NASCA Quantitative Literacy Materials Draft 1

## Unit 1: Number Formats and Conventions

### UNIT 1: NUMBER FORMATS AND CONVENTIONS

#### Learning Outcomes

By the end of the unit you should be able to:

* Understand and apply the skill of number formats and conventions.
* Understand and apply the skill of estimation and rounding in context.
* Solve practical real-life problems in context using estimation, rounding and accurate calculations.

### Introduction

In this unit you will learn about number formats and conventions. You will be required to write numbers in words and words as numbers. You will then learn the skills involved in rounding and estimation. You will also be expected to use these skills to solve problems in real-life contexts.

**Number formats**

Numbers can take on various formats. They could appear to look different or they could have several meanings depending on the context in which they are used. In the following example we will see how a number can be represented using three different formats. Some calculators insert what looks like a comma after every three digits:

Eg. 26,340,000,000

The comma in the above example in no way suggests that we are working with decimals but rather it enables us to read the number more easily. Similarly, the number may even be represented using the following format:

Eg. 26’340’000’000

Writing large numbers in words or digits is useful in real life, especially when discussing large groups of people or the country’s huge budget. For example, city populations are important: they are the topic of newspaper articles, classroom discussions, and government analysis.

When we, in Quantitative Literacy (QL), are writing numbers, we would generally include a small space where the comma would be. The digits must always be taken in groups of three from the right as follows:

Eg.R 26 340 000 000 The number alongside written in words will be:

 Twenty six billion three hundred and forty million rand.

Another example would be the use of a comma or a dot to show a decimal. For QL, a comma must be used to indicate a decimal.

Eg. 7,4 is exactly the same as 7.4, above but the comma is preferred.

**Activity for reflection**

**What you will do:**

When changing a number from digits to words, follow these steps:

* Work from left to right.
* Think in “threes.” Remember, a comma or space marks every three digits of a number, counting from the right. Write or say the number to the left of the first comma/space. Consider the number 736 123. Begin by saying “Seven hundred thirty-six.”
* When you reach the comma/space, write or say the place name of the last digit of the “three numbers”. Once again consider 736 123. The place value of the last digit before the comma is thousand. Say or write “Seven hundred thirty-six thousand.”
* When you reach the units (ones) place value, just say the number, not the place value. Just say “Seven hundred thirty-six thousand, one hundred twenty-three.” You wouldn’t say “units/ones.”

When writing a number in words follow the place values in the following table:

* Simply write the numbers in the table as you read them.
* Write zeros for missing places.
* Group numbers in threes with spaces between each three digits.

**Illustrative example**: Consider the example: Six million, four hundred and ninety four.

Use the table below to help you complete your answer as follows from right to left with zeroes inserted in empty places:

**Table for number formats**

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Trillions** | **Hundred Billions** | **Ten Billions** | **Billions** | **Hundred millions** | **Ten Millions** | **Millions** | **Hundred****Thousands** | **Ten****Thousands** | **Thousands** | **Hundreds** | **Tens** | **Ones** |
|  |  |  |  |  |  | **6** | **0** | **0** | **0** | **4** | **9** | **4** |

 Billions Millions Thousands Hundreds

**Activity for reflection:**

1 Write the following number in words:

* 1. An amount of R346 007 540.

**Guided reflection:**

Read the last three digits aloud.

What does the digit 5 represent?

*Five Hundred*

Now read the second last three digits aloud.

What does the digit 7 represent?

*Seven Thousand*

What does the zero next to the digit 7 tell you?

*There are no ten thousands*

*Notice the second three digits represent thousands, ten thousands and hundred thousands*

Now read the first three digits.

What do these digits represent?

*Millions*

Now use the table and start filling in the values from left to right.

Start with the last three digits and then slowly move to the left with the other digits.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Hundred** **millions** | **Ten** **Millions** | **Millions** | **Hundred****Thousands** | **Ten****Thousands** | **Thousands** | **Hundreds** | **Tens** | **Ones** |
| **3** | **4** | **6** | **0** | **0** | **7** | **5** | **4** | **0** |

Now write down the digits in words from the table.

**Solution:**

*Three hundred and forty six million, seven thousand, five hundred and forty rand.*

1.2 A population of 32 021 800.

Now read in groups of three starting from left to right.

Remember that the last three digits represent hundreds, tens and units.

What do the second last three digits represent?

*Thousands*

What do the first two digits represent?

*Millions*

Now write the answer in words.

 **Solution:**

 *Thirty two million, twenty one thousand , eight hundred.*

Now complete the following exercise by writing the digits in words.

1.3 The cost of a laptop computer valued at R13 728.

1.4 The cost of a small passenger vehicle priced at R200 914

1.5 The cost of a home in an upmarket area is priced at R3 050 425.

**Check your Answers**

*1.3 Thirteen thousand seven hundred and twenty eight rand*

*1.4 Two hundred thousand nine hundred and fourteen rand*

*1.5 Three million, fifty thousand, four hundred and twenty five rand*

**Activity for guided reflection**

Write the following using digits (numbers) only:

Four hundred and twenty six billion, two hundred and thirty five million and four hundred

and two thousand and four rand.

**Guided reflection:**

As you read the numbers in words identify the groupings in terms of billions, millions,

thousands and hundreds.

How many billions are contained in the question?

*426 billions*

How many millions are contained in the question?

*35 millions*

How many thousands are contained in the question?

*402 thousands*

How many hundreds are contained in the question?

*Zero hundreds.*

Now write out the numerical value:

*R426 035 402 004*

Now attempt the following questions. Write the following using numbers only:

1. A population of six million, eight hundred ninety-eight thousand seven hundred.
2. Four hundred and fourteen thousand, six hundred and three rand.
3. Sixteen million, five hundred sixteen thousand and ninety five rand.

Solution (Check also table that follows as an alternate solution)

1. 6 898 700
2. R414 603
3. R16 516 095

**O**r you may use the table to help you as follows:

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **Ten Millions** | **Millions** | **Hundred****Thousands** | **Ten****Thousands** | **Thousands** | **Hundreds** | **Tens** | **Ones** |
| 1. |  | 6 | 8 | 9 | 8 | 7 | 0 | 0 |
| 2. |  |  | 4 | 1 | 4 | 6 | 0 | 3 |
| 3. | 1 | 6 | 5 | 1 | 6 | 0 | 9 | 5 |

**Estimation**

The ability to estimate final solutions in context is of great importance for many applications of QL. Regrettably, estimation is a skill that is usually overlooked, as Mathematics educators don’t perceive estimation as its responsibility. Moreover, educators in other subject areas believe estimation skills form part of mathematics and so don’t teach it. Many students therefore find estimation difficult.

**Estimation:** Finding a number close enough to the exact answer.

In estimation the method that is often used is to guess the number of items that are in a portion of a collection to get a sense of the entire collection. The following pictures show how difficult it is to get the exact number quickly and easily. Therefore, estimation is necessary.



When estimating:

* You are not seeking to get an accurate answer.
* You intend getting an answer that is close enough quickly.
* You can save time, money and effort.

**Illustrative example saving you money:**

**Example**: You want to buy five pencils that cost R1,95 each. When you pay at the till the cost demanded is R12.25. Is that right?

Let’s estimate the cost of one pencil to the nearest whole number?

Is R1,95 closer to R1 or R2?

*R2*

Therefore, what will the estimated cost of 5 such pencils be?

*Multiply your estimate by 5 = R10.*

*It is clear from our estimate that we have been overcharged.*

*Therefore, ask for the total to be checked as this will save you money.*

**Illustrative example saving you time:**

**Example**: You want to plant a single row of flowers. The row is 68,3cm long. Each plant should be 7cm apart. Determine how many plants you will require to complete the single row.

Let’s estimate the length of the row to a number that is easier to work with.

What number will be appropriate, that is, will be easier to divide by?

*70 cm as 68,3 is closer to 70.*

Now how many 7cm can we get from 70 cm?

*70* $÷$*7 = 10*

Therefore, approximately how many plants will we be able to plant in this single row?

*10 plants*

This will therefore save you valuable time.

**Illustrative example saving you on effort:**

**Example**: You use your calculator to solve 105 $×$46, and your calculator shows an answer of 690. Is this answer correct?

Let’s start by estimating both our numbers to whole numbers that will be easier to multiply without using a calculator.

The first number 105 will be easier to use 100 and 46 will be replaced with 50 as it is easier to work with multiples of 10.

When you multiply by 10, what do we do to get out answer? We simply add a zero to the number we multiplying by.

Therefore, 50 $×$ 10 = 500. Add a zero to 50 to get 500.

We can therefore see that 690 was not a correct answer as it is much more than our estimated answer.

This will therefore save you much effort.

**Activity for reflection**

Can you estimate how many glasses of cool drink can be poured from a 2 litre bottle of Coke?

Reflection:

How many millimetres are there in 2 litres?

*2 000 ml*

What will half this quantity be?

*1 000 ml*

Hence, find out what will half the previous quantity be?

*500 ml*

About how many glasses of cool drink will you be able to get from 500 ml?

*2 glasses*

Now find out how many glasses can you get from 1 000 ml?

*4 glasses*

Hence, find out how many glasses will we be able to get from 2 000 ml of cool drink?

*8 glasses.*

## Rounding

A simple form of estimation is rounding. Rounding is a critical skill required to quickly estimate a number. This is where you make a long messy number simpler by rounding, or expressing it in terms of the nearest unit, ten, hundred, tenth, or a certain number of decimal places. Sometimes you may be in a shop and want to know whether you have sufficient money to purchase items, you will use rounding to roughly work out the total value of your purchase and whether you can afford the amount of the purchase.**Rules for rounding**:

**Rounding** means making a number simpler but keeping its value close to what it was

* If the digit you are rounding is followed by 5, 6, 7, 8, or 9, then round the digit up .Example: 46 rounded to the nearest ten is 50.
* If the digit you are rounding is followed by 0, 1, 2, 3, or 4, then round the digit down. Example: 23 rounded to the nearest ten is 20.
* Remember when rounding, all the digits to the right of the digit you are rounding becomes 0.
* In rounding off decimals, look only at the number in the place you are rounding to and the number that follows it. Similar rules apply.

|  |
| --- |
| ***Find the digit,******Look next door******Five or higher,******Add one more******Four or less,******Let it rest!*** |

**The rounding rhyme: Sing along**

**Illustrative examples**:

1. The number 5 718 rounded to the nearest:
* Ten is 5 720
* Hundred is 5 700
* Thousand is 6 000
1. The number 6,7198 rounded off to:
* One decimal place (or tenth) is 6,7
* Two decimal places (or hundredth) is 6,72
* Three decimal places (or thousandth) is 6,720

**Rounding in context**

Sometimes contexts dictate the type of rounding for practicality reasons.

When we are working in everyday real-life situations, we must consider what the results of rounding off will be. The answer must be practical so that it is not only correct, but it also makes sense in the context.

**Illustrative example**: When we want to determine the number of carpet tiles to cover an area that is 10,1 square metres one will have to buy 11 square metres and not 10 as 10 square metres will result in a portion of the floor without carpeting. This is rounding “up” in context.

**Illustrative example**: When we want to determine the number of people able to drink from a barrel containing 50 litres of beer if each person drinks 1,5 litres of beer, we will have to round down as rounding up will result in the last person not drinking the full quota. The accurate answer is 33,33 people which will be rounded down to 33 people and not 34.

Activity: Number formats (digits and words), rounding and estimations in context

**Purpose**

Sometimes, you do not need to know an exact value. Instead, you need to know an approximate amount, or about how many. Other times you may be in a shop and want to roughly know the amount you have to pay. You can find this information by rounding, or changing, a number to the nearest ten, hundred, thousand, ten thousand, hundred thousand, million, or so on.

**What you need:**

* The rounding rhyme
* Calculator for checking
* Table for number formats.

When rounding numbers, keep the following in mind:

* If the digit to the right of the rounding place is less than 5, then round down.
* If the digit to the right of the rounding place is 5 or greater, then round up.
* When you finish rounding, the digit(s) to the right of the rounding place will always be zero(s).

When rounding in context:

* Sometimes you will need to round up, for example, buying tiles, bricks, etc since you cannot ignore parts thereof.

**Illustrative example**. Tiling a floor area of 9,1 m2, one will need 10 m2 as 9 m2 won’t complete the job.

* Other times you will need to round down, for example, placing items in boxes, as rounding up will result in items not fitting.

**Illustrative example**: Consider the following example: Determine the number of people able to drink from a barrel containing 50 litres of beer if each person drinks 1,5 litres of beer.

The accurate answer would be 50 divided by 1,5 = 33,333. Your final answer would need to be rounded down to 33 as 34 people would not be able to drink 1,5 litres each. Therefore, you would have had to **ROUND DOWN** in this context.

**Practice Activity 1 (15 minutes)**

1. Complete the table below by following the instructions in each column.

 The first example has been done for you.

|  |  |  |  |
| --- | --- | --- | --- |
| **NUMBER** | **ROUND OFF TO….** | **WHAT DO YOU LOOK AT** | **ANSWER** |
| a. 2635 | Nearest 10 | Units digit 5 | 2640 |
| b. 20635 | Nearest 100 |  |  |
| c. 39645 | Nearest 1 000 |  |  |
| d. 9,36541725 | 3 decimal places |  |  |
| e. 9,36541725 | 4 decimal places |  |  |
| f. 4,3333333 | 1 decimal place |  |  |
| g. 7,6666666 | 2 decimal places |  |  |
| h. 708,49999 | Nearest whole number |  |  |
| i. 349 495 678  | Nearest million |  |  |

**Reflection**

Before looking at the answers to the exercise think about the following questions and then check your answers.

1. In question b) you are asked to round off 20635 to the closest 100.

Is the number 20635 close to 20600 or 27000?

Remember you round up to the next hundred if the last two digits (the tens digit) is above 50 and round down when the last two digits is less than 50.

2. Write down the steps you went through when rounding off 9,36541725 to 3 decimal places.

 What number do you look at?

*The 5 or 4*

 Is the number less than 5 or equal to 5 or bigger than 5?

 What did you do to complete the rounding process?

**Please check the answers on page 15 before moving on to the next exercise**

1. Complete the table below by following the instructions in each column.

 The first example has been done for you.

|  |  |  |  |
| --- | --- | --- | --- |
|  | **CONTEXT** | **ROUND UP OR****ROUND DOWN** | **ANSWER** |
| a. | On average 134,1 travel on a bus from the station to the airport at 6:00am. What is the minimum number of seats you need to have available, if the number of people that travel on a certain day is close to the average?  | Round up | 135 |
| b. | You require carpet tiles to cover 9,01 m2. How many m2 will you order? |  |  |
| c. | How many cars with length 5 m each can be parked in a single file if the road is 59 m long? |  |  |
| d. | You have a book shelf that is 100 cm long. How many books with width 3 cm each can be placed width wise on the shelf? |  |  |
| e.. | Your retailer rounds off all amounts to the nearest 5 cents. What will the amount R34,06 be rounded to? |  |  |

**Reflection**

In the example involving transport the average value is 134,1.

Is it possible to have 0,1 of a person?

If we reserved 134 seats, will it be sufficient for the average number of people travelling?

The average is more than 134, therefore 134 may not be sufficient as there may be days when there are more than 134 people travelling.

Hence, we reserve more than 134, namely 135 seats to ensure we have sufficient seats for any day.

**Please check the answers on page 15 before moving on to the next exercise**

3. Estimate the following then check your answer using a calculator: $\frac{168×5}{225}$ =\_\_\_\_\_\_.

**Reflection**

The number 168, is it closer to 150 or 200? It is closer to 150

Therefore let’s replace 168 by 150.

Therefore the estimate answer for the numerator is 150 $×$ 5 = 750

Let’s look at the number 225. Is it closer to 225 or 250?

It is the same distance apart. Since we short-changed the numerator we will therefore choose

250 as our estimate for the denominator.

We therefore have an estimate of 750 in the numerator divided by 250 in the denominator

which will give us an estimate as follows: 750 $÷$250 = 3.

**Please check the answers on page 15 before moving on to the next exercise**

**Practice Activity 2 (20 minutes)**

1. The figure below shows a wall which is being built. Estimate the total number of bricks required to complete the entire wall.



**Guided reflection**

What pattern is used to build up the wall?

Does each row have the same number of bricks?

Did you count the number of bricks in the bottom (first) row?

Did you count the number of bricks in the second row?

Count the number of bricks in the third and fourth rows?

What do you notice about the pattern?

Is the pattern the same in every alternate rows?

Did you count the number of incomplete rows?

Now establish an easier method to estimate the number of bricks that will be needed to build this entire wall.

**Please check the answers on page 16 before moving on to the next exercise**

1. The figure below shows “vetkoek” spread out on a baking tray. Estimate the total number of vet-koek” in 100 such baking trays.

**Guided reflection**

You will notice a kind of pattern that is almost similar across the tray. Describe this pattern.

How many columns of vetkoek do we have?

How many rows of vetkoek do we have?

Now estimate how many vetkoek altogether.

**Please check the answers on page 16 before moving on to the next exercise**

1. The figure below shows a book rack which is able to fit 11 books in one compartment. Estimate the total number of similar books required to fill up this book rack.



**Please check the answers on page 16 before moving on to the next exercise**

1. Round off the following prices to the nearest R5:
* R12 for the price of a pen

**Guided reflection**

 IsR12 closer to R15 or R20?

* R38 for the price of 3 loaves of bread

**Guided reflection**

Is R38 closer to R35 or R40?

* R83 for the price of shaving gel

**Please check the answers on page 15 before moving on to the next exercise**

**Summary Assessment 1 (20 minutes)**

Choose the alternative that best describes the statement for questions 1 to 9. Show all working details.

|  |  |
| --- | --- |
| 1. | Estimate the difference by first rounding each number to the nearest ten: 733 – 75 |
|  A.. 700 |  B. 657 |  C. 650 |  D. 660 (2) |
| 2. | Round 7065 to the nearest ten, nearest hundred, and nearest thousand:  |
|  | A. Ten 7060; Hundred 6000; Thousand 7000 |  |
|  | B. Ten 7070; Hundred 6000; Thousand 6000 |   |
|  | C. Ten 7060; Hundred 6100; Thousand 6000D. Ten 7070; Hundred 6100; Thousand 6000  |   (2) |
| 3. | 0,006472867 rounded to three decimal places is: |
|  | A. 0,00647 | B. 0,006 | C. 0,00867 | D. 0,647 (2) |
|  |  |  |  |  |
| 4. | 234 467 895 rounded off to the nearest million is: |
|  | A. 234 000 000 | B. 235 000 000 | C. 234 400 000 | D. 234 500 000  (2) |
|  |  |  |  |  |
| 5. | Sugar is sold only full kilograms. You require 1,32 kg of sugar. How much must you buy? |
|  | A. 1 kg  | B. 2 kg | C. 1,5 kg | D. 1,30 (2) |
|  |  |  |  |  |
| 6. | Seven million and seventy six thousand nine hundred and thirty eight in number format will be:  |
|  | A. 776 938 | B. 7 076 938 | C. 7 076 983 | D. 7 076 938  (2) |
|  |  |  |  |  |
| 7. | Round off the quotient of $\frac{10256}{32548}$ to three decimal places: |
|  | A. 0,315 | B. 0,316 | C. 0,318 | D. 0,3 (2) |
|  |  |  |  |  |
| 8. | 823km rounded off to the nearest 100km is: |
|  | A. 800km | B. 900km | C. 950km | D. 750km (2) |
|  |  |  |  |  |
| 9. | Solve the following by estimation. In 1999, the population of City A was 7 255 831 and the population of City B was 3 942 652. Round each population to the nearest hundred-thousand to estimate the difference in the populations of the two cities.  |
|  | A. 3 500 000 | B. 3 400 000 | C. 3 313 000 | D. 3 300 000 (3) |
| 10. | The pictures below show two similar sized bottles filled with the same amount of Smarties. If the bottle on the right has 16 smarties removed, estimate the total number of smarties in the bottle on the left. (3) |
|  |   |

11. Nonthando is planning her wedding reception. The following costs are budgeted for:

* She budgets R30 000 for food and drinks
* The cost for providing meals is R280 per person
* The budget for hiring tables is R2 500
* The cost to hire tables is R150 per table

Study the above information and then answer the questions that follow.

11.1 Calculate the maximum number of guests Nonthando can invite to her wedding. (4)

11.2 Determine the maximum number of tables she is able to hire. (4)

11.3 Calculate the minimum number of people that will be able to be seated in each of the tables calculated in question 11.3 above to ensure all the guests are seated. (4)

11.4 Nonthando has provided for juice to be served to each of the invited guests.

Juices are only available in 5 litre containers. If each guest is provided with one

250 ml glass of juice, calculate how many 5 litre containers of juice must be bought. (4)

**Guided reflection**

Each exercise provided you an opportunity to assess three important basic skills: Number formats (digits to words and words to digits); rounding (including in context) and estimations. All of these skills are essential in real life as it helps us to make quick decisions and also help us to save time, money and effort.

**Please check the answers on pages 16 - 17 before moving on to the next exercise**

**Solutions for Practice Exercises**

**Practice exercise 1**

1.

|  |  |  |  |
| --- | --- | --- | --- |
| **NUMBER** | **ROUND OFF TO….** | **WHAT DO YOU LOOK AT** | **ANSWER** |
| a. 2 635 | Nearest 10 | Units digit 5 | 2640 |
| b. 20 635 | Nearest 100 | Tens digit 3 | 20 600 |
| c. 39 645 | Nearest 1 000 | Hundredths digit 6 | 40 000 |
| d. 9,36541725 | 3 decimal places | 4th decimal place 4 | 9,365 |
| e. 9,36541725 | 4 decimal places | 5th decimal place 1 | 9,3654 |
| f. 4,3333333 | 1 decimal place | 2nd decimal place 3 | 4,3 |
| g. 7,6666666 | 2 decimal places | 3rd decimal place 9 | 7,67 |
| h. 708,49999 | Nearest whole number | 1st decimal place 4 | 708 |
| 1 349 495 678  | Nearest million | Hundredth thousandth digit 4 | 1 349 000 000 |

2.

|  |  |  |  |
| --- | --- | --- | --- |
|  | **CONTEXT** | **ROUND UP/DOWN** | **ANSWER** |
| a. | On average 134,1 travel on a bus from the station to the airport at 6:00am. What is the minimum number of seats you need to have available, if the number of people that travel on a certain day is close to the average? | Round up | 135 |
| b. | You require carpet tiles to cover 9,01 square metres. How many square metres will you order? | **Round up** | **10 m2** |
| c. | How many cars with length 5 m each can be parked in a single file if the road is 59 m long? | **Round down** | **11** |
| d. | You have a book shelf that is 100 cm long. How many books width 3 cm each can be placed width wise on the shelf? | **Round down** | **33** |
| e.. | Your retailer rounds off all amounts to the nearest 5 cents. What will the amount R34,06 be rounded to? | **Round down** | **R34,05** |

1. (150 × 5) $÷$ 250 = 750 ÷ 250 = 3

Calculator answer = 3,7

**Practice exercise 2**

1. In the first two rows there is an average of 11 bricks, so an estimate will be to multiply 11 by the number of rows: Therefore an estimation will be 11 bricks per row × 25 rows = 275 bricks
2. In the bottom row it seems that the vetkoek are arranged in 6 groups per row and that there are 4 rows. In each group there is 14 vetkoek found.

$∴ $estimation for each row will be: 6 groups × 14 vetkoek = 84 vetkoek

Therefore, the total estimate in one tray will be: 84 vetkoek × 4 rows = 336 vetkoek.

Finally the number of vetkoek in 100 trays will be: 336 vetkoek × 100 trays = 33 600 vetkoek.

1. There are 5 book compartments in each row. Each row can accommodate 11 books.

Therefore each row will have a total of: 11 books × 5 compartments = 55 books per row.

There are 5 rows, therefore final estimate will be: 55 books per row × 5 rows = 275 books.

1. R12 to the nearest R5 will be R10 as the units digit is closer to R10 than R15 in multiples of R5.

R38 to the nearest R5 will be R40 as the units digit is closer to R40 than R35 in multiples of R5.

R83 to the nearest R5 will be R85 as the units digit is closer to R85 than R80 in multiples of R5.

**Summary Assessment 1**

1. Remember the rounding rhyme.

733 to the nearest 10 $\rightarrow $ look at the units (ones) digit $\rightarrow $drop the digit and replace with

0 = 730

75 to the nearest 10 $\rightarrow $ round up as units (ones) digit is 5 = 80

$∴$ 730 – 80 = 650

$∴$ C ✓✓

1. 7 065 to the nearest ten = 7 070

7 065 to the nearest hundred = 7 100

7 065 to the nearest thousand = 7 000

$∴$ D ✓✓

1. Look at the 4th decimal $\rightarrow $ 4, therefore replace with zeros

$∴$ B ✓✓

1. Look at the Hundred thousand digit $\rightarrow $ 400 000 which is less than 500 000

$∴$ Drop 4 and replace all with 0

$∴$ 234 000 000

$∴$ A ✓✓

1. 1,32 kg rounded in context will be 2 kg as you require more than 1 kg and sugar is available in full kilograms

$∴ $B ✓✓

1. Use the number format table

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Ten Millions | Millions | HundredThousands | TenThousands | Thousands | Hundreds | Tens | Ones |
|  |  | 7 | 7 | 6 | 9 | 3 | 8 |

Insert digits starting from left to right $∴$ A ✓✓

1. 1 0256 ÷ 32 548 = 0,3151038466 = 0,315 to three decimal places $∴ $A ✓✓

1. 823 km rounded to the nearest 100 km = 800 km $∴ $A ✓✓
2. 7 255 831 rounded to nearest hundred thousand = 7 300 000

3 942 652 rounded to nearest hundred thousand = 3 900 000

Therefore difference = 7 300 000 – 3 900 000 = 3 400 000 $∴$ B ✓✓

1. The total space occupied by the smarties in one bottle is approximately 7 times the smarties removed from the bottle.

Therefore the total number of smarties in the left bottle = 7 × 16 = 112 smarties. ✓✓✓

11.1 Maximum guests = Total food budget ÷ Cost per person ✓

 = R30 000 ÷ R280 = 107,1428571✓✓

$ ∴$ round in context will be 107 people as 108 people it will be above the budget. ✓

11.2 Maximum number of tables = R2 500 ÷ R150 = 16,667 ✓✓

 $∴$ 16 tables as hiring 17 tables will be above the budget. ✓✓

11.3 Minimum number of people per table = 107 people ÷ 16 tables = 6,6875✓

 $∴$ 15 tables will have 7 people while the 16th table will seat 6 people ✓✓

 (15 tables × 7 guests) + (1 table × 2 guests)= 105 + 2 = 107 guests ✓

11.4 Total amount of juice to be purchased (ml) = 250 × 107

 = 26 750 ml ✓

 = 26, 750 litres ( 1 000ml = 1 litre) ✓

 $∴$Number of 5 litre containers = 26,750 litres ÷ 5 litres

 = 5,35 containers ✓

 $≈$ 6 containers. ✓

## UNIT 2: RATIO, RATE, PROPORTION & FOREIGN EXCHANGE

#### Learning Outcomes

By the end of the unit you should be able to:

* Understand and apply the concepts of ratio, rate and proportion.
* Understand and perform calculations involving foreign exchange.
* Interpret graphs and tables involving direct and indirect proportions.
* Solve practical real-life problems involving ratio, rate, proportions and foreign exchange

### Introduction

We use ratios, proportions and rate in daily activities often without knowing we are making mathematical calculations: example, you divide a pizza among four people, proportioning it equally. We talk about our car giving us 9 km per litre, which is a rate.

**Ratios**

A ratio is used to compare two or more quantities that have the same unit of measure. The parts in a ratio are separated by a colon (:) read as “is to”. Ratios are often used in the real world, when grocery shopping you are comparing prices of similar products); calculating the exact amount for ingredients in recipes or calculating how long a car trip might take.

To simplify a ratio:

**Illustrative example:**

12 : 16 = 3 : 4 We divide both numbers of the ratio by the same value (the highest common factor), which is 4 in this case.

**Illustrative example:**

Alice has 60 cents while Boysie has R1,20. Simplify the ratio of the amount of Alice : Boysie

To simplify the ratio of their amounts, we need to convert both amounts to the smaller unit, namely cents.

Therefore the ratio will be 60 cents : R1,20 = 60 cents : 120 cents – We then divide both parts of the ratio by the highest common factor, 60 in this case $\rightarrow $ 60 cents : 120 cents = 1 : 2. Note that there are no units shown in the final answer as they cancel each other out.

When writing a ratio in its simplest form it may result in one of the numbers being equal to 1. This is called a unit ratio.

**Illustrative example:**

The ratio of males to females is given as 100 : 150.

This ratio simplified as a unit ratio will be 100 : 150 = 1 : 1,5. We divide both sides by whatever number is found on the left side of the ratio, in this case by 100.

**Illustrative examples:**

1. Share R120 between Tom and Jones in the ratio 2 : 3.

**Guided reflection:**

How many parts are there in the ratio altogether?

Can you calculate the value of one part?

How many parts of the total parts does Tom get?

How many parts of the total parts does Jones get?

Now check to see that if you add both shares you get a sum of R120.

1. Juhi’s monthly salary to Jenny’s monthly salary is given in the ratio 2 : 7. If Jenny receives a monthly salary of R35 000, calculate Juhi’s monthly salary.

**Guided reflection:**

Is the R35 000 the total for both salaries?

What is the amount of Jenny’s salary?

How many parts of the ratio does Jenny have?

Calculate the value of one part of the ratio using Jenny’s salary.

How many parts of the ratio does Juhi have?

Now calculate the Juhi’s total salary using the value of one part calculated for Jenny.

1. A recipe for 8 cupcakes requires 2 cups of flour. Determine how many cups of flour will be required for 10 cupcakes.

**Guided reflection:**

How many cups of flour is required for 8 cupcakes?

Now calculate how many cups of flour are required for one cupcake.

Hence, calculate how many cups of flour are required for 10 cupcakes.

**Solutions**

1. The ratio 2 : 3 means there are (2 + 3) parts in the ratio, which is 5 parts.

Therefore it means 5 parts = R120

We calculate the value of 1 part by dividing both sides by 5

$$\frac{5 parts}{5}=\frac{R120}{5}$$

$$∴1 part=R24$$

$$∴2 parts=R24×2=R48$$

$$∴3 parts=R24×3=R72$$

$$Tom will receive R48 while Jones will receive R72$$

1. This example is different from the previous one as the value of the total parts are not given. However the value of one person’s share is given, namely Jenny’s share (7 parts) which is R35 000

$$ \frac{7 parts}{7}=\frac{R35 000}{7}$$

$$∴1 part=R5 000$$

$$∴Juhi^{'}s share which is 2 parts will be=R5 000×2=R10 000 $$

1. 8 cupcakes = 2 cups of flour

$∴1 cupcake=0,25 cups of flour$ (Divide both sides by 8)

$∴10 cupcakes=0,25×10=2,5 cups of flour$ (Multiply both sides by 10)

**Rate**

Rate, like a ratio, is also a comparison between two numbers or units of measure but rate has different units for the given numbers. In fact rate is a special kind of ratio. When we need to use rate, we use words like, “per”, “each” or “at”.

Examples of rate include cost rates, eg. petrol cost R 12,15 per litre or 12,15 R/l) and speed, eg. a car travels at 120 km/h.

You will notice some of the calculations in ratios required us to find the rate of one item before calculating the cost of the other part of the ratio.

To calculate rate, we always divide by the second value, which will thus enable us to calculate the amount per one unit.

We are often required to make a choice as to which quantity of an item is cheaper, rate is often used to make a decision.

**Illustrative examples**

1. A 7 kg bag of potatoes cost R49,00 while a 2 kg bag cost R21. Which is cheaper?
2. Linda, a star athlete, runs 200 m in 30 seconds. What is his speed in metres per second?

If he was able to keep running at this speed, how long would he take to cover 1 km?

1. Shaukat is able to type 106 words in 2 minutes on his laptop while Pilodia is able to type 314 words in 7 minutes. Determine who is able to type at a faster speed.

**Solutions**

1. 7 kg bag cost R49 2 kg bag cost R 21

$ ∴$ 1 kg bag = R49 ÷7 = R7/kg $∴$ 1 kg = R21 ÷2 = R10,50/kg

Therefore the 7 kg bag is much cheaper per kg.

1. 200 m = 3 seconds

$∴$100 m = 1,5 seconds Divide both sides by 2

$∴ $1m = 0,015 seconds Divide both sides by 100

$ $ We know 1km = 1 000 m

 $∴$ 1 000m = 15 seconds Multiply both sides by 1 000

1. Shaukat 106 words = 2 minutes

$ ∴$ In this case it would be better to compare minutes per word

 Therefore we will re-write the sum as:

 2 minutes = 106 words

 $∴$1 minute = 53 words Divide both sides by 2 to get one minute.

Pilodia 7 minutes = 314 words

 $∴$ 1 minute = 314 ÷ 7 words

 $∴$ 1 minute = 44,

Therefore, Shaukat is faster as he types 53 words per minute compared to Pilodia

 who is able to type only 44,86 words per minute.

## Proportion

A proportion is a statement that shows ratios are equal, eg. 1 : 2 = 3 : 6.

When solving problems relating to proportions we state the ratios as fractions, setting the two fractions equal to each other, [cross-multiplying](http://www.purplemath.com/modules/ratio2.htm#crossmultiply), and solving the resulting equation for the unknown.

**Illustrative example**: Find the unknown value in the proportion:  2 : x = 3 : 9.

**Solution:** 2**:** *x* = 3**:** 9

First, convert the colon-based ratios into fractional form:

 

 Then solve the proportion by cross multiplying and finding the unknown:

 
 9(2) = x(3)
 18 = 3x
 6 = x

**Illustrative example**: If twelve inches correspond to 30,48 centimetres, how many centimetres are there in thirty inches?

**Solution**: Set up your ratios as follows:

 Inches : centimetres = inches : centimetres

$∴$12 inches : 30,48 cm = 30 inches : ? cm (Let c be the ?)

 Writing the ratios in fractional form.

 $\rightarrow \frac{12}{30,48}=\frac{30}{c}$

$\rightarrow 12×c=30,48×30$ Cross multiply both sides
 $∴$12c = 914,4
c = 76,2

NB. One can use rate instead of proportions to solve the same sum as follows:

12 inches = 30,48 cm

$∴$ 1 inch = 30,48 cm ÷ 12 Dividing by 12 on both sides.

$∴$ 1 inch = 2,54 cm

$∴$ 30 inches = 2,54 cm × 30 = 76,2 cm This maybe easier.

When a proportion increases by the same ratio (factor), we call this a DIRECT PROPORTION.

(See table below each column (hours : payment) results in the same ratio 1: 85)

**Illustrative example:**

If you work and get paid by the hour, then the more hours you work the more you will get paid. If your payment per hour is R85,00, then the following table shows your payment for the different hours worked:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Hours worked | 1 | 4 | 6 | 9 | 12 | 20 |
| Payment in rand | 85 | 340 | 510 | 765 | 1 020 | 1 700 |
| Ratio | 1:85 | 1:85 | 1:85 | 1:85 | 1:85 | 1:85 |

You will notice from the table as the time increases the payment increases in the same ratio as well.

If a graph had to be drawn for a direct proportion it would be a straight line graph.

This straight line graph will pass through the origin, like the one immediately below, show that the quantities on the graph are in direct proportion. This graph states, therefore, that A is directly proportional to B. It also states that B is directly proportional to A, but we will work with the statement A is directly proportional to B.



When a proportion increases by the same product, we call this an **INDIRECT or INVERSE PROPORTION**.

(See table below – constant product of 12). As one increases the other decreases.

**Illustrative example:**

A motorist travelling at a speed of 100 km/h can complete a trip in 1 hour. If his speed increase he will take a shorter time to complete the same trip. Likewise if he reduces his speed he will take a longer time to complete the same journey. See the table below:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Speed in km | 1 | 2 | 4 | 5 | 10 | 20 | 50 |
| Time in hours | 100 | 50 | 25 | 20 | 10 | 5 | 2 |
| Product | 100 | 100 | 100 | 100 | 100 | 100 | 100 |

You will notice from the table as the speed increases the time decreases and the product in each column is 100.

If a graph for the above table had to be sketched (indirect or inverse proportion) it will be a curve graph called a hyperbola. See sketch below.



## Exchange rates

Exchange rate 9or rate of exchange) is the price of a country’s currency in terms of another currency.

Most countries often compare their currency with the US dollar (USD) to evaluate the strength of their own currency.

Therefore a rand dollar exchange will mean the amount of rands we require to buy one US dollar.

A weak rand will imply that more rands will be required to purchase 1 USD.

A strong rand will imply that less rands will be required to purchase 1 USD.

Everybody wants a weaker currency to encourage exports and to remain competitive. However, a strong currency also has some advantages example: lower inflation: a strong currency lowers the cost of imported goods, enabling lower prices for consumers.

Various factors can cause the exchange rate to change, namely, interest rate, inflation rate, political instability, the general state of the economy, etc.

**Illustrative example**

The table that follows shows the exchange rate between the rand and the major currencies around the world on 1 September 2016. Use the table to complete the questions which follow:

| South African Rand | 1 rand is worth ? units | 1 unit is worth ? rands |
| --- | --- | --- |
| US Dollar | [0,068847](http://www.x-rates.com/graph/?from=ZAR&to=USD) | [14,524946](http://www.x-rates.com/graph/?from=USD&to=ZAR) |
| Euro | [0,061340](http://www.x-rates.com/graph/?from=ZAR&to=EUR) | [16,302687](http://www.x-rates.com/graph/?from=EUR&to=ZAR) |
| British Pound | [0,051840](http://www.x-rates.com/graph/?from=ZAR&to=GBP) | [19,289949](http://www.x-rates.com/graph/?from=GBP&to=ZAR) |
| Indian Rupee | [4,605301](http://www.x-rates.com/graph/?from=ZAR&to=INR) | [0,217141](http://www.x-rates.com/graph/?from=INR&to=ZAR) |
| Australian Dollar | [0,091589](http://www.x-rates.com/graph/?from=ZAR&to=AUD) | [10,918305](http://www.x-rates.com/graph/?from=AUD&to=ZAR) |
| Canadian Dollar | [0,090275](http://www.x-rates.com/graph/?from=ZAR&to=CAD) | [11.077271](http://www.x-rates.com/graph/?from=CAD&to=ZAR) |
| Singapore Dollar | [0,093718](http://www.x-rates.com/graph/?from=ZAR&to=SGD) | [10.670258](http://www.x-rates.com/graph/?from=SGD&to=ZAR) |
| Swiss Franc | [0,067080](http://www.x-rates.com/graph/?from=ZAR&to=CHF) | [14,907530](http://www.x-rates.com/graph/?from=CHF&to=ZAR) |
| Malaysian Ringgit | [0,283942](http://www.x-rates.com/graph/?from=ZAR&to=MYR) | [3,521842](http://www.x-rates.com/graph/?from=MYR&to=ZAR) |
| Japanese Yen | [7,011683](http://www.x-rates.com/graph/?from=ZAR&to=JPY) | [0,142619](http://www.x-rates.com/graph/?from=JPY&to=ZAR) |

 Source <http://www.x-rates.com/table/?from=ZAR&amount=1>

Use the above table to answer the questions that follow.

1. Explain the table.
2. Which country will be the cheapest for a South African to visit? Give a reason for your answer.
3. Which country will be the most expensive for a South African to visit? Substantiate your answer.
4. Determine how many euros will I be able to get for R2 500.
5. Calculate how many rands I will be able to get for my 300 rupees.

**Solutions:**

1. The above table shows the rate of exchange for 10 countries with respect the value of the South African ZAR (rand). The second column shows the value of 1 ZAR (rand) in terms of the other currencies while the third column shows the value of I unit of other countries’ currencies in terms of the South African ZAR (rand). In other words the third column shows us how much of rands we require to purchase one unit of the other country’s currency.
2. Japan – You will be able to get more Japanese yen for your rands. The table shows that R1 has a value of 7,011693 yen or I yen is equivalent to 0,142619 rand, which is far less than 1 rand. Therefore Japan is cheapest currency for South Africa than any other currency in the table.
3. England/Britian – You need the most rands to purchase one British pound. The South African R1 is only worth 0,051840 British pounds whereas I British pound is worth a staggering R[19,289949](http://www.x-rates.com/graph/?from=GBP&to=ZAR).
4. The best way to set this exchange rate question is to use unit rates given in the table.

R1 = [0,061340](http://www.x-rates.com/graph/?from=ZAR&to=EUR)

$∴R2 500= $R2 500 × 0,061340 Multiplying both sides by R2 500.

 = 159,34 euros

Sometimes you may not be given column 2 but only column 3. Then set the answer as a ratio and proportion example as follows

(Remember when you equate two ratios it is called a proportion).

 1 euro = R[16,302687](http://www.x-rates.com/graph/?from=EUR&to=ZAR)

 ? = R2 500

Set like currencies underneath each other

 Thereafter you cross multiply and equate as follows

 1 euro × R2 500 = ? × R16,302687

 Then you solve for the missing part as follows

 ? = $\frac{1euro ×R2 500}{R16,302687}$

 = 153,34 euros

1. Similar to the previous example we can use unit rates or ratios and proportions

1 rupee = R[0,2171410,217141](http://www.x-rates.com/graph/?from=INR&to=ZAR) As given in the table

$∴$ 300 rupess = 300 × R0,217141 Multiply both sides by 300

$ $= R65,14

 OR using ratio and proportions

 1 rupee = R0,217141

 300 rupees = ?

Cross multiply and equate as follows:

 1 rupee × ? = 300 rupees × R0, 217 141

 ? = $\frac{300 rupees ×R0,217 141}{1 rupee}$

 = R65,14.

Activity: Ratios, rate, proportions and exchange rates in context

**Purpose**

Understand the significance that ratios, rates, proportions and rate of exchange plays in everyday life. When we go shopping we need to shop wisely: ratios and rates help simply purchase decisions. When we go overseas on holiday we need to know the value of our currency in terms of the country we visiting.

Students must be able to use graphs and tables to determine direct and inverse proportions.

**What you need:**

Know the definitions of the different concepts: ratio, rate, proportions (direct and inverse/indirect) and understand unit currencies and apply appropriate skills to convert currencies.

Know how to identify direct and inverse proportions from tables and graphs.

Be able to interpret graphical representations of direct and inverse proportions.

**Suggested time: (2 hours)**

**What you will do:**

You will be required to use your skills involving ratios, rate, proportions and rate of exchange to solve problems in context.

Recall ratio – is the comparison of two or more quantities have the same measure of unit.

Rate is a special kind of ratio having different units. Important for use in comparing cheaper products by working out unit cost.

Proportion is an equation formed when two ratios are set equal to each other.

Use direct and inverse proportions to solve practical everyday problems in context.

**Practice Exercise 4 (45 minutes) – Ratio and Rate**

1. Sandy’s boss pays her R50,00 per hour for normal hours worked from Monday to Friday.

Normal hours are from 08:00 to 17:00. Any time beyond these hours are classified as overtime hours and the rate will be time and a half. Weekends and public holidays are classified as overtime hours.

* 1. If Sandy worked the entire week (Mon to Fri) plus 3,5 hours overtime, then calculate her wages at the end of the week.

**Guided reflection:**

How many normal hours will Sandy work in one day?

How many normal hours in a working week (Monday to Friday)?

What is the rate per hour for a normal day worked?

Calculate the total amount earned for the normal hours worked in a week. (Normal rate $×$ number of normal hours worked).

How many overtime hours did Sandy work?

Calculate the total amount earned for overtime hours worked. (Overtime rate $×$ number of overtime hours worked).

Hence, calculate the total amount earned which includes normal hours and overtime hours.

* 1. If Sandy was paid R1 500 for a certain week and she did not work any overtime hours, calculate how many normal hours she worked.

**Guided reflection:**

How many normal hours in a working week?

What is the rate per hour for normal hours worked?

Now calculate the number of hours. (Total weekly wage $÷$ rate per normal hour).

* 1. If Sandy worked the entire month consisting of 4 full weeks (normal time) and 5,5 overtime hours for the month, then calculate her total wage for the month.

**Guided reflection:**

How many normal hours will there be a working month? (Exclude weekends)

Calculate the total amount earned for normal hours worked.

Calculate the total amount earned for overtime hours worked.

Hence calculate her total monthly salary including overtime hours worked.

* 1. On one normal Friday her boss told her that she needed to work from 08:00 until 19h30 as they had an important order to be shipped off the next morning. It was also pay day that Friday. How much did Sandy get paid at the end of that week if the other 4 days in the week she worked the full normal hours?

**Guided reflection:**

How many normal hours did she work for the entire working week?

How many overtime hours did she work for on Friday if normal hours stop at 17:00?

Now calculate the total amount she received at the end of the week.

* 1. Sandy was unhappy with her wage and approached her boss for an increase. Her boss agreed to give her an additional R10 per normal hour worked and her new overtime rate will be new normal time and a half.
1. Calculate her new overtime rate.

**Guided reflection:**

What is her new overtime rate?

Now add R10 to this rate to calculate the new overtime rate.

1. Calculate how much she will now earn if she worked a full week from Monday to Friday working normal hours and 6 hours on Saturday.

**Guided reflection:**

How many normal hours are there in a working week?

Calculate the amount earned for the normal hours worked for the week.

Calculate the amount earned for the overtime hours worked.

Hence, calculate the total amount earned for the entire week including the overtime hours worked.

1. Thabo requires the following ingredients to bake a cake which feeds 4 people: The main ingredients for the cake are:

 **Recipe for 4 people**

**2 cups flour 1 cup milk 1 cup sugar**

**2 eggs 100 ml butter 1 tablespoon baking powder**

$\frac{1}{2}$ **teaspoon Vanilla essence**

**Table for conversions**

|  |  |  |  |
| --- | --- | --- | --- |
| **Measure** | **Millilitres (ml)** | **Same as…** | **In .ounces** |
| ¼ teaspoon | 1.25 ml | ‘a pinch’ | 0.04 |
| 1 teaspoon | 5 ml |  | 0.17 |
| 1 tablespoon | 15 ml | 3 teaspoons | 0.5 |
| 2 tablespoons | 30 ml |  | 1.05 |
| 4 tablespoons | 60 ml | ¼ cup | 2.1 |
| 1 cup | 250 ml |  | 8.8 |

Use the recipe given and the table to answer the question that follows.

Calculate what amount of each ingredient Thabo will now require if he wants to bake a cake for 10 people?

**Guided reflection:**

For how many people is the recipe catering for?

Can you calculate the rate per of ingredient per person? Eg. 2 cups flour required for 4 people. How many cups of flour will be required for 10 people? Multiply the rate per person by 10.

In the same way we calculate the quantity of the other ingredients.

1. The graph shown below represents the time taken (in seconds) by Flick to walk around the park.

The distance covered is given in metres. Study the graph and then answer the questions which follow.

* 1. Write down the type of proportion represented in the graph above giving a reason for your answer.

**Guided reflection:**

Remember the two types of proportions? One type has both variables increasing in the same ratio while the other type has both variables moving in opposite directions but resulting in the same product.

You need to write down the name of the type of proportion.

* 1. Determine the time taken if Flick walked 28m.

**Guided reflection:**

The horizontal axis is called the x-axis which is the “Time” taken while the vertical axis is called the y-axis which represents the “Distance” covered.

Which axis will you look at to answer this question?

Once you identified this locate the value and follow that value until it intersects the graph, then read the value that corresponds this value on the graph. For example: If a distance of 8 metres is taken then the corresponding time will be 4 seconds.

* 1. Determine the distance walked if Flick took 23 seconds to complete his walk.

**Guided reflection:**

Which axis value is now given to you?

Now find the corresponding value that will answer the question.

* 1. Calculate how long it would take Flick to walk 100m.

Now remember the graph does not go to 100 metres, so you will have to work with ratios to get your answer.

* 1. Flick has covered a distance in a time of 1 hour 30 minutes. Calculate the distance covered.

**Guided reflection:**

How many seconds in a minute?

How many minutes in an hour?

Hence, find out how many seconds in an hour to answer this question.

* 1. Flick intends walking a distance of 1,5 km. Calculate the time (in hours) it would take him.2
1. The table below shows the number of days it takes a builder to complete a house. Study the table and then answer the questions which follow.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| No. of builders | 1 | B | 5 | 10 | 75 | 150 |
| No. of days | A | 25 | 15 | C | D | 0,5 |

* 1. What type of proportion is represented in the table above? Give a reason.

**Guided reflection:**

Look at the two completed columns and find a pattern.

(Hint: Multiply and check if they give the same product or Divide to see if they give the same quotients)

* 1. Calculate the missing vales A, B, C and D.

**Guided reflection:**

Use the property of the type of proportion to calculate the missing values.

* 1. Represent the above table by means of a graph.

**Guided reflection:**

Choose a suitable scale for the horizontal (x-axis) and the vertical axis (y-axis) that will cover all the values for number of builders and number of days.

Remember to plot a point (5 ; 15) means start with 5 on the x-axis and then locate 15 on the y-axis and where the two lines meet will be the point (5 : 15)

Make sure you plot your points accurately.

* 1. Describe the graph you have drawn.

**Guided reflection:**

Go back to your answer in Question 4.1 above. This will help you identify the type of graph.

**Practice Exercise 5 (20 minutes) – Ratios, Rate and foreign exchange**

1. If 2 litres of fuel cost R22,95, then calculate the value of 5 litres of fuel.

**Guided reflection:**

What do we have to calculate?

So get the unit cost of the quantity you have to calculate.

Hence, to get the answer we merely multiply the unit cost by the number of units required.

1. Jali is able to walk 4 km per hour. Walking at the same pace how many km’s would she have covered in 4 minutes?

**Guided reflection:**

The first part gives you the distance walked in km per hour.

You notice that the question requires an answer in km per minute.

What conversion do you think is necessary?

Now calculate the distance covered (in km) in one minute.

Hence multiply your answer by the required number of units.

1. A restaurant with 40 tables can seat 700 people. This restaurant has only two types of tables, 10-seater and 20-seater tables. Determine the ratio of the 10-seater tables to the 20-seater tables.

**Guided reflection:**

This question requires a starting point which is trial and error.

Do you think we will require more 20-seater tables than 10-seater tables for the 700 guests?

If we choose twenty 10-seater tables, then how many people will we be able to be seat?

How many 20-seater tables will be left over to seat the other guests?

Will these tables all add up to the maximum number of 40 tables available?

Will all the guests be seated using this arrangement?

If the answer is no, then we carry on trying other combinations until we get one that seats all the guests.

1. A dripping tap wastes 50ml of water per minute. Calculate the amount of water wasted in a day.

**Guided reflection:**

How much of water will be wasted in an hour?

How many hours in a day?

Hence, calculate the amount of water wasted in a day.

1. Study the exchange rate table below and then use it answer the questions which follow:

| South African Rand | 1 rand is worth ? units | 1 unit is worth ? rands |
| --- | --- | --- |
| US Dollar (($) | [0,068847](http://www.x-rates.com/graph/?from=ZAR&to=USD) | [14,524946](http://www.x-rates.com/graph/?from=USD&to=ZAR) |
| Euro (€) | [0,061340](http://www.x-rates.com/graph/?from=ZAR&to=EUR) | [16,302687](http://www.x-rates.com/graph/?from=EUR&to=ZAR) |
| British Pound (£) | [0,051840](http://www.x-rates.com/graph/?from=ZAR&to=GBP) | [19,289949](http://www.x-rates.com/graph/?from=GBP&to=ZAR) |
| Indian Rupee (http://www.xe.com/themes/xe/images/pages/curSymbols/curSymbolINR.gif) |  [4,605301](http://www.x-rates.com/graph/?from=ZAR&to=INR) | [0,217141](http://www.x-rates.com/graph/?from=INR&to=ZAR) |
| Australian Dollar AUD($) | [0,091589](http://www.x-rates.com/graph/?from=ZAR&to=AUD) | [10,918305](http://www.x-rates.com/graph/?from=AUD&to=ZAR) |
| Canadian Dollar CAD($) | [0,090275](http://www.x-rates.com/graph/?from=ZAR&to=CAD) | [11.077271](http://www.x-rates.com/graph/?from=CAD&to=ZAR) |
| Singapore Dollar SGD($) | [0,093718](http://www.x-rates.com/graph/?from=ZAR&to=SGD) | [10.670258](http://www.x-rates.com/graph/?from=SGD&to=ZAR) |
| Swiss Franc (CHF) | [0,067080](http://www.x-rates.com/graph/?from=ZAR&to=CHF) | [14,907530](http://www.x-rates.com/graph/?from=CHF&to=ZAR) |
| Malaysian Ringgit (RM) | [0,283942](http://www.x-rates.com/graph/?from=ZAR&to=MYR) | [3,521842](http://www.x-rates.com/graph/?from=MYR&to=ZAR) |
| Japanese Yen (¥) | [7,011683](http://www.x-rates.com/graph/?from=ZAR&to=JPY) | [0,142619](http://www.x-rates.com/graph/?from=JPY&to=ZAR) |

* 1. A student bought a pair of sunglasses in the USA for $35.50. In England, an identical pair of sunglasses costs £26.99. In which country were the sunglasses cheaper, and by how much (in rands)? Show all your working.

**Guided reflection:**

Ideally you should find a common unit to convert both currencies to, like rands.

You need to look at the correct column namely if it is USA dollars you converting then you need to use the column which states $1 USA =? rands.

Once converted it becomes easier to compare which is cheaper.

* 1. In the USA Rob buys a shirt for $67. In Australia the same shirt costs AUD 77.50 calculate the difference between the cost of the shirt in the USA and in Australia.

Give your answer in rand.

**Guided reflection:**

Once again we can only compare them if they have the same unit of currency.

So you need to convert countries’ currencies to rands.

**SUMMARY ASSESSMENT 2**

Choose the alternative that best describes the statement or answers the question. Show all working details.

1. It takes one hour to drive 80 km. How far can you drive in 20 minutes at that speed?
2. 20 km B. 25 km C. 27 km D. 30 km (3)
3. The ratio of 1 hour to 300 seconds is:
4. 1 : 12 B. 12 : 1 C. 1 : 5 D. 5 : 1 (3)
5. The length and width of a field are in the ratio 5 : 3. If the length of the field is 70 m then its

width is:

1. 112 m B. 43,75 m C. 26,25 m D. 42 m (3)
2. A recipe for dessert requires 3 cups sugar and 4 cups cocoa powder.

If you use 12 cups of cocoa powder, how much sugar will you need?

1. 12 cups B. 16 cups C. 9 cups D. 6 cups (3)
2. A car travels 200 km using 7 litres of petrol. At this rate, how far can the car travel using 35

litres of petrol?

A. 1,000 km B. 1,200 km C. 900 km D. 1,500 km (3)

 6. You can buy 8 cupcakes for R4,64. What is the unit price for each cupcake?

A. R0,60 B. R1,72 C. R0,58 D. R1,25 (2)

7. Jerry bought bananas, apples and pears. The ratio of bananas to apples was 3 : 5 and the ratio

of apples to pears was 4 : 2. If he bought 12 bananas, how many pears did he buy?

A. 10 B 12 C. 20 D. 42 (4)

 8. Which team has the best record?

A. Chiefs: 17 wins in 26 games. C. Pirates: 14 wins in 21 games.

1. Sundowns: 21 wins in 30 games. D. Wits: 15 wins in 22 games. (4)

 9. Which ratio is equivalent to 14 : 20?

1. 14,20 : 1 B. 0,7 : 1 C. 1 : 0,7 D. 1,4 : 1 (2)

10. Refer to the exchange rate table in Question 8 of Practice Activity 5 to answer

 this question.

The price of a cup of coffee is ₤2,30 in Great Britain and a similar cup of coffee costs

$3,66 (AUD) in Australia. In which country will you pay the most in ZAR for a cup of coffee?

1. Great Britain B. Australia (3)
2. Urishka works at a hotel as a porter. She receives tips from her guests who travel from around the world. The guests however, remunerates her in their currency.

Study the tables showing the total tips she received in a week and exchange rates and then answer the questions that follow.

**Table 1: Tips received for the month of December 2018.**

|  |  |
| --- | --- |
| **Country of origin of Guest** | **Amount received** |
| Japanese  | **¥84,50** |
| British | **₤6,50** |
| Brazil | **39,50BRL** |
| Indian | **₨100** |
| American | **$6,50** |
| Germany | **€9,60** |
| Australian | **9,80 AUD** |
| South African | **R65,00** |

**Table 2: Exchange rates as at May 2018**

| **South African Rand** | **1.00 ZAR** | **inv. 1.00 ZAR** |
| --- | --- | --- |
| US Dollar ($) | [**0.080099**](http://www.x-rates.com/graph/?from=ZAR&to=USD) | [**12.484558**](http://www.x-rates.com/graph/?from=USD&to=ZAR) |
| Euro (€) | [**0.066945**](http://www.x-rates.com/graph/?from=ZAR&to=EUR) | [**14.937717**](http://www.x-rates.com/graph/?from=EUR&to=ZAR) |
| British Pound (₤) | [**0.059203**](http://www.x-rates.com/graph/?from=ZAR&to=GBP) | [**16.891175**](http://www.x-rates.com/graph/?from=GBP&to=ZAR) |
| Indian Rupee (Rs) | [**5.352933**](http://www.x-rates.com/graph/?from=ZAR&to=INR) | [**0.186813**](http://www.x-rates.com/graph/?from=INR&to=ZAR) |
| Australian Dollar (AUD) | [**0.106244**](http://www.x-rates.com/graph/?from=ZAR&to=AUD) | [**9.412267**](http://www.x-rates.com/graph/?from=AUD&to=ZAR) |
| Japanese Yen (¥) | [**8.740066**](http://www.x-rates.com/graph/?from=ZAR&to=JPY) | [**0.114416**](http://www.x-rates.com/graph/?from=JPY&to=ZAR) |
| Brazilian Real (BRL) | [**0.282809**](http://www.x-rates.com/graph/?from=ZAR&to=BRL) | [**3.535955**](http://www.x-rates.com/graph/?from=BRL&to=ZAR) |

11.1 Determine, in rand, the highest amount Urishka received in tips. (7)

11.2 Which country’s guest tipped her the lowest amount? (2)

11.3 Which countries have a weaker currency when compared to South Africa?

Justify your answer. (3)

* 1. Determine which country has the strongest currency in relation to South Africa. (2)
	2. Calculate, in rand, the total value she received in tips. (2)
1. Sketch a graph to represent a direct proportion and an indirect proportion.

Give one property of each type of proportion. (4)

**Solutions for Practice Exercises**

**Practice exercise 4**

* 1. Normal hours worked per day = 9 hours at R50 per hour

 = 9 × R50

 = R450

 Therefore normal hours for 5 days = R450 × 5

 = R2 250

 Overtime hours worked = 3,5 hours at R75 ( Time and a half means normal rate

 Plus half the normal rate, which is R50 + R25 = R75)

 = 3,5 × R75

 = R262,50

 Therefore total wage for the week = R2 250 + R262,50 = R2 512,50.

* 1. Normal hourly rate = R50

 Therefore number of hours in R1 500 = R1 500 ÷ R50 = 30 normal hours.

* 1. Normal weekly wage as calculated in 1.1 above = R2 250

 Therefore, wages for 4 weeks at normal rate = R2 250 × 4 = R9 000

 Wages for 5,5 overtime hours at overtime rate = 5,5 × R75 = R412,50

 Therefore, her total monthly wage was = R9 000 + R412,50 = R9 412,50.

* 1. Wages for 5 full days = R2 250 (See question 1.1 above for calculation)

 Therefore the overtime hours = 19h30 – 17h00 = 2h 30minutes

 = 2,5 (30 minutes = half an hour) × R75

 = R187,50

 Total wage for the week = R2 250 + R187, 50 = R2 437,50.

 1.5 (a) New normal rate hourly rate = R50 + R10 = R60

 Therefore new overtime hourly rate = R60 + R30 + R90.

 (b) New daily normal wage = R60 × 9 hours = R540

 New normal full week = R540 × 4 = R2 160

 Overtime worked = 6 hours × R90 = R540

 Total weekly wage = R2 160 + R540 = R2 700

1. Recipe for 10 people = ingredients for 4 people ÷ 4 = ingredients for 1 part:

Thereafter × 10 = ingredients for 10 people

Flour = 2 cups ÷ 4 × 10 = 5 cups

Milk = 1 cup ÷ 4 × 10 = 2,5 cups

Sugar = 1 cup ÷ 4 × 10 = 2,5 cups

Eggs = 2 ÷ 4 × 10 = 5

 Butter = 100ml ÷ 4 × 10 = 250 ml = 1 cup

 Baking Powder = 15 ml ÷ 4 × 10 = 37,5 ml

 Vanilla essence = 5 ml ÷ 4 × 10 = 12,5 ml.

 OR You could take 10 ÷ 4 = 2,5 and multiply each ingredient by 2,5, to get your

 new ingredients.

* 1. Direct proportion – straight line graph starting from zero or constant ratio 1:2
	2. Ratio is 1:2 means a person takes 1second to complete 2 metres. Therefore, reading from the graph it would take 14 seconds to walk 28 metres. Ratio 1:2.
	3. 23 seconds will cover 46 metres. Read from graph and check with ratio.
	4. 100 metres will take 50 seconds. Read from graph and check with ratio.
	5. 1 hr 30 min = 90 minutes = 540 seconds. Using ratios 540 sec : 1 080 metres.

Therefore Flick will cover 1 080 metres or 1,08 km (1080 ÷ 1000) in 1h30m.

* 1. 1,5 km = 1 500 metres. Using ratios 750sec : 1 500 metres. Therefore it will take 750 seconds or 12,5 minutes (750 ÷ 60) to walk 1,5 km.

4.1 Inverse proportion. As the number of builders increase the number of days

 decrease or there is a constant product (5 $×$ 25) = (150 $×$ 0,5) = 75.

4.2 Constant product must be 75. This means every column must multiply to = 75.

$∴$ A = (75 $÷$ 1) = 75; B = (75 $÷$ 25) = 3; C = (75 $÷$ 10) = 7,5 and D = (75 $÷$ 75) = 1

To check your answers, simply multiply the no. of builders by the no. of days in each column and this total must be 75.

 4.3

* 1. Curve graph or hyperbola.

**Practice exercise 5**

1. 2 litres = R22,95

⸫ 1 litre = R22,95 ÷ 2 = R11,48 (Divide both sides by 2 to get 1 litre)

1. 4 km = 60 minutes (I hour = 60 minutes)

⸫ 1 minute = 4 km ÷ 60 (Divide both sides by 60 to get one minute)

 ⸫ 4 minutes = (4 ÷ 60) × 4 = 0,27 km. (Multiply both sides by 4)

1. Trial and error. Since there are only 40 tables and 700 guests, there must be more 20 seater tables.

We try 20 (10 seater) + 20 (20 seater) = 200 + 400 = 600 guests $\ne $ 700 guests

 15 (10 seater) + 25 (20 seater) = 150 + 500 = 650 guests - getting closer

 10 (10 seater) + 30 (20 seater) = 100 + 600 = 700 guests = got it!!!

$ ∴$ ratio = 10 : 30 = 1 : 3

1. 1 day = 24 hours = 24 × 60 = 1 440 minutes (60 minutes in an hour)

⸫ 50ml = 1 minute

⸫ 1 440 minutes = 50 ml $×$ 1 440 minutes = 72 000 ml = 72 litres ( 1 000 ml = 1 litre)

 5.1 Convert both currencies to rands as it will be easier to compare

$35,50 = R[14,524946](http://www.x-rates.com/graph/?from=USD&to=ZAR) × 35,50 = R515,64

£26.99 = R[19,289949](http://www.x-rates.com/graph/?from=GBP&to=ZAR) × 26,99 = R520,64

Therefore it was cheaper in USA by R5,00.

 5.2 USA = $67 = R[14,524946](http://www.x-rates.com/graph/?from=USD&to=ZAR) × 67 = R973,17

Australia = AUD ($)[10,918305](http://www.x-rates.com/graph/?from=AUD&to=ZAR) × 77,50 = R846,17

Therefore difference = R973,17 – R846,17 = R127.

**SUMMARY ASSESSMENT 2 – Ratio, rate, proportion and foreign exchange**

1. hour = 60 minutes = 80 km

⸫ 1 minute = 80km ÷ 60

⸫ 20 minutes = (80 ÷ 60) × 20 = 26,67 km.

⸫ C ✓✓✓

1. 1 hour = 60 minutes = 3 600 seconds

$\rightarrow $ 3 600 seconds : 300 seconds

$\rightarrow $12 : 1 (Divide both sides by 300)

 ⸫B. ✓✓✓

1. Length : Width

 5 : 3

 70m : ?

? = (70 m × 3) ÷ 5 = 42 m

OR 5 parts = 70 m

 ⸫ 1 part = 70 ÷ 5

 = 14 m

⸫ 3 parts (width) = 3 × 14 = 42 m

⸫D ✓✓✓

1. 3 : 4

4 cups cocoa × 3 = 12 cups cocoa

⸫3 cups sugar also × 3 = 9 cups sugar

⸫ C ✓✓✓

1. 200 km = 7 litres

1 litre = 200 ÷ 7

⸫ 35 litres = (200 ÷ 7) × 35 = 1 000 km

⸫A ✓✓✓

1. 8 cupcakes = R4,64

⸫ 1 cupcake = R4,64 ÷ 8 = R0,58

⸫C ✓✓

1. B : A

3 : 5

12 : ?

⸫ 3 parts bananas = 12

 1 part = 4 (Divide by 3 both sides by 3)

⸫ Pears = 5 × 4 = 20

A : P

4 : 2

20 : ?

⸫ Pears = (20 ÷ 4) × 2 = 10

⸫A ✓✓✓✓

1. Chiefs 17 wins : 26 games = 1 win in 1,53 games

Sundowns 21 wins : 30 games = 1 win in 1,43 games

Pirates 14 wins : 21 games = 1 win in 1,5 games

Wits 15 wins : 22 games = 1 win in 1,47

Therefore Sundowns has the best record

⸫C ✓✓✓✓

1. 14 : 20

0,7 ; 1 (Divide both sides by 20)

⸫B ✓✓

1. Great Britain price = ₤2,30 × R[19,289949](http://www.x-rates.com/graph/?from=GBP&to=ZAR) = R44,37

Australia = $3,66 (AUD) × [10,918305](http://www.x-rates.com/graph/?from=AUD&to=ZAR) = R39,96

Therefore cheaper in Australia.

⸫B ✓✓✓

11.

|  |  |  |
| --- | --- | --- |
| Country of origin of Guest | Amount received | Amount in rand |
| Japanese  | ¥84,50 | ¥84,50 × [0.114416](http://www.x-rates.com/graph/?from=JPY&to=ZAR)  | R9,67 |
| British | ₤6,50 | ₤6,50×[16.891175](http://www.x-rates.com/graph/?from=GBP&to=ZAR) | R109,79 |
| Brazil | 39,50BRL | 39,50BRL ×[3.535955](http://www.x-rates.com/graph/?from=BRL&to=ZAR) | R138,67 |
| Indian | ₨100 | ₨100×[0.186813](http://www.x-rates.com/graph/?from=INR&to=ZAR) | R18,68 |
| American | $6,50 | $6,50×[12.484558](http://www.x-rates.com/graph/?from=USD&to=ZAR) | R81,15 |
| Germany | €9,60 | €9,60×[14.937717](http://www.x-rates.com/graph/?from=EUR&to=ZAR) | R143,40 |
| Australian | 9,80 AUD | 9,80 AUD×[9.412267](http://www.x-rates.com/graph/?from=AUD&to=ZAR) | R92,24 |
| South African | R65,00 | R65 | R65 |
| TOTAL |  |  | R658,60 |

11.1 Highest amount received from German = R143,40 ✓✓✓✓✓✓✓

11.2 Japanese = R9,67 ✓✓

11.3 India and Japan - These countries 1 unit is worth less than the SA rand or you get more rupees and yen for R1. ✓✓✓

11.4 British pound ✓✓

11.5 See table total = R658,60 ✓✓

12.1 Direct proportion \_ Staright line graph – constant ratio ✓✓

Inverse/Indirect proportion = Hperbola or curve graph – constant product ✓✓



## UNIT 3: INTEREST CALCULATIONS

#### Learning Outcomes

By the end of the unit you should be able to:

* Solve problems by calculating percentage.
* Increase or decrease values in a given percentage.
* Calculate discounted amount using a given percentage.
* Calculate percentage discount.
* Calculate original amounts.
* Value Added Tax (VAT).
* Calculate the VAT amount for given exclusive and inclusive prices.
* Calculate the exclusive price for a given inclusive price.
* Use simple interest formulae to determine the interest, the interest rate, the period of investment and the final amount.
* Use compound interest factor tables to determine the initial amount, the final amount and the interest amount.
* If the formula is used then the period must not exceed 2 years.
* Calculate annual inflation / deflation for given products over multiple periods.
* Calculate and compare annual inflation deflation rates.

### Introduction

This section deals predominantly with calculation of discounts, simple interest, compound interest, inflation and deflation. In order to understand these afore-mentioned calculations a concept review will be foregrounded whereby the foundational skills required for percentages will be outlined. The basic skill of percentage increase and percentage decrease will be explained showing how this concept is related to profit or loss percent.

### Percentages

Definition of percent: Percent means “per 100” or “out of 100”. The symbol % is used to represent a percent. For example 37% is the same as $\frac{37}{100}$ .

To convert a percentage into a common fraction or decimal fraction, we divide by 100.

**Illustrative example:**

For example 20% = $\frac{20}{100}$ = $\frac{1}{5} $(common fraction – divide the numerator and denominator by the highest common factor) = 0, 2 (decimal fraction).

Remember when dividing by 100 (two zeroes) we shift the decimal comma two places to the left; or when dividing by 10 (one zero) shift the decimal comma 1 place to the left.

However, when multiplying by 10, 100, 1000, etc shift the decimal comma in the opposite direction, that is, to the right depending how many zeroes in the multiplier.

**Illustrative example:**

We can convert between decimal fractions, common fractions and percentages:

For example: Converting decimal to percentage: 0,07$ × \frac{100}{100}= \frac{7}{100}=7\%$.

**Illustrative example**: Convert from a percentage to decimal:

For example: 23,3% = $\frac{23,3}{100}=0,233$ - First express over 100 (percent) and then move the decimal comma two places to the left to express as a decimal.

**Illustrative example**: Convert a fraction to percent:

For example: $\frac{13}{30}=0,4333= 0,433×100\%= 43,3\%$ - To convert to a percentage we multiply the value by 100%. When we multiply by 100% we are actually multiplying by 1.

**Illustrative example**: Convert percentage to fraction:

For example: 40% = $\frac{40}{100}=\frac{2}{5}$ - Firstly we divide by 100, then we divide the numerator and denominator by the highest common factor to simplify.

**Illustrative example**: To express one quantity as a percentage of another, both require the same unit:

For example: Express 60 cents as a percentage of R4,00 $\rightarrow $ $\frac{60 c}{ 400 c}×100\%=15\%$. Ensure that units are the same when converting into a percentage.

**Illustrative example**: To calculate a certain percentage of a quantity:

For example: Calculate 15% of R400 $\rightarrow $ $\frac{15}{100} × R400 =R60$.

Given the answer to a percentage of a quantity, how do we calculate the original quantity?

This could be an amount given after discount or an amount representing the VAT amount, etc.

**Illustrative example:**

For example: We paid R15,50 which was the 15% VAT amount of a shirt. What was the original price of the shirt, excluding VAT?

We can use one of three Methods to obtain the answer:

**Method 1**: We can use ratios as follows:

 15% = R15,50

 $∴1\% =R15,50÷15$

= R1,03333..

 $∴$100% *=* R103,33

**Method 2**: We can use proportions to solve as follows:

 VAT R15,50 = 15%

 Original Price ? = 100%

 We cross multiply and divide: $\frac{R15,50×100\%}{15\%}$ = R103,33.

**Method 3**: We take the quantity given and divide by the given percentage as follows:

$$ \frac{R15,50}{15\%}=R103,33$$

**Illustrative example:** Lucy paid R240,00 for a jersey after receiving a 6% discount. Calculate the original price of the jersey before discount.

**Hint:**  R240 represents how many percent? 100% ­­­­­­­­­­­­­­­­­­­­­­­­­­­­­ – 6% = 94%

 What percent is the original price? 100%

 First find 1% (divide original price by 94), then multiply answer to get 100%, the original price

 (R255,32)

**Increasing** a quantity/value by a certain percent, like adding a mark up to a cost or adding the VAT amount.

**Illustrative example:**

For example: Increase R56 by 12%

Once again we can use one of three methods to obtain the answer:

**Method 1**: We can use ratios as follows:

 R56 is the original price, therefore it is 100%

 Therefore the increased price will be (100% + 12% = 112%)

 Calculate 1%, then multiply by 112% to find the increased price.

 100% = R56

 $∴$1% = R56 $÷$ 100 = R0,56

 $∴$112% = R0,56 $×$ 112 = R62,72

**Method 2**: We can use proportions to solve as follows:

 Original price R56 = 100%

 New price ? = 112% (100% +12%)

 We cross multiply and divide: $\frac{R56×112\%}{100\%}=R62,7$

**Method 3:** We can use the following method:

 R56 (original price) + $\left(12\%×R56\right) increased percentage$ =

 = R56 + R6,72 = R62,72

We can also use the same methods to calculate a **percentage** **decrease** or offering a discount – This requires decreasing a quantity/value by a given percent.

**Illustrative example:**

For example: The marked price of a belt is R325. The retailer offers you the belt at the price less 5% discount. What price did you pay for the belt?

Once again we can use one of three methods to obtain the answer:

**Method 1:** We can use ratios as follows:

 100% = R325 Original price

 $∴$1% = R3,25 Divide by 100

 $∴$ 95% = R3,25 $×$ 95 = R308,75

 Notice to get the new price we take 100% – 5% = 95%

**Method 2:** We can use proportions to solve as follows:

 Original price R325 = 100%

 New price ? = 95% (100% - 5%)

 We cross multiply and divide: $\frac{R325×95\%}{100\%}=R308,75$

**Method 3:** We can use the following method to decrease:

 R325 (original price) – $\left(5\%×R325\right) decrease percent$ = R325 – R16,25 = R308,75.

**Calculating percent INCREASE or percent DECREASE:**

To calculate a percent change (increase or decrease) the following formula may be used:

 Percent change (Increase or Decrease) = $\frac{Change in price (New Price -Old Price)}{Starting Point (Old Price)}×100\%$

The above formula is not usually given to you. Therefore, you need to memorise this formula.

**Illustrative example:**

For example: Calculate the percentage increase in Urishka’s monthly salary if it increased from R8500 to R9 200 per month.

Change in price = R9 200 – R8500 = R700.

Percent change (Increase) =$ \frac{R700}{R8500}×100\%=8,24\%$

**OR** Alternatively we can work out the new percentage price and then minus the original price to get the percentage increase or decrease. If we get a negative answer it means it’s a decrease.

**Solution:** New Price percent **=** $\frac{New price}{Old Price}×100\%$

$=\frac{R9 200}{R8 500}×100\%$

 $=108,24\%$

Therefore the percentage increase = 108,24% (New price percent) – 100%(Original price percent)

 = 8,24%

**Illustrative example:**

For example: An item marked at R250 was sold for R200. Calculate the percentage decrease.

Change in price = R200 – R250 = – R50. (We ignore the sign as a (–) will mean a decrease.

Percent change (Decrease) = $\frac{R50}{R250}×100\%=20\%$

**OR** Alternatively we can work out the new percentage price and then minus the original price to get the percentage increase or decrease. Remember if we get a negative answer it will be a decrease.

**Solution:** New Price percent **=** $\frac{New price}{Old Price}×100\%$

$=\frac{R 200}{R250}×100\%$

 $=80\%$

Therefore the percentage decrease = 80% (New price percent) – 100%(Original price) = – 20%. Therefore, a decrease of 20%.

Percentage **PROFIT** or percentage **LOSS**.

Calculating these percentages are similar to percentage increase and percentage decrease. However, the only difference is the use of different terminology, namely when dealing with profit or loss, the new price is called the SELLING PRICE, the old price is now called the COST PRICE while the increase is called PROFIT and a decrease is called a LOSS.

The formula will thus change as follows:

 Percent profit/loss = $\frac{Change in price (Selling Price -Cost Price)}{Starting Point (Cost Price)}×100\%$

 Remember if it is a loss the Cost price will be greater than the Selling price.

**Illustrative example:**

For example: Shivan is a street vendor who sells hotdog rolls at the local flea market. His total cost, including labour, ingredients and other overheads works out to be R5,50 per hotdog roll. He sells each hotdog roll for R10,00. Calculate his profit.

Profit amount (increase) = Selling Price – Cost Price

 = R10,00 – R5,50 = R4,50

Percent profit = $\frac{Profit (Selling Price -Cost Price)}{Starting Point (Cost Price)}×100\%$

 = $\frac{R4,50}{R5,50}×100\%$

 = 81,8%

 OR you could use any of the other methods illustrated under percentage increase.

**Illustrative example:**

For example: Juhi makes and sells bracelets at a local crafts market. Her cost to make each bracelet works out to be R8,50. She could not sell them at the original price, so she decided to get rid of the stock and sell each one for R5. Calculate the percentage loss.

 Loss amount (decrease) = R8,50 – R5,00 = R3,50

 Percent loss = $\frac{Loss(Cost Price -Selling Price)}{Starting Point (Cost Price)}×100\%$

 = $\frac{R3,50}{R8,50}×100\%$

 = 41.2%

 OR you could use any of the other methods illustrated under percentage decrease.

**Purpose**

Understand the basic foundational skills involved when using percentages as these skills will be incorporated in real life problems involving finances. Students must be able to apply the different skills relating to percentages to complete calculations embedded in contexts.

**What you need:**

Know the different conversions, namely from percent to decimal to fraction. Know how to apply percentage increase/decrease formula and relate this to percentage profit and loss. Students should be able to perform calculations involving VAT and discounts and working the reverse calculations.

**What you will do:**

You will be required to use your skills percentages to complete problems in context. Students will be required to perform calculations involving VAT, discounts, percentage increase and decrease and also percentage profit or loss. Calculations involving reverse processes will also be assessed.

**Practice exercise 7**

1. A cellphone bill excluding 15% VAT amounts to R325,00. Calculate the total bill with VAT.

**Guided reflection:**

What percent does the original price make up excluding VAT? 100%

What percent does Vat make up? 15%

What percent will the price including VAT be? 115%

Therefore, to calculate the price with VAT, we calculate the percent with VAT.

1. A clothing retailer offers a discount of 10% on all marked prices instore. If a shirt is marked at R325, calculate the discounted price.

**Guided reflection:**

What percent is the shirt before discount? 100%

What percent is the shirt after 10% discount? 90%

Therefore, to calculate the discounted price we calculate the discounted percent of the original price.

1. The number of students at a school in 2016 has increased by 15% since 2015. If 2015 had a student population of 820, calculate the student population in 2016.

**Guided reflection:**

How many percent was the number of students during 2016? 100%

What is the new percent after the increase? 115%

Therefore, calculate the increased percent of the original number.

1. The price of bread has increased from R11,40 to R12,00 because of inflation. Calculate the percentage increase.

**Guided reflection:**

What was the original price of bread?

What is the new price of bread?

Calculate the difference in price.

Now use the formula for percentage increase to complete the solution.

1. The price of a T-shirt inclusive of 15% VAT costs R250,00. Calculate the cost of the T-shirt excluding VAT.

**Guided reflection:**

What was the original percentage of the T-shirt excluding VAT? 100%

What is the percentage of the T-shirt including VAT? 115%

Now calculate 1% of the price including VAT by dividing by 1,15.

Then calculate the original price by multiplying the previous answer by 100.

1. Calculate the percentage discount (decrease) for an item marked at R250,00 but sold for R200,00.

**Guided reflection:**

This calculation is similar to the calculation involving percentage decrease.

You need to find the change in price and then divide by the starting price, thereafter multiply by 100%.

1. The average inflation rate in 2015 was 4,51% and 2016 was 6,54%. If the price of bread in 2014 was R9,80, calculate the price of bread in 2016.

**Guided reflection:**

Which year represents the base (starting) year?

What was the inflation rate for that year?

Calculate the increased priced for 2015 by using the previous year’s inflation rate and price.

What was the inflation rate for 2016?

Now calculate the increased price for 2016.

1. Riyaad purchased a laptop for R9 600. He received R400 discount. Calculate the percentage discount he received.

**Guided reflection:**

What was the original value of the laptop before the discount?

Now calculate the discount as a percent of the original amount.

1. Lynn receives R200 pocket money every month. She spends 90% of this amount she receives every month. Calculate the amount of money (in rands) she saves every month.

**Guided reflection:**

What percent does she save every month?

Now calculate the value she saves every month.

1. A fruit vendor sells bananas at R30 per kg. He makes a profit of 45% on the sale of bananas. Calculate how much a kg he paid for the bananas.

**Guided reflection:**

What percent will the selling price be? 145%

Now use the selling price to calculate 1%.

Hence, calculate the original price, namely 100%.

**Solutions**

**Practice exercise 7**

1. Original price = R325 = 100%

Price including VAT = 100% + 15% = 115%

100% = R325

1% = R3,25

 115% = R3,25 $×$ 115 = R373,75

OR

Increase R325 by 15% as follows:

R325 + (15% $×$ R325) = R325 + R48,75 = R373,75

OR

Calculate 115% of R325373,75

1. Original price = R325 = 100%

Discounted price = 100% - 10% = 90%

100% = R325

 1% = R3,25

90% = R3,25 $×$ 90 = R292,50

OR

Decrease R325 by 10% as follows:

R325 – (10% $×$ R325) = R325 – R32,50 = R292,50

OR

Calculate 90% of R325 = R292,50

1. In 2015 number = 100% = 820

In 2016 number = 100% +15% (increase) = 115%

$∴$ Number in 2016 = 115% of 820 = 943 students.

1. Old price = R11,40

New Price = R12,00

Change in price = R12,00 – R11,40 = R0,60

Percentage increase = $\frac{change in price}{Old price}×100\%$

 = $\frac{R0,60}{R11,40}×100\%$

 = 5,26%

1. R250 with VAT = 100% + 15% = 115%

We need to work out the original price which is 100%.

$∴$ 115% = R250

$∴$1% = R250 $÷$ 115 = R2,173913043

$∴$100% = R217,39

OR

Price before VAT = $\frac{Price with VAT}{115}×100$

 = $\frac{R250}{115}×100$

 = R217,39

1. Original price = 100% = R250

Selling price = ? % = R200

Discount (Change) = R250 – R00

= R50

$∴$Percentage discount (decrease) = $\frac{change in price}{Original price}×100\%$

 = $\frac{R50}{R250}×100\%$

 = 20%

OR

Calculate the selling price as a percentage of the marked price

Selling price % = $\frac{R200}{R250}×100\%$

= 80%

$∴$ Discount % = Original price % – Selling Price %

 = 100% – 80%

 = 20%

1. In 2014 the price was 100% = R9,80

In 2015 the price increased by 4,51% = 104,51%

$∴$ Price in 2015 = 104,51% $×$ R9,80 = R10,24198

In 2016 the price increased by 6,54% from the previous year (new 100%)

$∴$Price in 2016 = 106,54% $×$ R10,24198 = R10,91.

1. Original price (price before discount) = R9 600 + R400 = R10 000

$∴ $Percentage discount (decrease) = $\frac{change in price}{Original price}×100\%$

 = $\frac{R400}{R10 000}×100\%$

 = 4%

OR

Percentage price after discount = $\frac{R9 600}{R10 000}×100\%$

 =96%

$∴$ Dicount % = 100% – 96% = 4%

1. She saves 10% every month (100% – 90% = 10%)

Amount saved = 10% $×$ R200 = R20.

1. Selling price = 100% (Cost) + 45% (Profit) = 145%

$∴$145% = R30

$∴$100% (Cost) = $\frac{R30}{145}×100$

 =R20,69 kg

**Simple Interest**

Interest is the amount paid for the use of money – this interest can be expressed as a percent or as a value.

Simple interest is interest paid on the original principal (amount you initially invest or borrow) only.  With simple interest the amount of interest is fixed over a period of time.

**Illustrative example**

For example: Thabile deposits R250 in a bank account which pays 5% per annum simple

interest. Calculate how much interest Thabile will earn over 4 years.

Solution: Simple interest is the same interest received over a fixed period

 Therefore, interest in year 1 = 5% $×$ R250 = R12,50

 Thabile will receive the same interest amount (R12,50) in year 2, 3 and 4.

 This implies that the total simple interest earned will be **R12,50** $×$ **4 = R50.**

 In summary to calculate simple interest we can use the following formula;

SI = $\frac{P×i×n}{100}$

 Where: SI = Simple Interest

 *P* = Principal (Original amount invested/borrowed)

 *i* = rate of interest

 *n* = period in years.

 Going back to the previous calculation we can use the formula as follows:

 SI = $\frac{P×i×n}{100}$ = $\frac{R250×5×4}{100} $= R50

Sometimes, the question requires an accumulated value for the investment. Therefore, refer to the previous example where Thabile invested R250 at 5% p.a simple interest for 4 years. How to we calculate the accumulated amount after 4 years?

Accumulated value = Principal + Interest earned = R250 + R50 = R300.

Alternatively, we can use a formula to get the accumulated value as follows:

$$A=P\left(1+i×n\right), $$

$$ where A=Accumulated amount; P=Principal; i=interest rate; $$

$$ n=time period in years$$

Therefore, accumulated value can be calculated as follows using the formula:

$$A=P\left(1+i×n\right), $$

 $=$ R250( 1 + $\frac{5}{100}×4)$

 $ = $R300.

We can also use this formula to calculate the interest amount as follows:

Interest = Accumulated amount – Principal = R300 – R250 = R50.

Sometimes, a question will require you to calculate the rate of interest.

**Illustrative example**

For example: Calculate the rate of interest if R210 simple interest is earned over 3 years on an investment of R1 500.

Solution: R200 is earned over 3 years

 $∴$ Interest earned in one year = R210 $÷$ 3 = R70

 $∴ $Rate of interest (Finding a %) = $\frac{R70}{R1500}×100\%=4,67\%.$

**OR** Using the Simple Interest formula

 Interest = P $×i×n$

 R210 = R1 500 $×i ×3$

 R210 = R4 500 $×i$

$$∴i=R210÷R4 500=0,0467=4,674\%$$

Sometimes we are unable to afford to purchase items for cash, so we buy them on a **hire purchase agreement** (loan). Hire purchase (HP) agreements use simple interest calculations.

Most hire purchase agreements require that a deposit be paid before the product can be taken by the customer. The principal amount of the loan is therefore the cash price minus the deposit. The total loan amount is then divided into monthly payments over the period of the loan.

**Illustrative example**

For example: Ashraf wants to buy a laptop computer for R10 500 on a hire purchase agreement. He has enough to pay for the deposit of 10%. Calculate the following:

1. The deposit amount.
2. The total amount he would have paid for his laptop computer, if interest is calculated at 11,5% per annum for 4 years?

Solution:

1. Deposit amount = 10% $×$ R10 500 = R1 050.
2. The hire purchase agreement amount will be calculated as follows:

R10 500 (Original price) – R1 050 (deposit) = R9 450 (Principal owing)

Therefore A = P (1 + i$ ×n)$ = R9 450 (1 + 11,5% $×4)$ = R13 797

The total cost paid for the laptop = R1 050 + R13 797 = R14 847.

**Compound Interest**

Compound interest is the interest earned not only on the original principal, but also on all interests earned previously. In other words, at the end of each year, the interest earned is added to the original amount and the money is reinvested. In other words its “interest on interest”.

Interest can be calculated more than once a year, example, half yearly, quarterly or monthly. In this instance the annual interest rate would be divided by the number of times the interest is calculated in a year. See the table which shows interest calculations for a year for an amount of R10 000 at 10% per annum compounded as follows:

|  |  |  |  |
| --- | --- | --- | --- |
| **Compounded period** | **Number of periods** | **Values of *I* and *n*** | **Total with interest** |
| Annually | 1 | *i*= 10% *n* = 1 (int cal once a year) | R11 000,00 |
| Half-yearly | 2 | *i*= 10%$÷2=5\%$*n* = 2 (int cal2 times a year) | R11 025,00 |
| Quarterly | 4 | *i*= 10%$÷4=2,5 \%$*n* = 4 (int cal 4 times a year) | R11 038, 13 |
| Monthly | 12 | *i*= 10%$÷12=0,83 \%$*n* = 12 (int cal 12 times a year) | R11 047,13 |

**Illustrative example**

For example: Let’s look at the previous example but use compound interest calculation. Thabile deposits R250 in a bank account which pays 5% per annum interest

compounded annually. Calculate how much interest Thabile will earn over 4 years.

We can calculate the compounded amount using THREE different methods:

**Method 1:** Long method which involves repeated calculation of interest each period and finding the new principal for each period and then repeating the process.

 Year 1: Interest = 5% $×$ R250 = R12,50

 $∴$ New principal = R250 + R12,50 = R262,50

 Year 2: Interest = 5% $×$ R262,50 (new principal) = R13,13

 $∴$ New principal = R262,50 + R13,13 = R275,63

 Year 3: Interest = 5% $×$ R275,63 = R13,78

 $∴$ New principal = R275,63 + R13,78 = R289,41

 Year 4: Interest = 5% $×$ R289,41 = R14,47

 $∴$ New principal = R289,41 + R14,47 = R303,88

 Therefore Interest = R303,88 – R250 = R53,88

**Method 2:** Using the Compound Interest formula: A = P (1 + *i*)*n*, where A is the accumulated amount with interest; P the initial amount or principal; *i* the periodic interest rate and *n* the number of times interest is calculated or number of payments made.

 Using the formula: A = ?; P = R250; I = 5% and n = 4

$$A=P(1+i)^{n}$$

 $=R250(1+5\%)^{4}$

 $=R303,88$

 $∴$ Interest = A – P = R303,88 – R250 = R53,88.

**Method 3:** Use a factor table, which most financial institutions use.

**Factor table method**

Table showing compounded future value factors for a single amount of R1,00

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Year** | ***i* = 2%** | ***i* = 3%** | ***i* = 4%** | ***i* = 5%** |
| 1 | 1,020000 | 1,030000 | 1,040000 | 1,050000 |
| 2 | 1,040400 | 1,060900 | 1,081600 | 1,102500 |
| 3 | 1,061208 | 1,092727 | 1,124864 | 1,157625 |
| 4 | 1,082432 | 1,125509 | 1,169859 | 1,215506 |
| 5 | 1,104081 | 1,159274 | 1,216653 | 1,276282 |
| 6 | 1,126162 | 1,194052 | 1,265319 | 1,340096 |
| 7 | 1,148686 | 1,229874 | 1,315932 | 1,407100 |
| 8 | 1,171659 | 1,266770 | 1,368569 | 1,477455 |
| 9 | 1,195010 | 1,304773 | 1,423312 | 1,551328 |
| 10 | 1,218994 | 1,343916 | 1,480244 | 1,628895 |

**Note: Factors are rounded off to 6 decimal places.**

The table above shows the Future Values with interest compounded for a single amount of R1,00 at various interest rates. Note that factors are rounded to six decimal places which could impact on final answers because of early rounding.

A = Principal Amount $×$ factor given as per table = R250 $×$1,215506 = R303,88

Therefore Interest = R303,88 – R250 = R53,89.

Sometimes we have to calculate the value of an item that decreases in value, for example, when we buy a motor vehicle it decreases (depreciates) in value from year to year. Therefore the formula we use is called a compound decrease formula.

**Illustrative example**

For example: Find the value of a vehicle in 3 years’ time if its value now is R200 000 and depreciation is at the reducing balance method of 20% per annum. (Note the word reducing balance implies a compound decrease).

Formula now is modified as follows:

 $A=P\left(1-i\right)^{n}$

$$ =R200 000(1-20\%)^{3}$$

$$ =R102 400$$

**Inflation and Deflation**

Inflation is the average increase in the price of goods annually and is given as a percentage. It is the measure of the annual growth in the price of a product or service, computed as the percentage change in price from the previous year. In simple language, you get less for your money than you used to get.

**Illustrative example**

For example: You buy a loaf of bread for R12,00. A year later you go and buy the same loaf of bread, but it is now R12,80. You still only have R12,00, but the price of bread is gone up. We say that the price of bread has been inflated, that is, inflation is at work. You spend more money to buy the same things that you were able to buy for less.

**Illustrative example**

For example: If the price of petrol rose from R14.00 a litre to R14.80 per litre in one year, the inflation rate for petrol for that year would be calculated like a percentage increase:

Inflation rate (percentage increase) = $\frac{R14,80-R14,00}{R14,00}×100\%=5,71\%$

Since the rate of inflation increases from year to year, it is calculated using the compound interest formula.

Inflation results in the loss of the value of money, because with the same amount of money you can buy less goods in the next year, than you could have bought in the previous year.

Deflation occurs when the general level of prices decreases. A negative inflation rate.

Deflation results in the increase in the value of money, because you can buy more goods in the next year than you could have bought in the previous year.

Deflation occurs when too many goods are available or when there is not enough money circulating to purchase those goods. For example, if a particular type of car becomes highly popular, other manufacturers start to make a similar vehicle to compete. Soon, car companies have more of that vehicle style than they can sell, so they must drop the price to sell the cars.

Deflation can adversely affect the economy in significant ways, example layoffs, unemployment increases as companies cannot sell their goods or don’t make enough profit.

The following tabulates the differences between Inflation and deflation.

|  |  |
| --- | --- |
| **INFLATION** | **DEFLATION** |
| Prices are generally rising | Prices are falling  |
| Rich get richer and poor get poorer | Decreases production, output and income of the entire industry – everybody loses |
| Can be controlled easily | Difficult to control |
| Moderate inflation is considered healthy for an economy | Not healthy for an economy as it affects income levels and output. |

**Reflections**

**Principal** ‐ The principal is the amount of money upon which interest is paid.

**Annual Rate of Interest** ‐ The percentage an investor will earn on an investment each year.

**Interest** ‐ For the saver, interest is the price a financial institution pays for using a saver’s money and is normally expressed as a percentage of the amount saved.

**Simple Interest** – The amount of interest earned on the principal only.

**Compound Interest** – The interest that is earned on the principal and the interest already earned.

**Inflation –** the average increase in the price of goods annually.

**Practice exercise 8**

1. Junaid invested R2 800 in a bank offering 5,5% per annum simple interest. Calculate the interest amount earned after 5 years.

**Guided reflection:**

What amount of interest is earned in year one?

Is the same interest received for all 5 years?

Hence, calculate the total interest earned over five years.

1. Musi invested R500 for 9 months in a bank offering 6% per annum simple interest. Calculate the accumulated amount Musi will have after 9 months.

**Guided reflection:**

What part of a year is 9 months?

Calculate the simple interest for a year.

Now find the interest for 9 months which you converted to years.

You can now find the accumulated value.

1. John buys a laptop for R5 300. He takes a loan from a bank that charges simple interest at 14% simple interest per annum for 24 months. At the end of the first year he pays off R2 400. Calculate the amount John will be owing at the end of the second year.

**Guided reflection:**

What amount of interest will John be paying over the 24 months?

What effect will the payment of R2 400 have on the accumulated amount owing?

1. Rory buys a DVD player for R2 500. He has an option of paying a 10% deposit and then making 24 equal monthly payments using a hire purchase agreement, where interest is calculated at 7,5% p.a. Calculate what Rory's monthly payments will be.

**Guided reflection:**

What type of interest is associated with hire purchase agreements?

Will the deposit be added to the hire purchase agreement?

What is the value of the hire purchase agreement?

Now calculate the monthly instalment.

1. Sia buys a TV set on hire purchase agreement. The cash price of the TV is R10 000. She pays 15% deposit and chooses to pay off the balance over 54 equal monthly instalments at an interest rate of 21% p.a. An insurance premium of R12,50 is added to her monthly payments.
	1. Calculate her monthly instalments.

**Guided reflection:**

What effect does the deposit have on the amount owing?

How much will she be owing after interest is added?

Remember months need to be converted to years for simple interest calculation.

Is the insurance going to be added before calculating the monthly instalment or after?

Now calculate the total monthly instalment.

* 1. Calculate the total amount she paid for the TV set.

**Guided reflection:**

Will the deposit be added to obtain the total value of the TV set?

1. Lydia borrowed R6 000 for 2 years at 5% interest compounded annually from Mari. Calculate the interest paid by Lydia to Mari.

**Guided reflection:**

What is the difference between compound interest and simple interest?

What is the formula to calculate compound interest?

Will Lydia be paying the same interest each year?

1. Find the balance on a deposit of R818, earning 4% interest compounded semi-annually for 2 years.

**Guided interest:**

What does semi-annually mean?

What effect will semi-annually have on the interest rate and the time period?

How will this change the formula for calculating compound interest?

1. Milk currently costs R15,50 for 2 litres. Calculate the cost of the same 2 litres in 4 years’ time if the inflation rate remains constant at 5,5%.

**Guided reflection:**

What type of interest calculation do we associate inflation with?

1. A loaf of bread costs R13,50 today. Calculate the cost of the same loaf 3 years ago if the average rate of inflation rate was 6,5% p.a.

**Guided reflection:**

Is this an example of a compound increase or a compound decrease?

**Solutions**

**Practice exercise 8**

1. Principal (P) = R2 800; interest rate (i) = 5,5; time (n) = 5

SI = $\frac{P×i×n}{100}$

 $=\frac{R2 800×5,5×5}{100}$

$$ =R770$$

1. 9 months converted to years = $\frac{9}{12}$

A = ? P = R500; i = 6%; n = $\frac{9}{12}$

A = P (1 + i$ ×$ n)

 = R500 (1 + 6%$ ×$ $\frac{9}{12}$ )

 = R500 (1 + 0,045)

 = R522,50

1. A = P (1 + i $×$ n) – R2 400

 = R5 300 (1 + 14% $×$ 2) – R2 400

 = R5 300 (1 + 0,28) – R2 400

 = R6 784 – R2 400

 = R4 384

1. Deposit: 10% $×$ R2 500 = R250

Amount owing = R2 500 – R250 = R2 250

Total amount of loan: A = P (1 + i $×$ n)

 = R2 250 (1 + 7,5% $×$2)

 = R2 250(1,15)

 = R2 587,50.

$∴$ Monthly instalments = R2 587,50 $÷$ 24 = R107,81

1. Deposit: 15% $×$ R10 000 = R1 500

Amount owing = R10 000 – R1 500 = R8 500

5.1 A = P (1 + i $×$n)

 = R8 500 (1 + 21% $×$ $\frac{54 }{12}$)

 = R8 500 (1 + 0,945)

 = R16 532,50

 Monthly Instalments = R16 532,50 $÷$ 54 + Insurance

 = R306,16 + R12,50

 = R318,66

* 1. Total amount Paid = Deposit + Instalment $×$ 54

 = R1 500 + R318,66 $×$ 54

 = R18 707,64

1. Compound interest annually

$A=P(1+i)$*n*

 = R6 000 (1 + 5%)2

 = R6 000 (1 + 0,05)2

 = R6 000 (1,10250

 = R6 615

1. Semi-annually = half-yearly

$∴$ Interest rate = 4% $÷$ 2 = 2% and n = 2 $×$ 2 = 4

$A=P(1+i)$*n*

 = R818 ( 1 + 2%)4

 = R818 (1,02)4

 = R818 (1,0824)

 = R885,43

1. Compound increase

$A=P(1+i)$*n*

= R15,50 (1 + 5,5%)4

 = R15,50 (1,055)4

 = R15,50 (1,23882)

 = R19,20

1. Compound decrease

$A=P(1-i)$*n*

 = R13,50 (1 – 6,5%)3

 = R13,50 (0,935)3

 = R13,50 (0,817400)

 = R11,03

**SUMMARY ASSESSMENT 3 – Interest calculations**

Choose the alternative that best describes the statement or answers the question. Provide full working details.

1. A business sells 150 computers in July and then 212 computers in August.

What was the percent increase in sales from July to August?

A. 50,7% B. 41,3% C. 70,7% D. 25% (2)

 2. 40% of a class failed a test, and 80 percent of those who passed earned less than an

A (80%). What percentage of the students earned an A?

A. 20% B. 12% C. 60% D. 2% (2)

3. The price of bread increases by 6%. The original price was R8,50 for a loaf of bread. What is the new price of a loaf of bread?

 A. R9,01 B. R0,51 C. R7,99 D. R9,10 (2)

4. The price of fresh chips increased from R12,00 to R15,50. Calculate the percentage increase to the nearest whole number:

 A. 29% B. 0,29% C. 23% D. 0,23% (2)

 5. A 500g packet of salt costs R2.50. Assuming inflation averages 2.8% per annum

 over each of the next 3 years, how much will the salt cost in three years time?

A. R2,64 B. R2,72 C. R2,79 D. R2,87 (2)

6. Cathy invested R8200 at the rate of 4.5% p.a. simple interest. If R738 interest was

earned what was the period of the investment:

A. 6 months            B. 1 year               C. 2 years             D. 2,5 years (2)

 7. The difference between the compound interest on R5 000 for 1,5 years at 4% p.a compounded annually and semi-annually is:

 A. R2,04 B. R3,06 C. R4,80 D. R8,308. (2)

8. An amount invested at simple interest amounts to R815 in 3 years and to R854 in 4 years. The amount invested is therefore:

A. R850  B. R790 C. R698  D. R800 (2)

 9. A country is said to be experiencing inflation when:

A. prices of most goods and services are rising over time.

B. prices of most goods and services are falling over time.

C. total output is rising over time.

D. total output is falling over time. 2)

10. If the price of an item in 2015 was R100 and in 2016 it was R102 in 2017, the inflation rate will be:

A, 102% B. 20% C.2% D. 0.2% (2)

**Solutions**

**Summary Assessment 3**

1. Percentage increase = $\frac{Difference in number}{Original number}×100\%$

 $=\frac{212-150}{150}×100\%$

 = 41,3%

$∴$ B

1. If 40% failed, then 60% would have passed.

Therefore those who earned an A is 20% $×$ 60% = 0,12 = 12%

$∴$ B

1. Original price is R8,50 = 100%

Therefore, increased price will be 100% + 6% = 106%

Work out 1% as follows:

 100% = R8,50

 $∴$1% = R8,50 $÷$ 100

 = R0,085

 $∴$106% = R0,085$ ×$ 106

 = R9,01

$∴$ A

1. Percentage increase = $\frac{Difference in price}{Original price}×100\%$

 $=\frac{R15,50-R12}{R12}×100\%$

 = 29,2%

 $≈$ 29% nearest whole number

$∴$ A

1. This is an inflation question, therefore it involves compound interest.

Using the compound interest formula

A = P ( 1 + i )n

 = R2,50 (1 + 2,8%)3

 = R2,72

$∴$ B

1. This is a simple interest question.

Use the following formula:

SI = P $×$ i $×$ t

R738 = R8 200 $×$ 4,5% $×$ t

$∴$ t = $\frac{R738 }{R8 200×4,5\%}$

 = 2

$∴$ C

1. Compounded annually will be:

A = P ( 1 + i )n

= R5 000 ( 1 + 4%)1,5

 = R5 302,98

 Compounded semi-annually is the same as half-yearly

Therefore the rate (i) will be halved and the period (n)will be doubled (opposite effect). This means i = 2% and n = 3

 A = P (1 + i )n

 = R5 000 (1 + 2%)3

 = R5 306,04

 Therefore difference = R5 306,04 – R5 302,98 = R3,06

$∴$ B

1. The simple interest for 1 year = R854 – R815 = R39

Therefore the simple interest for 3 years = R39 $×$ 3 = R117

This implies that the P = A – SI = R815 – R117 = R698

$∴$ C

1. Inflation is experienced when the price of goods and services keep rising over time.

$∴$ A

1. Percentage increase = $\frac{Difference in price}{Original price}×100\%$

 $=\frac{R102-R100}{R100}×100\%$

 = 2%

$∴$ C

## UNIT 4: Personal Income Tax

#### Learning Outcomes

By the end of the unit you should be able to:

* Define terminology relating to personal income tax;
* Calculate personal income tax using given tax tables;
* Identify taxable and non-taxable deductions;
* Identify tax brackets and rebates;
* Calculate annual gross and net salaries;
* Calculate taxable amount after non-taxable deductions;
* Calculate tax due after deduction of the rebate.
* Calculate UIF contributions and unemployment benefits.
* Analyse documents
* Perform calculations involving stepped tariffs.

### Introduction

Have you ever wondered why we have to pay taxes? Well the government needs revenue to provide its citizens with infrastructure, health-care, schools, services, security, etc. The money needed for these services comes from the people themselves, by paying taxes.

Every person in South Africa who is employed and earns a certain minimum amount, has to pay tax on his or her income. The tax on income is calculated as a percentage of the income.

## Concepts and terminology

It’s important that you understand the terminology used in the context of income tax.

**Gross Salary:** Total amount earned in a month. This includes all types of salary (e.g. salary, overtime, commission, bonuses, etc.)

**Deductions:** Amounts that need to be subtracted from the gross salary before money is deposited into the employee’s bank account. These include UIF, Pension, Medical Aid, Trade Union Fees, Loan repayments, Tax, etc.

**Net Salary:** Also known as ‘take home pay’. Amount that is deposited into an employee’s bank account. It is calculated as follows: Net Salary = Gross Salary – Deductions.

**Personal Income Tax**: This is a tax on all sources of income (e.g. salary, interest income, rental income, etc.). It is calculated on the taxable income.

**Taxable Income**: This is different from Net Salary although the calculation looks similar.
Taxable Income = Gross Income – Tax-deductible Deductions

**Gross Income**: This is different from gross salary (above) because it includes all forms of income, e.g. salary, rental income, royalties, etc.

**Tax deductible deductions**: These are specific deductions that are subtracted from the gross income before tax is calculated. There are two types of taxable deductions:

**Salary-based deductibles**: subtracted from the gross salary by the employer before the salary is paid. These include: UIF, Pension fund contributions, etc.

**Non-Salary deductibles**: These may be paid out of an employee’s take-home
pay, e.g. donations to charities, certain medical expenses. There are limits placed on deductibles, e.g. the maximum amount that can be
deducted for pension is 7,5% of the gross salary.

**Non-tax deductible expens**e: The majority of expenses are not tax deductible. These are generally living expenses, e.g. food, rent, fuel, entertainment, etc. Only tax deductible deductions reduce the amount of taxable income owed.

**Taxable Deductions:** Some deductions subtracted from an employee’s payslip are taxable. Although the employee receives less money they still have to pay tax on the larger amount of money that they earned. Examples include: loans from an employer, a garnishee order, and monthly payments to the employer for services rendered, etc.

**Tax Bracket:** A range of taxable income intervals that are charged according to a set rate of tax. The amounts are representing annual taxable amounts. The higher the bracket, the higher the tax rate for that portion of the taxable income.

**Tax Rebate:** An amount deducted from your tax payable after your tax rate has been calculated. Only people who pay tax are eligible for a rebate. There are three rebates. Everyone is eligible for the primary rebate. Taxpayers who are 65 years and older qualify for the additional secondary rebate. Tax payers who are 75 years and older qualify for all three rebates.

**Tax Threshold:** This is the minimum salary a person must earn before tax is charged. Below the threshold, the person’s tax will be cancelled by the tax rebate.

**SARS** (South African Revenue Services): Issues tax tables which are used when determining tax payable by individuals.

## Calculations involving personal income tax

**Illustrative example**

For example: Using the tax table below, calculate how much tax a 60-year old person

 should pay monthly if his monthly taxable income is R14 800,00.

|  |
| --- |
| **Statutory rates individuals** |
| **Taxable annual Income (R**) | **Rates of tax(R)** |
| 0 – 160 000 | 18% of each R1 |
| 160 001 – 250 000 | 28 800 + 25% of the amount above R160 000 |
| 250 001 – 346 000 | 51 300 + 30% of the amount above 250 000 |
| 346 001 – 484 000 | 80 100 + 35% of the amount above 346 000 |
| 480 001 – 617 000 | 128 400 + 38% of the amount above 484 000 |
| 617 001 and above | 178 940 + 40% of the amount above 617 000 |
| **Tax rebates individuals** |
| Primary rebate | R11 440 |
| Secondary rebate(65 years and older) | R6 390 |
| Tertiary rebate (75 years and older) | R2 130 |
| **Tax thresholds individuals** |
| Persons under 65 years | R63 556 |
| Persons 65 years and older | R99 056 |
| Persons 75 years and older | R110 889 |

**Solution:**

**Step 1:** Calculate the annual taxable income if not given

 The annual taxable income = R14 800 × 12 = R177 600.

**Step 2:** Identify the correct tax bracket

 Tax bracket 2:

Annual tax

= R28 800 + 25% of the amount above R160 000
= R28 800 + 25 ÷ 100 × (R177 600 – R160 000)
= R28 800 + 25 ÷ 100 × (R17 600)
= R28 800 + R4 400,00
= R33 200,00

**Step 3:**  Identify the applicable rebate and deduct accordingly

 60 year old only qualifies for Primary rebate

 Total tax payable = R33 200 – R11 440 = R21 760 annually

**Step 4:**  Calculate his monthly tax payable

 Monthly tax payable = R21 760 ÷ 12 = R1 813,33.

**Illustrative example**

For example: Mr Lala is 45 years of age and earns a monthly salary of R44 857.

 The following deductions are taken off his monthly salary:

* Pension: R4187,55 (max of 7,5% of gross income is tax deductible)
* UIF: 1% of his gross salary
* Medical Aid: R5 423,00
* Repayment of a personal loan from employer: R3 500,00 per month

Calculate:

1. Mr Lala’s monthly tax payable.
2. His net monthly salary.

**Solution:**

1. Calculate the tax deductible items: Pension = 7,5% × R44 857 = R3 364,28

 UIF = 1% × R44 857 = R448,57

Calculate monthly taxable income = Gross Income – tax deductions

 = R44 857 – (R3 364,28 + R448,57)

 = R44 857 – R3 812,85

 = R41 0044,15

Therefore annual taxable amount = R41 044,15 × 12 = R492 529,80

Total annual tax before rebate = R128 400 + 38% × (R492 529,80 – R484 000)

 = R128 400 + R3241,32

 = R131 641,32

Annual tax after rebate = R131 641,32 – R11 440 = R120 201,32

Monthly tax payable = R10 016,78

1. \*Note: In addition to the standard rebate, SARS also gives each tax payer a medical tax credit. The total monthly medical tax credit is the total of the following amounts: R230 for the tax payer + R230 for the spouse + R154 per child.

Mr Lala’s total medical tax credit = R230 × 2 (Mr & Mrs Lala) + R154 × 2 (children)

 = R768,00

Net monthly salary(take home pay)

= Gross income – Total deductions + Med Tax Cr

= R44 857 – (R4 187,55 + R448,57 + R5 423 + R3 500 + R10 016,78) + R768

= R44 857 – R23 575,90 + R768

= R22 049,1

**Practice exercise 10**

**Use the following tax table to answer the questions that follow.**

|  |
| --- |
| **Statutory rates individuals** |
| **Taxable annual Income (R**) | **Rates of tax(R)** |
| 0 – 160 000 | 18% of each R1 |
| 160 001 – 250 000 | 28 800 + 25% of the amount above R160 000 |
| 250 001 – 346 000 | 51 300 + 30% of the amount above 250 000 |
| 346 001 – 484 000 | 80 100 + 35% of the amount above 346 000 |
| 480 001 – 617 000 | 128 400 + 38% of the amount above 484 000 |
| 617 001 and above | 178 940 + 40% of the amount above 617 000 |
| **Tax rebates individuals** |
| Primary rebate | R11 440 |
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| Tertiary rebate (75 years and older) | R2 130 |
| **Tax thresholds individuals** |
| Persons under 65 years | R63 556 |
| Persons 65 years and older | R99 056 |
| Persons 75 years and older | R110 889 |

1. Feroz works for an IT company and earns a gross salary of R27 500 per month. The following deductions are shown in his gross monthly salary advice:

Medical Aid – 25% of R4 678

Pension Fund – 7,5% of gross salary

UIF – 1% of gross salary

* 1. Complete the table below showing Feroz’s salary deductions

|  |  |
| --- | --- |
| Medical Aid |  |
| Pension Fund |  |
| UIF |  |
| Total deductions |  |

1.2 Complete the following table to calculate how much tax Feroz needs to pay each month.

|  |  |
| --- | --- |
| Gross Salary - Deductions |  |
| Monthly taxable income |  |
| Gross annual taxable income |  |
| Tax bracket |  |
| Tax before rebate |  |
| Less rebate |  |
| Annual tax payable |  |
| Monthly tax payable. |  |

* 1. Calculate Feroz’s net salary.
	2. Explain the term rebate in context.
	3. Give TWO reasons why payment of taxes are important.
1. Gregory, a 34 year old male, earns a gross monthly salary of R30 800. Use the given tax table to answer the questions that follow.
	1. Explain why people over 65 years old receive an additional rebate.
	2. Calculate Gregory’s monthly tax, if his monthly taxable income is R26 832.

**Solutions to Practice exercise 10**

**1.1**

|  |  |
| --- | --- |
| Medical Aid | 25% $×$ R4 678 = R1 169,50 |
| Pension Fund | 7,5% $×$ R27 500 = R2 062,50 |
| UIF | 1% $×$ R27 500 = R275 |
| Total deductions | R3 507 |

1.2

|  |  |
| --- | --- |
| Gross Salary - Deductions | R27 500 – R3 507  |
| Monthly taxable income | R23 993 |
| Gross annual taxable income | R23 993 $×$ 12 = R287 916 |
| Tax bracket | 3rd interval/bracket |
| Tax before rebate | 51 300 + 30% (R287 916 – R250 000)= R51 300 + R11 374,80= R62 674,80 |
| Less rebate | R11 440 |
| Annual tax payable | R51 234,80 |
| Monthly tax payable. | R51 234 80 $÷$ 12 = R4 269,57. |

* 1. Net salary = Gross Salary – Deductions

 = R27 500 – (R3 507 + R4 269,57)

 = R19 723,43

* 1. Rebate is an amount deducted from your tax payable after your tax rate has

 been calculated.

* 1. It is your legal duty to do so.

 Government needs the revenue to provide services to its citizens.

2.1 With the mandatory retirement age in South Africa sitting at 65 it would hardly be fair for taxpayers to be prejudiced for not being allowed to work anymore by paying more tax on their savings or retirement income. SARS calls these retirement benefits the secondary and tertiary rebates.

2.2 Gross annual taxable income = R26 832 $×$ 12 = R321 984

 Annual tax payable = R51 300 + 30% (R321 984 – R250 000) – Rebate

 = R51 300 + R21 595,20 – R11 440

 = R61 455,20

 Monthly tax payable = R61 455,20 $÷$12

= R5 121,27

**SUMMARY ASSESSMENT 3 – Personal Tax**

1. Linda started a new job on 1 March 2015 at ABC Bearings with a annual starting salary of R268 000. Study her incomplete salary advise below and then answer

the questions which follow:

|  |
| --- |
| ABC Bearings **Salary advice** Tax number: 30654321Employer: Linda Mkhize Date employed: 01/03/2010Pay period: 01/03/2016 – 31/03/2016 ID number: 7704020035081 |
|  | **Earnings** | **Deductions** |
| Gross salary | **A** |  |
| Pension FundUIF employee contributionNet tax payableInternal loanTotal deductions |  | 1 675,00 |
| **B** |
| **C** |
| 650,00 |
| **D** |
| **Net salary (R)** |  | **E** |

1.1 State Linda’s surname. (2)

 1.2 How many years to date is Linda working for this company? (2)

 1.3 Name the company Linda is working for. (2)

 1.4 Calculate the missing value A, her gross monthly salary. (2)

 1.5 What percentage of her monthly salary is her pension fund contribution? (3)

 1.6 Calculate B, her UIF contribution which is 1% of her gross monthly salary. (3)

 1.7 Is R650 tax deductible? (2)

 1.8 What is Linda’s present age. (2)

1.9 Calculate C, her monthly tax payable by using the tax table given in the previous activity (8)

 1.10 Determine D, her total deductions (2)

* 1. Calculate E, her net salary. (2)
1. Maistry is 56 years old who earns an annual salary of R405 000 as an accountant.

He is married and has 3 dependent children.

The following deductions are made from his monthly gross salary:

1% of his gross monthly salary towards UIF.

5% of his gross monthly salary towards a Provident fund.

R5 256 towards medical aid.

Calculate the following:

2.1 Maistry’s gross monthly salary. (2)

2.2 His monthly tax payable before medical aid tax credit. (8)

2.3 His monthly medical aid tax credit. (R230 each per principal member

and spouse and R154 per child). (3)

2.4 His net take home pay per month. (2)

**Solutions to Summary Assessment 3**

1.1 Mkhize

 1.2 6 years

 1.3 ABC Bearings

 1.4 A = R268 000 $÷$ 12 = R22 333,33

 1.5 (R1 675 $÷ $R22 333,33) $×$ 100% = 7,5%

 1.6 B = 1% $×$ R22 333,33 = R223,33.

 1.7 No

 1.8 2018 – 1977 = 41

 1.9 R22 333,33 – (R1 675 + R223,33) = R20 435

 Therefore annual taxable income = R20 435 $×$ 12 = R245 220

 Annual tax payable before rebate = R28 800 + 25% (R245 220 – R160 000)

 = R49 305

 Annual tax payable after rebate = R49 305 – R11 440 = R37 865

 Monthly tax payable C = R37 865 $÷$ 12 = R3 155,42

* 1. D = R1 675 + R223,33 + R3 155,42 + R650 = R5 703,75
	2. E = R22 333,33 – R5 703,75 = R16 629,58
	3. Monthly salary = R405 000 $÷$ 12 = R33 750
	4. UIF = 1% $×$ R33 750 = R337,50

 Provident fund = 5% $×$ R33 750 = R1687,50

Medical aid = R5 256

Total deductions = R7 281

Taxable monthly income = R33 750 – R7 281 = R26 469

Annual taxable income = R26 459 $×$ 12

= R317 628

Annual tax payable before rebate = R51 300 + 30% (R317 628 – R250 000)

 = R51 300 + R20 288,40

= R71 588,40

Annual tax payable after rebate = R71 588,40 – R11 440 = R60 148,40

Monthly tax payable before med credit = R60 148,40 $÷$ 12 = R5 012,37

* 1. Medical aid tax credits = R230 + R230 + 3(R154) = R922.
	2. Tax payable after med aid tax credit = R5 012,37 – R922 = R4 090,37

Net monthly salary = R33 750 – Deductions

 = R33 750 – R7 281 – R4 090,37 = R22 378,63

## UNIT 5: Profit and Loss

#### Learning Outcomes

By the end of the unit you should be able to:

* Define terminology relating to profit and loss.
* Calculate profit or loss using a given formula.
* Calculate cost price and selling price.
* Understand and apply break-even concept in context.
* Make economic decisions involving options available.

### Introduction

In this unit you will learn about personal income and expenditure and how they relate to one another.

You will learn how to identify and perform calculations involving income, expenditure and profit and loss values and make economical choices given various options.

## Concepts and terminology

**Income**is money that an individual or business receives in exchange for providing goods or services or through investing capital.

**Profit or loss** is calculated by taking the total revenue derived from an activity and taking away the total expenses.

**Profit and loss = total income – total expenses**

If the resulting answer is negative, you have made a loss, or if it is positive, you have made a profit.

**Variable costs** are those costs that vary depending on a company's production volume, eg.  raw materials, packaging, and labour directly involved in a company's manufacturing process.

**Fixed costs** are costs that do not change with an increase or decrease in the amount of goods or services produced or sold examples are rent, advertising, insurance and office supplies, which tend to remain the same regardless of production output.

**Total costs** comprise fixed costs and variable costs.

**Break-even point** is the point at which neither a gain nor loss is made, nor neither benefit nor detriment is experienced.

Break-even point

In order for a business to be successful they need to be in control of their income as well as their

expenses. Income – Expenses = Profit. The goal of every business is to make the maximum profit.

If the income is more than the expenses, the business makes a profit.

If the income is less than the expenses, the business makes a loss.

If the income is exactly the same as the expenses, the business breaks even.

Break-even point is a point reached when neither a profit nor a loss is made. Simply it is a point where income and expense is the same.

**Illustrative example:**

1. Sipho operates a car wash business in his neighbourhood. He charges his clients R35,00 per car for wash and vacuum. His operating costs are made up as follows:

Salaries R1 500,00

Water, detergent and electricity R5,00 per car washed.

* 1. Write down a formula for the total costs.

Solution: Total costs(C) = Fixed Costs + Variable Cost

C = 1 500 + 5 × *n* where *n* represents number of cars washed.

* 1. Write down a formula for the income received for each car washed.

Solution: Income (I) = 35 × *n*  where *n* represents number of cars washed.

* 1. Complete the following table by finding the missing values:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Number of cars washed | 0 | 2 | 4 | 6 | 8 | **B** | 15 |
| Total cost (C) in rand | 1500 | 1510 | 1520 | 1530 | **A** | 1570 | 1575 |
| Total income (I) in rand | 0 | 70 | 140 | **C** | 280 | **D** | 525 |

 Solution: A = 1 500 + 5 × 8 = R1 540.

B = 1 570 – 1 500 ÷5 = 14

 C = 35 × 6 = R210

 D = b × 35 = 14 × 35 = R490.

* 1. Sipho washed 60 cars on a particular day. State whether Sipho made a profit or a loss and then calculate this profit//loss amount.

Solution: Income (60) = 35 × 60 = R2 10

Total cost (60) = 1 500 + 5 × 60 = R1 800

Therefore Profit = R2 100 – R1 800 = R300

* 1. Calculate the number of cars that need to be washed in order to break even.

Solution: For Sipho to break even his Total Income must equal his Total costs

Therefore 35 × *n* = 1 500 + 5 × *n*

35*n* = 1 500 + 5*n*

 By trial and error *n* = 50.

* 1. Draw the Income and Cost graphs on the same system of axes.

Solution:

* 1. Where in the graph will the break-even point be shown?

Solution: At the point of intersection of the two graphs. (50; 1750)

**Practice exercise 12**

1. Phumla makes duvet sets, which she sells at the local street market at R150,00 per set (including VAT).

If she makes 50 or less duvet sets per month, her production costs are R100,00 per set.

If she makes more than 50 duvet sets per month, her production costs are reduced by 15% per set.

Phumla has to pay R8 400 annually for the rental of her stall and she has weekly transport costs of R75.

Phumla prepares a monthly budget.

1.1 Show that her fixed cost for the month of February is R1 000,00. (3)

**Guided reflection:**

What costs are regarded as fixed costs and why?

1.2 How does her fixed cost for February compare to her average monthly fixed costs?

 Show ALL calculations. (5)

**Guided reflection:**

Does the month of February always have the same number of weeks as the other months?

1.3 Calculate the production cost per duvet set if 90 sets are made per month. (3)

**Guided reflection:**

Remember that if she makes more than 50 sets her production costs change.

Will her production costs change for the quantity made in this question?

* 1. The table below shows Phumla’s production cost for different quantities of duvet sets made in February.

Table showing cost of duvet sets made in February

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Number of duvet sets | 0 | 30 | 50 | 51 | 56 | 60 | 70 | D |
| Total cost per month (R) | 1 000 | 4 000 | 6 000 | 5 335 | 5 760 | 6 100 | C | 7 800 |

The formula used to calculate the total cost is:

Total cost = Fixed monthly cost + (number of duvet sets $×$ cost per set)

Use the above formula to determine the missing values C and D. (5)

 **Guided reflection:**

 Remember if 50 or less is made then then production cost per unit is R100 per set.

 If more than 50 sets are made then the production cost will be ? per set.

* 1. Phumla draws two graphs to represent her income and expenses for different quantities of duvet sets. The graph showing her Income for different quantities

of duvet sets has already been drawn on the grid that follows.

Use the values from the table to draw a graph on the same grid for the total expenses.

Use your graphs to answer the questions that follow:

 1.5.1 How many duvet sets must Phumla sell to break even? (2)

 **Guided reflection:**

What does break-even mean?
 Where in the graph will both costs (total income and total costs) be exactly the

 same?

 1.5.2 What profit will she make if all 80 duvet sets are sold? (3)

 **Guided reflection:**

What is the income for 80 sets? Read from your graph and check with calculation.

 What is the total expense for 80 sets? Read from your graph and check with

 calculation.

 How do we calculate profit?

 1.5.3 Suppose Phumla makes 80 duvet sets, but only sells 70 of them.

 Calculate her profit for February. (3)

 **Guided reflection:**

How does the difference in quantities affect her costs if any?



**Solution for Practice Exercise 12**

* 1. Fixed monthly rental cost = $\frac{R8 400}{12}$

= R700 ✓

 Fixed monthly transport cost = R75 $×$ 4

 = R300 ✓

 Total fixed monthly costs = R700 + R300

 = R1 000 ✓

 1.2 Annual transport costs = R75 $×$ 52 (weeks in a year)

 = R3 900 ✓

 Total annual costs = R8 400 (rent) + R3 900 (transport) ✓

 = R12 300 ✓

 $∴$ Average monthly costs =$ \frac{R12 300}{12}$

 = R1 025 ✓

 $∴$ The fixed costs for February is R25 less than the average monthly fixed costs. ✓

1.3 15% reduction if more than 50 is produced = 100% – 15% = 85% ✓

 Therefore new production costs = 85% $×$ R100

 = R85 ✓

 Therefore costs for 90 sets = 90 $×$ R85

 = R7 650 ✓

1.4 70 is greater than 50, therefore cost is R85 per duvet set

 Therefore total cost of duvet = Fixed cost + (no. of duvet sets $×$ cost per set)

 = R1 000 + 70 $×$ R85

 = R1 000 + R5 950 ✓

 = R6 950

 $∴$ C = R6 950 ✓

$ $ To work out D we reverse the formula: We minus R1 000 and the divide by R85

 Therefore D = (R7 800 – R1 000) $÷$ R85 ✓✓

 = 80 ✓

 OR R1 000 + 80 $×$ R85 = R7 800

 1.5 Graphs on next page

 1.5.1 20 duvet sets ✓✓

 1.5.2 Profit = Income – Expenses

 = R12 000 – R7 800

 = R4 200 ✓✓✓

 OR Using the formula: Income for 80 duvet sets = R150 $× $80

 = R12 000

 Expenses for 80 duvet sets = R1 000 + R85 $×$ 80

 = R1 000 + R6 800

 = R7 800

 Profit = Income – Expenses

 = R12 000 – R7 800 = R4 200

1.5.3 Profit = Income for 70 sets – Expenses for 80 sets

 = R150 $×$ 70 – [R1 000 + 80 $×$ 85] ✓

 = R10 500 – R7 800 ✓

 = R2 700 ✓

1.5

 

 Plotting given points from table ✓✓ Plotting calculated points C and D ✓✓

 Joining points with straight line ✓ Plotting (51; 5 335) ✓

 Joining points up to (80; 7 800) ✓

## **Budgeting for Economical Choices**

Renting or hiring a car often presents challenges, like whether to take unlimited kilometres and a fixed rate or rate per kilometre.

Sometimes an option will depend on the number of kilometres travelled.

|  |  |  |
| --- | --- | --- |
| **Rental Company** | **Standard daily rate in rand** | **Plus cost per km travelled in rand** |
| Company A | 200 | 2,10 |
| Company B | 250 | 1,75 |
| Company C | 100 | 3,00 (first 50 km free daily) |

Eg.Juhi and her family rents a vehicle to travel to the South Coast which is approximately 150 km one way.

1.1 Using the above table determine the most cost effective option they can choose assuming they all offer the same class of vehicle and they return the same day.

**Solution:** Company A: 300 km × R2,10 + R200 = R830,00

 Company B: 300 km × R1,75 + R250 = R775,00

 Company C: (300 km –50 km) × R3,00 + R100 = R850,00

Therefore Company B is the most cost effective.

 1.2 Write down a formula to calculate the daily rental fee for Company C.

 **Solution:** Daily rental fee (in rand) = 100 + 3(number of daily kms – 50)

**Practice Exercise 13**

1. The Grade 11’s of Alena Park Secondary are planning a matric farewell function for the grade 12’s. They have a choice of either choosing their school hall or the Coastal Hotel as a venue.

If their school hall is used, then meals, table decorations as well as the music will cost R110 per person.

TABLE 1: Cost of using the school hall

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **No of tickets sold** | **0** | **10** | **25** | **40** | **80** | **100** |
| **Cost in rand** | **0** | **1 100** | **2 750** | **4 400** | **8 800** | **11 000** |

The Coastal Hotel charges a basic fee of R2 400 which covers the cost of the music and

decorations. An additional charge of R50 per person will be levied for meals.

TABLE 2: Cost of using the Coastal Hotel

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **No. of tickets sold** | **0** | **10** | **20** | **40** | **50** | **100** |
| **Cost in rand** | **2 400** | **2 900** | **3 400** | **4 400** | **4 900** | **7 400** |

Use TABLE 1 and TABLE 2 to answer the questions that follow.

1.1 Determine the total cost if 50 tickets are sold for the function in their school hall. (3)

1.2 Write down the following:

1.2.1 Number of tickets sold when the cost for the two venues is the same. (2)

1.2.2 The corresponding cost for your answer in Question 1.2.1. (1)

1.3 Calculate the number of tickets sold if the cost of hiring the Coastal Hotel is

R8 400. (3)

* 1. Draw two straight line graphs on the same system of axes using the data in

TABLE 1 and TABLE 2. (6)

1.5 If there were 30 Grade 12’s and each one brings an escort to the function,

which hall should they use? Motivate your answer. (2)

**Solution for Practice Exercise 13**

1.1 Cost for 50 using school hall = R110 $×$50 ✓✓ = R5 500 ✓

1.2.1 40 tickets ✓✓

1.2.2 R4 400 ✓

1.3 Number of tickets at Coastal Hotel = (R8 500 – R2 400) $÷$ R50 ✓✓

 = R6 100 $÷$ R50

 = 122 ✓

1.4

* 1. 60 people will be cheaper at Coastal Hotel ✓

School Hall for 60 = R110 $×$ 60 = R6 600

Coastal Hotel for 60 = R2 400 + R50 (60) = R5 400

Therefore cheaper at Coastal by R1 100. ✓

**SUMMARY ASSESSMENT – Practice Exercise 14**

1. Millicent lives in Durban and is planning on selling fried mealies in tubs in order to pay for her tertiary education. She has obtained the following information about her planned business:

 Expenses:

* Monthly rental R1500
* Cost of mealies per tub R2,50
* Cost for serviette and spoon per tub R0,50
* Cost of seasoning per tub R0,50
* Gas required for frying R4,00 per tub

 Income:

* Sale of each tub of mealies R20,00 per tub.
	1. List Millicent’s fixed monthly cost.
	2. Write down her variable costs.
	3. Calculate the variable costs for the frying of 30 tubs of mealies per day.
	4. Determine a formula for the Total Costs and the Income.
	5. Calculate the total costs for producing 30 tubs of mealies.
	6. Complete the table below by finding the missing values A, B and C.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **No of tubs** | 0 | 40 | 80 | 120 | 200 | C |
| **Income (R)** | 0 | 800 | **A** | 2 400 | 4 000 | 5 000 |
| **Total Cost (R)** | 1500 | 1 800 | 2 100 | **B** | 3 000 | 3 375 |

* 1. Use the table above to draw TWO graphs on the same system of axes.

 Label each graph properly as Income and Total Cost.

* 1. Determine how many tubs Millicent has to sell to break even.
	2. How much will the number of tubs stated in question 1.8 above be?
	3. If she wants to make a profit of R1 000, how many tubs must she sell?
	4. On a particular day she produced 130 tubs, but was able to sell only 120.

 Determine whether she has made a profit or loss. Calculate this amount.

1. Sbu needs to visit his aunt in Port Shepstone which is 120 km from his home.

He has three options to choose from in order to rent a vehicle. Refer to the table and find the most economical option:

|  |  |
| --- | --- |
| **Options** | **Rental Costs** |
| **A** | Only R5 per km |
| **B** | Basic of R200 plus R2,00 per km |
| **C** | Basic of R300 plus 100 kms free daily thereafter R2,50 per km.  |

**Solution for Practice Exercise 14**

* 1. Rental
	2. Cost of producing each tub = Mealies, Serviette and spoon, Seasoning, Gas
	3. Variable costs = R2,50 + R0,50 + R0,50 + R4,00 = R7,50

Therefore, variable costs for 30 tubs = 30 $×$ R7,50 = R225.

* 1. Total costs = Fixed costs + Variable costs

 C = 1 500 + 7,50n, where n represents number of tubs of mealies produced.

Income I = 20 n , where n represents number of tubs of mealies sold.

* 1. Total costs for 30 tubs = 1 500 + 7,50 (30) = R1 725
	2. A = Income for 80 = 80 $×$ R20 = R1 600

B = Total cost for 120 = 1 500 + 7,50 (120) = R2 400

C = Number of tubs = Selling price $÷$ Selling price per tub

 = R5 000 $÷$ R20 = 250

OR: Number of tubs = (Total cost – R1 500) $÷ $R7,50

 = (R3 375 – R1 500) $÷$ R7,50 = 250

1.8 120 tubs – point of intersection of two graphs

1.9 R2 400

1.10 Read from table: At 200 the difference between Income and Total expenses = R1 000

OR: Profit (R1 000) = Income – Total costs

 $ ∴$ 1 000 = 20n – (1 500 + 7,50n)

 $ ∴$ 1 000 = 20n – 1 500 – 7,50n

 $ ∴$ 1 000 + 1 500 = 20n – 7,50n

 $ ∴$ 2 500 = 12,50n

 $∴$ n = 2500 $÷$12,50

 = 200

* 1. Income for 120 = 120 $×$ R20 = R2 400

Total expenses for 130 = R1 500 + R7,50(130) = R2 475

Profit = Income – Expenses

 = R2 400 – R2 475

 = – R75,00

Therefore a loss of R75 was made because negative answer.

2. Option A: 120 $×$ R5 = R600

 Option B: R200 + R2(120) = R440

 Option C: R300 + (120 – 100)$ ×$ R2,50 = R350

 Therefore Option C is the cheapest.

**Practice Exercise 15**

It costs a publishing company R650 000 to print and bind books. The R650 000 represents the fixed cost or a cost that cannot change. To help the publishing company sell the books, a marketing company charges R50 for each book sold. If the publishing company charges R180 per book, how many books should they sell to break even?
**Guided reflection:**

What are your costs total?

What are your variable costs total?

How will you arrive at your total costs?

What is the value of your income?

Draw tables to represent the income and total costs.

Look at your tables to look where the total costs and income are exactly the same.

If you are unable to find this value then it means extending your table.

**Solution for Practice exercise 15**
Step 1: Start off with a formula representing the total cost:

 Total cost (R)= Fixed cost + Variable cost

 = 650 000 + 50 (number of books)

Step 2: Draw a table to represent the formula above;

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Number of books | 0 | 1 000 | 2 000 | 3 000 | 4 000 | 5 000 | 6 000 |
| Total cost in R | 650 000 | 700 000 | 750 000 | 800 000 | 850 000 | 900 000 | 950 000 |

Step 3: Now establish a formula for the income

 Income (R) = 180 $×$ number of books sold.

Step 4; Draw a table to represent the formula for income:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Number of books | 0 | 1 000 | 2 000 | 3 000 | 4 000 | 5 000 | 6 000 |
| Total income in R | 0 | 180 000 | 360 000 | 540 000 | 720 000 | 900 000 | 1080 000 |

The break-even point is to sell 5 000 books.

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