examples of these types of reasoning and reflection questions include:

  - using calculations to compare income and expenditure values for a business in order to determine whether the business is in a healthy financial position;
  - comparing bank charges on two different types of accounts for various transactions and making a decision about the most suitable account for an individual with particular needs;
  - constructing a table to model a loan scenario, taking into account the interest calculated on the loan, the monthly repayment and the closing balance on the loan every month;
  - using the model of the loan scenario to investigate the effect of changes in the interest rate on the loan and the impact of increasing the monthly repayment on the real cost of the loan;
  - designing two different types of boxes for packaging an item, comparing the boxes in terms of wasted space (volume) and materials (surface area), and making a decision about the most cost-effective box for packaging the item.
    (Adapted from DBE, 2012)

TOPIC 1: NUMBERS AND OPERATION IN FINANCE

Introduction

While a fair amount of the content included in this component comprises some elementary foundational mathematical skills which students ought to have already been exposed to in Grade 9 (e.g. different number formats and conventions, ratios, rates, rounding, estimations, calculating percentages, drawing graphs from tables of values, and so on), these skills are embedded in contexts whereby applications are then required. The inclusion of these essential foundational skills affords tutors/educators with an opportunity to revise these important concepts and skills as a sound grasp of these skills are necessary for making sense of the content and contexts outlined in the other main components as well.

Numbers and operations in Finance – Content Structure

Topic Heading	Topic (with Approximate Instructional Time)
Numbers Formats and conventions	1. Estimation and rounding (1 hour) 2. Ratios (1 hour) 3. Rate (1 hour) 4. Proportions (1 hour) 5. Exchange rates (2 hours)
Interest Calculations	6. Percentages (1 hour) 7. Simple Interest (2 hour) 8. Compound Interest (2 hours) 9. Inflation/Deflation (1 hour)
Personal Income Tax	10. Terminology associated with personal tax (1 hour) 11. Calculations of personal income tax (2 hours)
Profit and Loss	12. Concepts and Terminology 13. Break-even-point 14. Budgeting

Number Formats and conventions

Introduction

Quantitative Literacy, as mentioned earlier, is embedded in real-life contexts. However, the requisite basic foundational skills must first be fore grounded before engaging with calculations in context. We start off with number formats and then gradually proceed to operations involving numbers. You will be introduced to two representations of numbers which basically comprise the use of two scales used worldwide to represent large and small numbers, namely the long scale and the short scale. In the United States, the small scale is used whereas in the United Kingdom, the large scale is used. In South Africa, and for the purposes of Quantitative Literacy (QL), we will use the short scale where for example a billion is represented by 9 zeros instead of the long scale where a billion is indicated by 12 zeros. Table 1 and Table 2 shown later reflect a comparison of the two scales relating to large and small numbers. Students will also be required to write numbers embedded in context in word format and vice versa.

Number formats

Numbers can take on various formats or they could appear to look different or they could have several meanings depending on the context in which they are used. In the following example we will see how a number can be represented using three different formats. Some calculators insert what looks like a comma after every three digits:

Eg. 26,340,000,000

The comma in the above example in no way suggests that we are working with decimals but rather it enables us to read the number more easily. Similarly, the number may even be represented using the following format:

Eg. 26’340’000’000

When we, in QL, are writing the number, we would generally include a small space where the comma would be. The digits must always be taken in groups of three from the right as follows:

Eg.R 26 340 000 000 The number alongside written in words will be:
Twenty six billion three hundred and forty million rand.

Another example would be the use of a comma or a dot to show a decimal. For QL, a comma must be used to indicate a decimal.

Eg. 7,4 is exactly the same as 7.4, above but the comma is preferred.

In Tables 1 and 2 which follow, a comparison of the 2 different scales used in the US and UK are shown. In South Africa, and for QL purposes, the short scale must be used.

TABLE 1 showing a comparison of large numbers in short and long scale

Large Numbers	Description using (Short Scale)	Description using (Long Scale)
1,000,000,000,000,000,000,000,000	Septillion	Quadrillion
1,000,000,000,000,000,000,000	Sextillion	Thousand Trillion/Trilliard
1,000,000,000,000,000,000	Quintillion	Trillion
1,000,000,000,000,000	Quadrillion	Thousand Billion/Billiard
1,000,000,000,000	Trillion	Billion
1,000,000,000	Billion	Thousand Million/Milliard
1,000,000	Million	Million
1,000	Thousand	Thousand
100	Hundred	Hundred
10	Ten	Ten

TABLE 2 showing a comparison of small numbers in short and long scale

Small Numbers	Description using (Short Scale)	Description using (Long Scale)
0.1	Tenth	Tenth
0.01	Hundredth	Hundredth
0.001	Thousandth	Thousandth
0.000 001	Millionth	Millionth
0.000 000 001	Billionth	Thousand Millionth
0.000 000 000 001	Trillionth	Billionth
0.000 000 000 000 001	Quadrillionth	Thousand Billionth
0.000 000 000 000 000 001	Quitillionth	Trillionth
0.000 000 000 000 000 000 001	Sextillionth	Thousand Trillionth
0.000 000 000 000 000 000 000 001	Septillionth	Quadrillionth

Estimation and Rounding

Learning outcomes:

When you have completed this unit, you should be able to:

- Solve problems in different authentic real-life contexts by using estimation and accurate calculation skills (including basic calculator skills)

Estimation

The ability to estimate final solutions in context is of great importance for many applications of QL. Regrettably, estimation is a skill that is usually overlooked, as Mathematics educators don't perceive estimation as its responsibility. Moreover, educators in other subject areas believe estimation skills form part of mathematics and so don't teach it. Many students therefore find estimation difficult.

> **Estimation:** Finding a number close enough to the exact answer.

When estimating:

- You are not seeking to get an accurate answer.
- You intend getting an answer that is close enough quickly.
- You can save time, money and effort.

Illustrative example saving you money:

Example: You want to buy five pencils that cost R1,95 each. When you pay at the till the cost demanded is R12.25. Is that right?

Solution: No. Using estimation, 5 at R1,95 is approximately 5 times 2, which is approximately R10,00 so R12.25 seems too much. Ask to have the total checked.

Illustrative example saving you time:

Example: You want to plant a single row of flowers. The row is 68,3cm long. Each plant should be 7cm apart. Determine how many plants you will require to complete the single row.

Solution: 68.3 is close to 70, and 70 divided by 7 is 10, so 10 plants will be sufficient.

Illustrative example saving you on effort:

Example: You use your calculator to solve 105 ×46, and your calculator shows an answer of 690. Is this answer correct?

Solution: 105 × 46 is approximately 100 × 50, which is 5 000. This tells us that you have incorrectly keyed in 15 instead of 105 in your calculator. Therefore estimating your answer would indicate whether your answer is more or less correct.

Rounding

A simple form of estimation is rounding. Rounding is a critical skill required to quickly estimate a number. This is where you make a long messy number simpler by rounding, or expressing it in terms of the nearest unit, ten, hundred, tenth, or a certain number of decimal places.

Rules for rounding:

- If the digit you are rounding is followed by 5, 6, 7, 8, or 9, then round the digit up .Example: 46 rounded to the nearest ten is 50.
- If the digit you are rounding is followed by 0, 1, 2, 3, or 4, then round the digit down. Example: 23 rounded to the nearest ten is 20.
- Remember when rounding, all the digits to the right of the digit you are rounding becomes 0.
- In rounding off decimals, look only at the number in the place you are rounding to and the number that follows it. Similar rules apply.

Illustrative examples:

Example: The number 5 718 rounded off to the nearest:

- Ten is 5 720
- Hundred is 5 700
- Thousand is 6 000

Example: The number 6,7198 rounded off to:

- One decimal place (or tenth) is 6,7
- Two decimal places (or hundredth) is 6,72
- Three decimal places (or thousandth) is 6,720

Rounding in context:

- Consider the following problem:

  Determine the number of tiles you require to tile a floor which has an area of 10,1 square metres.

  From a QL perspective, your final answer would need to be rounded up to 11 as the store will not sell you 0,1 square metres of tiles. Furthermore, if you buy 10 square metres you will still have a small portion that will remain untiled. Therefore you would have had to **ROUND UP** in this context.

- Consider the following problem:

  Determine the number of people able to drink from a barrel containing 50 litres of beer if each person drinks 1,5 litres of beer.

  The accurate answer would be 50 divided by 1,5 = 33,333. From a QL perspective, your final answer would need to be rounded down to 33 as 34 people would not be able to drink 1,5 litres each. Therefore, you would have had to **ROUND DOWN** in this context.

If the digit you are rounding is followed by 5, 6, 7, 8, or 9, then round the digit up.
If the digit you are rounding is followed by 0, 1, 2, 3, or 4, then round the digit down.
You need to round appropriately according to the context: Round Up or Round Down.

Activity 1

1. Complete the table below by following the instructions in each column. The first example has been done for you.

NUMBER	ROUND OFF TO….	WHAT DO YOU LOOK AT	ANSWER
a. 2635	Nearest 10	Units digit 5	2640
b. 20635	Nearest 100		
c. 39645	Nearest 1 000		
d. 9,36541725	3 decimal places		
e. 9,36541725	4 decimal places		
f. 4,3333333	1 decimal place		
g. 7,6666666	2 decimal places		
h. 708,49999	Nearest whole number		
i. 349 495 678	Nearest million		

2. Complete the table below by following the instructions in each column. The first example has been done for you.

	CONTEXT	ROUND UP/DOWN	ANSWER
a.	You require transport for 134,1 people. How many people will you cater for?	Round up	135
b.	You require carpet tiles to cover 9,01 square metres. How many square metres will you order?		
c.	How many cars with length 5 m each can be parked in a single file if the road is 59 m long?		
d.	You have a book shelf that is 100 cm long. How many books width 3 cm each can be placed with width wise on the shelf?		
e..	Your retailer rounds off all amounts to the nearest 5 cents. What will the amount R34,06 be rounded to?		

3. Write 346 007 540 in words. ____

4. Write the following in number format

Four hundred and twenty six billion, two hundred and thirty five million and four hundred and two thousand and four rand. ________________________________

5. Estimate the following then check your answer using a calculator: $\frac{168\times5}{225}$ =________

Summary Assessment 1: **Rounding, estimation and formats**

MULTIPLE CHOICE.

Choose the alternative that best describes the statement or answers the question.

1. Estimate the difference by rounding each number to the nearest ten:

733 – 75

A. 700 B. 657 C. 650 D. 660

2. Round 7065 to the nearest ten, nearest hundred, and nearest thousand:

A. Ten 7060; Hundred 6000; Thousand 7000

B. Ten 7070; Hundred 6000; Thousand 6000

C. Ten 7060; Hundred 6100; Thousand 6000

D. Ten 7070; Hundred 6100; Thousand 6000

3. 0,006472867 rounded to three decimal places is:

A. 0,00647 B. 0,006 C. 0,00867 D. 0,647

4. 234 467 895 rounded off to the nearest million is:

A. 234 000 000 B. 235 000 000 C.234 400 000 D. 234 500 000

5. Sugar is sold only full kilograms. You require 1,32 kg of sugar. How much must you buy?

A. 1 kg B. 2 kg C. 1,5 kg D. 1,30 kg

6. Seven million and seventy six thousand nine hundred and thirty eight in number format will be:

A. 776 938 B. 7 076 938 C.7 076 983 D. 7 076 938

7. Round off the quotient of $\frac{10256}{32548}$ to three decimal places:

A. 0,315 B. 0,316 C. 0,318 D. 0,3

8. 823km rounded off to the nearest 10km is:

A. 800km B. 900km C. 950km D. 750km

My Notes

Use this space to write your own questions, comments or key points.

Ratios, rates, proportions and foreign exchange

Learning outcomes:

When you have completed this unit, you should be able to:

- Perform calculations using given ratios and rates.
- Perform calculations using proportions (both direct and indirect)
- Solve problems by calculating ratio, rate or proportion.
- Use ratio, rate, proportions and percentages to solve problems.
- Understand terms "strong and weak" relating to currency exchanges.
- Perform currency conversions using given conversion rates/tables

Ratios

- Ratio is used to compare two or more quantities that have the same unit of measure.
- The numbers in a ratio are separated by a colon (:) (read as "is to").
- To simplify a ratio 12 : 16 we divide both parts of the ratio by the highest common factor, therefore 12 : 16 = 3 : 4 or it can be expressed as ¾
- If the quantities or units of measure are different we need to convert to the same unit of measure before doing any calculations.

  Eg. Alice has 60 cents while Boysie has R1,20. To simplify we need to convert both to the smaller unit cents and then divide each side of the ratio by the highest common factor.

  Solution: 60 cents : R1,20 = 60 cents : 120 cents = 1:2.
  Notice that is no units in the final answer because they cancel each other out.
- Writing a ratio in its simplest form may result in one of the numbers being equal to 1. This is called a unit ratio.
  Eg. The ratio of 10 males to 150femalesin simplified form will be 10 : 150 = 1 : 15.
- Sometimes a unit ratio may not be the simplest form, for example, 7 : 9 can be written as 1 : 1,2857 which is a unit ratio, but not in its simplest form. Therefore to calculate the unit form, we simply divide both numbers by the smaller number, so 7÷7 : 9÷7=1:1,2857.

Illustrative Examples

1. Share R120 between two people in the ratio 2:3.
2. Juhi's salary to Jenny's salary is in the ratio 2:5. If Jenny receives R30 000 a month, calculate Juhi's monthly salary.
3. A recipe for 4 people needs 6 cups of flour. How many cups of flour are needed for 10 people?

Solutions

1. The ratio 2:3 means there are (2 + 3) parts in the ratio, which is 5 parts. Therefore to calculate one part we take R120 and divide by 5 parts to get the value of one part, which is R24. Therefore the first person will get 2 parts × R24 = R48 and the second person will get 3 parts × R24 = R72.
2. The second example is different as the total value of all parts are not given but Jenny's share which is 5 parts is given. We therefore use Jenny's salary to calculate one part. We take R30 000 and divide by 5 parts = R6 000 for one part.
   Juhi therefore receives 2 × R6 000 = R12 000.
3. 4 people = 10 cups
   1 person = 10 cups ÷ 4 = 2,5 cups
   ∴ 10 people = 2,5 cups × 10 = 25 cups.

Rate

- Rate, like a ratio, is also a comparison between two numbers or units of measure but rate has different units for the given numbers.
- Examples of rate include cost rates, eg. petrol cost R 12,15 per litre or 12,15 R/l) and speed, eg. a car travels at 120 km/h.
- You will notice some of the calculations in ratios required us to find the rate of one item before calculating the cost of the other part of the ratio.
- To calculate rate, we always divide by the second value, which will thus enable us to calculate the amount per one unit.
- We are often required to make a choice as to which quantity of an item is cheaper, rate is often used to make a decision.

Illustrative Examples

1. A pocket of potatoes (7kg) costs R49,00 while 2 kg cost R21. Which is cheaper?
2. Linda, a star athlete, runs 200 m in 30 seconds. What is his speed in metres per second?
   If he was able to keep running at this speed, how long would he take to cover 1 km?
3. Shaukat is able to type 106 words in 2 minutes on his laptop while Pilodia is able to type 314 words in 7 minutes. Determine who is able to type at a faster speed.

Solutions

1. 7 kg = R49 2 kg = R 21

   ∴ 1 kg = R49 ÷7 = R7/kg ∴ 1 kg = R21 ÷2 = R10,50/kg

Therefore the 7 kg pocket is much cheaper per kg.

2. 200 m = 3 seconds

   100 m = 1,5 seconds

   1m = 0,015 seconds

   ∴ 1 km = 1 000 m = 1,5 seconds × 10 = 15 seconds.

3. Shaukat 106 words = 2 minutes

   ∴ 1 word = 2 minutes ÷ 106 = 0,019 minutes

   Pilodia 314 words = 7 minutes

∴ 1 word = 7 minutes ÷ 314 = 0,022 minutes

Therefore, Shaukat is faster.

Proportions

- A proportion is a statement that shows ratios are equal, eg. 1:2 = 3:6.
- When solving problems relating to proportions we state the ratios as fractions, setting the two fractions equal to each other, cross-multiplying, and solving the resulting equation for the unknown.

Example 1: Find the unknown value in the proportion: 2 : *x* = 3 : 9.

Solution: 2 **:** *x* = 3 **:** 9

First, convert the colon-based ratios to fractional form:

$$\frac{2}{x} = \frac{3}{9}$$

Then solve the proportion by cross multiplying and finding the unknown:

$$\frac{2}{x} = \frac{3}{9}$$

9(2) = x(3)

18 = 3x

6 = x

Example 2: If twelve inches correspond to 30,48 centimeters, how many centimeters are there in thirty inches?

Solution: Set up your ratios as follows: inches: centimetres = inches :centimetres

∴12 inches : 30,48 cm = 30 inches : ? cm (Let c be the ?)

$$\frac{\text{inches}}{\text{centimeters}} : \frac{12}{30.48} = \frac{30}{c}$$

$$\frac{12}{30.48} = \frac{30}{c}$$

12*c* = (30)(30,48)

12*c* = 914,4

c = 76,2

NB. One can use rate instead of proportions to solve example 2 as follows:

12 inches = 30,48 cm

∴ 1 inch = 30,48 cm ÷ 12 = 2,54 cm

∴ 30 inches = 2,54 cm × 30 = 76,2 cm

- When a proportion increases by the same factor, we call this a direct proportion. (see table below each column results in the same ratio 1/85)

Example: If you work and get paid by the hour, then the more hours you work the more you will get paid. If your payment per hour is R85,00, then the following table shows your payment for the different hours worked:

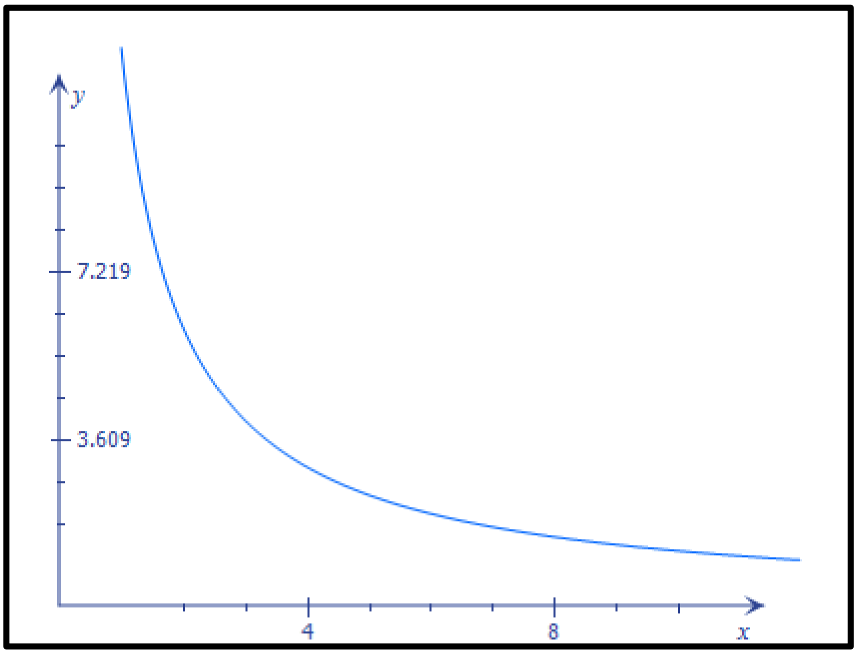
Hours worked	1	4	6	9	12	20
Payment in rand	85	340	510	765	1 020	1 700

- If a graph had to be drawn for a direct proportion it would be a straight line graph.
- When a proportion increases by the same product, we call this a indirect or inverse proportion. (see table below – constant product of 12)

Example: It takes 12 men 2 days to build a wall, how long will it take 4 men to build the same wall. You will notice that as the number of men increases then the number of days decrease. The following table shows the progression.

No of men	1	2	3	4	6	12
No of days	12	6	4	3	2	1

- If a graph for the above table had to be sketched (indirect or inverse proportion) it will be a curve graph called a hyperbola. See sketch below.



Activity 2

1. Sandy's boss pays her R50,00 an hour for normal hours worked from Monday to Friday.

   Normal hours are from 08:00 to 17:00. Any time beyond these hours are classified as overtime hours and the rate will be R75 an hour.

   Weekends and public holidays are classified as overtime hours.

   1.1 If Sandy worked the entire week plus 3,5 hours overtime, calculate her wages at the end of the week.

   1.2 If Sandy was paid R1 500 for a week and she did not work any overtime hours, calculate how many normal hours she worked.

   1.3 If Sandy worked the entire month consisting of 4 full weeks and 5,5 overtime hours for the month, calculate her wage for the month.

   1.4 On one normal Friday her boss told her that she needed to work till 19h30 to have an important order shipped off the next morning. It was also pay day that Friday. How much did Sandy get paid if the other 4 days in the week she worked the full normal hours.

1.5 Sandy was unhappy with her wage and approached her boss for an increase. Her boss agreed to give her an additional R10 per normal hour worked and her new overtime rate will be new normal time and a half.

(a) Calculate her new overtime rate.

(b) Calculate how much she will now earn if she worked a full week with normal hours and 6 hours on Saturday.

2. Thabo requires the following ingredients to bake a cake which feeds 4 people:

2 cups flour

1 cup milk

1 cup sugar

What amount of each ingredient will Thabo need if he wants to now bake a cake for 10 people?

Exchange rates

- Exchange rate is the price of a country's currency in terms of another currency.
- Most countries often compare their currency with the US dollar (USD) to evaluate the strength of their own currency.
- Therefore a rand dollar exchange will mean the amount of rands require to buy one US dollar.
- A weak rand will imply that more rands will be required to purchase 1 USD.
- A strong rand will imply that less rands will be required to purchase 1 USD.
- Various factors can cause the exchange rate to change, namely, interest rate, inflation rate, political instability, the general state of the economy, etc.

Illustrative examples

The table that follows shows the exchange rate between the rand and the major currencies around the world on 1 September 2016. Use the table to complete the questions which follow:

South African Rand	1 rand is worth ? units	1 unit is worth ? rands
US Dollar	0,068847	14,524946
Euro	0,061340	16,302687
British Pound	0,051840	19,289949
Indian Rupee	4,605301	0,217141
Australian Dollar	0,091589	10,918305
Canadian Dollar	0,090275	11.077271

South African Rand	1 rand is worth ? units	1 unit is worth ? rands
Singapore Dollar	0,093718	10.670258
Swiss Franc	0,067080	14,907530
Malaysian Ringgit	0,283942	3,521842
Japanese Yen	7,011683	0,142619

Source http://www.x-rates.com/table/?from=ZAR&amount=1

1. Which country will be the cheapest for a South African to visit? Why?
2. Which country will be the most expensive for a South African to visit?
3. Determine how many euros will I be able to get for R2 500.
4. Calculate how many rands I will be able to get for my 300 rupees.

Solutions:

1. Japan – You will be able to get more Japanese yen for your rand.
2. England/Britian – You need more rands to purchase one British pound.
3. R2 500 × 0,061340 = 159,34 euros
4. 300 × 0,217141 = R65,14

Activity 3

1. If 2 litres of fuel cost R22,95, what will 5 litres of fuel cost?
2. Jali is able to walk 4 km per hour. Walking at the same pace how many km's would she have covered in 4 minutes?
3. A gallon contains 8 pints of water. If each pint contains 2 cups, calculate how many cups of water will be found in 10 gallons.
4. A lawn can be mowed by 8 people in 6 hours. How long will it take 5 people to mow the same lawn?
5. If 4,5 kg of sugar costs R36, what will 2,5 kg of sugar cost?
6. A restaurant with 40 tables can seat 700 people. This restaurant has only two types of tables, 10-seater and 20-seater tables. Determine the ratio of the 10-seater tables to the 20-seater tables.
7. Study the exchange rate table below and then use it answer the questions which follow:

South African Rand	1 rand is worth ? units	1 unit is worth ? rands
US Dollar (($)	0,068847	14,524946
Euro (€)	0,061340	16,302687
British Pound (£)	0,051840	19,289949
Indian Rupee (₹)	4,605301	0,217141
Australian Dollar AUD($)	0,091589	10,918305
Canadian Dollar CAD($)	0,090275	11.077271
Singapore Dollar SGD($)	0,093718	10.670258
Swiss Franc (CHF)	0,067080	14,907530
Malaysian Ringgit (RM)	0,283942	3,521842
Japanese Yen (¥)	7,011683	0,142619

7.1 A student bought a pair of sunglasses in the USA for $35.50. In England, an identical pair of sunglasses costs £26.99. In which country were the sunglasses cheaper, and by how much? Show all your working.

7.2 Fadia went on holiday to India. While on holiday she bought a belt and a hat. The belt cost 125 rupees, while the hat cost 5 rupees. Give the total cost in rands.

7.3 In the USA Rob buys a shirt for $67. In London the same shirt costs £47.50 calculate the difference between the cost of the shirt in the USA and in London. Give your answer in rands.

Summary of key learning: There following key concepts were covered in this unit:

- Ratios – comparison of two or more numbers of the same unit.
- Rate – comparison of two numbers each with different units.
- Proportion – is a statement showing two ratios are equal.
- Direct proportion – has the same ratio. (divide to get same answer)
  Graph of a direct proportion is a straight line.
- Indirect or inverse proportion – has the same product (multiply to get same answer)
  Graph of an inverse proportion is curved called hyperbola.
- Exchange rate – the price of a country's currency in terms of another country.

My Notes

Use this space to write your own questions, comments or key points.

Suggested sources of additional information

Read more: Exchange Rate Definition | Investopedia http://www.investopedia.com/terms/e/exchangerate.asp#ixzz4K2ne5rk1

https://www.khanacademy.org/math/cc-sixth.../cc-6th-ratios-prop-topic
https://www.khanacademy.org/math/pre-algebra/pre-algebra-ratios-rates

Summary Assessment 2: Ratio, Rate, Proportions and Exchange Rate

MULTIPLE CHOICE.

Choose the alternative that best describes the statement or answers the question.

1. It takes one hour to drive 80 km. How far can you drive in 20 minutes at that speed?
   A. 20 km B. 25 km C. 27 km D. 30 km

2. The ratio of 1 hour to 300 seconds is:
   A. 1 : 12 B. 12 : 1 C. 1 : 5 D. 5 : 1

3. The length and width of a field are in the ratio 5 : 3. If the length of the field is 70 m then its width is:
   A. 112 m B. 43,75 m C. 26,25 m D. 42 m

4. A recipe for dessert requires 3 cups sugar and 4 cups cocoa powder.
   If you use 12 cups of cocoa powder, how much sugar will you need?
   A. 12 cups B. 16 cups C. 9 cups D. 6 cups

5. A car travels 200 km using 7 litres of petrol. At this rate, how far can the car travel using 35 litres of petrol?
   A. 1,000 km B. 1,200 km C. 900 km D. 1,500 km

6. You can buy 8 cupcakes for R4,64. What is the unit price for each cupcake?
   A. R0,60 B. R1,72 C. R0,58 D. R1,25

7. Jerry bought bananas, apples and pears. The ratio of bananas to apples was 3 : 5 and the ratio of apples to pears was 4 : 2. If he bought 12 bananas, how many pears did he buy?
   A. 10 B 12 C. 20 D. 42

8. Which team has the best record?
   A. Chiefs: 17 wins in 26 games. C. Pirates: 14 wins in 21 games.
   B. Sundowns: 21 wins in 30 games. D. Wits: 15 wins in 22 games.

9. Which ratio is equivalent to 14 : 20?
   A. 14,20 : 1 B. 0,7 : 1 C. 1 : 0,7 D. 1,4 : 1

10. Refer to the exchange rate table in Question 7 of Activity 2 to answer this question.
    The price of a cup of coffee is £2,30 in Great Britain and a similar cup of coffee costs $3,66 (AUD) in Australia. In which country will you pay the most in ZAR for a cup of coffee?
    A. Great Britain B. Australia

Interest Calculations

Introduction

This section deals predominantly with calculation of discounts, simple interest, compound interest, inflation and deflation. In order to understand these afore-mentioned calculations a concept review will be foregrounded whereby the foundational skills required for percentages will be outlined.

Content – Interest Calculations

Learning outcomes:

When you have completed this unit, you should be able to:

- Solve problems by calculating percentage.
- Increase or decrease values in a given percentage.
- Calculate discounted amount using a given percentage.
- Calculate percentage discount.
- Calculate original amounts.
- Value Added Tax (VAT).
- Calculate the VAT amount for given exclusive and inclusive prices.
- Calculate the exclusive price for a given inclusive price.
- Use simple interest formulae to determine the interest, the interest rate, the period of investment and the final amount.
- Use compound interest factor tables to determine the initial amount, the final amount and the interest amount.
- If the formula is used then the period must not exceed 2 years.
- Calculate annual inflation / deflation for given products over multiple periods.
- Calculate and compare annual inflation deflation rates.

Percentages

- Percent means "per 100" or "out of 100". The symbol % is used to represent a percent. For example 37% is the same as $\frac{37}{100}$.
- To convert a percentage into a common fraction or decimal fraction, we divide by 100. For example 20% = $\frac{20}{100}$ = $\frac{1}{5}$(common fraction) = 0, 2 (decimal fraction),
- We can convert between decimal fractions, common fractions and percentages:

  Eg. Converting decimal to percentage: $0{,}07 \times \frac{100}{100} = \frac{7}{100} = 7\%$

  Converting percentage to decimal: $23{,}3\% = \frac{23{,}3}{100} = 0{,}233$

  Converting a fraction to percent: $\frac{13}{30} = 0{,}4333 = 0{,}433 \times 100\% = 43{,}3\%$

  Converting percentage to fraction: $40\% = \frac{40}{100} = \frac{2}{5}$
- To express one quantity as a percentage of another, both require the same unit:

  Eg. Express 60 cents as a percentage of R4,00: $\frac{60\ c}{400\ c} \times 100\% = 15\%$

- To calculate a certain percentage of a quantity:

  Eg. Calculate 15% of R400: $\frac{15}{100} \times$ R400 = R60.

- Given the answer to a percentage of a quantity, how do we calculate the original quantity?

  Eg. We paid R15,50 which was the 14% VAT for a shirt. What was the total cost of the shirt including VAT?

  We can use one of two Methods to obtain the answer:

  Method 1:

  We can use proportions to solve as follows:

  R15,50 = 14%

  Original Price ?= 100%

  We cross multiply and divide: $\frac{\text{R}15{,}50 \times 100\%}{14\%}$ = R110,71.

  Method 2:

  We simply take the quantity given and divide by the given percentage as follows:

  $$\frac{\text{R}15{,}50}{14\%} = \text{R}110{,}71$$

- Increasing a quantity/value by a certain percent, like adding a mark up to a cost:

  Eg. Increase R56 by 12%

  Once again we can use one of two methods to obtain the answer:

  Method 1:

  We can use proportions to solve as follows:

  R56 = 100%

  New ? = 112% (100% +12%)

  We cross multiply and divide: $\frac{\text{R}56 \times 112\%}{100\%}$ = R62,72

  Method 2:

  We can use the following method:

  R56 + (12% × R56) = R56 + R6,72 = R62,72

- Decreasing a quantity/value by a certain percent, like offering a discount:

  Eg. The marked price of a belt is R325. However, the retailer offers you the belt at the price less 5% discount. What price did you pay for the belt?

  Once again we can use two methods to obtain the answer:

  Method 1:

  We can use proportions to solve as follows:

  R325 = 100%

  New ? = 95% (100% - 5%)

  We cross multiply and divide: $\frac{\text{R}325 \times 95\%}{100\%}$ = R308,75

Method 2:
We can use the following method:
R325 – (5% × R325) = R325 – R16,25 = R308,75

- Calculating percent **INCREASE** or percent **DECREASE:**
  To calculate a percent change (increase or decrease) the following formula may be used:

Percent change (Increase or Decrease) = $\frac{\textbf{Change (New Price} - \textbf{Old Price)}}{\textbf{Starting Point (Old Price)}} \times \mathbf{100\%}$

Eg. Calculate the percent increase in Urishka's salary if it increased from R8500 per month to R9 200 per month.

Change = R9 200 – R8500 = R700.

Percent change (Increase)=$\frac{\boldsymbol{R700}}{\textbf{R8500}} \times \mathbf{100\%} = \mathbf{8,24\%}$

Eg. An item marked at R250 was sold for R200. Calculate the percent decrease:

Change = R200 – R250 = – R50. (We ignore the sign as a (–) will mean a decrease.

Percent change (Decrease) = $\frac{\boldsymbol{R50}}{\textbf{R250}} \times \mathbf{100\%} = \mathbf{20\%}$

Simple Interest

- Simple interest is calculated only on the principal (initial) amount for each year of the investment – this means the same amount of interest will be earned for the duration of the investment.

Eg. An amount of R2 000 is invested for 3 years at a simple interest rate of 5,5% per annum. Calculate the accumulated amount after 3 years.
You may use the following formula:
Accumulate amount (A) = Initial Amount (1 + interest rate × time period in years)
ie. A = P (1 + $i \times n$)
A = R2 000 (1 + 5,5% × 3) = R2 330
You will notice the interest earned over 3 years is R330, which is R110 per year (5,5% × R2 000 = R110)

Eg. Calculate the rate of interest if R200 is earned over 3 years on an investment of R1 500 if simple interest is used.

Interest = P × $i \times n$

R200 = R1 500 × i × 3

R200 = R4 500 × i

$\therefore i = R200 \div R4\ 500 = 0{,}0444 = 4{,}44\%$

- Sometimes we are unable to afford to purchase items for cash, so we buy them on a **hire purchase** agreement (loan). Hire purchase agreements use simple interest calculations.
- Most **hire purchase** agreements require that a deposit is paid before the product can be taken by the customer. The principal amount of the loan is therefore the cash price minus the deposit. The total loan amount is then divided into monthly payments over the period of the loan.

Eg. Ashraf wants to buy a laptop computer for R10 500 on a hire purchase agreement. He has enough to pay for the deposit of 10%. Calculate the following:
(a) The deposit amount.
(b) What is the total amount he would have paid for his laptop computer, if interest is calculated at 11,5% per annum for 4 years?

(a) Deposit amount = 10% × R10 500 = R1 050.
(b) The hire purchase agreement amount will be calculated as follows:
R10 500 – R1 050 = R9 450
Therefore A = P $(1 + i \times n)$ = R9 450 (1 + 11,5% × 4) = R13 797
The total cost paid for the laptop = R1 050 + R13 797 = R14 847.

Compound Interest

- Compound interest involves calculating interest and adding it to the principal. This new amount becomes the new principal upon which interest is then calculated. In other words its "interest on interest".
- Interest can be calculated more than once a year example, half yearly, quarterly or monthly. In this instance the annual interest rate would be divided by the number of times the interest is calculated in a year. See the table which shows interest calculations for a year for an amount of R10 000 at 10% per annum compounded … :

Compounded period	Number of periods	Values of *i*and*n*	Total with interest
Annually	1	i= 10% n = 1 (intcal once a year)	R11 000,00
Half-yearly	2	i= 10%÷ 2 = 5% n = 2 (intcal2 times a year)	R11 025,00
Quarterly	4	i= 10%÷ 4 = 2,5 % n = 4 (intcal 4 times a year)	R11 038, 13
Monthly	12	i= 10%÷ 12 = 0,83 % n = 12 (intcal 12 times a year)	R11 047,13

- We can calculate the compounded amount using THREE different methods:
Method 1: Long method which involves calculating interest each period and finding the new principal for each period and then repeating the process.

Method 2: Using a formula: A = P $(1 + i)^n$, where A is the accumulated amount with interest; P the initial amount or principal; i the periodic interest rate and n the number of times interest is calculated or number of payments made.

Method 3: Use a factor table, which most financial institutions use.

Illustrative example showing calculations involving the three methods.

Eg. Calculate the accumulated amount for an investment of R5 000,00 invested for 2 years at 5% per annum compounded annually.

Method 1: Long method

Year 1: Interest = 5% × R5 000 = R250,00

Therefore the accumulated amount at the end of year 1 which becomes the Principal for year 2 = R5 000,00 + R250,00 = R5 250.

Year 2: Interest = 5% × R5 250,00 = R262,50

Accumulated amount at end of two years = R5 250,00 + R262,50 = R5 512,50.

Method 2: Formula method

$$A = P(1 + i)^n = R5\,000(1 + 5\%)^2 = R5\,512{,}50$$

Method 3: Factor table method

Table showing compounded future value factors for a single amount of R1,00

Year	*i* = 2%	*i* = 3%	*i* = 4%	*i* = 5%
1	1,020000	1,030000	1,040000	1,050000
2	1,040400	1,060900	1,081600	1,102500
3	1,061208	1,092727	1,124864	1,157625
4	1,082432	1,125509	1,169859	1,215506
5	1,104081	1,159274	1,216653	1,276282
6	1,126162	1,194052	1,265319	1,340096
7	1,148686	1,229874	1,315932	1,407100
8	1,171659	1,266770	1,368569	1,477455
9	1,195010	1,304773	1,423312	1,551328
10	1,218994	1,343916	1,480244	1,628895

Note: Factors are rounded off to 6 decimal places.

The table above shows the Future Values with interest compounded for a single amount of R1,00 at various interest rates. Note that factors are rounded to six decimal places which could impact on final answers because of early rounding.

A = Principal Amount × factor given as per table = R5 000 ×1,102500 = R5 512,50.

- Sometimes we have to work out the value of an item that decreases in value, for example, when we buy a motor vehicle it decreases (depreciates) in value from year to year. Therefore the formula we use is called a compound decrease formula.
  Eg. Find the value of a vehicle in 3 years' time if its value now is R200 000 and depreciation is at the reducing balance method of 20% per annum. (Note the word reducing balance implies a compound decrease).
  Formula now is modified as follows:
  $$A = P(1 - i)^n = R200\,000(1 - 20\%)^3 = R102\,400$$

Inflation and Deflation

- Inflation is the average increase in the price of goods annually and is given as a percentage.
- Since the rate of inflation increases from year to year, it is calculated using the compound interest formula.
- Inflation results in the loss of the value of money, because with the same amount of money you can buy less goods in the next year, than you could have bought in the previous year.
- Deflation occurs when the general level of prices decreases. A negative inflation rate.
- Deflation results in the increase in the value of money, because you can buy more goods in the next year than you could have bought in the previous year.
- Deflation occurs when too many goods are available or when there is not enough money circulating to purchase those goods. For example, if a particular type of car becomes highly popular, other manufacturers start to make a similar vehicle to compete. Soon, car companies have more of that vehicle style than they can sell, so they must drop the price to sell the cars.
- Deflation can adversely affect the economy in significant ways, example layoffs, unemployment increases as companies cannot sell their goods or don't make enough profit.
- The following tabulates the differences between Inflation and deflation.

INFLATION	DEFLATION
Prices are generally rising	Prices are falling
Rich gets richer and poor get poorer	Decreases production, output and income of the entire industry – everybody loses
Can be controlled easily	Difficult to control
Moderate inflation is considered healthy for an economy	Not healthy for an economy as it affects income levels and output.

Inflation and **deflation** relate to a change in the general level of prices.

Inflation occurs when the general level of prices increases.

Deflation occurs when the general level of prices decreases.

Summary of key learning: There following key concepts were covered in this unit:

- **Percentages – conversions between decimal, common fractions and percentages; calculations involving increase and decrease and solving real life problems, examples discounts and VAT calculations**
- **Simple interest calculations including hire purchase agreements**
- **Compound interest calculations including compound decrease and period changes**
- **Inflation and deflation and its effect on the economy**

My Notes

Use this space to write your own questions, comments or key points.

Suggested sources of additional information

Read more: [What is the difference between inflation and deflation? | Investopedia](http://www.investopedia.com/ask/answers/111414/what-difference-between-inflation-and-deflation.asp#ixzz4KcU7NeMA) http://www.investopedia.com/ask/answers/111414/what-difference-between-inflation-and-deflation.asp#ixzz4KcU7NeMA

Activity 4 – **Interest calculations**

1. A cellphone bill excluding 14% VAT amounts to R325,00. Calculate the total bill with VAT.
2. A clothing retailer offers a discount of 15% on all marked prices instore. If a shirt is marked at R325, calculate the discounted price.
3. The number of students at a school in 2016 has increased by 15% since 2015. If 2015 had a student population of 820, calculate the student population in 2016.
4. The price of bread has increased from R11,40 to R12,00 because of inflation. Calculate the percentage increase.
5. The price of a T-shirt inclusive of 14% VAT costs R250,00. Calculate the cost of the T-shirt exclusive of 14% VAT.
6. Calculate the percentage discount (decrease) of an item marked at R250,00 but sold for R200,00.
7. Which is a better investment:
   7.1 R12 500,00 invested at Bank A for 2 years at 5,5% p.a simple interest
   7.2 R12 500,00 invested at Bank B for 2 years at 5,3% p.a compound half-yearly.
8. The average inflation rate in 2015 was 4,51% and 2016 was 6,54%. If the price of bread in 2014 was R9,80, calculate the price of bread in 2016.
9. Explain the difference between inflation and deflation.
10. State with reasons whether deflation has a positive impact on the state of a country's economy.
11. Sipho wants to buy a fridge which is advertised at R8 500,00. The store manager offers Sipho to buy the fridge on hire purchase agreement. The agreement states Sipho must pay a deposit of 10% and repay the balance in 60 monthly instalments. If the rate of interest is 13,75%, calculate Sipho's monthly instalment.
12. Refer to the factor table below and then answer the question which follow:

Table showing compounded future value factors for a single amount of R1,00

Year	*i* = 2%	*i* = 3%	*i* = 4%	*i* = 5%
1	1,020000	1,030000	1,040000	1,050000
2	1,040400	1,060900	1,081600	1,102500
3	1,061208	1,092727	1,124864	1,157625
4	1,082432	1,125509	1,169859	1,215506
5	1,104081	1,159274	1,216653	1,276282
6	1,126162	1,194052	1,265319	1,340096
7	1,148686	1,229874	1,315932	1,407100
8	1,171659	1,266770	1,368569	1,477455
9	1,195010	1,304773	1,423312	1,551328
10	1,218994	1,343916	1,480244	1,628895

Note: Factors are rounded off to 6 decimal places.

Using firstly the above factor table and then the compound interest formula, perform two calculations to find the final amount if R12 345 is invested for 3 years at 4% per annum compounded annually.

Summary Assessment 3 – **Percentages and Interest calculations**

MULTIPLE CHOICE.

Choose the alternative that best describes the statement or answers the question.

1. A business sells 150 computers in July and then 212 computers in August What was the percent increase in sales from July to August?

   A. 50,7% B. 41,3% C. 70,7% D. 25%

2. 40% of a class failed a test, and 80 percent of those who passed earned less than an A (80%), what percentage of the students earned an A?

   A. 20% B. 12% C. 60% D. 2%

3. The price of bread increases by 6%. The original price was R8,50 for a loaf of bread. What is the new price of a loaf of bread?

   A. R9,01 B. R0,51 C. R7,99 D. R9,10

4. The price of fresh chips increased from R12,00 to R15,50. Calculate the percentage increase to the nearest whole number:

   A. 29% B. 0,29% C. 23% D. 0,23%

5. Find the principal amount which yields a simple interest of R20 and compound interest of R21 in two years, at the same percent rate per annum.

   A. R520 B. R480 C. R420 D. R200

6. The compound interest on an amount invested for 2 years is R832 and the simple interest on the same amount at the same rate for the same period is R800. What is the rate of interest?

   A. 6% B. 8% C. 10% D. 12%

7. The difference between the compound interest on R5 000 for 1,5 years at 4% p.a compounded annually and semi-annually is:

   A. R2,04 B. R3,06 C. R4,80 D. R8,308.

8. An amount invested at simple interest amounts to R815 in 3 years and to R854 in 4 years. The amount invested is therefore:

   A. R850 B. R790 C. R698 D. R800

9. A country is said to be experiencing inflation when:

   A. prices of most goods and services are rising over time.
   B. prices of most goods and services are falling over time.
   C. total output is rising over time.
   D. total output is falling over time.

10. If the price of an item in 2015 was R100 and in 2016 it was R102 in 2018, the inflation rate will be:

    A, 102% B. 20% C.2% D. 0.2%

Personal Income Tax

Learning outcomes:

When you have completed this unit, you should be able to:

- Define terminology relating to personal income tax;
- Calculate personal income tax using given tax tables;
- Identify taxable and non-taxable deductions;
- Identify tax brackets and rebates;
- Calculate annual gross and net salaries;
- Calculate taxable amount after non-taxable deductions;
- Calculate tax due after deduction of the rebate.
- Calculate UIF contributions and unemployment benefits.
- Analyse documents
- Perform calculations involving stepped tariffs.

Concepts and terminology

It is important that you understand the terminology used in the context of personal income tax.

- **Gross Salary:** Total amount earned in a month. This includes all types of salary (e.g. salary, overtime, commission, bonuses, etc.)
- **Deductions:** Amounts that need to be subtracted from the gross salary before money is deposited into the employee's bank account. These include UIF, Pension, Medical Aid, Trade Union Fees, Loan repayments, Tax, etc.
- **Net Salary:** Also known as 'take home pay'. Amount that is deposited into an employee's bank account. It is calculated as follows:

  Net Salary = Gross Salary - Deductions
- **Personal Income Tax:** This is a tax on all sources of income (e.g. salary, interest income, rental income, etc.). It is calculated on the taxable income.
- **Taxable Income:** This is different from Net Salary although the calculation looks similar.

  Taxable Income = Gross Income – Tax-deductible Deductions
- **Gross Income:** This is different from gross salary (above) because it includes all forms of income, e.g. salary, rental income, royalties, etc.
- **Tax deductible deductions:** These are specific deductions that are subtracted from the gross income before tax is calculated. There are two types of taxable deductions:

Salary-based deductibles: subtracted from the gross salary by the employer before the salary is paid. These include: UIF, Pension fund contributions, etc.

Non-Salary deductibles: These may be paid out of an employee's take-home
pay, e.g. donations to charities, certain medical expenses.There are limits placed on deductibles, e.g. the maximum amount that can be deducted for pension is 7,5% of the gross salary.

- **Non-tax deductible expense:** The majority of expenses are not tax deductible. These are generally living expenses, e.g. food, rent, fuel, entertainment, etc. Only tax deductible deductions reduce the amount of taxable income owed.
- **Taxable Deductions:** Some deductions subtracted from an employee's payslip are taxable. Although the employee receives less money they still have to pay tax on the larger amount of money that they earned. Examples include: loans from an employer, a garnishee order, and monthly payments to the employer for services rendered, etc.
- **Tax Bracket:** A range of taxable income intervals that are charged according to a set rate of tax. The amounts are representing annual taxable amounts. The higher the bracket, the higher the tax rate for that portion of the taxable income.
- **Tax Rebate:** An amount deducted from your tax payable after your tax rate has been calculated. Only people who pay tax are eligible for a rebate. There are three rebates. Everyone is eligible for the primary rebate. Taxpayers who are 65 years and older qualify for the additional secondary rebate. Tax payers who are 75 years and older qualify for all three rebates.
- **Tax Threshold:** This is the minimum salary a person must earn before tax is charged. Below the threshold, the person's tax will be cancelled by the tax rebate.
- **SARS** (South African Revenue Services): Issues tax tables which are used when determining tax payable by individuals.

Calculations involving personal income tax

Examples on tax calculations:

Refer to the tax table which follows and then answer the questions which follow:

Statutory rates individuals	
Taxable annual Income (R)	**Rates of tax(R)**
0 – 160 000	18% of each R1
160 001 – 250 000	28 800 + 25% of the amount above R160 000
250 001 – 346 000	51 300 + 30% of the amount above 250 000
346 001 – 484 000	80 100 + 35% of the amount above 346 000
480 001 – 617 000	128 400 + 38% of the amount above 484 000
617 001 and above	178 940 + 40% of the amount above 617 000
Tax rebates individuals	
Primary rebate	R11 440
Secondary rebate(65 years and older)	R6 390
Tertiary rebate (75 years and older)	R2 130
Tax thresholds individuals	
Persons under 65 years	R63 556
Persons 65 years and older	R99 056
Persons 75 years and older	R110 889

Eg 1.Using the above tax table, calculate how much tax a 60-year old person should pay monthly ifhis monthly taxable income is R14 800,00.

Solution:

Step 1: Calculate the annual taxable income if not given

The annual taxable income = R14 800 × 12 = R177 600.

Step 2: Identify the correct tax bracket

Tax bracket 2:

Annual tax

= R28 800 + 25% of the amount above R160 000
= R28 800 + 25 ÷ 100 × (R177 600 – R160 000)
= R28 800 + 25 ÷ 100 × (R17 600)
= R28 800 + R4 400,00
= R33 200,00

Step 3: Identify the applicable rebate and deduct accordingly

60 year old only qualifies for Primary rebate

Total tax payable = R33 200 – R11 440 = R21 760 annually

Step 4: Calculate his monthly tax payable

Monthly tax payable = R21 760 ÷ 12 = R1 813,33.

Eg 2: Mr Lala is 45 years of age and earns a monthly salary of R44 857.

The following deductions are taken off his monthly salary:

- Pension: R4187,55 (max of 7,5% of gross income is tax deductible)
- UIF: 1% of his gross salary
- Medical Aid: R5 423,00
- Repayment of a personal loan from employer: R3 500,00 per month

Calculate:

(a) Mr Lala's monthly tax payable.
(b) His net monthly salary.

Solution:

(a) Calculate the tax deductible items: Pension = 7,5% × R44 857 = R3 364,28

UIF = 1% × R44 857 = R448,57

Calculate monthly taxable income = Gross Income – tax deductions

= R44 857 – (R3 364,28 + R448,57)

= R44 857 – R3 812,85

= R41 0044,15

Therefore annual taxable amount = R41 044,15 × 12 = R492 529,80

Total annual tax before rebate = R128 400 + 38% × (R492 529,80 – R484 000)

= R128 400 + R3241,32

= R131 641,32

Annual tax after rebate = R131 641,32 – R11 440 = R120 201,32

Monthly tax payable = R10 016,78

(b) ***Note**: In addition to the standard rebate, SARS also gives each tax payer a medical tax credit. The total monthly medical tax credit is the total of the following amounts: R230 for the tax payer + R230 for the spouse + R154 per child.

Mr Lala's total medical tax credit = R230 × 2 (Mr & Mrs Lala) + R154 × 2 (children)

= R768,00

Net monthly salary(take home pay) = Gross income – Total deductions + Med Tax Cr

= R44 857 – (R4 187,55 + R448,57 + R5 423 + R3 500 + R10 016,78) + R768

= R44 857 – R23 575,90 + R768

= R22 049,1

Activity 5 – **Personal Income Tax**

Refer to the tax table below and then answer the questions which follow:

	Rates of tax(R)
0 – 160 000	18% of each R1
Statutory rates individuals	28 800 + 25% of the amount above R160 000
Taxable annual Income (R)	51 300 + 30% of the amount above 250 000
346 001 – 484 000	80 100 + 35% of the amount above 346 000
480 001 – 617 000	128 400 + 38% of the amount above 484 000
617 001 and above	178 940 + 40% of the amount above 617 000
Tax rebates individuals	
Primary rebate	R11 440
Secondary rebate(65 years and older)	R6 390
Tertiary rebate (75 years and older)	R2 130
Tax thresholds individuals	
Persons under 65 years	R63 556
Persons 65 years and older	R99 056
Persons 75 years and older	R110 889

1. Feroz is 65 years old and earns R27 000 per month as a lecturer. He pays R2 025 towards a pension fund. Note: Only 7,5% of gross income is tax deductible.
   1.1 Calculate his annual taxable income.
   1.2 Calculate his monthly tax.
   1.3 Calculate his take-home monthly pay.

2. Maistry is 56 years old who earns an annual salary of R405 000 as an accountant.
   He is married and has 3 children.
   The following deductions are made from his monthly gross salary:
   - 1% of his gross monthly salary towards UIF.
   - 5% of his gross monthly salary towards a Provident fund.
   - R10 256 towards medical aid.
   - Bond repayment of R6 500.

   Calculate the following:

   2.1 Maistry's gross monthly salary.
   2.2 His monthly tax payable.
   2.3 His monthly medical aid tax credit. (R230 each per principal member and spouse and R154 per child).
   2.4 His net take home pay per month.
   2.5 What percent of his gross monthly salary does he pay for medical aid?

Summary Assessment 4 - **Personal Income Tax**

1. Linda started a new job on 1 March 2015 at ABC Bearings with a annual starting salary of R268 000. Study her incomplete salary advise below and then answer the questions which follow:

ABC Bearings	Salary advice	Tax number: 30654321
Employer: Linda Mkhize		Date employed: 01/03/2015
Pay period: 01/03/2016 – 31/03/2016		ID number: 7704020035081
	Earnings	**Deductions**
	A	
Pension Fund		4 547,07
UIF employee contribution		**B**
Net tax payable		**C**
Total deductions		**D**
Net salary (R)		**E**

1.1 State Linda's surname.
1.2 On which date of every month is Linda getting paid.
1.3 Name the company Linda is working for.
1.4 Calculate the missing value A, her monthly salary.
1.5 What percentage of her monthly salary is her pension fund contribution?
1.6 Calculate B, her UIF contribution which is 1% of her gross monthly salary.
1.7 Calculate D, her total salary deductions.
1.8 Determine E, her net monthly salary.
1.9 Use the tax table in the previous activity to calculate her monthly tax payable.

2. Linda's total household water consumption for the month of March 2016 amounted to 28,2 kl. Use the stepped tariff rates shown in the table below to answer the questions which follow: The first example is done for you.

Kilolitre (kl) used	**Tariff per kl (excl VAT)**
6kl or less	Free
6,1 kl - 10 kl	R4,86
10,1 kl - 15 kl	R6,82
15,1 kl - 20 kl	R8,79
20, 1 kl – 30 kl	R10,64

Eg. If a person consumed 18,4 kl then calculation as follows: 6kl free, therefore 12,4 kl need to be accounted for, Second interval has 4kl × R4,86 = R19,44

Third interval has 5kl × R6,82 = R34,10; Fourth interval will be 3,4 × R8,79 = R29,87. Therefore total bill will be R19,44 + R34,10 + R29,87 = **R83,43** (excl)

Now calculate Linda's total bill payable including VAT for March 2016.

Summary of key learning: There following key concepts were covered in this unit:

- Definitions of important terminology related personal income tax.
- Calculation of UIF and Pension Fund contributions.
- Calculation of Medical aid tax credit.
- Calculation of personal income tax payable.
- Calculation of gross salary and net salary
- Calculations involving stepped tariffs.
- Performing calculations given a document to analyse.

My Notes

Use this space to write your own questions, comments or key points.

Suggested sources of additional information

Read more: Via Afrika ›› Mathematical Literacy Gr 12

Profit and Loss

When you have completed this unit, you should be able to:

- Define terminology relating to profit and loss;
- Calculate profit or loss using a given formula;
- Calculate cost price and selling price;
- The break-even concept in context.

Concepts and terminology

- Income is money that an individual or business receives in exchange for providing goods or services or through investing capital.
- Profit or loss is calculated by taking the total revenue derived from an activity and taking away the total expenses.

Profit and loss = total income – total expenses

If the resulting answer is negative, you have made a loss,or if it is positive, you have made a profit.

- Variable costs are those costs that vary depending on a company's production volume, eg. raw materials, packaging, and labour directly involved in a company's manufacturing process.
- Fixed costs are costs that do not change with an increase or decrease in the amount of goods or services produced or sold examples are rent, advertising, insurance and office supplies, which tend to remain the same regardless of production output.
- Total costs comprise fixed costs and variable costs.
- Break-even point is the point at which neither a gain nor loss is made, or neither benefit nor detriment is experienced.

Break-even point

- It is a point reached when neither a profit nor a loss is made. Simply it is a point where income and revenue is the same.

Illustrative example:

1. Sipho operates a car wash business in his neighbourhood. He charges his clients R35,00 per car for wash and vacuum. His operating costs are made up as follows:

   Salaries R1 500,00

   Water, detergent and electricity R5,00 per car washed.

   1.1 Write down a formula for the total costs.

   Solution: Total costs(C) = Fixed Costs + Variable Cost

   $C = 1\,500 + 5 \times n$ where n represents number of cars washed.

1.2 Write down a formula for the income received for each car washed.

Solution: Income (I) = 35 × *n* where *n* represents number of cars washed.

1.3 Complete the following table by finding the missing values:

Number of cars washed	0	2	4	6	8	**B**	15
Total cost C in rands	1500	1510	1520	1530	**A**	1570	1575
Total income I in rands	0	70	140	**C**	280	**D**	525

Solution: A = 1 500 + 5 × 8 = R1 540.

B = 1 570 – 1 500 ÷5 = 14

C = 35 × 6 = R210

D = b × 35 = 14 × 35 = R490.

1.4 Sipho washed 60 cars on a particular day. State whether Sipho made a profit or a loss and then calculate this profit//loss amount.

Solution: Income (60) = 35 × 60 = R2 100

Total cost (60) = 1 500 + 5 × 60 = R1 800

Therefore Profit = R2 100 – R1 800 = R300

1.5 Calculate the number of cars that need to be washed in order to break even.

Solution:

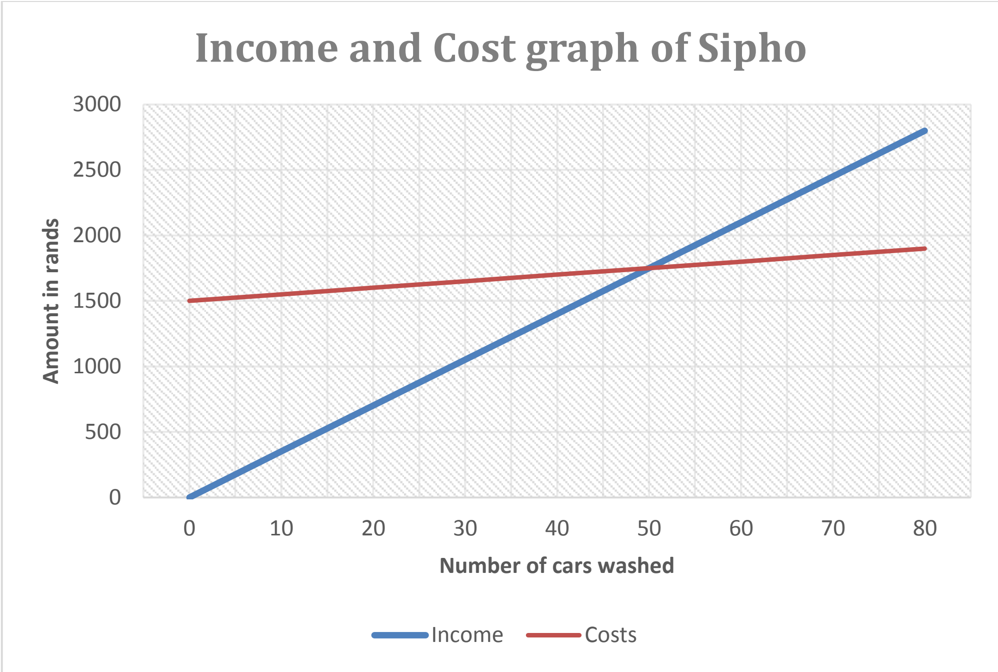
For Sipho to break even his Total Income must equal his Total costs

Therefore 35 × *n* = 1 500 + 5 × *n*

35*n* = 1 500 + 5*n*

By trial and error *n* = 50.

1.6 Draw the Income and Cost graphs on the same system of axes.
Solution:



1.7 Where in the graph will the break-even point be shown?
Solution: At the point of intersection of the two graphs. (50; 1750)

Budgeting for Economical Choices

- Renting or hiring a car often presents challenges, like whether to take unlimited kilometres and a fixed rate or rate per kilometre.
- Sometimes an option will depend on the number of kilometres travelled.

Rental Company	Standard daily rate in rands	Plus cost per km travelled in rands
Company A	200	2,10
Company B	250	1,75
Company C	100	3,00 (first 50 km free daily)

Eg.Juhi and her family rents a vehicle to travel to the South Coast which is approximately 150 km one way.

1.1 Using the above table determine the most cost effective option they can choose assuming they all offer the same class of vehicle and they return the same day.

Solution: Company A: 300 km × R2,10 + R200 = R830,00
Company B: 300 km × R1,75 + R250 = R775,00
Company C: (300 km –50 km) × R3,00 + R100 = R850,00
Therefore Company B is the most cost effective.

1.2 Write down a formula to calculate the daily rental fee for Company C.
Solution: Daily rental fee (in rand) = 100 + 3(number of daily kms – 50)

Activity 6 – **Profit and Loss**

1. Millicent lives in Durban and is planning on selling fried mealies in tubs in order to pay for her tertiary education. She has obtained the following information about her planned business:
   Expenses:
   - Monthly salaries and wages R1500
   - Cost of mealies per tub R2,50
   - Cost for serviette and spoon per tub R0,50
   - Cost of seasoning per tub R0,50
   - Gas required for frying R4,00 per tub

   Income:
   - Sale of each tub of mealies R20,00 per tub.

   1.1 Name Millicent's fixed monthly costs.
   1.2 Write down her variable costs.
   1.3 Calculate the variable costs for the frying of 30 tubs of mealies per day.
   1.4 Determine a formula for the Total Costs and the Income.
   1.5 Calculate the total costs for producing 30 tubs of mealies.
   1.6 Complete the table below by finding the missing values:

No of tubs	0	4	8	15	20	C
Income (R)	0	80	**A**	300	400	600
Total Cost (R)	1500	1530	1560	**B**	1650	1725

   1.7 Use the table above to draw TWO graphs on the same system of axes. Label each graph properly as Income and Total Cost.
   1.8 Determine how many tubs Millicent has to sell to break even.
   1.9 How much will the number of tubs stated in question 1.7 above be?
   1.10 If she wants to make a profit of R1 000, how many tubs must she sell?
   1.11 On a particular day she produced 130 tubs, but was able to sell only 120. Determine whether she has made a profit or loss. Calculate this amount.

2. Sbu needs to visit his aunt in Port Shepstone which is 120 km from his home.
   He has three options to choose from in order to rent a vehicle. Refer to the table and find the most economical option:

Options	**Rental Costs**
A	Only R5 per km
B	Basic of R200 plus R2,00 per km
C	Basic of R300 plus 100 kms free daily thereafter R2,50 per km.

Summary Assessment 5 - **Profit and Loss**

1. Refer to the cell phone rates below and then answer the questions which follow:

VCMT Cell Phone Service Provider		
	Peak time cost	Off peak costs
	Rands per minute or part thereof	
Calls to same networks	1,80	0,80
Calls to other networks	2,30	1,10
Calls to landlines	1,80	0,80

Anton's bill for the current month shows the following usage:

VCMT Cell Phone Service Provider		
	Peak time cost	Off peak costs
	Rands per minute or part thereof	
Calls to same networks	1500 seconds	13 minutes
Calls to other networks	5,5 minutes	15 minutes
Calls to landlines	3 minutes	30 minutes

Her total bill for the current month will be:

A. R2770,10 B. R115,10 C. R116,41 D. R2768,95

2. Two car rental companies charges are as follows:

Car Rental Company A charges R170 basic plus R10 for every km travelled

Car Rental Company B has no basic charge but charges R15 per km travelled

Determine after how many kms the rates of both companies will be the same:

A. 34 km B. 510 km C. 17 km D. 11,3 km

3. Moses operates a hotdog stand outside a famous soccer stadium in Durban. He sells each hot-dog at R10,00 each. His costs are made up as follows:

- Rental of Hot-dog stand with fittings R450 per day
- Rolls at R7,50 for 6
- Vienna Sausages at R50,00 for 20
- Tomato sauce at R25 for 2 litres. (20 ml required for each hotdog)

Determine how many hot-dogs he must sell in a day to cover his expenses.

Summary of key learning: There following key concepts were covered in this unit:

- Definitions of important terminology related to profit
- Calculation of Break-even point
- Drawing of graphs to find breakeven point
- Calculation of most economical car rental

My Notes

Use this space to write your own questions, comments or key points.

Suggested sources of additional information

Read more: Via Afrika ›› Mathematical Literacy Gr 12

TOPIC 2: MEASUREMENT

Introduction

Our introduction to measurement begins on the day we are born and continues for the rest of our lives. On the day we are born your length is measured, the size of your head (circumference) and you are weighed (mass). As you become more aware of yourself you learn "how old you are" (time). Eventually we learn to understand two dimensional (area) and three dimensional (volume) measurement. How about how hot or cold (temperature) it is?

Measurements surround you on all sides and they matter! They are here for a reason. We measure and keep track of time every minute of the day, just to plan how when you want to watch your favourite your television show. Likewise, you need to know when the examination is, so as to prepare for the examination. Measurements like weight, height, pulse and blood pressure tell a doctor a lot about your health. In the kitchen you cannot cook or bake without understanding measurements. Travelling on the road we need to keep within the speed limit, again with a need to understand speed, distance and time.

Using the correct measurement and checking and re-checking can benefit us in many areas of our life.

Just reflect on one day in your life and you will see how important understanding and using the correct unit of measure will make your life run smoothly.

Measurement – Content Structure

Topic Heading	Topic (with Approximate Instructional Time)
Measurement calculations.	1. Convert units of measurement (length, mass and time) within the metric system (1 hour) 2. Convert units of measurement in context (0,5 hour) 3. Convert between the metric and imperial systems (1 hour) 4. Convert between degrees Fahrenheit to degrees Celsius using a given formula (1 hour) 5. Time calculations, both digital and analogue (1 hour) 6. Distance, time and average speed calculations using a given formula
Solve problems in 2-dimensional and 3-dimensional contexts by estimation, measurement and calculation of values.	7. Lengths, distances and perimeters (circumference) 8. Areas of triangle, square, rectangle and circle including combinations of these shapes using given formulae 9. Surface areas of right prisms (cube, rectangular and triangular) and right cylinders using given formulae 10. Volumes of right prisms (cube, rectangular and triangular) and right cylinders using given formulae

Check validity of measurements in solutions against the contexts in terms of suitability and degree of accuracy.	11. Real life application using measurement

At the end of this chapter you will be able to:

- Solve 2-dimensional and 3-dimensional problems using measurement and calculations.
- Solve problems in 2-dimensional and 3-dimensional in contexts by estimation, measurement and calculation of values.
- Check validity of measurements in solutions against the contexts in terms of suitability and degree of accuracy.

Convert units of measurement (length, mass and time) within the metric system

The metric system of measuring length

The **metric system** is an internationally agreed decimal system of measurement. In the metric system, multiples and submultiples of units follow a decimal pattern.

The prefix *kilo*, for example, is used to multiply the unit by 1000, and the prefix *milli* is to indicate a one-thousandth part of the unit. Thus the *kilogram* and *kilometre* are a thousand grams and metres respectively, and a *milligram* and *millimetre* are one thousandth of a gram and metre respectively. These relations can be written symbolically as:

1 mg = 0,001 g

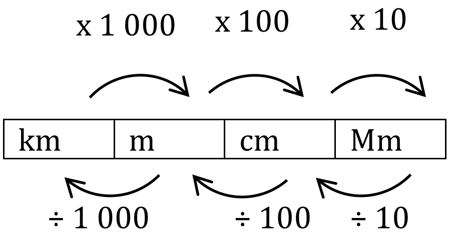
1 km = 1 000 m

Standard unit of length is the metre, but small objects are measured in centimetres (cm) and millimetres (mm), while larger distances or objects are measured in kilometres (km).

To understand length, let us look at a standard ruler. The small marks on the ruler are mm and the bigger markings are cm.

Hence: 10 mm =**1 cm** and 100 cm = **1m** ; 1 000 m **=** **1 km**; 1000 000 mm = **1 km**

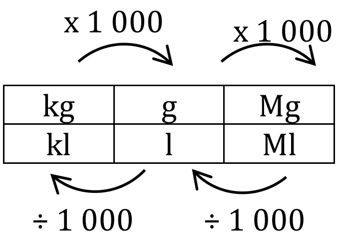
Converting from a smaller to a bigger unit we divide.
Converting from a bigger to smaller unit we multiply.
To convert from **mm to cm** (divide by 10); but cm to m (multiply by 10)



For square units, e.g. m^2, the multiplication and division factors must also be squared.

For cubic units, e.g. m^3, the multiplication and division factors must also be cubed.

Converting units of mass in the metric system takes the same format as units of length:



Illustrative examples:

Example: The length1 526 780 mm:

- In cm is 152 678 cm
- In m is 1 526,78 m
- In km is 1, 52678 km $\approx$ 1,53 km (rounded off to the nearest 2 decimal places)

Example: The distance 5,43 km

- In metres (m) is 5 430 m
- In centimetres (cm) is 543 000 cm
- In millimetres (mm) is 5 430 000 mm

Example: The area 120 m^2

- In km^2 is 0,00012 km^2(dividing by $(1000)^2$)
- In cm^2 is 1 20 000 cm^2(multiplying by $(100)^2$)
- In mm^2 is 120 000 000 mm^2(multiplying by $(1000)^2$)

Convert units of measurement in context

One of the basic skills of measurement is knowing which unit is the most appropriate unit of measure to use.
The table below shows you some common objects and the size of the object.

Object and its size	**Object and its size**
Head of a pin	2mm

Flea	2,5mm
Soccer ball	254mm
Cat (average length)	0,45m
Driveway (average length)	15,2m
Football (Soccer) field length	110 m
Airport runway	3,35km

Convert between the metric and imperial systems

In South Africa we use the metric system, but in some other countries the imperial system is used. The Metric System uses the measuring units such as meters and grams and adds prefixes like kilo, milli and centi to count orders of magnitude. The Imperial system measures in feet, inches and pounds.

In quantitative literacy, you must be able to convert within the same system and between the two systems.

The table below shows how imperial units relate to the metric units:

Distance (length)		**Mass**		**Capacity (liquid volume)**	
Imperial	**Metric**	**Imperial**	**Metric**	**Imperial**	**Metric**
1 mile (mi)	1,6 km	1 short ton (ST)	907 kg	1 gallon (gal)	3,8 litre
1 yard (yd)	0,91 m	1 pound (lb)	453 g	1 pint (pt)	473 ml
1 foot (ft)	30 cm	1 ounce (oz)	28 g	1 fluid ounce (floz)	29,6 ml
1 inch (in)	2,54 cm				

To convert between Metric measure and Imperial measure, we can use the **table below:**

Measurement Conversions

LENGTH : Imperial to Metric

1 inch(in)	2.54cm	25.4mm
6 inches	15.24cm	152.4mm
1 Foot	30.48cm	304.8mm
1 Yard	91.44cm	914.4mm
1 Foot	30.48cm	0.3048m
6 Feet	182.88cm	1.828m
12 Feet	365.76cm	3.657m
30 Feet	914.40cm	9.144m
50 Feet	1524.00cm	15.240m

LENGTH : Metric to Imperial

1 centimetre	0.39370 in
5 centimetres	1.98850 in
10 centimetres	3.93700 in
1 metre	3.2808 ft
3 metres	9.8425 ft
5 metres	16.4042 ft
10 metres	32.8083 ft
15 metres	49.2120 ft
25 metres	82.0200 ft

AREA : Imperial to Metric

1 yard^2	0.836 metres^2
2 yards^2	1.672 metres^2
5 yards^2	4.180 metres^2
10 yards^2	8.361 metres^2
25 yards^2	20.902 metres^2

AREA : Metric to Imperial

1 metre^2	1.959 yards^2
2 metre^2	2.391 yards^2
5 metre^2	5.979 yards^2
10 metre^2	11.960 yards^2
25 metre^2	29.900 yards^2

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Illustrative examples:

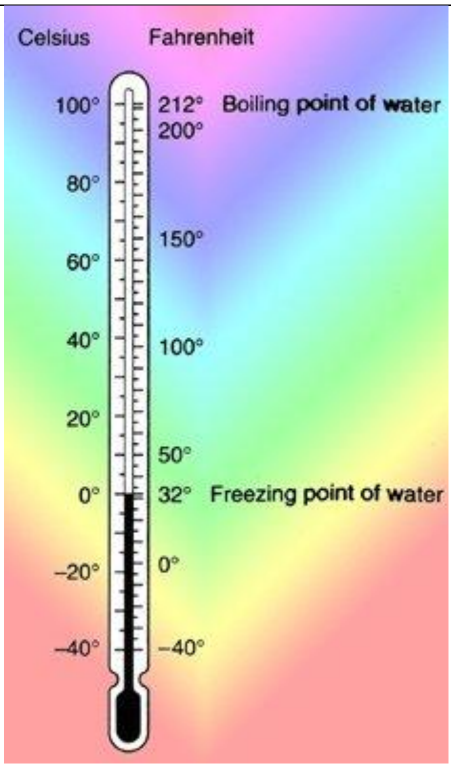
Example: Do each of the following conversions:

1. 3 feet into cm.
2. 1,5 yard into metre
3. 2,5 metres into feet
4. 15 inches into cm

Answers:

1. 3 feet = 3 × 30,48 cm = 91,44 cm
2. 1,5 yard = 1,5 × 91,44 cm = 137,16 cm = 1,3716 m
3. 2,5 metres = 2,5 × 3,28 feet = 8,3 ft
4. 15 inches = 15 ÷ 2,54 cm = 5,91 cm

Convert between degrees Fahrenheit to degrees Celsius using a given formula

	Many devices have been invented to accurately measure temperature. It all started with the establishment of a temperature scale. This scale transformed the measurement of temperature into meaningful numbers. The Fahrenheit scale sets the freezing point of water at 32° and the boiling point at 212° The Celsius scale sets the freezing temperature for water to be 0° and the boiling temperature 100°. The relationships between the different temperature scales are: $^oC = (\frac{5}{9}) \times (^oF-32)$ $^oF = (\frac{9}{5}) \times {}^oC+32$ http://coolcosmos.ipac.caltech.edu/

Illustrative examples:

Example 1: Convert each of the following temperatures to temperature in °C:

(a) 212 °F

(b) 450 °F

(c) 1 650 °F

Example 2: Convert each of the following temperatures to temperature in ^{o}F

(a) 120 ^{o}C

(b) 250 ^{o}C

(c) – 5 ^{o}C

Answers: 1(a) 212^{o}F $= \frac{5}{9} \times (212^{o} - 32^{o}) = \frac{5}{9} \times 180\ ^{o}C = 100\ ^{o}C$

1(b) 450^{o}F $= \frac{5}{9} \times (450^{o} - 32^{o}) = \frac{5}{9} \times 418\ ^{o}C = 232{,}22\ ^{o}C$

1(c) 1 650 ^{o}F $= \frac{5}{9} \times (1\,650^{o} - 32^{o}) = \frac{5}{9} \times 1\,618\ ^{o}C = 1\,078{,}67\ ^{o}C$

2(a) 120 ^{o}C$=(\frac{9}{5} \times 120^{o} + 32^{o})F = 248\ ^{o}F$

2(b) 250^{o}C $=(\frac{9}{5} \times 250^{o} + 32^{o})F = 482\ ^{o}F$

2(c) – 5^{o}C $=(\frac{9}{5} \times -5^{o} + 32^{o})F = 23\ ^{o}F$

Time calculations, both digital and analogue

For Time conversions we need to **MEMORISE**the following:

60 seconds = 1 minute

60 minutes = 1 hour

24 hours = 1 day

7 days = 1 week

365 days = 1 year

366 days = 1 leap year (every year divisible by 4, e.g. 2016, 2020 etc)

12months = 1 year

1 decade = 10 years

1 century = 100 years

Days in every month:

Jan = 31	Feb = 28 (29 in leap year)	Mar = 31
Apr = 30	May = 31	June = 30
July = 31	Aug = 31	Sep = 30
Oct = 31	Nov = 30	Dec = 31

We can calculate elapsed time by subtraction; by counting or by using a calculator

Illustrative Example	Time elapsed from 08:45 to 14:22 the same day	Time elapsed from 22:10 to 4:15 the following day
By subtraction	14h22min ⟶ 13h82min 08h45min 08h45min 05h37min	24h00min ⟶ 23h60min 22h10min 22h10min 01h50min Plus04h15min Total time elapsed → 5h 65 min → 6h 5 min
By counting	From 08:45 to 09:00 → 0 h 15 min From 09:00 to 14:22 →5 h 22 min Total time elapsed → 05 h 37 min	From 22:10 to 23:00 → 0 h 50 min From 23:00 to 24:00 → 1 h 00 min From 24:00 to 04:15 → 4 h 15 min Total time elapsed → 5 h 65 min → 6 h 5 min
With a calculator	Use the °' " key. Key in: 14 °' " 22 °' " minus 8°' " 45°' "	

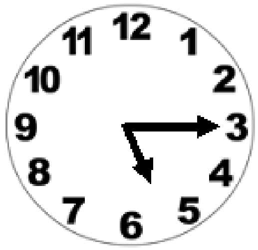
Analogue time uses the face of the clock to indicate a time.

Below is an illustration of the relationship between an analogue clock and the corresponding digital time.

But the analogue clock only shows a 12-hour day. Therefore each for the times above could also be:

21:25 **22:15** **23:40**

When reading the analog clock, we need to know the meaning of some common words and phrases like:**o'clock**; Past; ; **quarter pass**; **half past**; **to** ; **quarter to**

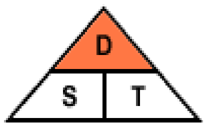
o'clock	Past	quarter pass
 12 o'clock	 22 minutes past 8	 22 minutes past 8
half past  Half past ten	to  Ten **to** two	*Example: quarter to* Illustrate quarter to three below.

Distance, time and average speed calculations using a given formula

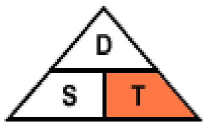
In South Africa we have different speed limit for different areas. In residential areas the speed limit is 60 km per hour. This means that a car is allowed to travel a maximum of 60 km in an hour.

If an object is moving at varying speeds, then the relationship between speed, distance and time can only be used to calculate the average speed.

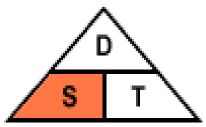
FORMULA



Distance = speed × time



Time = $\frac{\text{distance}}{\text{speed}}$



Speed = $\frac{\text{distance}}{\text{time}}$

Illustrative examples:

Example 1: Calculate the speed (in km per hour) of a car which travelled 50 km in 36 minutes

Answer: Time = $\frac{36}{60}$ h = 0,6 h

$$\text{Speed} = \frac{\text{distance}}{\text{time}} = \frac{50}{0,6}\text{ km per hour} = 83{,}33\text{km per hour}$$

Example 2: A train travels at an average speed of 45 km per hour for 3 hours 30 min. Calculate the distance travelled by the train.

Answer: Time = 3hours 30 min = 3,5 h

Distance = speed × time

= 45 km per hour × 3,5 hour

= 157,5 km

Activity 7

Have you understood your reading. This activity is based on conversions and time calculations.

Nolwazi is baking Vanilla Cup-Cakes or the school fete. She uses the following ingredients.

Ingredients:*(makes 16 cakes)*

1 ½ cups flour
¾ cup sugar
1 Tsp baking powder
½ tsp salt
½ cup milk
½ cup oil
2 eggs
1 tsp vanilla essence

Conversion Table

5 mℓ = 1 teaspoon (tsp)

1 tablespoon (Tsp) = 12,5 mℓ

1 cup = 250 mℓ

Bake at 180 °C

Study the above information and answer the questions that follow.

1.1 Write down the ratio of salt : baking powder in the form **1 : ...**

1.2 Convert the amount of oil used to tablespoons.

1.3 Calculate how many mℓ of flour is need for 80 cup cakes

1.4 Calculate the total volume of liquid ingredients added to the cake mixture.

1.5 Determine how many teaspoons of vanilla essence Nolwazi will need to make 48 cup cakes.

1.6 The cup-cakes take 15 minutes to mix, are baked for 20 minutes and are cooled for 30 min and then iced. Nolwazi makes the icing while the cakes are baking. Nolwazi takes 10 minutes to ice the cakes.

Nolwazi starts mixing the cakes at 09:15. At what time will she finish icing the cakes? (write the time in digital form and analogue form).

1.7 Convert the time that the cakes must bake at to °F.

Solution for Activity 1:

1.1 salt : baking powder = ½ tsp : 1 Tsp
= 2,5 : 12,5 = 1 : 5

1.2 ½ cup= 125mℓ = 10 Tsp

1.3 1 ½ cups = 375 mℓ for 16 cup cakes

For 80 cup cakes we need = $\frac{80}{16}$ ×375 mℓ = 1 875 mℓ

1.4 liquid ingredients are milk, oil and vanilla essence
Volume = ½ cup +½ cup + 5 mℓ = 255mℓ

1.5 48 cup cakes= 3×16

Amount of vanilla essence = 3 ×5 mℓ = 15mℓ

1.6Total time taken = 15 min + 20 min + 30 min + 10min = 75 min = 1 h 15 min

She will end at 09:15 + 1 h 15 min = 10:30 or half past ten.

1.7 180°C =($\frac{9}{5}$ × 180° + 32°)F = 356 °F = 360°F

Activity 8

Have you understood your reading. This activity is based on conversions and time calculations.

Marie would like to use an old recipe of her grandmother to bake coconut tarts for the school bazaar.

The coconut tarts have to be baked at 350 °F for 20 minutes.

Coconut Tarts

(Makes 3 dozen)

INGREDIENTS
1 lb self-raising flour
9 oz margarine
$\frac{3}{4}$ cup of sugar
4 eggs
10 oz coconut
$\frac{1}{2}$lb apricot jam
1 teaspoon vanilla essence

Study the above information and answer the questions that follow.

1.1 Convert $\frac{1}{2}$lb to grams. (1 lb = 450 g)

1.2 Convert 9oz to grams. (1 oz = 30 g)

1.3 One cup of sugar is equal to 250 mℓ. How many mℓ of sugar are needed for this r recipe?

1.4 Convert 350 °F (degrees Fahrenheit) to °C (degrees Celsius) using the following formula: **Temperature in °C = (Temperature in °F – 32°)** $\times \frac{5}{9}$

Round off the answer to the nearest 10°.

1.5 How many eggs does Marie need to bake 72 tarts?

(Answers at the back of the guide)

Summary of key learning: There following key concepts were covered in this unit:

- Units of measure in the metric system – length, mass and time
- Conversions– metric unit to imperial units and imperial units to metric units
- Temperature – Conversions between Fahrenheit and Celsius (using a given formula)
- Time calculation – digital and analogue
- Distance, speed and time calculations – using a given formula

My Notes

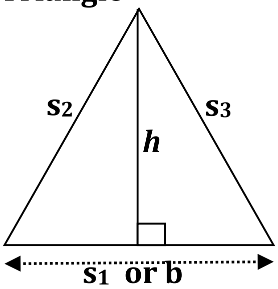
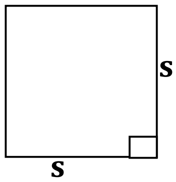
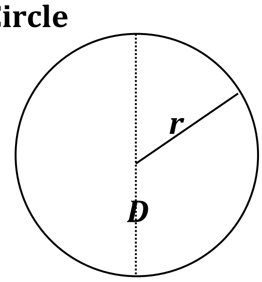
Use this space to write your own questions, comments or key points.

Suggested sources of additional information

Read more: Via Afrika ›› Mathematical Literacy Gr 12

Lengths, distances and perimeters (circumference)

A **perimeter** is a path that surrounds a two-dimensional shape. (*https://en.wikipedia.org/)*

SHAPE	VARIABLES/Definitions	PERIMETER (P)
Triangle  A three-sided figure	**Three sides – s_1; s_2; s_3 ..** **Base (b)** the length of the side of a triangle from which the perpendicular height is drawn Perpendicular height (h) the perpendicular drawn from the base to the apex of the triangle.	**$P = s_1 + s_2 + s_3$**
Square  A four-sided figure with all four sides equal in length and all four angles 90^{o}.	**Side (s)– measure of one side**	**$P = 4 \times s$** **OR** **$P = s + s + s + s$**
Rectangle  A four sided figure with all angles 90^{o} and opposite sides equal.	**Length (ℓ)** – measure of the longer side. **Breadth (b)** – measure of the shorter side. (In calculations the length and breadth can be interchanged)	**$P = 2 \times (\ell + b)$** **OR** **$P = \ell + b + \ell + b$**
Circle  A set of points equidistant from a centre.	**Circumference** (C) the outer edge of a circle **Radius** (r) – measurement from the centre of the circle to the circumference **Diameter** (D) – the length from one end of the circumference, through the centre to the other end.	**$P = 2 \times \pi \times r^2$** **where$\pi$ = 3,14** **$P = \pi \times D$**

Areas of triangle, square, rectangle and circle including combinations of these shapes using given formulae

The area is the inside shape or space measured in square units on a flat surface. In rectangles and in squares, a simple calculation of length multiplied by breadth will give the number of square units. The square units could be centimetres, metres or whatever the requested unit of measure asks for. The following formula can be used to calculate area of basic shapes (see the table on perimeter).

Area of a triangle $= \frac{1}{2} b \times \perp h$

Area of a square $= s \times s$

Area of a rectangle $= \ell \times b$

Area of a circle $= \frac{1}{2} \times \pi \times r^2$

Illustrative examples

Example 1: Calculate the perimeter and area of a triangle with side length equal to 3 m, 5 m and 7 m and a perpendicular height of 1,86 m from base = 7 m

Answer: $P = s_1 + s_2 + s_3 = 3\text{ m} + 5\text{ m} + 7\text{ m} = 25\text{ m}$

$A = \frac{1}{2} b \times \perp h = \frac{1}{2} \times 1{,}86\text{ m} \times 7\text{ m} = 6{,}51\text{ m}^2$

Example2: Calculate the perimeter and area of a square with side length equal to 8 m.

Answer: $P = 4 \times 8\text{ m} = 32\text{ m}$

$A = 8\text{ m} \times 8\text{m} = 64\text{ m}^2$

Example 3: Calculate the perimeter and area of a rectangle with length of 100 m, and a breadth of 45 m

Answer: $P = 2 \times (\ell + b) = 2 \times (100\text{ m} + 45\text{ m}) = 290\text{ m}$

$A = \ell \times b = 100\text{ m} \times 45\text{ m} = 4\,500\text{ m}^2$

Example 4: Calculate the circumference and area of a circle with radius of 4,5 m.

Answer: Circumference $= P = 2 \times \pi \times r = 2 \times 3{,}14 \times 4{,}5\text{ m} = 28{,}26\text{ m}$

$A = \frac{1}{2} \times \pi \times r^2 = \frac{1}{2} \times 3{,}142 \times (4{,}5\text{ m})^2 = 31{,}81275\text{ m}^2 \cong 31{,}81\text{ m}^2$

Surface areas of right prisms (cube, rectangular and triangular) and right cylinders using given formulae

Surface area is the sum of the area of each part that makes up a three-dimensional shape known as a right prism. Examples of right prisms are square-based and rectangular based prisms, triangular prisms and cylindrical prism. A real life example of each of the prisms is:

Cube – a dice

Rectangular prism – opened or closed boxes

Triangular prism – triangular-shaped box.

Cylinder – a coffee/ tin.

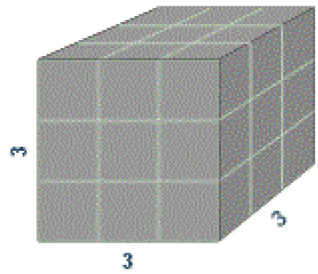
The formula for surface area is given in the table under volume.

Volumes of right prisms (cube, rectangular and triangular) and right cylinders using given formulae

Volume is a measure of how much space an object takes up (the capacity of a container).

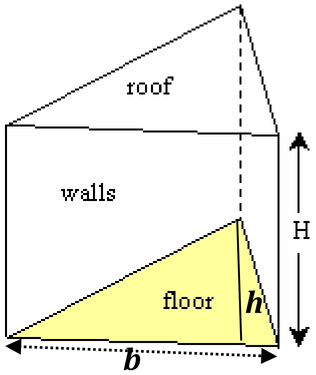
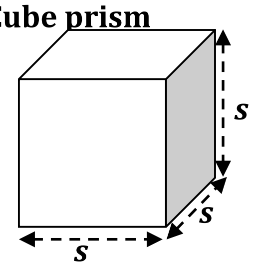
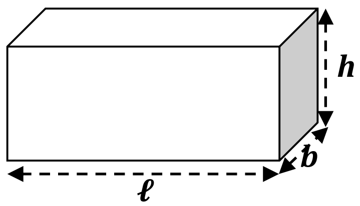
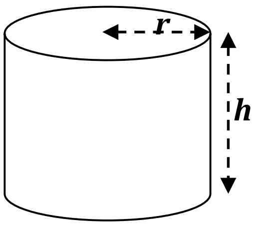
Example:

In the **cube** on the right, the volume is 3×3×3 or 27. If you count the small cubes you will find there are 27 of them. The volume of solid objects is measured in cubic units. For example in the cube alongside, if the sides are 3 meters long,then the volume is 27 cubic meters (27 m^3).



http://www.mathopenref.com/volume.html

The table below shows the formula for the surface area and volume of some right prisms.

PRISM	SURFACE AREA (SA)	VOLUME (V)
Triangular prism  	**$SA = b \times h + (s_1+s_2+s_3) \times H$**	**$V = \frac{1}{2} b \times \perp h \times H$**
Cube prism  	**$SA = 6 \times s \times s$**	**$V = s^3$** OR **$V = s \times s \times s$**
Rectangular prism  	**$SA = 2\ell \times b + 2\ell \times h + 2b \times h$** OR **$SA = 2(\ell \times b + \ell \times h + b \times h)$**	**$V = \ell \times b \times h$**
Cylinder  	**$SA = 2\pi \times r^2 + 2\pi \times r \times h$** OR **$SA = 2\pi r\,(r + h)$**	**$V = \pi \times r^2 \times h$**

The most important thing to remember when calculating perimeter, area, surface area and volume is that: ***All the dimensions must be in the same units.***

In the examination questions may also be set on combinations of shapes.

Illustrative examples

Example: Calculate the surface area and volume of each of the following prisms:

(a) A triangular prism with a base = 7 m , h = 1,86 m, the other two sides of length 3 m and 5 m, and with H = 6 m.

(b) a square with side length equal to 8 m and a height of 3,5 m

(c) a rectangle with length of 1 m, and a breadth of 45 cm and a height of 70 cm.

(d) a cylinder with a radius of 45 cm and a height of 30 cm

Answers:

(a) SA = $b \times h$ + (s_1+s_2+s_3)× H

= 7 m × 1,86 m + (7 m + 3 m + 5 m) × 6 m = 103,02 m^2

V = $\frac{1}{2} b \times \perp h \times H = \frac{1}{2} \times 7\text{ m} \times 1{,}86\text{ m} \times 6\text{ m} = 39{,}06\text{ m}^3$

(b) SA = 6 × s × s

= 6×8 m×8 m = 384m^2

V = $s \times s \times$ h= 8m × 8 m × 3,5 m = 224 m^3

(c) First convert all the measurements to the same unit. Length = 1 m = 100 cm

SA = $2\ell \times b + 2\ell \times h + 2b \times h$

= 2× 100 cm × 45cm + 2 × 100 cm ×70cm +2 × 45 cm × 70cm

= 29 300 cm^2 = 2,93 m^2

V = $\ell \times b \times$ h=100 cm × 45cm×70cm = 315 000 cm^3 = 0,315 m^3

(d) SA= 2π r (r+ h)

= 2× 3,14×45 cm(45 cm + 30cm) = 21 195 cm^2

V = π × r^2× h= 2× 3,14× (45 cm)^2 ×30 cm = 381 510 cm^3

Check validity of measurements in solutions against the contexts in terms of suitability and degree of accuracy.

Always reflect on the answer and check that the unit of the answer is in line with the problem.

Applications of measurement in real-life problem solving

Real life calculations based on measurement are an integral part of our life. If we want to paint of house or tile our floors we need to know how much it would cost, the amount of material we need to buy and all of this requirements some form of measurement.

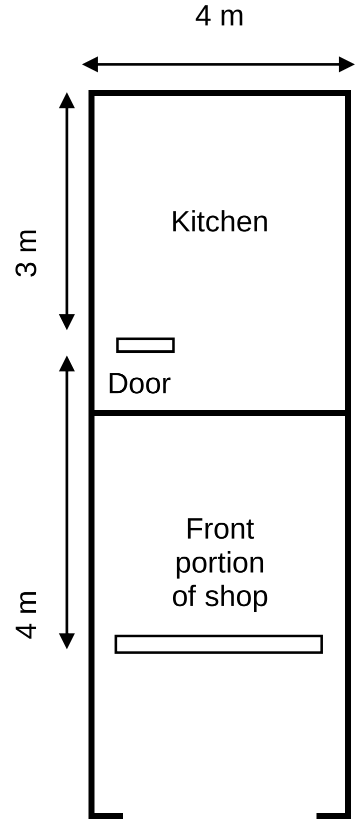
The activities below are examples of measurement in real life.

Worked Example 1

1.1 Mrs. Zwane has a diploma in small business management. She is hired to run the school tuck shop in her area. The tuck-shop has two rooms: a kitchen behind the front portion of the shop where learners will be served. The measurements of the two rooms are shown in the diagram alongside.

Mrs. Zwane decided to paint only the inner walls of the front portion of her shop and not the kitchen walls. The walls of the shop are all 3 m high. The surface area of the front door is 9 m^2.

The measurements of the door leading to the kitchen is 0,8 m by 2,04 m.



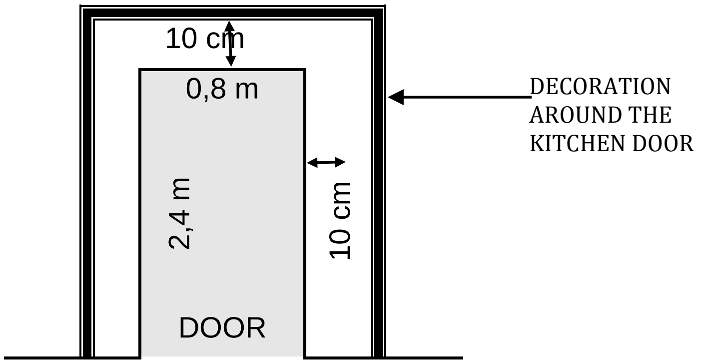
Front Door

1.1.1 Calculate the inner surface of the walls that must be painted.

You may use the formula: **Area of a rectangle = length × breadth**

1.1.2 One 5 litre tin of paint covers 30 m^2 of surface area. Mrs Zwane would like to give the walls of the front of the shop TWO coats of paint.Determine how many 5 ℓ tins of paintshe needs to buy.

1.2 Mrs. Zwane wants an edge decoration around the frame of the door leading to the kitchen. This decoration must be 10 cm from the door frame.



1.2.1 Determine the minimum length of decoration edging she must buy.

1.2.2 If the edging cost R54,65, calculate how much Mrs Zwane will pay for the edging.

Answers:

1.1.1 Area of one wall $= 4\text{ m} \times 3\text{ m}$

$= 12\text{ m}^2$

Area of four walls $= 4 \times 12\text{ m}^2$

$= 48\text{ m}^2$

Area of door $= 0{,}8\text{ m} \times 2{,}04\text{ m}$

$= 1{,}632\text{ m}^2$

Area to be painted $= 48\text{ m}^2 - (9\text{m}^2 + 1{,}632\text{ m}^2)$

$= 37{,}368\text{ m}^2$

1.1.2 $\frac{37{,}368}{30} = 1{,}2456 = 2$

1.2.1 Perimeter of door frame $= 2{,}4\text{ m} + 0{,}8\text{ m} + 2{,}4\text{ m}$

$= 5{,}6\text{ m}$

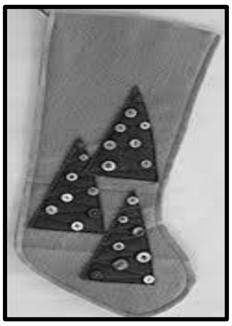
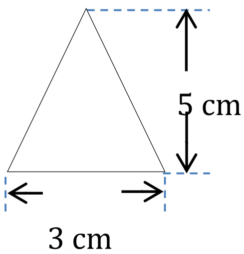
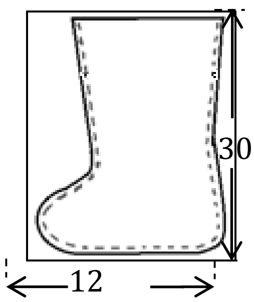
1.2.2 Length of decoration edge $= 5{,}6\text{ m} + 2 \times 0{,}1\text{ m}$

$= 5{,}8\text{ m}$

Cost of edging $= 5{,}8 \times$ R54,65= R316,97

Worked Example 2

2.1 Petru makes craft products that she sells at a craft market. She makes gift stockings (gift bags shaped like a sock)decorated with triangular shapes, as shown below. She sews three triangles onto each side of the stocking.

Photograph of a gift stocking	**Dimensions of the triangular pieces of fabric**	**Dimensions of a rectangular piece of fabric required for one side of a stocking**
		

[www.marthastewart.com]

2.1.1 The area of one side of a stocking (without the triangular pieces) is 355,25 cm².

Calculate the area of the fabric that is left over if Petru cuts ONE complete stocking from two rectangular pieces of fabric.

You may use the following formula: **Area of rectangle = l × w**

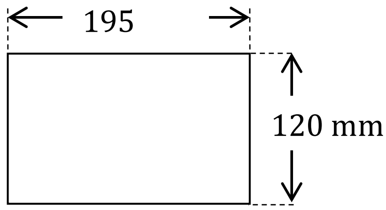
2.1.2 Calculate the total area of the triangular shapes needed to decorate ONE stocking.

You may use the following formula: **Area of triangle =** $\frac{1}{2}$ **× b × H**

2.1.3 It takes Petru 18 minutes to cut, decorate and hand-stitch one stocking.

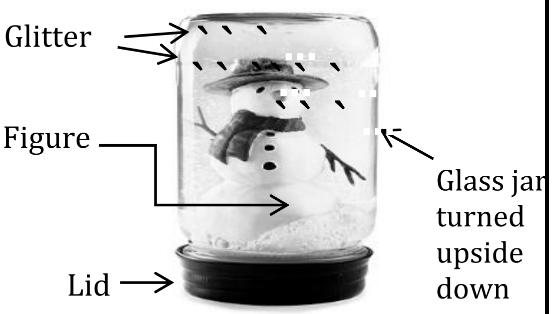
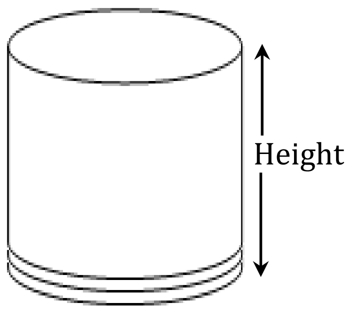
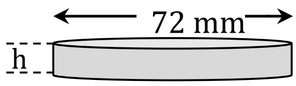
Determine at what time she will finish making NINE stockings if she starts at 08:25.

2.2 Petru buys rectangular boxes with reels of thread for stitching the stockings. The radius of a cylindrical reel is 11,5 mm.

Reels of thread	**A box with reels of thread**	**Top view showing dimensions of the box**
		

Determine the maximum number of reels of thread that will fit exactly into a rectangular box that is 120 mm wide and 195 mm long. Show ALL calculations.

2.3 Below is a photograph of aglassornamentthatPetru makes using250 mℓcylindrical glass jars.

Picture of a glass ornament	**250 mℓ cylindrical glass jar**
 [www.marthastewart.com]	 **Lid of the glass jar** 

The inside radius of the glass jar is 3,25 cm.

The outside diameter of the lid of the jar is 72 mm and the height (h) is 9 mm.

The exterior surface of the lid is painted red.

The jar is filled 75% with water and a pinch of glitter is added to the water. A dash of glycerine is also added to keep the glitter from sinking too quickly.

The figure is glued to the inside of the lid before the lid is placed on the jar. The jar is then turned upside down.

2.3.1 Calculate (to the nearest cm²) the exterior surface area of the lid that needs to be painted.

You may use the following formula:

Painted exterior surface area of lid= $\pi r(r+2h)$

Where π **= 3,142**; **r** is the radius and **h** is the height of the lid.

2.3.2 Determine (to the nearest cm) the height of the water in the jar before the lid is placed on the jar.

You may use the following formula:

Height of the water in the jar = $\dfrac{\text{volume of the water (in cm}^3)}{\pi \times (\text{radius})^2}$

1 cm³ = 1mℓ

2.3.3 Use the conversions below to answer the following questions.

1 pinch	=	$\frac{1}{16}$ teaspoon
2 pinches	=	1 dash
1 teaspoon	=	5 mℓ

Determine what fraction of a teaspoon equals ONE dash.

Answers:

2.1.1 Total area of a rectangular piece = 30 cm × 12 cm = 360 cm²

Off-cut piece = 360 cm² – 355,25 cm² = 4,75 cm²

Total off-cut piece for both sides = 4,75 cm² × 2 = 9,5 cm²

2.1.2 Area of a triangle = $\frac{1}{2}$ × 3 cm × 5 cm

= 7,5 cm²

Area of 6 triangles = 7,5 cm²× 6

= 45 cm²

2.1.3 Time taken = 9× 18 minutes

= 162 minutes

= 2h 42 min

Finishing time = 08:25 + 2h42

= 11:07

2.2 Number of reels along length = 195 mm ÷ 23mm

= 8,4782…

≈ 8

Number of reels along breadth = 120 mm ÷ 23mm

= 5,2173…

$\approx$ 5

Total = 5 × 8 = 40

2.3.1 Painted surface area of the lid= 3,142 × 3,6 cm(3,6 + 2 × 0,9) cm

$\approx$ 61 cm^2

2.3.2 Capacity = 75% × 250 mℓ

= 187,5 mℓ

Volume = 187,5 cm^3

Height of the water in the jar

$$= \frac{\text{Volume of the water (in cm}^3)}{\pi \times \text{radius}^2}$$

$$= \frac{187{,}5\,\text{cm}^3}{3{,}142 \times (3{,}25\,\text{cm})^2}$$

$$= \frac{187{,}5\,\text{cm}^3}{33{,}187375\,\text{cm}^3}$$

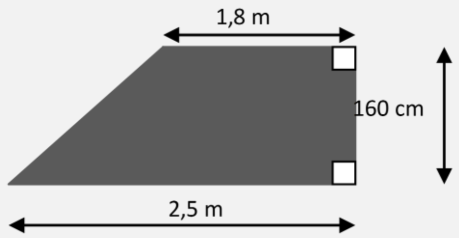
= 5,6497… cm

$\approx$ 6 cm

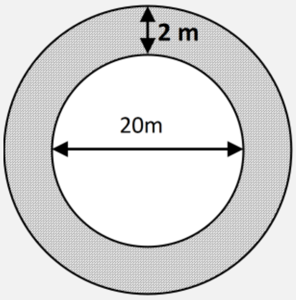
2.3.3 $2 \times \frac{1}{16} = \frac{1}{8}$

Activity 9

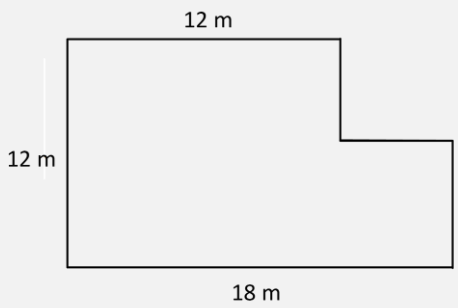
1. Calculate the circumference and the area of a circular flowerbed with diameter 2,5 m.
2. Calculate the perimeter and the area of the trapezium below.



3. A circular garden with a diameter of 20 m is surrounded by a 2 m gravel path as indicated below. Calculate the area of the path.



4. Determine the perimeter and the area of the garden shown in the diagram.

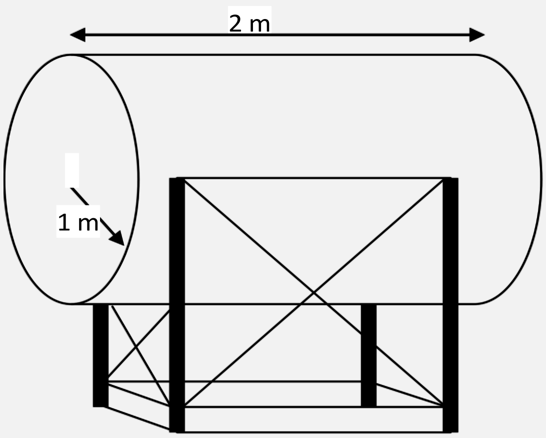


5. Calculate the volume and surface area of a tissue box with dimensions 22 cm long, 110 mm wide and 80 mm high.
6. Calculate the surface areas (in cm^2) of the rectangular seat cushions if the dimensions are: length 65 cm, width 60 cm and thickness (depth) 50 mm.

Summative assessment 6

1.

As a result of load shedding, Wayne, a chicken farmer, goes back to using a generator to provide dependable power for his chicken sheds and his farmhouse.



He buys a second-hand diesel tank with a radius of 1 m and a length of 2 m to store the fuel for the generator.

He decides to paint both the outside surface area of the tank and the stand on which it rests. The surface area of the stand is 1 m^2. It takes 1 ℓ paint to paint 3 m^2 of the surface area.

1.1 Calculate the surface area (SA) of the tank in m^2. Use the formula: **SA = 2 π *r*² + 2 π *r h***, where ***r*** = radius, ***h*** = height and use **π =** 3,14

1.2 Calculate the quantity of paint (in litres) needed to paint both the outside of the tank and the stand. Round off your answer to the nearest litre.

1.3 If a 1 ℓ tin of paint costs R23,63 and a 5 ℓ tin of paint costs R113,15, calculate the most economical way to purchase the amount of paint needed in QUESTION 1.1.2.

1.4 Calculate the capacity (volume) of the diesel tank in litres where 1 m^3 = 1 000 ℓ.

Use the formula: **V = *π r*²*h*,** where ***r*** = radius, ***h*** = height and use **π** = 3,14

2. A foreign tourist visits Durban and he buys traditional Zulu-styled clothing outfits. He needs to buy a suitcase for these extra goods. The luggage shop has two suitcase to choose from, with the following dimensions:
NOTE: The clothing outfits have a volume of 9 655 cm^3 and cam be folded and roll to fit in a tight space without gaps.

Dimensions	SUITCASE A	SUITCASE B
Length of the base	34,5 cm	61 cm
Width of the base	22,5 cm	30 cm
Height of the base	12,5 cm	12,5 cm

(a) Use the formula: **Volume = Length × Width × Height**, to calculate the volume of suitcase A and suitcase B.
(b) Which of the two do you think he should buy? Give a reason.

Summary of key learning: There following key concepts were covered in this unit:

- Length, distance and perimeter – measuring the outer edge of a object
- Area– triangle, square, rectangle and circle including combinations of these shapes using given formulae
- Surface areas – right prisms (cube, rectangular and triangular) and right cylinders using given formulae
- Volumes of right prisms – (cube, rectangular and triangular) and right cylinders using given formulae

My Notes

Use this space to write your own questions, comments or key points.

Suggested sources of additional information

Read more: Via Afrika ›› Mathematical Literacy Gr 12

TOPIC3: MAPS, PLANS AND MODELS

Introduction

The basic skills required in this section are understanding and working with ratio and proportion and a sense of direction. Reading maps and plans is an important tool to make sense of our world.

A map contains useful information and helps you to navigate where you want to go. Maps use symbols like lines and different colours to show features such as rivers, roads, cities or mountains. Symbols help us to visualise what things on the ground actually look like. Maps also help us to know distances so that we know how far away one thing is from another. We need to be able to estimate distances on maps because all maps show the earth or regions within it as a much smaller size than their real size. To do this we need to be able to read the scale on a map.

A plan is like a map. A house plans is a scaled diagram of the dimensions of a house.

In this section we will learn how to use maps, seating plans, layout plans and models to interpret and analyse spatial relations. The emphasis of this Exit-Level Outcome is on the development of spatial understanding and skills relating to non-contrived real-life contexts. A variety of applications are available in Design, Art, Geography and other fields to develop these spatial skills.

Maps, Plans and Models – Content Structure

Topic Heading	**Topic (with Approximate Instructional Time)**
Represent and identify views of scale drawings of plans, maps and models and calculate measurements.	1. Identify different features shown on maps and plans (1 hour) 2. Interpret different types of scales(1 hour) 3. Determine scales for maps, plans and models(1 hour) 4. Use and interpret scale drawings of plans, maps and models to identify views, .estimate and calculate values according to scale(2 hour) 5. Calculate actual length and distance using a given scale (*bar scale or number .scale*(1 hour)
Use maps, seating plans and layout plans to interpret and analyse spatial relations	6. Determine locations and grid references (1 hour) 7. Plan trips (2 hour) 8. Describe routes between two different locations (1 hours) 9. Describe relative positions(1 hour)

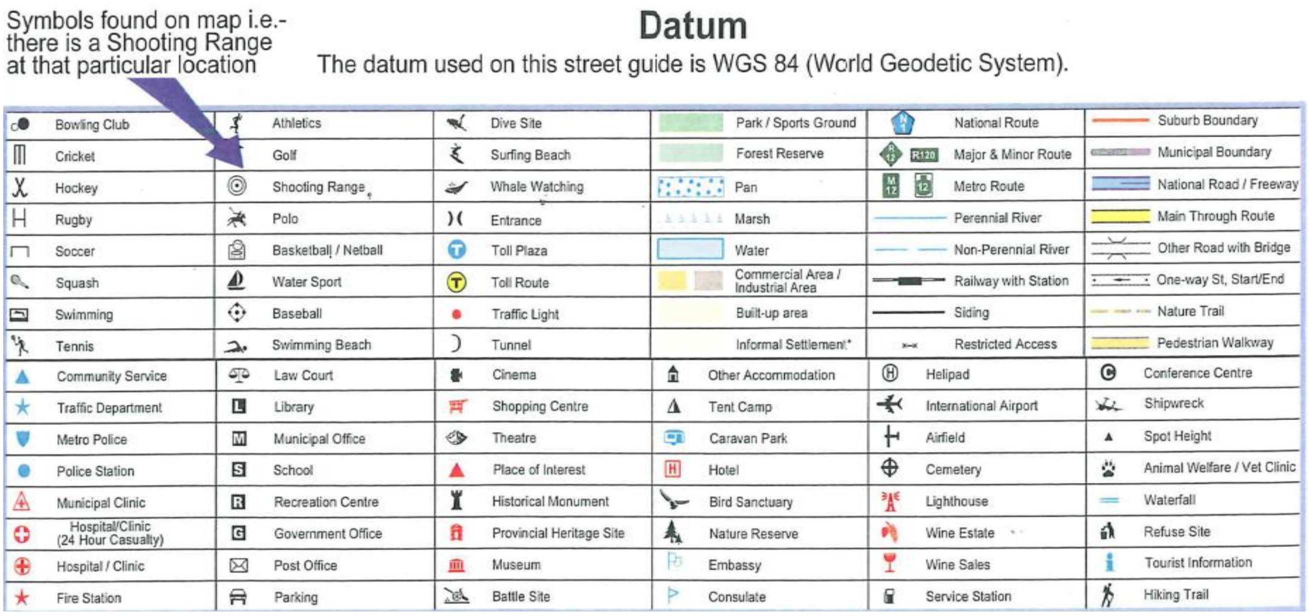
Identify different features shown on maps and plans

The various features shown on the map are represented by conventional signs or symbols. For example, colours such as blue can be used to indicate water such as rivers and ponds. Symbols such as N and R are used to classify roads. These signs are usually explained in the margin of the map, or on a separately published characteristic sheet or key.

Example

The Map Studio map book of Gauteng provides the following table explaining the various symbols used on the maps included in the map book.

(Source: *Map Studio Street Guide: Gauteng – The Complete Map Book*, 3rd Edition, p.12)



Bowling Club	Athletics	Dive Site	Park / Sports Ground	National Route	Suburb Boundary
Cricket	Golf	Surfing Beach	Forest Reserve	Major & Minor Route	Municipal Boundary
Hockey	Shooting Range	Whale Watching	Pan	Metro Route	National Road / Freeway
Rugby	Polo	Entrance	Marsh	Perennial River	Main Through Route
Soccer	Basketball / Netball	Toll Plaza	Water	Non-Perennial River	Other Road with Bridge
Squash	Water Sport	Toll Route	Commercial Area / Industrial Area	Railway with Station	One-way St, Start/End
Swimming	Baseball	Traffic Light	Built-up area	Siding	Nature Trail
Tennis	Swimming Beach	Tunnel	Informal Settlement*	Restricted Access	Pedestrian Walkway
Community Service	Law Court	Cinema	Other Accommodation	Helipad	Conference Centre
Traffic Department	Library	Shopping Centre	Tent Camp	International Airport	Shipwreck
Metro Police	Municipal Office	Theatre	Caravan Park	Airfield	Spot Height
Police Station	School	Place of Interest	Hotel	Cemetery	Animal Welfare / Vet Clinic
Municipal Clinic	Recreation Centre	Historical Monument	Bird Sanctuary	Lighthouse	Waterfall
Hospital/Clinic (24 Hour Casualty)	Government Office	Provincial Heritage Site	Nature Reserve	Wine Estate	Refuse Site
Hospital / Clinic	Post Office	Museum	Embassy	Wine Sales	Tourist Information
Fire Station	Parking	Battle Site	Consulate	Service Station	Hiking Trail

A key is always given on a map to explain each symbol used in the map.

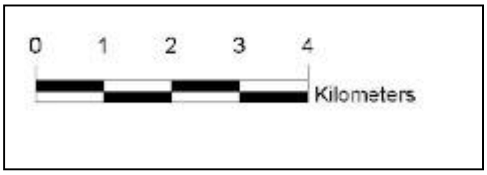
Interpret different types of scales

A map or plan cannot be of the same size as the area it represents. So, the measurements are **scaled down** to make the map/plan of a size that can be conveniently used by users such as motorists or builders. A scale drawing of a building has the same shape as the real building that it represents but a different size. Builders use scaled drawings to make buildings and bridges.
A map scale is a ratio of the distance on the map to the distance on the ground.
There are three different types of scale used in maps and plans.

Verbal scale uses units– example 1 cm on the map represents 5 km in real life.

Number scale or **ratio** also known as a representative fraction – example 1 : 200 000, can be read as 1 cm on the map or plan represents 200 000 cm in real life.
Therefore 1 cm on the map represents 2 000 m or 2 km in real life.

Bar scale (graphic) uses a picture.



The length of the full picture represents 4 km in real life.

In this scale you first have to measure the length of 0 – 1 in mm to know the correct number of mm length which on the scale drawing, represents 1 km in real life.

Determine scales for maps, plans and models

To calculate the scale used to draw a map or map you need both the measurement on the map or plan and then the real life (true) measurement. Next, you divide the true distance by the measured map distance. The simplified ratio is the scale used to draw the map.

Example: The distance between your home and your workplace is 2 km in real life. The distance on a map of the area between your home and your workplace is 5 cm. Determine the scale used to draw the map.

Solution 5 cm : 2 km
5 cm : 2 000 m
5 cm : 200 000 cm (convert to the same unit)
5 : 200 000 (drop the unit)
1 : 40 000 (simplify the ratio)

Example: Simplify the scale 4mm : 1 m.

Solution: 4 mm : 1 m = 4 mm : 1 000 mm
= 4 : 1 000
= 1 : 250

Example: Simplify the scale 4cm : 2 km.

Solution 4 cm : 2 km = 4 cm : 2 000 m
= 4 cm : 200 000 cm
= 4 : 200 000
= 1 : 50 000

Use and interpret scale drawings of plans, maps and models to identify views, estimate and calculate values according to scale

All scale drawings must be miniature replicas of the object (room; house; or picture) that they represent. Thus we can say that:

- A **scale drawing** of an object has the same shape as the object, but a different size.
- The **scale** of the drawing is the ratio length on the drawing : length on the actual object.
- A scale can be written as the ratio of two lengths, or as the ratio of two numbers. For example: scale = 1 cm : 5 m or scale = 1 : 500
- Matching angles are equal, and the ratio of matching lengths equals the scale.

Builders use scale drawing to have a picture of the building or room that needs to be built. Scale drawings also help to

Calculations using scale

Calculate actual length and distance using a given scale (*bar scale or number scale*)

If the scale is 1 : x, then multiply the map distance by x to calculate the actual distance.
To use a bar scale, first measure the bar and convert I to a ratio scale.

Example: A particular map shows a scale of 1 : 50 000. What is the actual distance if the map distance is 6 cm?

Solution: Scale of 1 : 50 000This means: 1 cm : 50 000 cm

Map distance = 6 cm. Let the actual distance be *a* cm.

6 : a = 1 : 50 000 (units in cm)

$$\frac{6}{a} = \frac{1}{50\ 000}$$

$$\frac{a}{6} = \frac{50\ 000}{1} \quad \text{(invert the fraction)}$$

$$\frac{a}{6} \times 6 = \frac{50\ 000}{1} \times 6 \quad \text{(multiply both sides by 6)}$$

a = 300 000 cm

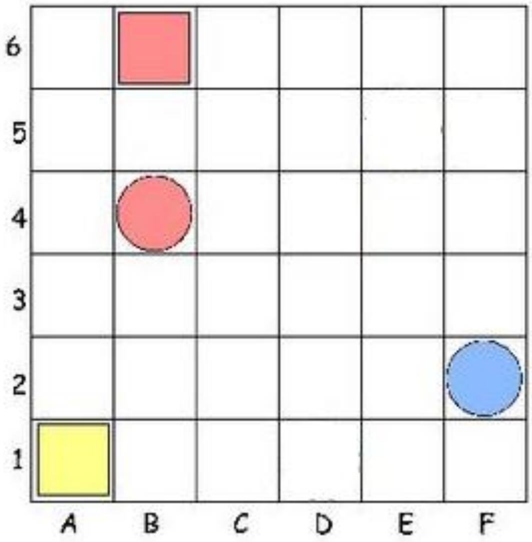
The actual distance is 300 000 cm = 3 000 m = 3 km

Determine locations and grid references.

Grid references define locations on maps using Cartesian coordinates. Grid lines on maps define the coordinate system, and are numbered to provide a unique reference to features. Grids may be arbitrary, or can be based on specific distances, for example some maps use a one-kilometre square grid spacing.

A grid reference locates a unique square region on the map. The precision of location varies, for example a simple town plan may use a simple grid system with single letters for Eastings (horizontal direction)and single numbers for Northings (vertical direction). A grid reference in this system, such as 'H3', locates a particular square rather than a single point, that is the square labeled “H” in the horizontal direction and “3” in the vertical direction.

Example: Sate the grid references for the square and circles indicated below.

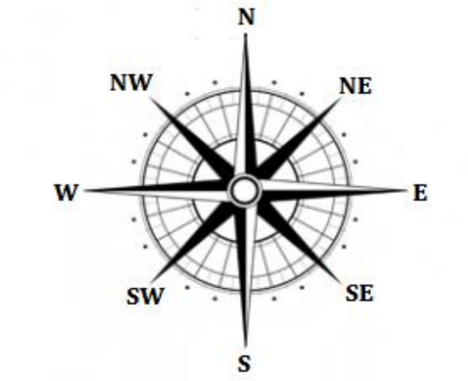


Answer: The squares are in A1 and B6. The circles are in B4 and F2.

The answers can also be written as 1A; 6B; 4B and 2F.

A reference point is needed when determining location or direction. The most common reference point is the cardinal points of a compass.

The compass alongside is used in giving direction where:



N indicates north

NE is north east (half-way between north and east)

E indicates east

SE is south east (half-way between north and east)

S indicates south

SW is south west (halfway between south and west)

W indicates west

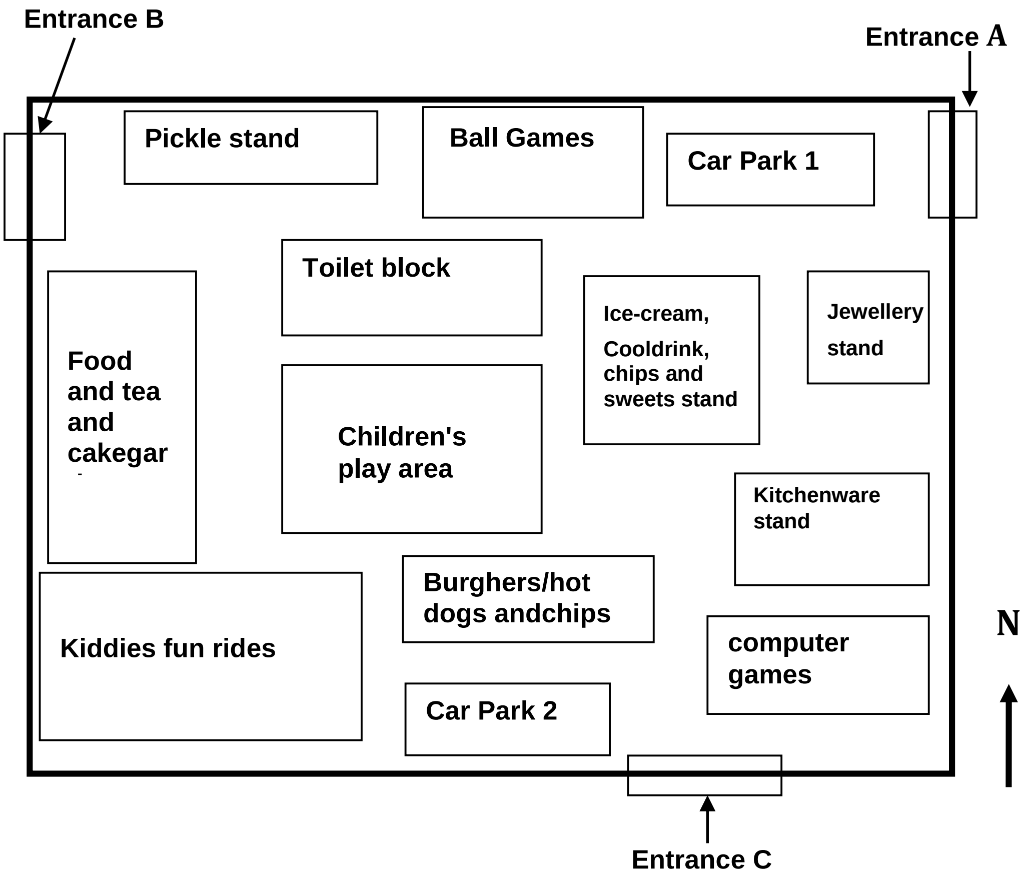
All maps and plans must include a directional arrow generally pointed in a northerly direction. From the given arrow the directions South (directly below north – 180° to the north line) can be determined.

Example 1:

The school governing body of Protea Primary School decides to hold a Fair on the school ground to raise funds. They erect a fence around the rectangular field.

The layout plan for the fair is illustrated below

LAYOUT PLAN OF THE SCHOOL FAIR



1. State the general direction from the Kiddies fun rides to Car Park 1.
2. Write down the number of stalls where games are played.
3. State one reason why Entrance B is for pedestrians only.
4. Name one item that could be sold at the jewellery stand.
5. The actual length of Car Park 2 is 32,4 m.

   Determine the scale used to draw this layout plan.

Answers:

1. North east
2. 2.
3. No car park near entrance B..
4. Earrings or rings or bracelets (any jewellery item)
5. The length of the car park on the map is 2,7 cm

   2,7 cm represents 32,4 m

The scale is 2,7 cm : 3 240 cm
= 2,7 : 3 240
= 1 : 1 200

Plan trips

Many people travel on trains, road and aeroplanes in South Africa. To plan for the trip one needs to understand the time table that governs the train, bus or aeroplane. The example below illustrates how to use a train time-tale to plan a trip.

Example: Below is the ShosholozaMeyl train time-table for travelling from Durban to Bloemfontein and then to Kimberley. The train departs from Durban on Wednesdays.

Town	Arrival	Departure	Time in minutes stopped at station
Durban		18:30	
Pietermaritzburg	20:53	21:10	17
Ladysmith	00:33		27
Harrismith	03:23	03:53	30
Bethlehem	05:20	05:40	20
Kroonstad	07:49	08:19	30
Hennenman	08:57	08:59	2
Virginia	09:17	09:19	2
Theunissen	09:50	09:52	2
Brandfort	10:25	10:27	2
Bloemfontein	11:15	11:45	30
Kimberley	14:50		

Study the time table and answer the questions that follow:

1. On which day of the week does this train from Durban arrive in Kimberley?
2. How long did the total journey take? Write your answer in hours?
3. At what time did the train leave Ladysmith?
4. Calculate the total time taken for stops between Durban and Kimberley. Give the answer in hours and minutes.
5. Not counting stops, the actual travel time for the train journey is 17 hours 36 minutes and the distance between Durban and Kimberley is 842 km. Calculate the average speed at which the train is travelling. Use the formula: **Speed =** $\frac{\textbf{Distance}}{\textbf{Time}}$
6. James travels from Durban to Brandfort on the same train. He needs to board a bus in Brandfort that is leaving the bus station at 11:00. It takes 5 minutes to walk from the train station to the bus station. Determine whether or not James will be in time to board the bus. Show ALL the necessary calculations.

Answers:

1. Train departs on Wednesdays 18:30 and arrives in Kimberley the next day , Thursday
2. 18:30 to 19:00 → 0 h 30 min

   19:00 to 24:00 → 5 h 00 min

   00:00 to 14:50 →14 h 50 min

   Total duration = 19 h 80 min

$= 20$ h 20 min

$= 20\frac{1}{3}$ hours (or 20,3333...h)

3. 00:33 + 27 minutes = 01:00
4. 17 + 27 + 30 + 20 + 30 + 2 + 2 + 2 + 2 + 30 = 162 minutes
   = 2 hours 42 minutes
5. Speed $= \frac{\textbf{Distance}}{\textbf{Time}}$

   $= \frac{842 \text{ km}}{17,6 \text{ h}}$

   = 47,84 km per h
6. Arrival time of the train = 10:25

   Assume the time lost between the train's arrival and getting off the train is negligible.

   Walking time = 5 minutes

   Arrival time at bus station = 10:30.

   ∴ James will be at the bus station in plenty of time.

Describe routes between two different locations

Many people use road maps to travel around the country. One of the skills needed to reduce travel cost is to use the shortest distance between two venues.

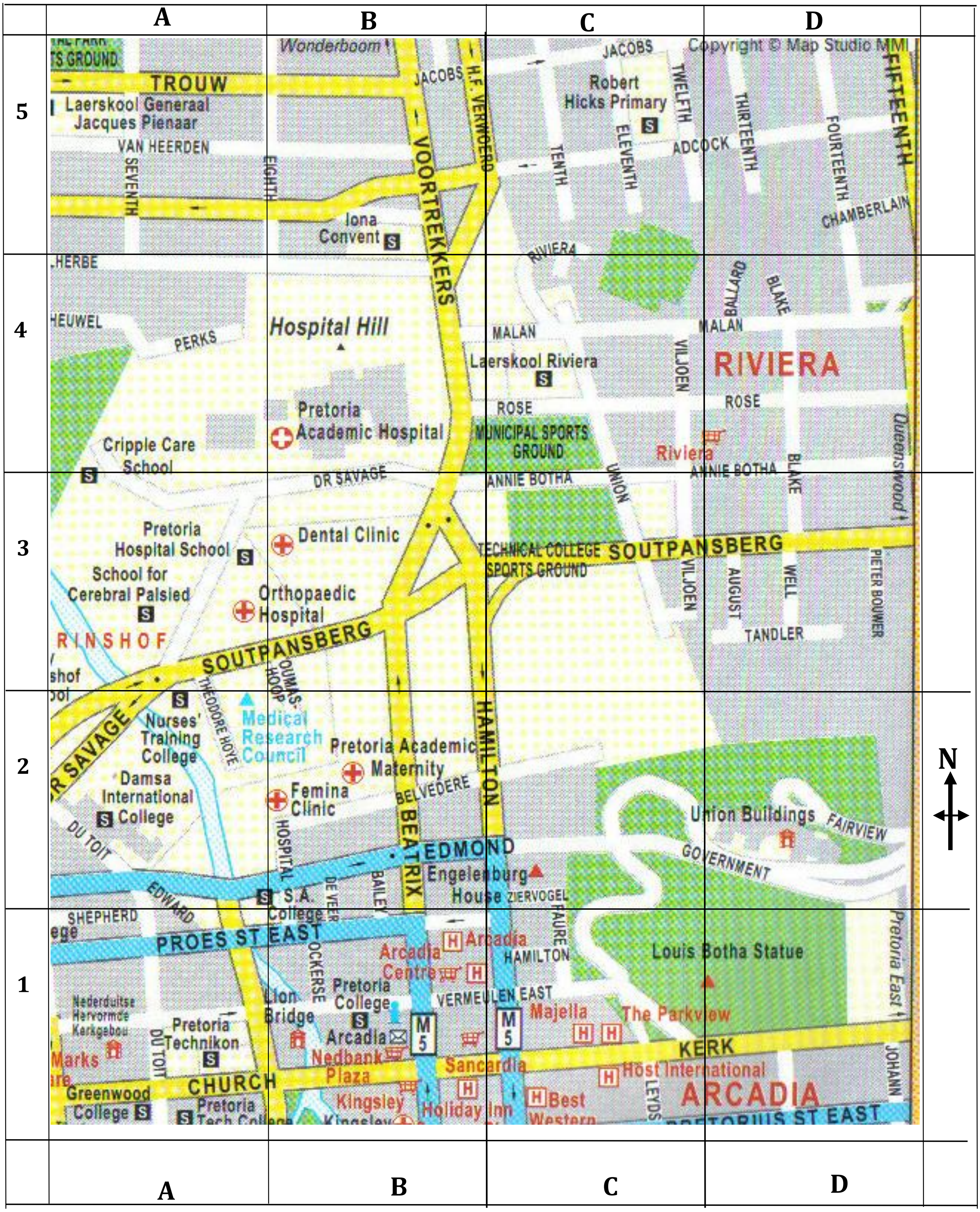
Describe relative positions

A relative position is the relationship between two objects based on a common reference point.

Example:

Dr.Nerisha Naidu is from India. She is spending three months visiting hospitals in Pretoria. She used the map of Pretoria on ANNEXURE B to aid her in finding her way around the town. Study the map below and answer the questions that follow.

THE MAP



1. Name the schools (labelled S) situated in grid reference B1.
2. Which hospital is situated north of the dental clinic situated in grid reference C2?

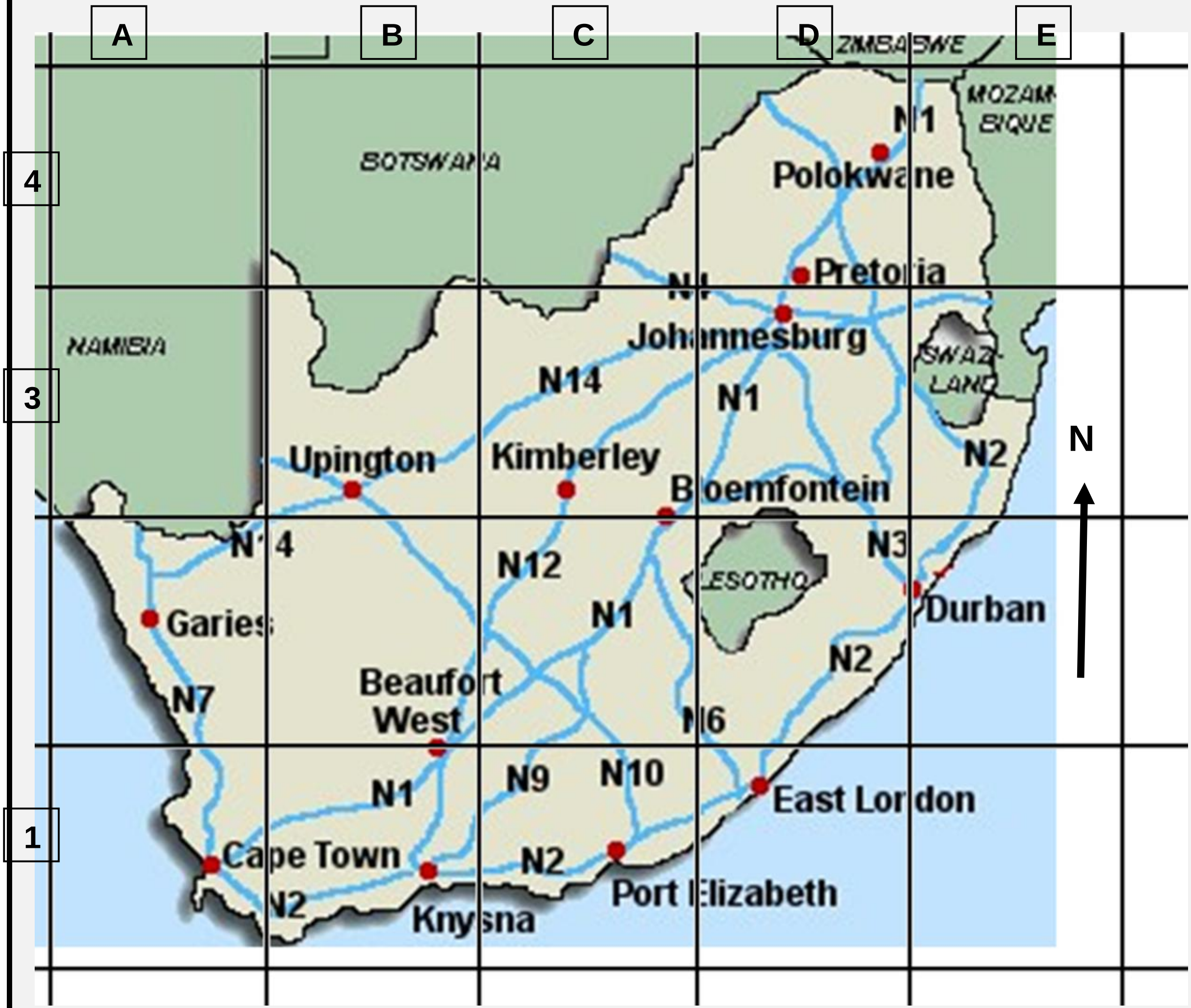
3. Write down the grid reference for the Union Buildings.
4. Write down the relative position of Pretoria College to Pretoria Hospital School.
5. Use the cardinal points of the compass to write down
   (a) The direction of the traffic flow in Beatrix Street.
   (b) The general direction that Pretoria Academic Maternity is from the Dental clinic.
6. The scale on the map is 1: 15 000. Dr. Naidu walked from the Orthopaedic hospital to the Dental Clinic. The distance on the map was 2,5 cm. How far did she walk in metres?

SOLUTIONS

1. Pretoria College.
2. Pretoria Academic hospital
3. D2 or 2D.
4. North West.
5.
   (a) North
   (b) South East
6. 1: 15 000 = 1 cm : 1 500 cm
   = 2,5 cm : 3 750 cm
   She walked 3 750 cm = 3,75 m

Activity 10

1 Ma Gumede is planning to travel from Johannesburg to Cape Town.



http://www.sacarrental.com/maps.htm

Study the map above and answer the questions which follow:

1. Name the national roads labelled N1, N2 etc. that she will travel on if she goes to Cape Town via:

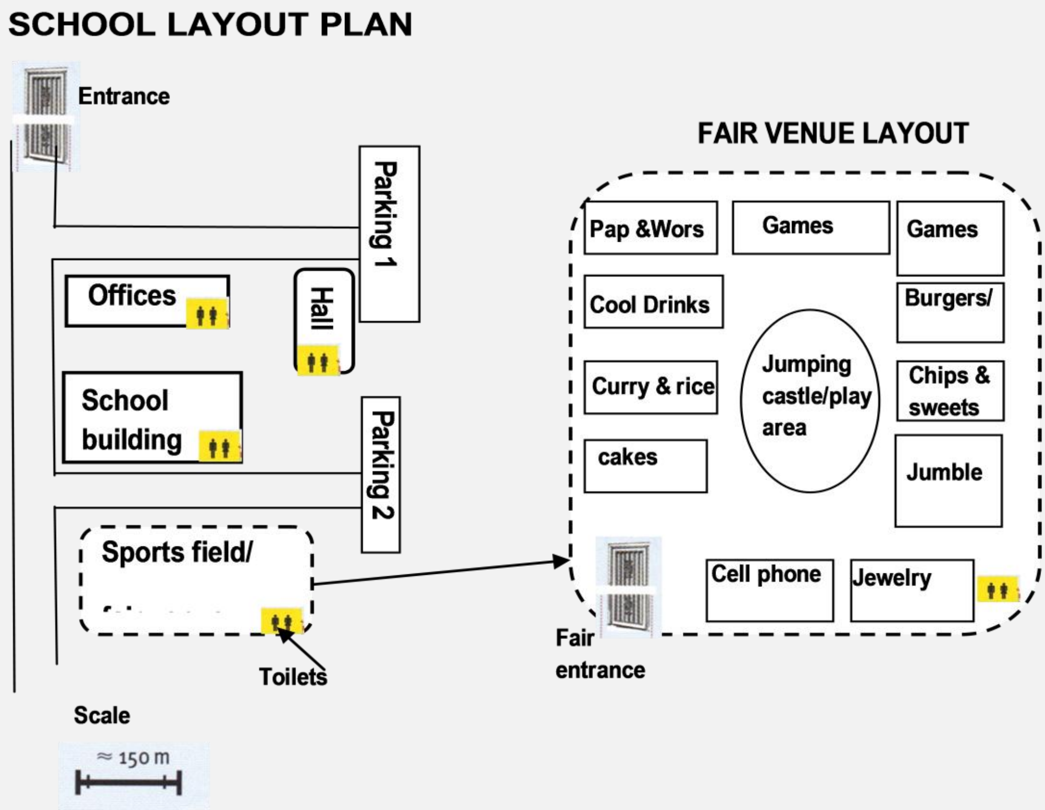
   1.1 Durban 1.2 Bloemfontein 1.3 Kimberley 1.4 Upington 1.5 Port Elizabeth

2. Name the shortest route between Johannesburg and Cape Town.
3. State one good reason why a map is an important tool in our lives.
4. Describe the relative position of East London from Durban.
5. Write down the grid reference of:

   5.1 Kimberley 5.2 Johannesburg 5.3 Upington 5.4 Cape Town

Summative assessment 8

New Dawn High School is having a fair on the sports grounds to raise funds. The layout plan below shows the school and the fair venue layout.



Study the plan above and answer the questions that follow:

1. Write down the number of sets of toilets represented on the school and sports field layout plan.
2. Why do you think the jumping castle/play area has been placed in the middle of the field?
3. Write down the names of the non-food stalls at the fair.
4. Which parking area is most suitable for visitors to the fair?
5. Use the scale on the given layout plan. Measure the length of the school building on the layout plan. Determine the actual length of the school building in metres.

Summary of key learning: There following key concepts were covered in this unit:

- Identify different features shown on maps and plans
- Interpret different types of scales
- Determine scales for maps, plans and models
- Use and interpret scale drawings of plans, maps and models to identify views, .estimate and calculate values according to scale
- Calculate actual length and distance using a given scale (*bar scale or number scale)*
- Determine locations and grid references
- Plan trips
- Describe routes between two different locations
- Describe relative positions

My Notes

Use this space to write your own questions, comments or key points.

Suggested sources of additional information

Read more: Via Afrika ›› Mathematical Literacy Gr 12

TOPIC 4: DATA HANDLING

Introduction

- The principles underlying this topic is primarily to develop student's ability to critically engage and communicate with data.
- Although some experience in collecting, organising and interpreting data is required, the focus in this topic is more on interpreting data rather than gathering or generating it.
- To develop an analytical and critical approach towards arguments based on data, students ought to know that data can be represented/misrepresented and interpreted/misinterpreted in different ways.

At the end of this chapter you should be able to:

- Identify examples of samples and populations;
- Use appropriate statistical methods to collect, classify, summarise, represent and interpret data;
- Calculate measures of central tendencies and measures of spread;
- Represent and analyse data;
- Critically evaluate and make recommendations using data.

Content Structure

Topic Heading	Topic (with Approximate Instructional Time)
Types of Data	1. Samples and Population (1 hour) 2. Quantitative and qualitative data (1 hour) 3. Data collection methods (0,5 hours)
Measures of central tendency	4. Mean (1 hours) 5. Median (0,5 hours) 6. Mode (0,5hours)
Measures of spread	7. Range (0,5 hours) 8. Quartiles (1 hour)
Representation of data	9. Frequency table (1 hour) 10. Box and whisker plot (2 hours) – only interpretation 11. Scatter plots (1 hour) 12. Misrepresentation of data (1 hour)

Samples and Populations

- A Sample is when we collect data just for selected members of the group.
- A Population is when we collect data for every member of the group, eg. Census, the whole population is involved.

- Example showing differences: It is not practical to count all the damaged oranges in an orchard (population), but it is possible to count the number damaged in a set of oranges (sample) taken from the orchard (population).

Illustrative examples:

Eg 1. Before an election, a news agency wanted to conduct a poll to determine who wouldwin.Which of the following represents a sample and which a population?

(a) A selection of voters of different ages
(b) All voters

Solution: (a) Sample (b) Population

Eg 2. A popular artist wanted to know what people thought about his painting. Which of the following will represent a sample and which a population?

(a) Every person who purchased the painting
(b) A selection of people who purchased the painting

Solution: (a) Population (b) Sample

Quantitative and Qualitative data

There are two types of data:

(a) Quantitative data:

- Is numerical information (numbers)
- Is made up two types: Discrete (countable items – whole numbers) and Continuous (measurements – decimals, fractions)
  Egs of discrete data: Number of products in a brouhure; Number of employees you have.
  Egs of continuous data: Dimensions of a specific product; Time taken to reach your destination.

(b) Qualitative data:

- Descriptive information, describing something.
- Are measures of categories or types and may be represented by name, symbol, category or type.
- Also referred to as categorical data. Eg. Favourite colour or pet.

Quantitative data is numerical information while **Qualitative** data is descriptive information.

Discrete data is counted while **continuous** data is measured.

Illustrative Examples

Classify the following as quantitative or qualitative data:

(a) How many pets do you have?
(b) In which city is the school located?
(c) How do you travel to work?
(d) How much do you earn?
(e) Which brand of cool drink is the most popular?
(f) Which country won the last soccer world cup trophy?

Solutions:

(a) quantitative data	(b) qualitative data	(c) qualitative data
(d) quantitative data	(e) qualitative data	(f) qualitative data

Data collection methods

Data can be collected using the following methods:

- Registers or registrations
- Questionnaires
- Surveys
- Interviews
- Direct observations
- Reporting

Measures of Central Tendency

- A measure of central tendency is a single value that tries to define a set of data by finding the middle position within that set of data.
- The mean, median and mode are valid measures of central tendencies.
- The **mean** is the average and is calculated by finding the sum of the data values and then dividing by the number of data values.
- The **median** is the middle value or the mean of two middle values when the data is arranged in numerical order.
- The **mode** is the value that appears the most number of times. It is possible to have more than one mode.
- An **outlier** is a value that "lies outside", that is, it is much smaller or larger than most of the other values in a set of data.
- Use **mean** to describe a set of data that**does not** have an outlier.
- Use **median** to describe a set of data that **has** an outlier.
- Use **mode** when asked to choose most **popular** item in a set of data.

Illustrative examples involving calculations of measures of central tendencies.

Eg. Pearl scored the following marks for her first 5 History tests:
100; 72; 86; 72 and 92. Find the mean, median and mode for her test scores.

Solution: Mean = (Sum of all the scores) ÷ (number of scores)

= (100 + 72 + 86 + 72 + 92) ÷ 5

= **84,4**

Median: Arrange the data from smallest to largest.

72 72 86 92 100

Therefore the middle value which is the median is **86**

Mode = **72**

Practice exercise

1. Find the mean, median and mode for the following set of data:
5; 15; 10; 15; 5; 10; 10; 20; 25; 15; 20; 15
2. On his first 6 English tests, Alfie received the following scores:
72; 68; 10; 92 and 77. What test score must Alfie get in his sixth test so that his mean score will be 80?

Measures of Spread and Percentiles

- A measure of spread refers to how the data within the set is spread out, or dispersed or scattered about the mean.
- Sometimes a set of data may have the same mean and median. Therefore to determine how these data sets are different requires us to calculate the spread of the data set, that is, how is the data spread out?
- We can use the range and the Interquartile range to measure the spread.
- Range is the difference between the largest data value and the smallest data value in the data set.
- The interquartile range is another form of range which divides the data set into four equal parts (quarters). The three values that form the four divisions are called quartiles: First quartile (Lower quartile or Q_1), Second quartile (Median or Q_2) and Third quartile (Upper quartile or Q_3).
- The Interquartile Range (IQR) is the difference between the third quartile and the first quartile (IQR = $Q_3 - Q_1$).
- The IQR represents 50% of the data which eliminates the influence of any outliers.
- Percentiles divide a data set into hundred parts. Commonly used percentiles are the 25^{th} (quartile 1), 50^{th} (median or quartile 2) and 75^{th} (quartile 3).

Illustrative examples involving calculations of measures of spread.

Eg. 1 Determine the quartiles, range and interquartile range for the following set of data: 5; 9; 8; 4; 3; 8; 6

Solution:Arrangethe data set in order; then divide them into quarters.

3 (4) 5 (6) 8 (8) 9

↓ Q_1 ↓ Q_2 ↓ Q_3

Quartile 1 = 4;

Quartile 2 = 6;

Quartile 3 = 8

Range = Highest value – Lowest value = 9 – 3 = 6

Interquartile range = $Q_3 - Q_1 = 8 - 4 = 4$

Eg. 2 Determine the quartiles, for the following set of data:

5; 9; 8; 4; 3; 9; 6; 6

Solution: Arrange the data set in order; then divide them into quarters.

3 4 5 6 6 8 9 9

$Q_2 = (6+6) \div 2 = 6$

$Q_1 = (4+5) \div 2 = 4,5$ **$Q_3 = (8+9) \div 2 = 8,5$**

Representation of data

- Ungrouped data can be represented by means of a frequency table.
- Data can be represented using bar graphs, pie charts, histograms, straight line graphs, broken line graphs, pictographs, frequency polygons and box-and-whisker plots.
- Tally marks are represented by | for each time a data value appears. When a data value is repeated 5 times, then 4 tally marks are used with the fifth going across, eg. 𝍸.

Illustrative examples:

Eg.1 A 6-sided dice was thrown 30 times and the following outcomes were observed:

1, 4, 2, 4, 6, 1, 2, 3, 6, 5, 4, 4, 3, 1, 1, 3, 1, 1, 5, 6, 6, 2, 2, 3, 4, 2, 5, 5, 6, 2

Draw a frequency table to represent the above data.

Solution:

NUMBER	TALLY	FREQUENCY
1	~~||||~~ |	6
2	~~||||~~ |	6
3	||||	4
4	~~||||~~	5
5	||||	4
6	~~||||~~	5

Eg.2 Thescores obtained by 40 students in an examination are given below:

8, 47, 22, 31, 17, 13, 38, 26,
3, 34, 29, 11, 22, 7, 15, 24,
38, 31, 21, 35, 42, 24, 45, 23,
21, 27, 29, 49, 25, 48, 21, 15,
18, 27, 19, 45, 14, 34, 37, 34

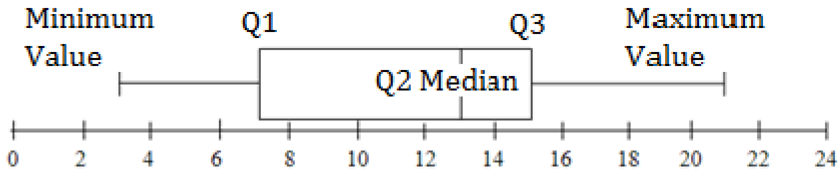
Draw up a frequency table to group the data using 5 class intervals, 0 -10, where 10 is excluded.

Solution

Scores	Frequency
0 – 10	**3**
10 – 20	**8**
20 – 30	**14**
30 - 40	**9**
40 – 50	**6**

The Box-and-whisker plot

- Candidates taking this course will **NOT** be required to draw the box-and-whisker plots but will rather be assessed on the interpretation of it.
- The box-and-whisker plot is a simple way of representing statistical data on a number line in which a rectangle is drawn to represent the second and third quartiles, usually with a vertical line inside to indicate the median value. The lower and upper quartiles are shown as horizontal lines on either side of the rectangle. See example below:

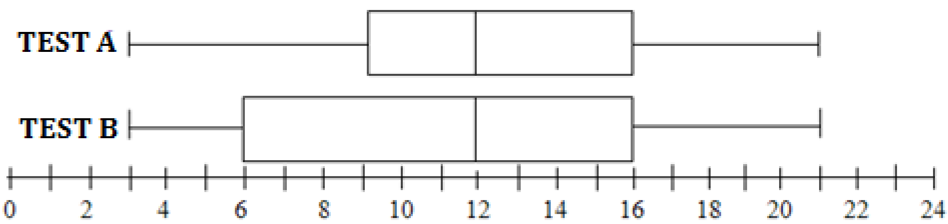


The above box-and-whisker shows a five number summary as follows:
(a) The minimum value of 3
(b) Q1 = Quartile 1 which is 7
(c) Q2 = Quartile 2 which is the median 13
(d) Q3 = Quartile 3 which is 15
(e) Maximum value is 21

- The 5 number summary of a set of observable data consists of the following:
  Maximum– the largest observation
  Upper Quartile (Q3) – a value that separates the largest 25% of the observations from the smallest 75%
  Median (Q2) – a value that separates the largest 50% of the observations from the smallest 50%
  Lower Quartile (Q1) – a value that separates the largest 75% of the observations from the smallest 25%.
  Minimum – the smallest observation

Practice exercise

1. Refer to the following box-and-whisker plots which represents two tests written by a grade 12 class and then answer the questions which follow:



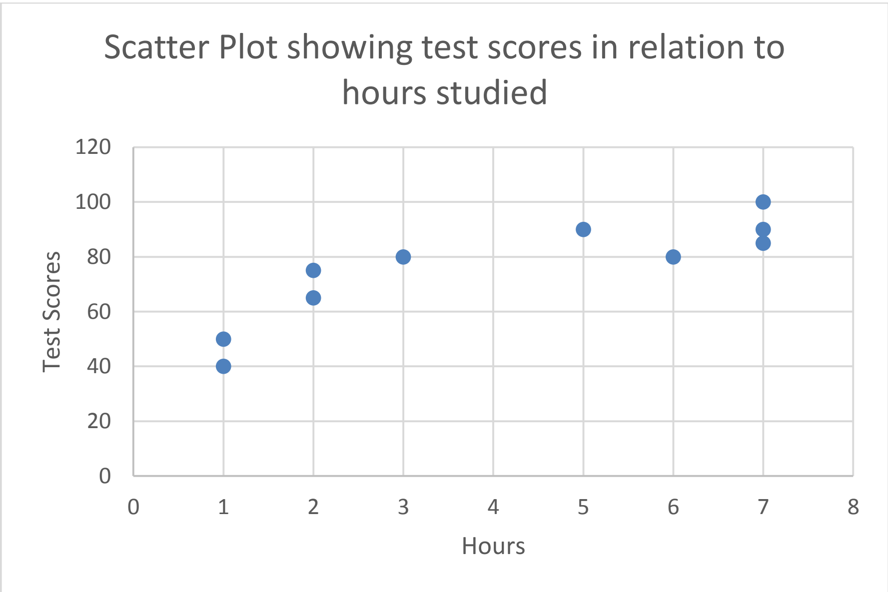
1.1 Determine the 5 number summary for each of the tests.
1.2 State with reasons in which test did the students perform better.
1.3 Determine the Interquartile range for each test.

Scatter Plots

- Candidates taking this course will **NOT** be required to draw scatter plots but will rather be assessed on the interpretation of it.
- A scatter plot is used to display two sets of data in order to find a relationship between them.
- A scatter plot shows trends.
- A scatter plot shows a positive trend if, as one set of data values increases, the other set tends to increase.
- A scatter plot shows a negative trend if, as one data set of values increases, the other set tends to decrease.
- A scatter plot shows no trend if the ordered pairs show no correlation.
- Correlation is made of co- ("together"), and relation – thus relationship.

Eg. The following represents test scores (as %) and the corresponding study hours.

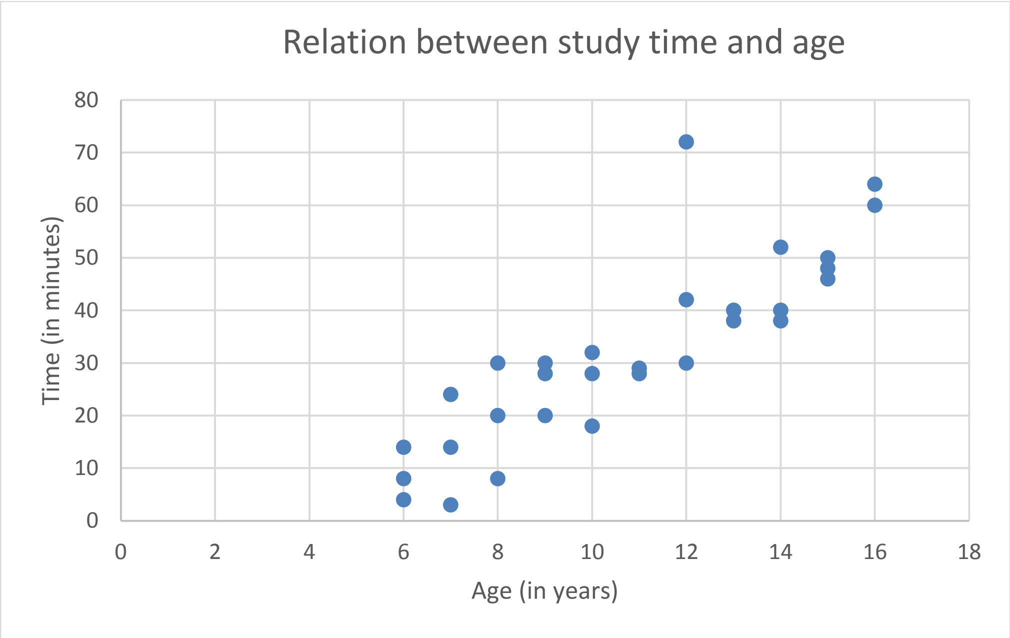
Hours studied	**3**	**5**	**2**	**6**	**7**	**1**	**2**	**7**	**1**	**7**
Test scores	**80**	**90**	**75**	**80**	**90**	**50**	**65**	**85**	**40**	**100**



- The above scatter plot shows a strong positive correlation.

Practice Exercise

Study the following scatter plot showing the relationship between study time and age and then answer the questions that follow:



(a) Does the above scatter plot represent a positive, negative or no association? Give a reason for your answer.

(b) Write down the coordinates of the outlier.

(c) Comment on the relationship between age and the amount of time spent studying.

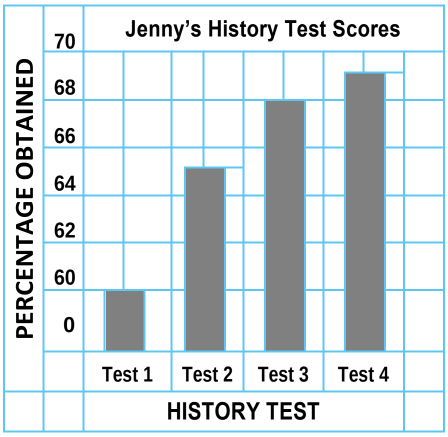
Misrepresentation of data

- Very often data is distorted (change the appearance) to intentionally mislead or advantage individuals.
- Some of the ways in which data can be distorted are:
  - Changing the intervals; or scale on a graph could be inconsistent; or not starting at 0.
  - Enhancing the visual representation of a graph, eg. 3D.
  - The size of the bars on a bar graph may be enhanced.
  - The sum of the sectors on a pie chart may not add to 100%.

Practice exercise

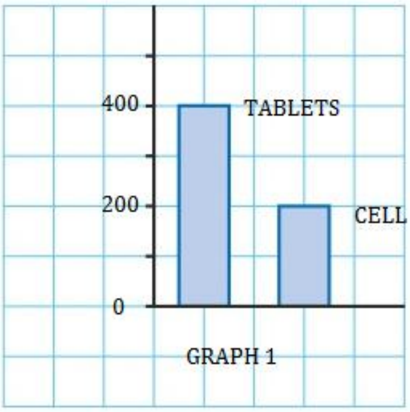
Refer to the following representations and then answer the questions which follow:

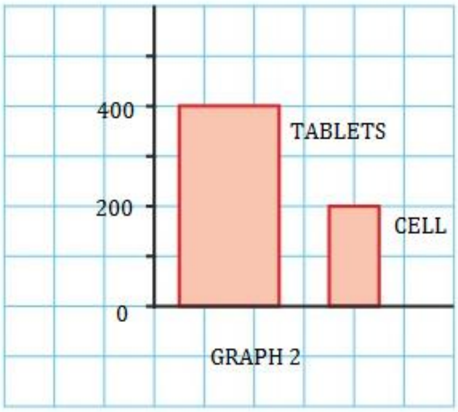
1. Jenny's History test scores are shown in the bar graph below:



Comment on the misleading test performance reflected by the graph above.

2. The following two graphs represent the same information relating to a service providers sales of mobile devices, namely, tablets and cell phones. Study the graphs and then answer the question that follows:

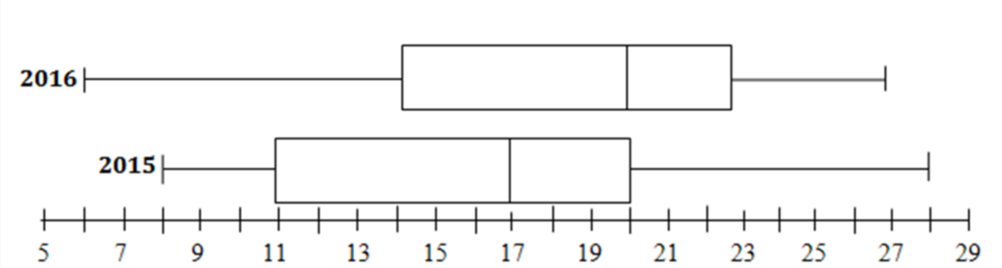




Give a reason why graph 2 is misleading.

Activity11 – Data Handling

1. The following box-and-whisker plots show the 5 number summaries of an entrance test of 2 different cohorts of students. Study the plots and answer the questions which follow:



1.1 Determine the IQR for 2016 and 2015.

1.2 State with reasons which cohort performed better.

2. Find the mean, median and mode of the following data set:

   {3, 7, 8, 5, 12, 14, 21, 15, 18, 14}.

3. State the difference between qualitative data and quantitative data.
4. Show with examples the difference between discrete and continuous data.
5. A similar test was given to Grade 12 learners in 2 different schools. The results were recorded as a percentage and summarised in the following table. Use the table to answer the questions which follow:

SCHOOL A **200 learners wrote the test**		**SCHOOL B** **170 learners wrote the test**	
	Mark		Mark
Minimum	0	Minimum	32
Lower quartile	0	Lower quartile	41
Median	33	Median	68
Upper quartile	57	Upper quartile	78
Maximum	66	Maximum	98

State whether the following statements are true or false. If false correct it:

5.1 50% of the learners in school A got no answers correct.

5.2 The range in both schools are identical.

5.3 75% of learners in school B obtained more than 78%.

5.4 Approximately 85 learners from school B scored higher than the best mark in school A.

5.5 School A had a better interquartile range.

5.6 School B performed better in the test.

5.7 The maximum mark of school A was found between the 25th and 50th

Summary Assessment 9 – Data Handling

1. Jerome scored the following percentages in his five English tests: 60%, 45%; 30%; 55% and 35%. If a pie chart is drawn to represent these marks, what angle would be represented by his last test result?

   A. 96° B.12° C.56° D. 48°

2. What type of data is represented by Joe's weight of 57,5 kg?

   A. Discrete B. Continuous C.Categorical D. Qualitative

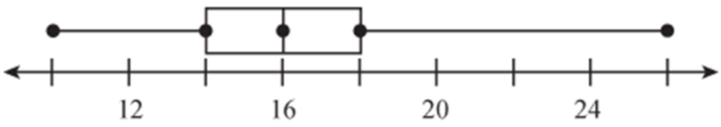
3. Which measure of central tendency is affected if the 2 outliers from both ends of ordered data are removed?

   A. Median B. Mode C. Mean D. None

4. Virat Kholi, a famous Indian cricketer scored the following runs in 8 innings: 76; 58; 35; 40; 45; 0; 46; 50. How many runs must he score in his next innings to keep is average score at 50?

   A. 50 B. 100 C. 200 D. 400

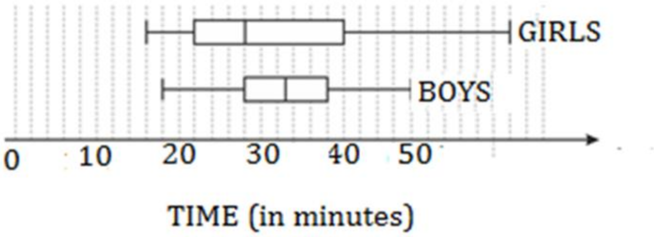
5. The box and whisker plot below shows the number of people visiting a hairdressing salon in the past 10 days.



What percentage of people lie in the14 and 18 interval?

A. 15% B. 25% C. 50% D. 75%

6. The box and whisker plots below shows the finishing times of girls and boys in an athletics race.



State whether the following statements are true or false giving reasons for each answer:

5.1 A girl was the first to cross the finishing line.
5.2 A boy had a larger median time.
5.3 Only a quarter of the boys completed the race when 50% of the girls completed the race.
5.4 The girls had the slowest finishing time.
5.5 The boys had a smaller third quartile time.
5.6 The girls had a higher interquartile range time than the boys.
5.7 The boys performed better in the race overall.
5.8 The data for the girls are negatively skewed.

Summary of key learning: There following key concepts were covered in this unit:

- Definitions of important terminology related to data handling concepts
- Quantitative and qualitative data
- Discrete and continuous data
- Measures of central tendency
- Measures of spread
- Methods of data collection
- Representation of data
- Misrepresentation of data
- Scatter plots

My Notes

Use this space to write your own questions, comments or key points.

Suggested sources of additional information

Read more: Via Afrika ›› Mathematical Literacy Gr 12

Probability

Learning objectives:

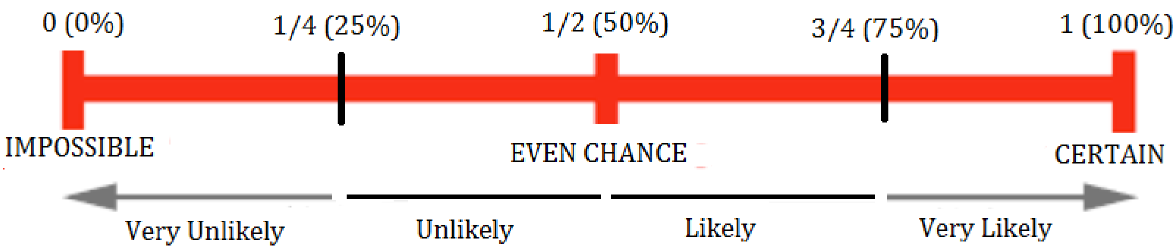
- Express probability values in terms of common fractions, decimal fractions and percentages;
- Use tree diagrams to determine the probability of dependent events;
- Effectively communicate conclusions and predictions that can be made from the analysis and representation of data, using appropriate terminology such as, trends, increase, decrease, constant, impossible, likely and even chance.

Expressions of probability

- Probability is the chance of an event occurring or how likely something is to happen.
- An experiment is a situation involving probability resulting in outcomes.
- An outcome is the result of a single trial of an experiment.
- An event is one or more outcomes of an experiment.
- Probability of an event happening =

$$(Number\ of\ favourable\ outcomes) \div (Total\ outcomes)$$

- The chances of any event can be shown on a probability scale from 0 to 1: • a probability of 0 tells us that the event will never happen - it's impossible • a probability of 1 tells us that the event is certain to happen • a probability of ½ tells us that the event has an even chance of happening
- Probability scale below shows possible outcomes:



- Probability can be expressed as a fraction, decimal, percentage or using words (see scale above).

Illustrative examples:

1. Determine the probability of rolling a four in a six sided die (singular for dice).
2. What is the probability of tossing a head on a coin?
3. In a bag there are 3 green balls, 5 red balls and 4 white balls. Determine the probability of selecting a white ball.

Solutions:

1. There is one four on a six sided die, therefore using the formula we get: Probability = (Favourable outcomes/Total outcomes) = 1/6 or 0,17 or 17%.
2. ½ or 0,5 or 50%.
3. 4/12 = 1/3 or 0,3 or 33%

Practice Exercise

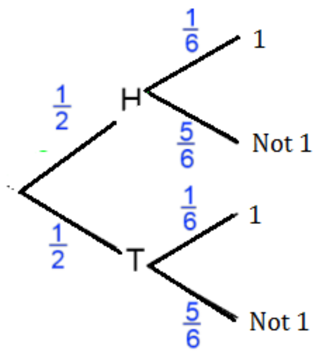
1. Determine the following probabilities expressing all your answers as a fraction, decimal and a percentage.
   - 1.1 Selecting an even number on a six sided die.
   - 1.2 Choosing a whiteSmarties from a box containing 8 red, 12 blue and 6 green Smarties.
   - 1.3 Selecting a picture card from a pack of 52 cards.
   - 1.4 Driving a silver car in an auto dealership which has the following cars on the floor: 10 white; 3 silver; 5 blue and 2 red cars.
   - 1.5 Selecting a boy from a class having 24 boys and 18 girls.

Tree Diagrams

- A tree diagram is a means of representing a sequence of events.
- Tree diagrams enable us to see all the possible outcomes of an event and calculate their probability.
- Each branch in a tree diagram represents a possible outcome.
- For each pair of branches the sum of the probabilities adds to 1.
- "And" only means multiply outcomes if events are independent, that is, the outcome of one event does not affect the outcome of another.
- "Or" means add the outcomes.

Illustrative examples:

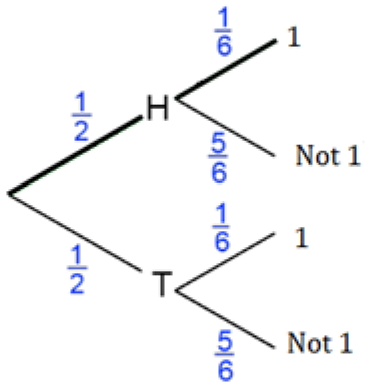
1. A coin is tossed and a six sided die is rolled. Use the tree diagram that follows to answer the questions which follow:



1.1 Determine the probability of tossing a Head **And** rolling a 1.

1.2 Determine the probability of tossing a Head **Or**rolling a 1**.**

Solution



P (H, 1) = $\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$

P (H, Not 1) = $\frac{1}{2} \times \frac{5}{6} = \frac{5}{12}$

P (T, 1) = $\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$

P (T, Not 1) = $\frac{1}{2} \times \frac{5}{6} = \frac{5}{12}$

Each path represents a possible outcome, and the fractions indicate the probability of travelling along that branch. For each pair of branches the sum of the probabilities adds up to 1.

Therefore, the probability of getting a Head **AND** a 1 is:

1.1 P (H and1) = "And" means multiply the outcomes, which is $\frac{1}{12}$

1.2 P (H or 1) = "Or" mean add all the outcomes that have a Head and a 1.

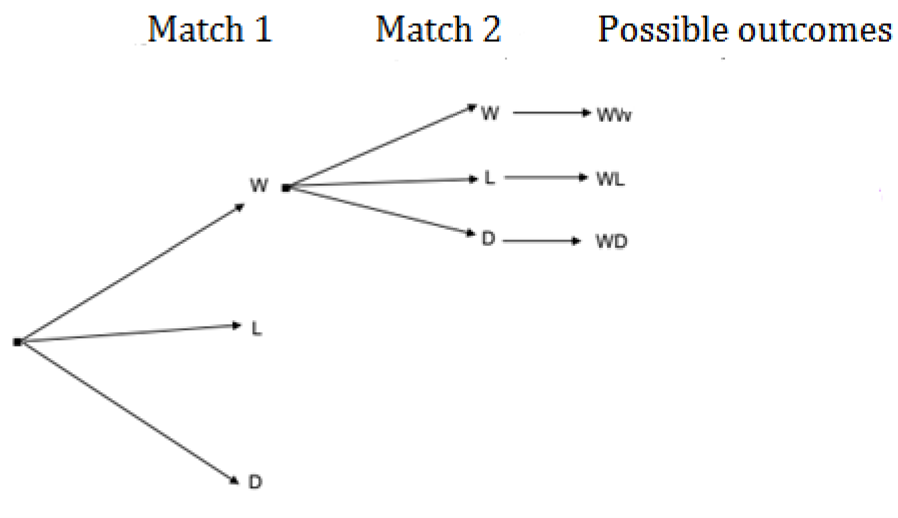
$$= \frac{1}{12} + \frac{5}{12} + \frac{1}{12} = \frac{7}{12}$$

Practice Exercise

1. Moondowns Football Club (FC) played two matches in May. There are THREE possible outcomes for each match: W (win), L (lose) or D (draw).

   Study the following tree diagram and then answer the questions which follow (Adapted from DBE, November, 2008, Paper 2)

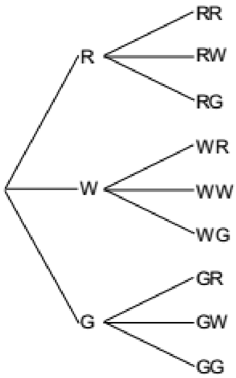
   1.1



ree diagram to show ALL the possible outcomes of the two matches.

1.2 Use the tree diagram to predict the probability that Moondowns FC will:
   a) Win both matches
   b) Win only one of the matches
   c) Draw at least one of the matches.

2. A bag contains the following coloured marbles: 5 red; 4 white and 3 green. Study the tree diagram that follows and then answer the questions which follow:



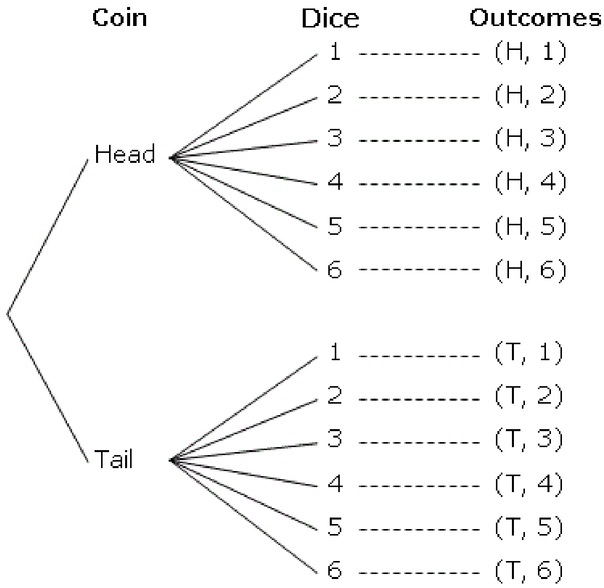
2.1 Sizwe picks a white marble from the bag but then places it back into the bag as he wanted a green marble. Determine the probability of choosing at least one green marble.

2.2 Sizwe picks a white marble and keeps it. He then picks another marble. What is the probability that he chooses a green marble.

3. A coin and a six sided die are thrown at random. Find the probability of:

3.1 Getting a Tail and an Even number.

3.2 Getting a Head and an Odd number.



4. Extracted from past Mathematical Literacy examination papers.

4.1 Determine (in simplest form) the probability of randomly selecting a boy out of a class with 18 boys and 24 girls.

4.2 If 18 May 2009 is on a Monday, what is the probability that 19 May 2009 is on Tuesday?

4.3 Fred, the oldest living goldfish, was exactly 41,5 years old on 15 March 2012. What is the probability that Fred was born on 15 September 1970?

4.4 What is the probability that Christmas day in South Africa is on 25 December?

Prediction

- We use probability to predict results and determine if something is impossible, certain, likely, or unlikely.
- A prediction is a statement about the way events will occur in the future, often but not always based on experience or knowledge.
- We have two types of probabilities, namely: theoretical probability and experimental probability.
- Theoretical probability is the ratio of the number of favourable outcomes to the number of total possible outcomes. Experimental probability is the ratio of the number of times an event occurs to the total number of trials.
- In simpler terms, theoretical probability is a ratio that describes what ***should*** happen, while experimental probability is a ratio that describes what ***actually*** happened.
- An example of theoretical probability is rolling a six sided die once to get a 5.
  Answer: Theoretical probability = $\frac{1}{6}$
- An example of experimental probability is rolling a six sided die 50 times and the number 5 appeared 10 times.
  Answer: Experimental probability = $\frac{10}{50} = \frac{1}{5}$ or 20%.

Practice exercise

1. The table below shows data for a sample of people who were interviewed about their consumption of cans of cool drink per day. Use it to answer the questions that follow:

Number of cans of cool drink per day	Age and gender of people interviewed						Totals
	18 - 28		29 - 39		40 - 50		
	Male	Female	Male	Female	Male	Female	
0	124	146	135	154	159	153	871
1	52	43	43	28	28	35	229
2	24	11	22	18	13	12	100
Totals	200	200	200	200	200	200	1 200

1.1 How many people were interviewed?

1.2 How many of the people interviewed in the 18 – 28 group were female?

1.3 How many people consumed more than one can of cool drink a day?

1.4 If a person was selected randomly, determine as a percentage, the probability of:

(a) Consuming two cans of cool drink per day.

(b) Selecting a male in the 29 – 39 category consuming one can of cool drinkper day.

(c) Choosing a male consuming no cool drink at all per day.

1.5 How can the data in the table be of use to the general public?

SOLUTIONS

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GlOSSARY OF TERMS

Association - a general term to describe the relationship between two variables. Two variables in bivariate data are *associated* or **dependent** if the pattern of frequencies of their bivariate values cannot be explained only by the frequencies of the univariate values. In contrast, two variables are *not associated* or **independent**, if the frequencies of bivariate values can be determined simply from the frequencies of the values of each variable.

Associative Law / Property - the property of an operation which allows for the operation to be carried out by grouping the terms differently (e.g. for addition of real numbers: $(a + b) + c = a + (b + c)$ and for multiplication $(a \times b) \times c = a \times (b \times c)$.

Bar Graph / Diagram - a diagram that uses horizontal or vertical bars to represent the **frequency** of classes (or groups or labels) in data consisting of observations of a categorical variable. The height or length of each bar is proportional to the frequency of the corresponding class, but the thickness of a bar has no meaning. The bars are not required to touch each other, and may be separated.

Bias - a distortion of the data in a set due to irregularities in the collection of the data; an unjustified tendency to favour a particular point of view.

Break-Even Point - the value of the independent variable at which the costs associated with various (two) pricing structures for a commodity become equal; the point at which expenditure and income are equal.

Circumference - the (measure of) the perimeter of a circle.

Commutative Law/Property - the property of an operation which allows for the order of the values operated with to be interchanged (e.g. for the addition of real numbers $a + b = b + a$ *and* for multiplication $a \times b = b \times a$.

Compass Direction - the direction indicated with reference to the globe of the earth as north, south, east or west; the direction in degrees from the northerly direction in an anticlockwise sense.

Compound Growth - the accelerated effect in the manner in which a quantity increases (or decreases) due to the factor causing the increase (or decrease) also acting on the increase (or decrease) in the amount itself (e.g. the growth in the amount invested when interest is calculated on interest, as in **Compound Interest**).

Compound Interest - the calculation of the new amount A when the original amount (the principal), P, of money is subjected to interest being calculated on interest at the end of a period.

Continuous Variable - a variable which ranges through all the real numbers on the interval applicable to it.

Data - items of information that have been observed and recorded; can be categorical (e.g. gender), numerical (e.g. age), are often arranged in a list or table.

Dependent Variable - the element of the range of a function which depends on the corresponding value(s) of the domain (e.g. in $y = f(x) = \pi x^2$ the area of a circle (y) depends of the radius, x. x is the **Independent Variable** and y the dependent variable).

Direct Proportion - two variables, x and y, which are related by the equation $y = kx$, are said to be in direct proportion.

Discrete Variables - variables for which the values do not take on all the real numbers within the range over which they vary; a discrete variable is often associated with a count and so takes the values of the counting numbers (e.g. 0, 1, 2, 3 …).

Event - any subset of all the possible outcomes of an experiment. An event *occurs* at a particular experimental **trial** if *any one* of its constituent outcomes is the outcome observed for that trial.

Exchange Rate - the price of a unit of the currency of one country in terms of the currency of another.

Experiment - a repeatable activity or process for which each repetition gives rise to exactly one outcome drawn from the sample space (statistical experiment); gives rise to **univariate data** on the outcome of each **trial** (e.g. the observed face of a die) (simple experiment). The number of trials observed is the sample size n.

Frequency - a count of the number of times a particular outcome or event was observed in data with a sample size n.

Frequency Table - a table reporting the groups into which data values were organised, and the **frequency** of each group.

Global Positioning System - a system using satellite and electronic technology, whereby a particular location on the earth's surface is determined in terms of its **latitude** and **longitude.**

Grid - a pattern of lines usually drawn at right angles to each other to form rectangles.

Grouped Data - data arising from organising n observed values into a smaller number of disjoint groups of values, and counting the frequency of each group; often presented as a **frequency table**.

Hire-Purchase - the system whereby goods are bought by putting down a deposit and then periodically paying off the balance of the purchase price plus interest. Simple interest usually applies while insurance costs are commonly also included.

Inflation Rate - a quantitative measure which indicates the rate at which the price of consumer goods is increasing over time.
Indirect/Inverse Proportion - two variables, *x* and *y*, which are related to each other by the equation $y = k/x$ are said to be in inverse proportion.

Latitude - the number of degrees that a location is north or south of the equator.
Longitude - the number of degrees that a location is east or west of a line passing through the poles and Greenwich in Britain.
Median - a value that splits the sample data of a numerical variable into two parts of equal size, one part consisting of all values less than the median and one part with all values greater than the median; most easily established if the data values are arranged in increasing or decreasing order.

Mode - the most frequently occurring observation in a set of data.
Mortgage Bond - a loan from a bank, usually for the purchase of property. The loan is subject to the payment of **compound interest** and is paid off in regular instalments which include interest and capital.
Outcome - the result of an experiment (in statistics) (e.g. the outcome of an experiment in which a dice is rolled can be any one of the natural numbers 1 through 6).
Percentiles - values of ranked data separated into one hundred groups of equal size, especially when sample size *n* is very large.

Polygon - a figure in a plane formed by some number of straight sides.
Probability - for equally likely outcomes, the number of favourable outcomes divided by the total number of possible outcomes of an experiment.
Qualitative Data - information or data arising from observations which are *not* numerical; can be categorical.
Quantitative Data - data with values that are numerical; can be discrete (counted) or continuous (measured).
Right Cylinder - a solid that has one axis of symmetry through the centre of the circular base and a uniform, circular cross-section.

Right Prism - a prism whose lateral sides are perpendicular to its base.

Sample - in statistics, a group of data chosen from all the possible data.

Tree Diagrams - a diagram in which the possible outcomes of trials involving one or more events are indicated by line segments.

Trial - each repetition of a statistical **experiment.**

EXEMPLAR(S)